HOW TO TEACH STATISTICAL CONCEPTS TO SLOW-LEARNING PUPILS

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1. Some general methodological remarks: The "Operative Method"

The so-called operative method has its roots in the work of J. Piaget. His concept of the "operation", both concrete and formal operation, and the psychological structure of "grouping" constitute the background of this methodological approach. It was developed by Aebli (1963) and Fricke (1970) in a special direction which we could call non-formal and non-deductive.

Some colleagues of the Pädagogische Hochschule Heidelberg[3] who have continued working on this level have furthermore modified Fricke's methodological principles, in order to make this methodological concept easier to put into practice at school level. They have elaborated the following five principles:

1. The Principle of Associativity
   Example: To pose a problem or a task which allows several solutions or (at least) several modes of finding the solution.

2. The Principle of Composing
   Example: To combine an (arithmetical) operation with others or with itself: The multiplication by two/and once more the multiplication by two equals the multiplication by four

3. The Principle of Reversion
   Example: When you are changing a square into a rectangle by doubling the one side and cutting the half of the other side the area of the figures remains constant.

4. The Principle of Transitivity
   Example: 5 + 3 = 8, therefore, <u>1</u>5 + 3 = <u>1</u>8, <u>2</u>5 + 3 = <u>2</u>8, and so on . . .

5. The Principle of Variation
   It is the most general of these principles. Usually this is used for the construction of easier adjacent tasks as a means for solving more difficult tasks, or we use the possibility of variation in the construction of task series.
   Example: Four friends make a trip by bicycle. Uwe does 30 km in 2 hours, Peter does 30 km in 1-1/2 hours, Jens does 40 km in 2 hours, Bernd does 40 km in 1-1/2 hours. The parents of Peter follow them by car. They are 3 times faster than Peter.

After these introducing remarks I want to point some didactic consequences: We have learned from psychology that the learning of operational concepts has to be linked with the attributes of the operative method above mentioned. This can be realized with the proven method of building up mathematical knowledge in three stages:

1. Learning through experience: the pupils solve problems, figure out carefully selected assignments, analyse questions, perform appropriate experiments.

2. Stage of concept formation: the pupils look for shared and essential characteristics of the objects within the scope of this experience: they begin to generalize.

3. Stage of representation: the pupils use graphic or symbolic representations and also learn to use more formal methods.

It is the experience of many teachers that pupils accept and make use of the operative attributes when problems are posed according to the operative principles, that pupils deal – sometimes – consciously with these attributes of tasks or problems, and that a special sort of active behaviour is developed during a long term learning process according to this didactical design.

2. Special information on the methodological approach

Pupils between 12 and 15 years of age – especially the "slow-learners" - have to solve a large number of assignments before they can begin and carry out a process of generalization.

One of the goals is to concretize learning through discovery; therefore formal solutions remain somewhat in the background, whereas more emphasis is placed upon the activities of the pupils and their dealing with variation in situations and tasks.

The following points should be given special attention:

- Dealing with the contents of the problems in a practical manner is definitely preferable to a formal treatment. A formal description of a set of facts or a technical treatment with formulas will generally bring more disadvantages for the student than advantages. This is why we only occasionally use the formal description form. On the contrary, an outstanding role is played by the heuristic, experimental approach – for example, with the introduction of the arithmetic mean - and systematically varied problem formulation – for instance, in the discussion of scatter (mean deviation) or in the treatment of the minimal property of the median value. Only this approach makes it possible to convey a vivid idea of these concepts and their interrelationships within the few teaching hours available for this topic.

- In order to implement this strategy, the teacher needs topical and stimulating assignment material. For this reason the assignments were selected in such a manner that the environment of the students or the general reporting section of the daily press provided the contextual background. Assignments dealing with games of fortune such as lotteries and so on are highly attractive to students. These problems fall under the topic of probability calculus and are to be dealt with there.

The slow-learning pupils cannot learn systematically the statistical concepts so that the official curriculum defines the following scope, for descriptive statistics: "Solving simple basic problems in the area of descriptive statistics". These basic problems can be roughly divided up into three general areas:

First area: "Collecting statistical material; arranging and representing it appropriately". This is primarily a matter of qualitative aspects of the statistical method of working. How does one take a "coincidental random sample"? What does random sample actually mean? Which graphic representations are especially suitable for depicting the information concisely and clearly? These and similar questions are dealt with in great depth in the first three sections of our proposals by means of examples and assignments. In addition to the highly varied use of different graphic possibilities, the use of random numbers deserves special emphasis in the classroom. Random numbers provide a much more convenient process than the usual drawing of lots to construct a real random selection from any populations.

Second area: Here the question is pursued as to how the above mentioned qualitative aspects can be quantified. Which "measurements", which characteristic values of a random sample allow for reasonable statements? The official curriculum guidelines include this objective among the following goals:

"Calculating mean values; interpreting frequency distributions; deciding whether certain statistical representations and compilations are appropriate, useful and clear". This task, among others, is dealt with in sections 4, 5 and 6 as well as, to a lesser degree, a few of the assignments in section 7. The crucial matter in teaching this material is the question as to how the individual characteristic values (highest, lowest, most frequent, least frequent, mean value, median value) change when the individual values of the random sample change; for only in this manner it is possible to illustrate what is achieved or not achieved by the characterization of a random sample by means of these values.

Third area: More in-depth qualitative and quantitative analysis of random samples. This objective can be especially well justified and achieved by comparing two random samples. This is done in section 7. This is where it becomes particularly clear that the subject matter of grade 8 and 9 are closely intertwined: in grade 8 the in-depth treatment of the abovementioned parameter values is limited to the mean value (arithmetic mean) and the median value. So the minimal property of the median value could be the basis of quite interesting problem formulations.

"Comprehending and interpreting the deviations from mean values" is defined as a special learning objective for the upper track of grade 8, with the recommendation "not to introduce the standard deviation". On the other hand, the treatment of this subject matter as the "calculation of mean values and deviations from mean values" is required for all students in ninth grade. Here it might very well be advisable to introduce the standard deviation by means of examples only in the upper track as well.

Since curriculum guidelines do not explicitly mention the concept of "sum frequency" and the procedure of simplified calculation of the mean value, it is probably a good idea not to deal with assignments in this area until ninth grade when the objective is "deepening knowledge in working with statistics".

The suggestions in section 5 (misleading representations) are meant to encourage teachers and students to analyze further examples of false or misleading reporting with "slanted" statistics. As a rule the material brought along by the students will be quite extensive and varied, so individual problems can be solved in the group teaching situation.

3. Some assignments and problems according to this approach

The methodological possibilities are elaborated within the context of the following contents:

a) What is a "random sample"? b) How do we use a "list"? c) A few diagrams d) Can you add mean values? e) The median. f) Comparison of random samples - mean deviation and standard deviation.

For the moment I will pick out the following example:

e) Comparison of random samples - mean deviation and standard deviation
Main objectives: There are several data corresponding to each random sample. It depends on the formulation of the particular question whether the highest value, the lowest value, the mean value, the median value, the (absolute) mean deviation, the standard deviation of a random sample are used to evaluate the sample.

Problems referring to mean deviation.

1. Thirteen children play croquet in two teams. These are the game results:

Team I: Child	Number of strokes
A	29
B	31
C	30
D	26
E	27
F	24
G	20

Team II: Child	Number of strokes
H	29
I	24
K	35
L	19
M	31
N	34

Which team has won? The children cannot agree.

(1) L: "I have fewer strokes than anybody else, so our team has won".

(2) G: "No, K has the most strokes of all, so your team has lost".

(3) F: "Even if you add up the highest and the lowest numbers for both teams and compare the sums, your team has still lost".

(4) H: "No, you have to add up all the points for each team. Then we've won".

(5) B: "But there were only six of you, we had seven on our team. Let's say all of the players in each team the same number of strokes. For us that's 187:7 and for you it's 172:6. So we've won".

a) Which team would have won if the children had taken the median value as the decisive criterion?

b) Evaluate the above suggestions for winning! – Which proposal most emphasizes the performance of the entire team; which gives priority to the performance of individual players?

2. Compare the last two math tests taken in a class by figuring out their average values (lists 1 and 2).

Grade	1	2	3	4	5	6
Number of grades	1	6	12	4	2	0

Grade	1	2	3	4	5	6
Number of grades	6	4	5	5	4	1

Result: Both tests have the same average value of 3. This means that it can be said that the grades were equally good on both. Nevertheless there are major differences. This can also be seen when the two corresponding block diagrams are compared (figure 1 and 2). What are these differences?

Figure 1

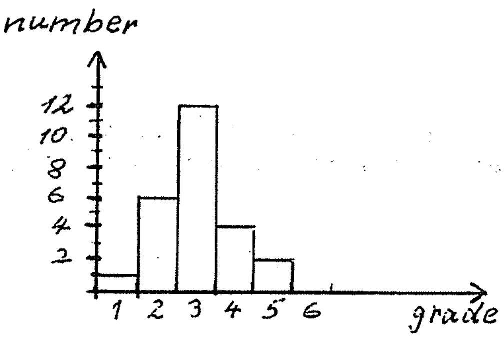
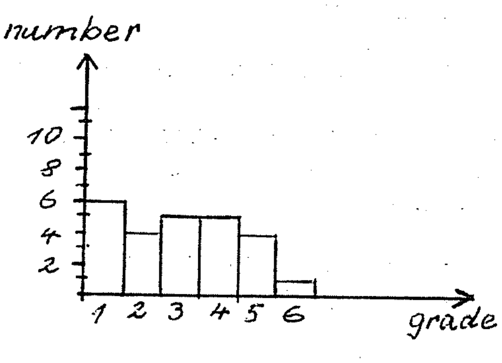


Figure 2



The distribution of the random sample values around each of the mean values differ as follows:

Qualitative evaluations:

Random sample 1: "Many" values are close to the mean value, "few" values are far away or: the values are accumulated around the mean value.

Random sample 2: "Few" values are close to the mean value, "many" values are far away or: the values are highly "scattered".

Quantitative evaluations:

Calculate for both random samples

1. the difference between the two boundary values in the ranking list (range),

2. the number of values that differ from the mean value,

3. the greatest deviation of the grades from the mean value,

4. the sum of all of the deviations of the grades from the mean value,

5. the mean deviation of the grades from the mean value.

   a) Systematically vary the data of the two random samples in such a manner that it is possible to make a decision as to the usefulness of the five statistical units introduced here to describe the qualitatively collected data.

   b) What would a random sample have to look like for the mean deviation of all of its values from the mean value to be zero?

References

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