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CHILDREN'S LEARNING PROGRESSIONS ON PROBABILITY AND SUGGESTIONS FOR CURRICULUM IMPROVEMENT

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This study focused on children's Learning Progressions (LPs) on probability. A sample of 174 children aged 6-14 was selected and a questionnaire involved 7 questions was designed for the study. The study showed that children's LPs on probability formed a sequence initiated by cognition of randomness to qualitative comparison followed by sample space. The following are suggestions for curriculum: (a) for children at concrete operational stage, curriculum objectives of probability should be set in a moderate level that does not have a high cognitive benchmark about quantitative comparison of probability, (b) curriculum should develop and organize certain combination-related knowledge as an enlightenment for sample space that will further help them understand quantitative comparison of probability.

1. Introduction

Probability literacy is one of the core mathematics literacy (Gal, 2002; 2005). Since the 1980s, there has been a burgeoning growth in curriculum activities (e.g. Australian Education Council, 1991; Department of Education and Science and the Welsh Office, 1991; National Council of Teachers of Mathematics, 2000) that introduce probability-related knowledge as a content branch in primary and middle school curricula. Compared with European and American schools, the introduction of probability in Chinese mathematics curriculum is done comparatively later. Curriculum Standards for School Mathematics in the Full-time Nine-year Compulsory Education (China Ministry of Education, 2001) published in 2001 organized probability related knowledge as a content branch in the compulsory curriculum and determined the cognitive objectives for the first time. "Curriculum Standards for School Mathematics in the Full-time Nine-year Compulsory Education (2011 Edition)" (China Ministry of Education, 2012) adjusted the cognitive objections and difficulty to satisfy students' cognitive level. For instance, students of stage 2 (grade 4-6) are only required to understand randomness, and stage 3 (grade 7-9) are required to master probability comparison. In terms of mathematics curriculum standards, however, the suitability between students' LPs and knowledge development is still an issue to be resolved. Hence, this study focused on how do children develop their knowledge of probability and what children's LPs on probability are? By conducting this study, we anticipated to formulate psychological basis for probability curriculum design to improve children learning about probability.

2. Method

2.1 Subjects

Gong (2012) proposed that probability cognition for children aged 6-14 experience the following 5 stages: slow development stage I (6-7 year old), rapid development stage I (8-9 year old), slow development stage II (10 year old), rapid development stage II (11-12 year old) and stagnant stage (13-14 year old). In this view, this study chose children at the highest age of each stage (namely, children aged 7, 9, 10, 12, and 14) as the subjects among which 79 were males and 95 were females.

Grade	Age	Number	Min	Max	Average	SD
1	7	34	6.7	8.4	7.349	.3532
3	9	37	8.7	10.3	9.368	.3742
4	10	35	9.6	11.3	10.356	.3357
6	12	36	11.7	13.4	12.422	.3814
8	14	32	13.8	15.4	14.358	.4024

Table 1 Profile of the subjects

2.2 Questionnaire

As a method of data collection, a questionnaire was designed. The questionnaire was based on "ball-box model" and involved 7 questions. Each question involved 3 cognitive tasks which referred to randomness, sample space and qualitative comparison of probability. The difficulty of these questions was gradually increased by controlling the 4 variables: number of the boxes, color types of the ball, total number of the balls, and the number of the balls taken out. In addition, some simple or essentially analogous questions were eliminated to simplify the questionnaire structure. A sample of such question is presented in Table 2.

Questions	Cognitive tasks
An opaque box contains 2 white balls and 2 black ones, and they are the same except for color. Close your eyes and shake the box, and then take out 2 balls simultaneously from the box, and please answer:	For the 2 balls have been taken out, Task 1: are they certainly a white ball and a black one? Task 2: how may possible situations totally? Please list all the possible situations. Task 3: please compare the probability of the situation of "1 black and 1 white" and 2 whites".

Table 2 Questionnaire for probability cognition (example)

The three questions (see the column on the right in Table 2) concerned the 3 cognitive tasks: randomness, sample space, and qualitative comparison. Cognition of randomness referred to children's understanding of the differences between the random events and certain ones and of the meaning of "certainly", "possible" and "impossible". Cognition of sample space referred to children's estimation of the all possibilities of random events (classical), and theoretically, it was the basic foundation for probability comparison. Cognition of qualitative probability referred to children's judgment for probability, which is not necessarily based on accurate calculation, but it rather focuses on children's understanding of the probability of random events.

2.3 Reliability and validity

Cronbach α coefficient among the 7 questions was 0.821, which showed that the questionnaire had comparatively high homogenous reliability.

Correlation coefficient among the 7 questions were all significant (0.041-0.672, $P<0.05$), which showed that the questionnaire had a comparatively high construct validity.

3. Analysis

3.1 Descriptive statistics of the accuracy of each cognitive task

This study analyzed the cognitive level (see Table 3) and its development (see Figure 1) on the 3 probability tasks for children aged 6-14.

Age	Cognitive tasks			Average level
	Randomness	Sample space	Qualitative comparison	
7	0.84	0.10	0.43	1.37
9	0.89	0.15	0.52	1.56
10	0.92	0.23	0.54	1.69
12	0.87	0.35	0.64	1.86
14	0.94	0.52	0.68	2.14

Table 3 Descriptive statistics of the accuracy of each cognitive task

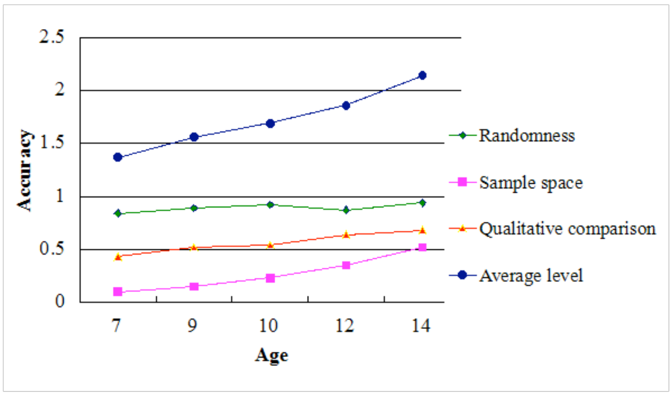

Figure 1 Cognitive development on probability for children aged 6-14

It can be concluded that children's cognitive level of probability increases with their age and the accuracy of the 3 probability tasks from high to low are: randomness, qualitative comparison, and sample space. According to the standard that 20%, 50% and 80% respectively serves as the description of preliminary understanding and mastery (Shen & Liu, 1984), it is suggested that children can master randomness at the age of 7 and conduct qualitative comparison of probability at the age of 9, while only get the level of understanding for sample space at the age of 14.

3.2 Learning Progressions on Probability

The above descriptive statistics indicates certain difference of accuracy among the 3 cognitive tasks. A further analysis of variance and multiple comparisons showed that the cognitive level is significantly different among the 3 tasks ($p < 0.01$). Thus, one can conclude that children's LPs on probability follows the sequence from cognition of randomness and qualitative comparison of probability to cognition of sample space. This further shows that sample space is a more complex cognitive task that develops in childrens' later age.

4. Summary and Discussion

In terms of the LPs on probability for children at concrete operational stage, curriculum objectives should be set at a moderate level that do not have a high cognitive benchmark about quantitative comparison of probability. Furthermore, one can conclude that the adjustments in difficulty and cognitive levels of probability in Chinese current curriculum standards are reasonable. Yet, although the accuracy is higher than 50% about qualitative comparison of probability for children at the age of 9, various studies showed that they just based on a set of non-standard or simple strategies such as equi-probability (Lecoutre, 1992), proportionality misconception (Shaughnessy, 2007), etc. Thus, although children can roughly compare probability, their strategies need to be standardized and appropriate teaching intervention should be taken. Sample space is the prior knowledge for probability comparison and for a reliable model of probability calculations. Additionally, probability cognition is closely related to combination operation (He & Gong, 2013), therefore curriculum should develop and organize certain combination-related knowledge as the enlightenment for sample space and to help children understand quantitative comparison of probability.

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