13th International Congress on Mathematical Education
Hamburg, 24-31 July 2016

MATHEMATIZATION OF UNCERTAINTY WITH THE AID OF COMPUTERS: A MODEL OF ACTIVITY IN HIGH SCHOOL

Maria Giovanna Frassia, Annarosa Serpe

Department of Mathematics and Computer Science, University of Calabria - ITALY

This paper presents a model of didactic activity centred on some random phenomena. The activity was designed privileging a game-like approach first in a real way, and then virtually through computer simulations in the programming environment MatCos. The reason for introducing the study of random phenomena through this model lies in the need to suggest different methods from the traditional approach to this topic in Italian schools. In most cases the calculus of probability is traditionally presented as a sum of properties and results to be applied to often limited contexts, which is insufficient to ensure the learners comprehend the true meaning of probability.

INTRODUCTION

In the schools of the III millennium, the mathematics of probability is at the basis of education because it helps to shape and channel the decisions and choices of future citizens (Bennet, 1998; Beltrami, 1999; Everitt, 1999; Gal, 2005). The teaching of probabilistic reasoning from an early age means educating to critical thinking, it means to train students to "bring order" to their ideas in order to approach events rationally and correctly interpret real-world phenomena. In Italian secondary schools, unfortunately, the calculus of probability is often neglected in every-day teaching. Such generalized ignorance of even the most basic notions leads to wrong convictions and/or misconceptions, which may even border on superstition (Fischbein, et al., 1991). The calculus of probability is actually one of the most stimulating branches of mathematics also because of its many applications (Gigerenzer, et al., 1989; Greer & Mukhopadhyay, 2005), and includes three different approaches: classic, frequentist and subjective. The classic approach to probability has long been dominant in Italian secondary schools. The majority of students find this approach based on combinatory calculus quite hard especially because of the calculations involved in solving the formulae. The difficulties in the classic approach have so far represented a real obstacle to classroom teaching and learning of the topic, not only in Italy but also in other countries (Batanero, et al., 2005). However, in recent years the growing interest for statistical methods and the use of information technology have contributed to the study of probability as a limit of stabilized frequency (Biehler, 1991). The modelling point of view has been adopted in recent years, linking probability teaching to statistical thinking. The introduction of efficient computers in secondary education enables the simulations of models resulting from statistical observations and the introduction of students to the larger field of statistical inference. The advantage of using simulations is that we can overcome much of the difficulty encountered by learners when using the formal rules. This contribution fits the above framework, and illustrates a teaching proposal based on the use of the computer as a tool for programming, and aimed at enhancing the debate around the meaning and interpretation of probability.

Activity model

The teaching action is aimed at students attending the final year of high school.

The approach is phenomenological-inductive and uses play as 'knowledge space' so that students can pose pertinent questions, can record their reflections, and eventually formulate a common synthesis (Frassia, 2015). The activity was set up as problem solving and uses computers as pedagogical tools within the MatCos programming environment (Costabile & Serpe, 2013). The basic model of the teaching action consists of a four-step process outlined below:

STEP 1	Research through questionnaire of students' awareness and application of problems involved in dice games.
STEP 2	Monitoring of questionnaires and specific intervention on misconceptions emerged through a problem posing-problem solving task. Real simulation of the game for 30 consecutive times and recording of data on a chart. Analysis and comparison of results of the real simulation carried out by the students in groups. Formulation of a common synthesis: need to stabilize the model.
STEP 3	Moving from the plane of reality to that of mathematics: re-configuration of aspects of sensible reality for analysis and study through the computer. Elaboration of the algorithm and the programming code in the MatCos environment. Execution of the algorithm for a growing number n of dice throws. Evaluation of results obtained in output and comparison with those of the real simulation.
STEP 4	Evaluation of results obtained in output and comparison with theoretical model. Compare and contrast theoretical and experimental probability based on a given event and collected during a classroom activity. Summarize the information and draw conclusion. The student understand and work with the basic concepts/formula for theoretical and experimental probability.

Table 1: Steps of activity model.

In STEP 1 the students fill in a questionnaire with three questions (Q) on dice games.

Q1: Giovanni and Francesco are playing dice. Throwing one die, Giovanni wins if he gets a six. What is the probability for Giovanni to win?

Q2: Giovanni and Francesco are playing dice. Throwing one die, Giovanni wins if he gets a six, while Francesco wins if he gets a five. Who has the highest probability of winning?

Q3: Giovanni and Francesco are playing dice. Throwing two dice, Giovanni wins if he gets two six, while Francesco wins if he gets a five and a six. Who has the highest probability of winning?

The table shows the results of the questionnaires:

Question	Right answers	Wrong answers
1	57	0
2	51	6
3	8	49

Table 2: Results of the questionnaire.

The analysis of the results shows that the students had difficulty solving and interpreting the third question. In particular, many students wrote:

> *The events 'Obtaining 5 and 6' and 'Obtaining 6 twice' when throwing two dice have the same probability to occur, that is 1 out of 12, because when throwing a dice the possible cases are 6 and so when throwing two dice the possible cases become 12 and each result has the same probability to occur.*

STEP 2: Eradicating students' wrong intuitions is the teacher's main objective. Once corrected the questionnaires, she or he will not give out any solutions but will set up an activity and get the students involved in the solution of a question very much like question 3 in the questionnaire. The group task is as follows:

> Giovanni and Francesco are playing dice. Throwing two dice, Giovanni wins if he gets a double three, while Francesco wins if he gets a three and a four. Who has the highest probability of winning?

The students are given some dice and take on the roles of Giovanni and Francesco, so that they have a real chance to experience learning without limitations to their access. However, they will soon realize from comparing the data obtained and recorded that it is different for different groups. At this point the teacher will start a guided discussion in order to stimulate reasoning and hypotheses. As a result of the practical experience, the need to make quite a high number of repeated trials within a reasonably short time will be apparent, and that is when a computer becomes an indispensable support to teaching. The MatCos software, already used in other mathematical teaching activities, finds here another useful application.

STEP 3: The use of the computer as programming tool is very important since it offers students the real chance to explore mathematical concepts, formulate conjectures to be validated or rejected, and thus continue the problem solving experience. The construction of the algorithm indeed represents an important and delicate phase in so far as the students have to plan the finite succession of steps which culminates in the writing of a program to be implemented in the MatCos environment. The writing of such a program implies the breaking down of complex notions into simpler concepts, and opportunities for personalized learning paths and individual re-elaboration arise as a result. The output of the game simulation algorithm implemented in the MatCos programming environment is shown in Figure 1.

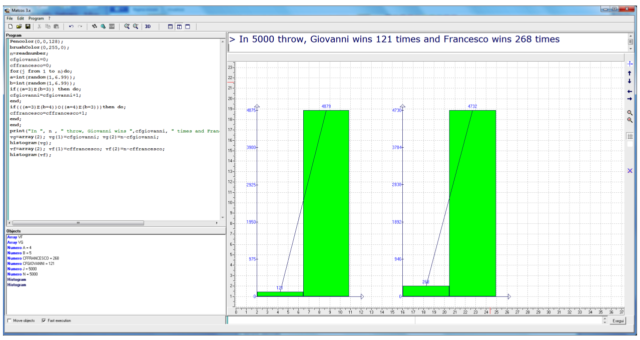

Figure 1: Output of the game for 5000 repeated trials.

After the virtual simulation of the game for a growing number of throws the students come to realize that Francesco's probability of winning is higher than Giovanni's.

STEP 4: Evaluation of the results obtained in output and comparison with the theoretical model.

The students proceed to the algebraic solution of the suggested game and compare the results with the numerical values of the relative frequencies obtained in the virtual simulation. That is how they realize that Francesco has twice the probability of winning compared to Giovanni, in perfect agreement with the experimental results obtained in output.

Conclusion

The teaching action presented here increases students' confidence on statistical methods by raising their awareness of random phenomena, thus preventing conceptual difficulties during the learning phases of the mathematics of uncertainty. Furthermore, computer-based virtual simulation through programming adds value to the learning experience because students can better comprehend the concept of probability of an event, and allocate it a 'degree of reliability' in predicting random phenomena. At the same time the learner has a real chance to learn about algorithms and constructive aspects of mathematics which are generally neglected in school education.

References

Batanero, C., Henry, M., Parzysz, B. (2005). The nature of chance and probability. In G. Jones (Ed.). *Exploring Probability in School: Challenges for Teaching and Learning.* Springer, New York, pp. 15-37.

Beltrami, E., (1999). *What is random? Chance and order in mathematics and life.* New York: Copernicus/Springer-Verlang.

Bennet, D. J. (1998). *Randomness.* Cambrige, MA: Harvard University Press.

Biehler, R., (1991). Computers in probability education. In: Kapadia, R., Borovcnik, M. (eds.). *Chance Encounters - Probability in Education. Dordrecht*: Kluwer, Vol. 12, pp. 169-211.

Costabile, F.A., & Serpe, A. (2013). Computer-Based Mathematics Instructions with MATCOS: A Pedagogical Experiment. In P. M. Pumilia-Gnarini et al. (Eds.), *Handbook of Research on Didactic Strategies and Technologies for Education: Incorporating Advancements, Information Science Reference*. IGI Global, Hershey, USA, pp. 724-738.

Everitt, B.S. (1999). *Chance rules: An informal guide to probability, risk and statistics.* New York: Copernicus/Springer-Verlang.

Fischbein, E., Nello, M.S., & Marino, M.S. (1991). Factors affecting probabilistic judgements in children and adolescents. *Educational Studies in Mathematics, 22*(6), 523-549.

Frassia, M.G., (2015). Games design with the programming environment MatCos for enhanced learning of probability theory: a didactic example for high schools. In *Proceedings EDULEARN 2015, International Conference on Education and New Learning Technologies.* IATED, Barcellona, pp. 1165-1172.

Gal, I. (2005). Towards “probability literacy” for all citizens: building blocks and instructional dilemas. In G. Jones (Ed.). *Exploring Probability in Schools: Challenges for Teaching and Learning*. Springer, New York, pp. 39-63.

Gigerenzer, G., Swijtink, Z., Porter, T., Daston, L., Beatty, J. & Kruger, L. (1989). The empire of chance: how probability changed science and everyday life. New York: Cambridge University Press.

Greer, B., & Mukhopadhyay, S. (2005). Teaching and learning the mathematization of uncertainty: Historical, cultural, social and political contexts. In G. Jones (Ed.). *Exploring Probability in School: Challenges for Teaching and Learning.* Springer, New York, pp. 297-324.