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THE TEACHING OF PROBABILITY IN CONTEXT THROUGH READING AND WRITING STRATEGIES AT SECONDARY EDUCATION

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This article aims to present a research on a methodology to contextualize teaching of Mathematics through Reading and Writing strategies, as recommended by the current São Paulo State Curriculum (São Paulo, 2010) and the current National Curricular Standards (Brasil, 2002). It is exemplified by the teaching of Probability at Secondary Education and consists of two types of strategies: "instruments" applied to all subjects (opening questions, glossary, attachments, closing letter) and "activities" that depend on the subject (for Probability: games, problem solving, learning objects, open-ended questions). The results allow us to affirm that the methodology has promoted the construction of probabilistic knowledge, enabling students to deepen, consolidate and systematize concepts beyond intuition, common sense and simple exercise solving.

INTRODUCTION

The approach between school and real world through *Contextualization* is one of the bases for all disciplines in the current São Paulo State Curriculum (São Paulo, 2010), implemented from 2008. In this research the *Contextualization* of Mathematics is understood as a practice that inserts facts of everyday life and practical applications of the subjects and relates them to other disciplines, other mathematical subjects and to the History of Mathematics, proposes exciting games and problems as well as encourages students to use technological resources. In recent years, *Reading and Writing* have been also emphasized in order to enable *Contextualization* of the contents and constitute another important basis of the current State Curriculum. Also, the current National Curricular Standards (Brasil, 2002) and the National Secondary Education Examination (ENEM, according to its initials in Portuguese) emphasize the use of *Reading and Writing* strategies for *Contextualization* of all curricular components as well.

However, even today many teachers teach Mathematics in a fragmented way, away from students' reality and through traditional approaches. One of the reasons is that theoretical frameworks (including Curriculum and Curricular Standards) not always make clear as *Contextualization* and *Reading and Writing* can be put into practice. This article deals with a research that has been developed emphasizing these two bases of the current State Mathematics Curriculum. It has been applied to several mathematical subjects of Secondary Education (Combinatorial Analysis, Arithmetic and Geometric Progressions, Trigonometry, 3D Geometry, Financial Mathematics) and is exemplified in this article by the teaching of Probability.

METHOD

The research was applied to four Secondary Education classes in a public school in Suzano, SP, Brazil, and consists of two types of strategies: *instruments* – applied to all subjects; *activities* – defined depending on the subject. The first *instrument* was the *opening questions*. The students answered in writing directed questions in order to prepare them for the subject that would be studied and to get signs of what activities should be developed (adapted from Santos, 2005). For Probability the *opening questions* were: – a) What is Probability? – b) Do you know any application of Probability? If so, what application? – c) What is a *random experiment*? Give examples – d) What are *certain event* and *impossible event*? Give examples of both.

The analysis of the responses revealed that the most of students could identify the presence of Probability in everyday life. However, it was noted they gave meaning to the words and mathematical expressions according to their historical-cultural context, showing the teacher needs to be very careful in order the students do the appropriate generalization (Pimm, 2002). This fact indicated that the development of Probability should not start in a formal way; greater formalization could be achieved gradually. Thus, the activity chosen for the introduction was the *head and tail game*. Each pair of students tossed a coin 30 times and noted the number of heads and tails. Upon completion of the results at the four classes they discussed the results of each pair of the class and the total of all classes (2,010 throws, with 996 heads and 1,014 tails). The conclusions of the discussions were recorded in writing.

Comparing the experience with the tossing of a dice and with the card game, the students achieved to Laplace's representation of Probability as a fraction and started to use it without many difficulties in *problem solving* on games and lotteries, quality control, weather forecast, Genetics, insurance, researches etc. When necessary, calculators could be used to transform the fraction into a decimal number and a percentage. The problems were based on National Secondary Education Examination (ENEM), which is much contextualized, requiring a lot of reading. The students were asked to check if the solutions were consistent with the situations presented, to prepare their own problems or to create other questions to the ones presented by the teacher. Examples of problems proposed to the students: – 1) According to Brazilian Institute of Geography and Statistics (IBGE, in its initials in Portuguese), from the total of 61.3 million of Brazilian households in 2011, 43% had at least one computer. However, 15% of these households had no access to Internet. Choosing at random a household in Brazil, what was the probability that it had a computer with Internet? – 2) A survey of 5,000 people revealed that 2,220 has the A antigen in the blood, 1,930 the B antigen and 1,550 none of them. What is the probability that one of these people, chosen at random, has both antigens?

Besides this kind of problems, in which data are provided by the text, it was also proposed other types, whose data were provided by tables or graphs. During the activity, it was commented that Probability had its origin precisely with games of chance, but now it has more *meritorious* applications, as in the other subjects dealt with in the problems. The biographies of mathematicians who created the bases of Probability Theory, especially of Fermat, Pascal and Laplace, were also commented.

The next activity was the *learning objects* available at the Teacher's Portal of Brazilian Ministry of Education (portaldoprofessor.mec.gov.br/fichaTecnicaAula.html?aula=28034 - retrieved July, 2013): – a) *Probabilistic Notions from a Popular Culture Game* software allowed the students to estimate empirically the possibilities of different sums of matchstick amounts, to understand the concepts of *equally likely events, union, intersection and difference of two events* and to interpret bar graphs and double-entry tables; – b) *Probability with Urns* software allowed them to investigate the *random behavior* of the balls extracted from an urn, to understand the concepts of *relative* *frequency, dependence and independence of events* and to interpret temporal evolution graphs. The material is self-instructive, the calculator of the *learning objects* can be used and it is necessary to answer questions in writing. During the activities the students discussed the relationship among Probability, Functions and Combinatorial Analysis and recorded in writing the discussion results.

Then, the students answered in writing *open-ended questions* to show understanding of concepts through the effort to explain and relate them with their own words, as in these examples: – A) Many people who bet on Mega-Sena Lottery consult graphs or frequency tables on the numbers already raffled, to combine 6 to 6 the more or the less frequent ones, what is called *closing* the chosen numbers, and so they believe can hit the *Sena* (the 6 winning numbers). Do you think this method is efficient? Why? – B) The soldier training for World War I and II recommended the soldiers took refuge in bomb holes because it was very unlikely another bomb could fall in the same place. What do you think of this? Why? – C) A lady who lives in the State of Minas Gerais, when comes to the State of São Paulo, buys here a National Lottery ticket, but does not do this in her State. Her explanation is that if she buys it in São Paulo, her chances are greater, since most of the winning numbers take place here. Do you think this reasoning is right? Why?[1] – D) Which of the two following statements is the most likely? Why?: a) Dilma Rousseff will win the 2014 election; b) Brazilian football team will win 2014 FIFA World Cup and Dilma Rousseff the election.[2]

During the development of the subject the students prepared an *instrument*, the *glossary*, in order to help them in constructing and assimilating mathematical knowledge (adapted from Santos, 2005). They did a survey of the terms or expressions considered the most important of the subject and gave its definition, applications, curiosities about them etc. To the *glossary* of Probability the terms and expressions were: *random, chance, disjoint sets, mutually exclusive events, independent events, games of chance, probability.*

It should also be produced another *instrument*, the *attachments*, which are self-produced or researched materials from Internet, books, newspapers, magazines etc., that could be articles, drawings, poems and others, on the contents studied. The students should add their own comments to material produced by other people. The purpose of the *attachments* is to provide *Contextualization*, by the student himself, of the contents studied in class, since mathematical knowledge has been historically built and in order to be learned by the student and this learning to constitute a source for his development, it is essential for him to rediscover and to build it again, in a particular way, according to Beatón (2005). By the way, for D'Ambrosio (2004) it is very important to give the student the opportunity of manifesting on the matters he thinks are related to the topic of the lesson, because he feels valued and this can also help enrich teacher's knowledge.

As Probability closing, the students elaborated the *closing letter*, addressed to a relative, friend, colleague etc., explaining the theme, clarifying the concepts they consider the most important and how they were developed, thinking over their importance in daily life, in other disciplines, in professions etc.. (adapted from Santos, 2005).

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[1] Brazil has 27 States and one Federal District; São Paulo is the most populated State, with 22% of the country population and Minas Gerais is the 2nd one, with 10%.

[2] Question proposed in 2013. Dilma Rousseff was re-elected President of Brazil in 2014, as expected, and Brazilian football team did not win the World Cup, which was not expected.

RESULTS

The students were monitored during the strategies (*formative assessment*), which were done in pairs to allow interaction and exchange of experiences. Upon closing of Probability there was also an individual test (*summative assessment*) to evaluate how they dealt with all the concepts studied, with very positive results. The two types of assessment showed the students were able to evolve during the construction of concepts towards probabilistic reasoning.

Analyzing each *instrument*, the *opening questions* proved to be a great way for verifying knowledge that the students already had brought from their historical-cultural context and how it could be deepened. Regarding the *glossary*, comparing the definitions with the answers of the *opening questions*, it was possible to verify that concepts evolved to their mathematical sense, either in the unknown terms and expressions or in the ones they already knew in their regular sense and could extend it to Mathematics. For example, in the *opening questions* many students defined the expression *random experiment* in its ordinary sense as: “experiment with no plan and purpose”. (Student No. 36, 2A). But in the *glossary*, which was done closer to the end of the theme development, the majority defined it in mathematical sense, as the same student: “casual experiment, which can give different results and we cannot guess”. One of the causes of this fact, according to Pimm (2002), are certain differences between mathematical register elements borrowed from natural language and their use in this language, a fact that has strong influence from the students’ historical-cultural context.

Thus, it seems the students overcome what the same author describes as incongruity and disruption of communication, caused by the lack of discrimination between both natural and mathematical languages, once that many mathematics classes are developed around a mix of the two language registers. It can be said that the definition of the terms with their own words has helped the students to organize information, to construct mathematical knowledge as well as to improve the perception they already had of the terms and the meaning they attributed to them, as stated by Santos (2005).

The research and production of *attachments* showing the presence of Probability in daily life, in professions and in other areas were very helpful for the consolidation of the concepts, favoring a positive attitude towards Mathematics. Many students attached texts from Internet on the relationship between Genetics and Probability. Some extracts from comments added to the texts:

> Probability acts on genetics: the chance of having the same grandfather’s genetic disease or what will be the color of the baby’s hair, eyes or skin. Everything can be calculated using the distribution of genes and the probability rules. (Student No. 9, 3B).
>
> Probability helps when it’s necessary to show compatibility among genes. There are ways to find criminals or to do paternity tests with 99.9% of certainty through DNA testing. (Student No. 31, 3B).

Similarly, some students searched texts from Internet about the History of Probability, adding to them some comments like:

> I didn’t know that former mathematicians enjoyed playing and calculating the probability of winning in games of chance, they were really very smart. (Student No. 19, 3A).
>
> How important is probability! It’s applied in so *crazy* things as games of chance, genetics, insurance, weatherman, quality control, medicine tests and a lot of other things of our day-to-day. (Student No. 4, 3A).

Motivated by the resolution in class of the probability of hitting the Mega-Sena, the greatest Brazilian lottery, many students became interested in researching more about lotteries and in analyzing the probabilities on the back of their tickets. They attached the tickets of various types of lotteries, calculated the probability of hitting some of them and did comments like:

> And the government has the effrontery to put on the Mega-Sena ticket that the chance to hit the winning numbers is 1: 50,063,860. It's because the government puts this very very small and on the back of the paper otherwise everyone would suspect. (Student No. 32, 3A).

> The calculation we did in classroom was totally correct. It's true that is one chance in more than 50 million to hit the *sena*. Math never lies. (Student No. 24, 3B).

One student attached a scratch card coming inside a snack pack and calculated the probability of winning the game. He managed to reach the correct result, which was very unfavorable to the consumer. The comment he added: "These traders like very much to delude people; it seems easy to win the game but it isn't. And we only win one more snack pack". (Student No. 22, 2B).

As an *attachment*, a student wrote the text below, demonstrating the concepts on the subject that had been assimilated by her:

> GUESS WHO I AM – I study at 2nd A class. In my class there are 35 students. Then the probability of *u* guessing who I'm is 1/35. I'm a girl. In my class there are 20 girls. Then the probability of *u* guessing who I'm is 1/20. My name begins with the letter A. In my class there are 8 girls whose names begin with the letter A. Then the probability of *u* guessing who I'm is 1/8. My name's Aline. In my class there are 3 girls named Aline. Then the probability of *u* guessing who I'm is 1/3. My name's Aline A. So *u* know who I'm because in my class there is only me with this name. That's not fair because this is a *certain event*, one thing that will happen for sure, as the teacher has taught. Did you like it? (Student No. 2, 2A).

The *closing letter* reinforced mathematical concepts assimilated by the students and learning aspects experienced by them, becoming an efficient communication channel between the students and the teacher (Santos, 2005). This type of recording allowed the students to use writing to express what they had learned and the difficulties that still remaining. Examples of *closing letters* presented by the students:

> Probability is summarized as the results of chances calculated from data collected through numbers, tables or graphs. It helps us to understand the chances of a part being defective, of a child having his father's eye color, etc. For example, if *u* have a number of chances in a total number of them, just divide the 1st number by the 2nd one and *u*'ll have the probability, which can be a fraction, a decimal number or a percentage. Very easy! (Student No. 10, 3B – fragment of a letter to a friend).

> I'm very happy these weeks because I've learned new things and found myself surprised with my progress in math on probability learning. Very *cool* can know the possibilities of the occurrence of events, especially in genetics and in games of chance! I'd some difficulties with interpretation of the problems, but I think I'm overcoming this with many exercises. I also have difficulties with graphs, but I also think I'm going to overcome all of this. (Student No. 7, 2A – fragment of a letter to his mother).

The *activities* for introduction and development of Probability also brought great results. *The head and tail game* showed to the students how mathematical concepts can be developed from an informal situation, as with many concepts along the History of Mathematics, demystifying the idea that the discipline deals only with abstract and complex situations. It was verified that students were able to observe that, even most of them having previously the notion that the probability of hitting head or tail is 50%, this does not mean that in 30 throws we must necessarily have 15 heads and 15 tails, since the results achieved in class typically ranged from 10 heads and 20 tails, going through

15 heads and 15 tails, reaching 20 heads and 10 tails (results as 21 heads and 9 tails were very rare, as expected). But for the large number of throws considered, virtually all the students noted that these numbers approach 50% of the total, as shown by the sum of the four classes results: total of 2,010 throws, with 996 heads (49.6%) and 1,014 tails (50.4%).

Solving problems that followed the game, incentivizing the student to analyze the responses, to develop other questions to the problems posed by the teacher, to create their own problems etc., involved the students a lot, stimulating curiosity and creativity, with very positive results for constructing mathematical knowledge. In most problems, the only difficulty faced by a few students was at the beginning of the resolution, that is, in determining the number of favoring results, due to difficulties in basic mathematical concepts, once that the problem data could be provided by the text itself or by tables or graphs. In preparing problems by themselves, several students proposed similar problems to the ones presented above, on the access to Internet and on blood antigens. One student, for example, searched on Internet the percentage of Brazilian people connected to social networks as well as to a given one and developed a problem on the probability of a person of the country, chosen at random, be connected to that specific network. Similarly, other students presented data on several subjects in the form of tables or graphs, taken from Internet, and elaborated questions on Probability. Some examples can be cited: a problem on vaccination against influenza, as well as another one on a blog visitors' opinion.

As in Kleine & Lopes (2012) the students established a warm relationship with the *learning objects*, reflecting and writing their conclusions as if they were *playing virtually,* what allowed them to advance from the representation of Probability as a quotient to its estimate through statistical data collection (Batanero, 2005). Another explanation for the success of these activities is the fact that they respect students' pace in the construction of knowledge, as they could repeat them, as well as establish another sequence of the various stages of each *learning object,* allowing the students to work independently and to develop self-confidence, as was also noted by Kleine & Lopes (2012).

The record of the discussion about the *activities* and on the relationship between Probability and other mathematical topics allowed again that students could use writing to express what they had learned and the difficulties still remained. For example, many students wrote that they came to the conclusion that games of chance actually do not depend on luck, but on Probability. Other students mentioned their difficulties with the basics of Mathematics, such as percentages, reading of tables and graphs etc., emphasizing that the *activities* have been important to solve or to minimize these doubts. In students' statements in general, we can note that, even when they presented difficulties in expressing themselves in writing, committing some grammatical or structural errors, using informal language or being repetitive, they succeed in communicating their ideas.

The *open-ended questions* showed how much they advanced, which confirms that expressing their own reasoning through *Reading and Writing* promotes Mathematics understanding. Regarding to question A (on Mega-Sena Lottery), few students fell into *gambler's fallacy*, indicating as efficient or the choice of the most frequent numbers or of the less frequent ones, possibly still influenced by popular beliefs related to games of chance:

> I believe that are the numbers that comes out less than others because since all the numbers have the same chance, if they've came out less they're more likely to win. (Student No. 35, 3A).

> If a number has come out many times, it's more likely to come out again. If the person can spend money it'll be possible *to close* the numbers that have come out many times and to get more chances to hit the *Sena*. (Student No. 8, 2B).

However, the majority of the students took into account the concepts covered in class:

> No, because it is a game of chance, and if the method was efficient there would be many winners and the *guys* who make the tables and graphs should win almost always. (Student No. 21, 3A).

> It's the same thing to bet the more or the less frequent numbers because the other ones *u* haven't chosen can always win, but it isn't easy *to close* the chosen numbers because the combinations are many and the bet will be very expensive or even impossible. (Student No. 29, 3A).

With respect to question B (on soldier training), the majority of the students understood that the probability of the first event does not reduce the probability of the following one: "The training is mistaken *'cause* in the same way the bomb falls once in a place it may fall another time; so, it's equal to the chance of falling elsewhere and the probability remains the same." (Student No. 35, 3A). Some students felt that the training was correct, but gave plausible explanation for their opinion: "Because the dangerous target was already achieved, so it's meaningless to throw another bomb in the same place." (Student No. 33, 2B). Similarly, a student has surprised the teacher with bringing the problem to current times, "If it was today, when we have *piles* of drones and radars, it was right, because the places of bombings are carefully chosen and they aren't going to waste time with targets already bombed." (Student No. 3, 3B).

Concerning question C (on National Lottery), very few students let themselves be influenced by the apparently well elaborated reasoning of the lady (this is a real happening). However, most of the students showed they have understood the issue:

> The truth is that in São Paulo there are more ticket buyers than in other regions so it's more likely the winner is here, but it makes no difference buying here or there. (Student No. 36, 2A).

> No, because all tickets are put together and there's one possibility among a *pile* of possibilities anywhere. They come out more here because there are more people here. (Student No. 12, 2B).

Regarding to question D (on election and World Cup), some students answer the question in a less rational manner, indicating that the most likely was the second statement, probably due to Brazilian people's collective desire to win the World Cup. However, many others realized it was the probability of simultaneous occurrence of two independent events:

> The most likely is Mrs. Dilma Rousseff to win the election because her single probability is considerably greater than the probability of the two things happening at the same time. (Student No. 37, 3A).

> Dilma be president because I'm saying only one thing to happen, but in the letter *b* there are two things to happen so it's easier to happen only one. (Student No. 35, 3A).

CONCLUSIONS

The results enable us to conclude that the application of the methodology has promoted the construction of probabilistic knowledge and has provided to students the practice of *interdisciplinarity*, a wide awareness of the importance of the theme in everyday life and a critical view on lotteries and other games of chance. There was also the opportunity to verify that mathematical contents are not insulated, to realize that mathematical knowledge reflects the historical moment of its creators, to systematize concepts beyond intuition, common sense and simple exercise solving as well as to develop creativity, autonomy, self-confidence and citizenship.

For example, in the following statements we can see changing in some students' perception regarding Probability:

> I've realized how probability is important in daily life, because when I'm going to do something that's possible to calculate its chance I'm already going to know the probability and I'm going to see if it's worthwhile or not to invest in such a thing. (Student No. 8, 2B).
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> My father usually bets on everything that's lottery. I'm going to tell him not to do this anymore because it's a waste of money because the chances to win are very small. (Student No. 27, 2A).

So, we can consider interaction among the students, the teacher and the strategies has had positive influence on deepening, consolidation and systematization of probabilistic knowledge, allowing the students to move from *intuitive meaning* of Probability to its *classical meaning*, towards *frequential meaning* thus creating the possibility for later, at Tertiary Education courses, to reach *subjective and axiomatic meanings*, according to the different meanings of Probability cited by Batanero (2005).

It is worth mentioning that other types of *activities* were applied for introducing and developing other Secondary Education mathematical contents, depending on their characteristics and on the analysis of the responses to the *opening questions* of each theme, but keeping the *instruments* used in Probability. These other *activities* could include: *supplemental related teaching texts*; *newspapers as a teaching resource* (Corrêa, 2005); *texts from other contexts* (Fonseca & Cardoso, 2005); *concept maps with explanatory texts* (Santos, 2005) etc.. The outcomes of the methodology application were also very positive for those contents (Combinatorial Analysis, Arithmetic and Geometric Progressions, Trigonometry, 3D Geometry, Financial Mathematics).

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