

## **THE RISE AND FALL OF PROBABILITY IN THE K–8 MATHEMATICS CURRICULUM IN THE UNITED STATES**

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*Probability is no longer featured as an important domain of study in the K–8 mathematics curriculum in the United States. I illustrate its decline via an overview of the various standards documents that have guided curriculum and instruction for the past 25 years. Also, I discuss the implications of this curricular shift for the research community and raise questions for the consideration of participants in the Teaching and Learning of Probability Topic Study Group.*

Probability has been recognized as an important topic in mathematics education, across the grade levels, since the 1950s (Jones, Langrall, & Mooney, 2007). For example, in the United States, the authors of the Cambridge Conference report (Educational Services Incorporated, 1963) described probability as a critical component of a liberal education that should be taught beginning in the elementary grades. During the era of “new math,” the School Mathematics Study Group developed textbooks for probability instruction in the primary and intermediate grades (Blakeslee et al., 1966a, 1966b). And more recently, Moore (1990) identified a set of key probability concepts to be developed beginning in the elementary grades. Additionally, research on the nature of probabilistic thinking and the teaching and learning of probability has laid a strong foundation to support the inclusion of probability at all levels of the school mathematics curriculum. For reviews of this robust body of research, see Shaughnessy (1992); Borovcnik and Peard (1996); Jones (2005); Jones, Langrall, and Mooney (2007); and Chernoff and Sriraman (2014).

In 2005, Graham Jones argued that probability had become an established strand within the mathematics curriculum, as evidenced by the inclusion of probability across the grade levels in national curriculum documents in Australia, the United Kingdom, and the United States. Indeed, a report of an analysis of state mathematics curriculum documents (Dingman & Tarr, 2011) indicated that as of 2005 most U.S. states included learning expectations for probability at each grade level, albeit with considerable variability in grade level placement. However, as of August 2015, the majority of U.S. states have adopted a mathematics curriculum that has eliminated probability as a topic of study in the elementary grades and narrowed its focus in the middle grades (Common Core State Standards Initiative, 2015b).

### **CURRICULUM STANDARDS FOR PROBABILITY**

The shift in status of probability in mathematics education in the United States can be seen in the standards that have been developed to guide the design of curriculum at state and local levels and to ultimately affect instruction in mathematics. The first of such standards documents was the *Curriculum and Evaluation Standards for School Mathematics* published by the National Council of Teachers of Mathematics (NCTM) in 1989. In that document, standards were organized according to grade bands; the elementary and middle grade levels (i.e., Kindergarten through Grade 8) were represented in two bands, Kindergarten–Grade 4 and Grades 5–8. For the Kindergarten–

Grade 4 band, probability was addressed in a standard with statistics and was aimed at exploring concepts of chance. The emphasis was specifically on inquiry and problem solving: “The study of statistics and probability highlights the importance of questioning, conjecturing, and searching for relationships when formulating and solving real-world problems” (National Council of Teachers of Mathematics [NCTM], 1989, p. 54). An example activity highlighted the importance of exploring aspects of probability in relation to collecting and analyzing data as well as providing opportunities for children to discuss events that are likely, unlikely, or certain and to consider the notion of luck.

At the Grades 5–8 band, probability had its own standard. It called for students to be engaged in explorations of probability in real-world contexts and detailed five specific expectations, which are listed in Figure 1. As in the earlier grades, the emphasis was on exploration and real-world problems. In fact, the document specifically stated that the study of probability “should not focus on developing formulas or computing the likelihood of events pictured in texts” (NCTM, 1989, p. 109). The idea of using probability to model situations was introduced at this level: “Students should actively explore situations by experimenting and simulating probability models . . . . Students should talk about their idea and use the results of their experiments to model situations or predict events” (NCTM, 1989, p. 109).

The *Curriculum and Evaluation Standards for School Mathematics* (hereafter 1989 Standards) set the bar for educational standards and they were widely influential in the mathematics education community; their publication prompted revisions of conventional textbooks to be “standards aligned” and a program of funding from the National Science Foundation was designated to support the development and dissemination of curriculum materials that followed the content and pedagogy outlined in the standards (Reys, 2008). In 2000, NCTM published an updated version of the standards, *Principles and Standards for School Mathematics* (hereafter 2000 Standards). One way in which these standards differed from their predecessor was that the same set of content standards (number and operations, algebra, geometry, measurement, and data analysis and probability) spanned the grade bands, which were restructured as Pre-Kindergarten–Grade 2 (Pre-K–2), Grades 3–5, Grades 6–8, and Grades 9–12. The data analysis and probability standard included four key components that were common across all grade bands, and each component had grade-band specific student expectations. One key component pertained to probability: “Instructional programs from prekindergarten to grade 12 should enable all students to . . . understand and apply basic concepts of probability” (NCTM, 2000, p. 48). For the Pre-K–2 band, no expectations were listed for the probability component. However, the text describing the standard states that

probability experiences should be informal and often take the form of answering questions about the likelihood of events, using such vocabulary as *more likely* or *less likely* . . . . Teachers should address the beginnings of probability through informal activities with spinners or number cubes that reinforce other Standards, primarily number. (p. 114)

The specific expectations for students at the Grades 3–5 and Grades 6–8 bands are presented in Figure 1.

Although probability was no longer a separate standard, it maintained its prominence across the grade bands in the 2000 Standards and was more explicitly defined for the upper elementary grades (i.e., Grades 3–5). However, there was less emphasis on modeling situations and real-world contexts than in the 1989 Standards. Probability measurements, which were evident in both

standards documents at the middle grades, were also included in the Grades 3–5 band in the 2000 Standards: “Students in grades 3–5 should begin to learn about probability as a measurement of the likelihood of events . . . they can begin to learn how to quantify likelihood” (NCTM, 2000, p. 181). Thus, the 2000 Standards placed greater focus on the use of appropriate terminology and computing probabilities, which aligned with the expectation that students would be able to apply basic probability concepts.

In 2006, NCTM published *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* (hereafter Curriculum Focal Points). This document was in response to the concern that the U.S. mathematics curriculum was “a mile wide and an inch deep” (Schmidt, McKnight, & Raizen, 1997) and aimed to address variation in the placement and emphasis of topics by grade levels in school systems across the country. Rather than referring to grade bands, focal points were identified at each grade level based on the following rationale:

A focused curriculum would allow teachers to commit more time each year to topics receiving special emphasis. At the same time, students would have opportunities to explore these topics in depth, in the context of related content and connected applications, thus developing more robust mathematical understandings. (NCTM, 2006, p. 4)

Three focal points were identified at each grade level as the content emphases for mathematics instruction. Also, several related topics were presented as possible connections to each focal point. These connections identified concepts or skills that might serve as a prerequisite or follow up to the focal points or identified “ways in which a grade level’s focal points can support learning in relation to strands that are not focal points at that grade level” (p. 8). The only mention of probability in the entire Curriculum Focal Points document was as a connection to the Grade 7 focal points; it is presented in Figure 1. This connection appears to fit the latter purpose stated above. One of the three focal points for Grade 7 pertains to understanding proportionality and similarly and applying these concepts to “solve single and multistep problems in numerous contexts” (p. 19). Apparently, the intent was that teachers should consider probability to be one such context.

Publication of the Curriculum Focal Points marked a significant shift in curricular recommendations that essentially erased probability from the elementary grades. The implications of this change were far reaching. The Curriculum Focal Points were “widely used by state mathematics content developers in designing their own standards and curricula” (Achieve, 2010, p. 1) and thus, influenced the most recent standards document to affect mathematics curriculum in the United States, the *Common Core State Standards for Mathematics* (CCSSM; National Governors Association Center for Best Practices [NGA] & Council of Chief State School Officers [CCSSO], 2010).

In contrast to the three standards documents previously described, which were initiated by NCTM, the CCSSM was a state-led initiative, directed by governors and chief state school officers (e.g., commissioners of education).

The Common Core [which includes standards for mathematics and English language arts] is informed by the highest, most effective standards from states across the United States and countries around the world. The standards define the knowledge and skills students should gain throughout their K–12 education in order to graduate high school prepared to succeed in entry-level careers, introductory academic college courses, and workforce training programs. (Common Core State Standards Initiative, 2015a, “About the Common Core State Standards,” para. 4)

Standards and Learning Expectations for Probability	Grade Levels								
	K	1	2	3	4	5	6	7	8
<b>Curriculum and Evaluation Standards for School Mathematics: Probability Standards</b> (NCTM, 1989, pp. 54, 109)									
• Explore concepts of chance									
• Model situations by devising and carrying out experiments or simulations to determine probabilities									
• Model situations by constructing a sample space to determine probabilities									
• Appreciate the power of using a probability model by comparing experimental results with mathematical expectations									
• Make predictions that are based on experimental or theoretical probabilities									
• Develop an appreciation for the pervasive use of probability in the real world									
<b>Principles and Standards for School Mathematics: Probability Expectations</b> (NCTM, 2000, p. 400)									
• Describe events as likely or unlikely and discuss the degree of likelihood using such words as <i>certain</i> , <i>equally likely</i> , and <i>impossible</i>									
• Predict the probability of outcomes of simple experiments and test the predictions									
• Understand that the measure of the likelihood of an event can be represented by a number from 0 to 1									
• Understand and use appropriate terminology to describe complementary and mutually exclusive events									
• Use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations									
• Compute probabilities for simple compound events, using such methods as organized lists, tree diagrams, and area models									
<b>Curriculum Focal Points: Connections for Grade 7 Focal Points</b> (NCTM, 2006, p. 19)									
• Students understand that when all outcomes of an experiment are equally likely, the theoretical probability of an event is the fraction of outcomes in which the event occurs. Students use theoretical probability and proportions to make approximate predictions.									
<b>Common Core State Standards for Mathematics</b> (NGA & CCSSO, 2010, pp. 50–51)									
• Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring.									
• Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.									
• Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.									
• Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.									

Figure 1: Grade level learning expectations for probability, designated by shaded cells.

Additionally, the CCSSM was intended to address the lack of coherence and focus in the mathematics curriculum as described in the Curriculum Focal Points. At the K–8 level, the CCSSM includes Standards for Mathematical Practice (common across grade levels) and Standards for Mathematical Content (specific to each grade level). The mathematical content standards include descriptions of three or four critical areas for study, and identify related content domains (e.g., statistics and probability) and specific standards that define what students should understand and be able to do. Grade 7 is the only grade that includes a standard for probability: “Investigate chance processes and develop, use, and evaluate probability models” (NGA & CCSSO, 2010, p. 50). The specific learning expectations for this standard are presented in Figure 1.

Following the Curriculum Focal Points, the CCSSM has narrowed the study of probability to one grade level essentially collapsing the learning expectations that were spread across the elementary and middle grade levels in the 2000 Standards. The rationale for doing so is not clear. The authors of the CCSSM document stated, “The development of these Standards began with research-based learning progressions detailing what is known today about how students’ mathematical knowledge, skill, and understanding develop over time” (NGA & CCSSO, 2010, p. 4). Yet the list of works consulted includes no research about the development of students’ probabilistic thinking, no evidence of research-based learning progressions for probability (Mooney, Langrall, & Hertel, 2014). In a plenary talk at a 2010 conference to examine curriculum issues associated with the CCSSM, Jere Confrey commented on aspects of the standards that warranted further consideration or revision. In reference to the statistics and probability standard, she stated: “They ignore significant empirical data on children’s ability to develop an understanding of variability, distributions, data use, statistical reasoning, and probability in the early grades; instead, these are added abruptly in sixth and seventh grade” (Confrey & Krupa, 2010, p. 4).

It is not known why the authors of the CCSSM disregarded the research literature on the teaching and learning of probability. Perhaps they viewed probability solely as a tool for statistics, as presented in the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report: A PreK–12 Curriculum Framework* (Franklin et al., 2007), which was included in the list of consulted work. This might explain the treatment of probability as a skill that could be adequately addressed in Grade 7. But the wisdom of that approach is questionable and the role of probability in the CCSSM certainly deserves further consideration.

## **IMPLICATIONS FOR RESEARCH**

Although the development and implementation of the CCSSM are deeply immersed in political and policy issues that are beyond the scope of discussion here, the apparent disregard of the research literature pertaining to the teaching and learning of probability in the development of both the Curriculum Focal Points and CCSSM should be of concern to the research community. However, the authors of the Curriculum Focal Points asserted that the document “represents an important, initial step in advancing collaborative discussions about what mathematics students should know and be able to do” (NCTM, 2006, p. 2). Likewise, the CCSSM has been described as a “living document” that should be subjected to “short-term fixes, medium-term adjustments, and long-term review and modification, as needed, based on expert advice and empirical evidence” (Confrey & Krupa, 2010, p. 9). Thus, we should consider how probability research might contribute to such collaborative discussions and provide research-based evidence to inform curriculum modifications.

What might be the ramifications of an elementary mathematics curriculum devoid of probability? Will students' experiences representing and interpreting data (basic learning expectations that are included in the CCSSM at the elementary grade levels) provide sufficient opportunities for them to develop intuitions about probability and to engage in probabilistic thinking? Is it reasonable to expect Grade 7 students to develop a robust understanding of fundamental probability concepts and skills that will prepare them for the formal study of probability and statistics in later years? Can the findings of extant research answer these questions? If not, what kind of research is needed?

In this commentary, my focus has been aimed narrowly at curriculum and instruction in the United States. However, there is evidence that the United States is not the only country to omit probability in the elementary grades mathematics curriculum (Kapadia, 2009; Schmidt & Houang, 2012; Watson, Jones, & Pratt, 2013). Thus, I believe this phenomenon and the types of questions I have posed above are important points for discussion in the Teaching and Learning of Probability Topic Study Group at ICME 13.

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