

The role of representations in the understanding of probabilities in tertiary education



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THEORETICAL FRAMEWORK

- There is an increasing recognition that statistical and probabilistic concepts are among the most important unifying ideas in mathematics.
- Nowadays probability and statistics are part of mathematics curricula for primary and secondary schools in many countries.
- The reasons for this development are related to the usefulness of statistics and probability for daily life (Chadjipadelis, 2003), its instrumental role in other disciplines, the need for basic stochastic knowledge in many professions, and its key role in developing critical reasoning (Batanero et al., 2004).



THEORETICAL FRAMEWORK

- Understanding of probabilistic and statistical concepts does not appear to be easy, given the diversity of representations associated to this concept, and the difficulties inherent in the processes of articulating systems of representation involved in probabilistic and statistical problem solving (Anastasiadou, 2007).
- Probability is difficult to teach for various reasons, including disparity between intuition and conceptual development even as regards apparently elementary concepts (Chadjipadelis & Gastaris, 1995).



THEORETICAL FRAMEWORK

- At primary and secondary levels, probability and statistics is part of the mathematics curriculum; yet primary school teachers and mathematics teachers frequently lack specific preparation in statistics education (Anastasiadou 2007, Chadjipadelis, 2003).
- According to Batanero e.a. (2005) probability is increasingly taking an important role in the school mathematics curriculum; yet most teachers have little experience with probability and share with their students a variety of probabilistic misconceptions (Chadjipadelis, 2003).



THEORETICAL FRAMEWORK

- Since an education that only focuses on technical skills is unlikely to help teachers overcome their erroneous beliefs, it is important to find new ways to teach probability to them, while at the same time bridging their content and pedagogical knowledge (Batanero e.a. 2005).
- There is general consensus in the mathematics education community that teachers need a deep and meaningful understanding of any mathematical content they teach (Estrada e. a. 2004).
- Biehler (1990) suggests that teachers require meta-knowledge about probabilities and statistics, including a historical, philosophical, cultural and epistemological perspective on statistics and its relations to other domains of science.



THEORETICAL FRAMEWORK

- The need for a variety of semiotic representations in the teaching and learning of probability is usually explained through reference to the cost of processing, the limited representation affordances for each domain of symbolism and the ability to transfer knowledge from one representation to another (Duval, 1987).
- A representation is defined as any configuration of characters, images, concrete objects, etc., that can symbolize or “represent” something else (DeWindt-King & Goldin, 2003; Goldin, 1998; Kaput, 1985).



THEORETICAL FRAMEWORK

- In the field of statistics and probability instruction, representations play an important role as an aid for supporting reflection and as a means for communicating probabilistic ideas. In this study, we revisited the role of representations in an effort to understand the nature and structure of representations in developing probability concepts.
- We investigated the ability to use multiple representations and translate from one representation to another.



PURPOSE OF THE STUDY

The focus of this study is to evaluate the approach pre-service teachers use in order to solve simple probability tasks.

It is of interest to know whether these teachers are flexible in using algebraic, graphical and verbal representations in probabilistic problems. Most of the teachers in our study use an algebraic approach in order to solve simple probabilistic tasks.

This study analyzes the role of different modes of representation on understanding of some basic probabilistic concepts. Teachers' performance is investigated in two aspects of probabilistic understanding: the flexibility in using multiple representations and the ability to solve the problems posed.



PARTICIPANTS – TASKS – DATA ANALYSIS

- The sample consisted of 243 pre-service students from the University of Western Macedonia.
- For the analysis of the collected data we use the statistical implicative analysis. For this study's needs, a similarity and an implicative diagram are produced.
- The tasks consist of 12 exercises related to the concept and definition of probability, to Venn diagrams, and to probability problem solving.

PARTICIPANTS – TASKS – DATA ANALYSIS



- Task 1. Given two events A and B of a chance experiment and with the help of set theory we have the following event

$$A' \cap B' .$$

Present this event by a Venn diagram (encoded as V1sg).



PARTICIPANTS – TASKS – DATA ANALYSIS

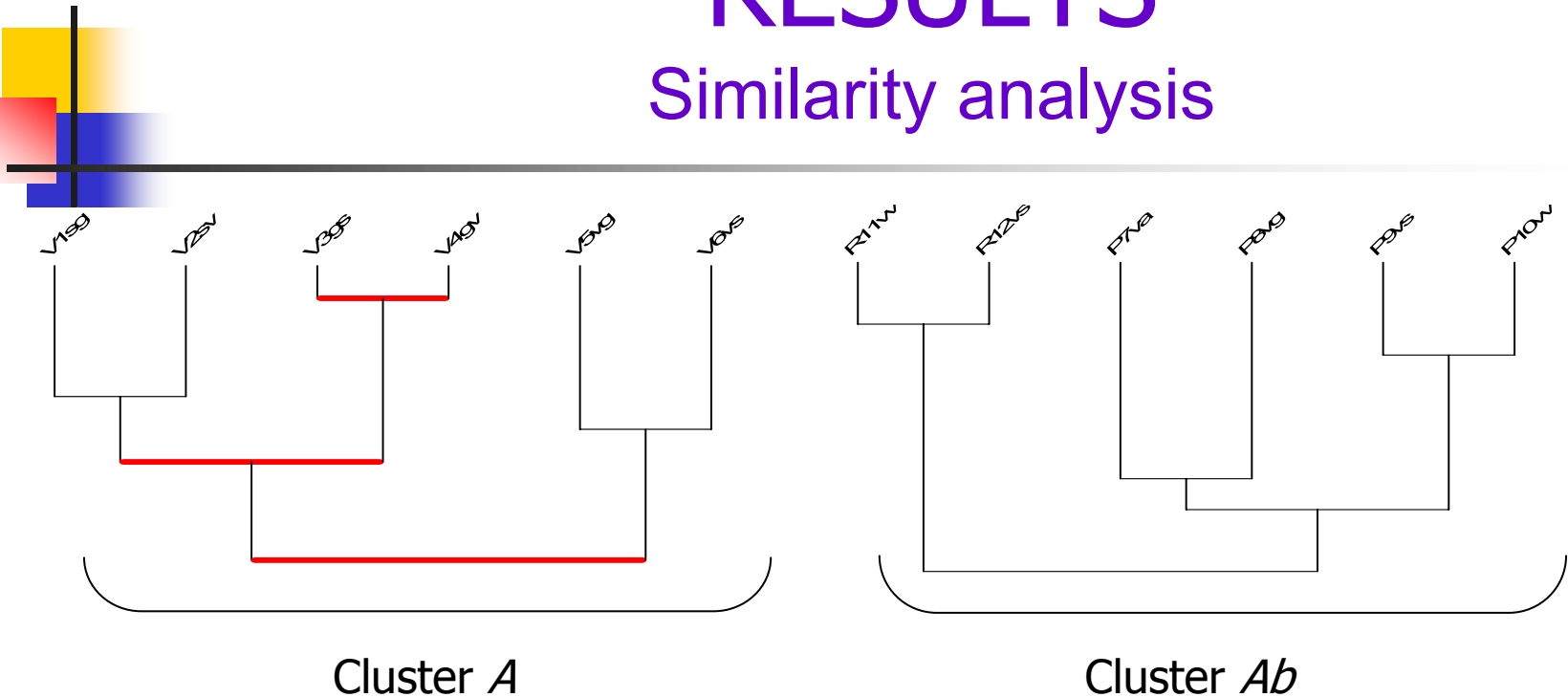
- Task 2. Given two events A and B of a chance experiment; with the help of set theory we define the event

$$(A' \cap B) \cup (A \cap B')$$

Express this event verbally (encoded as V2sv).

RESULTS

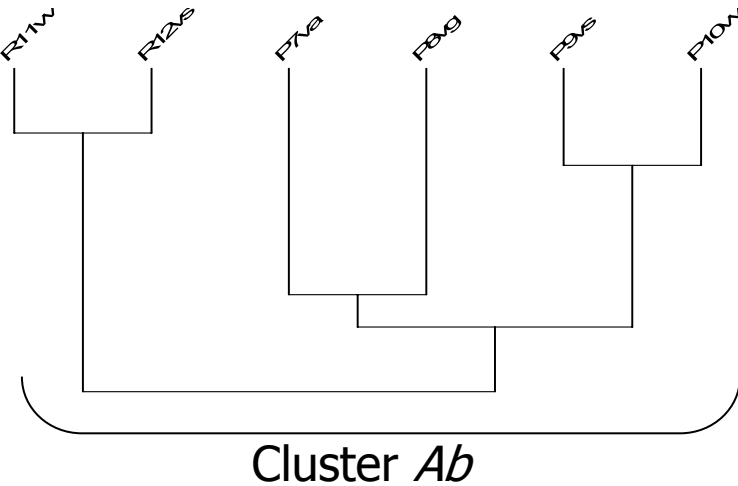
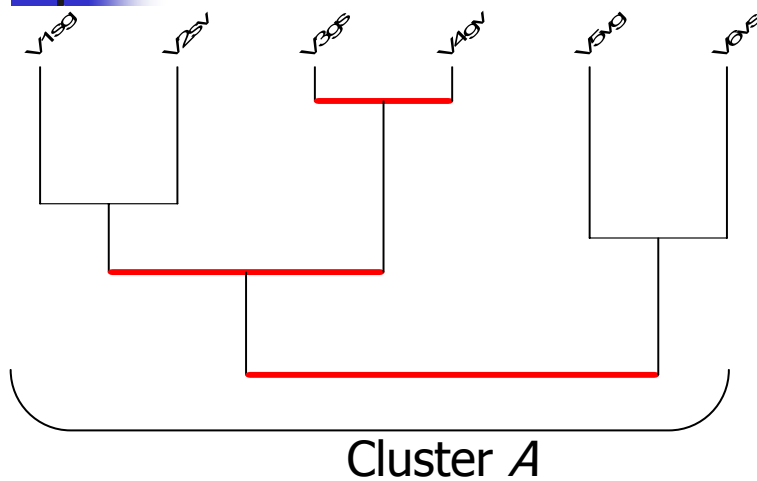
Similarity analysis



The similarity diagram allows the arrangement of the responses to the tasks into groups according to their homogeneity. The similarity lines with red colour are significant at a significance level of 99%.

RESULTS

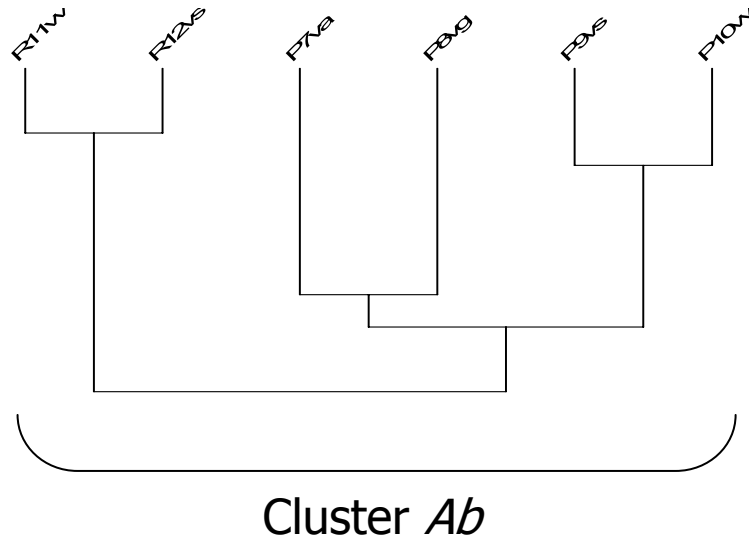
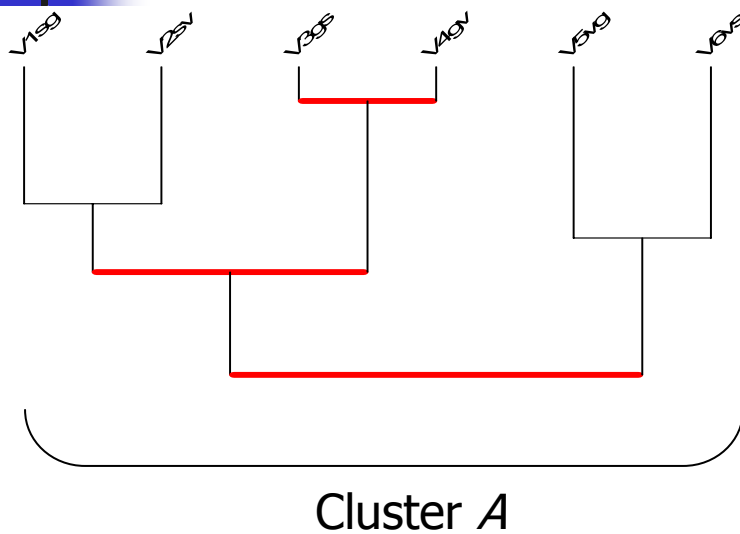
Similarity analysis



Two clusters *A* and *B* of variables are identified in the similarity diagram of pre-service students' responses as shown in the diagram. Cluster *A* involves three pairs of variables V1sg-V2sv, V3gs-V4gv, V5vg-V6vs and comprises representations of events with the aid of Venn diagrams. Cluster *B* involves also three pairs of variables, namely R11v- R12vs, P7va-P8vg, P9vs-P10vv and involves variables relating to probability problem solving. This grouping suggests that students dealt similarly with the conversions involving probability problems of the same cluster.

RESULTS

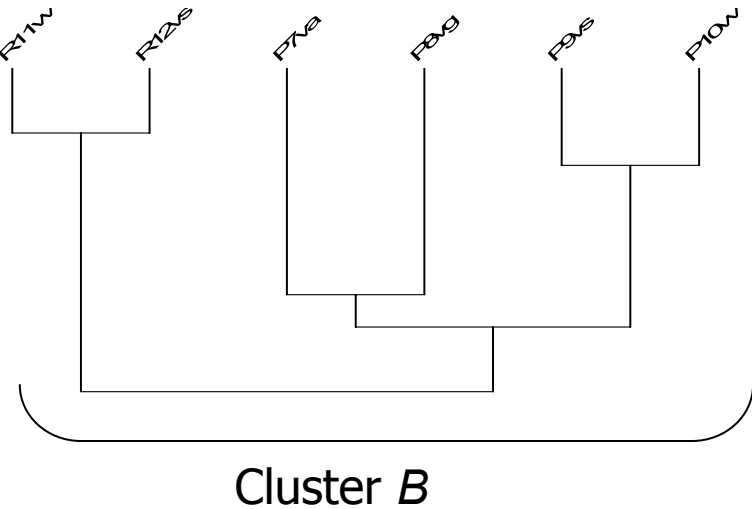
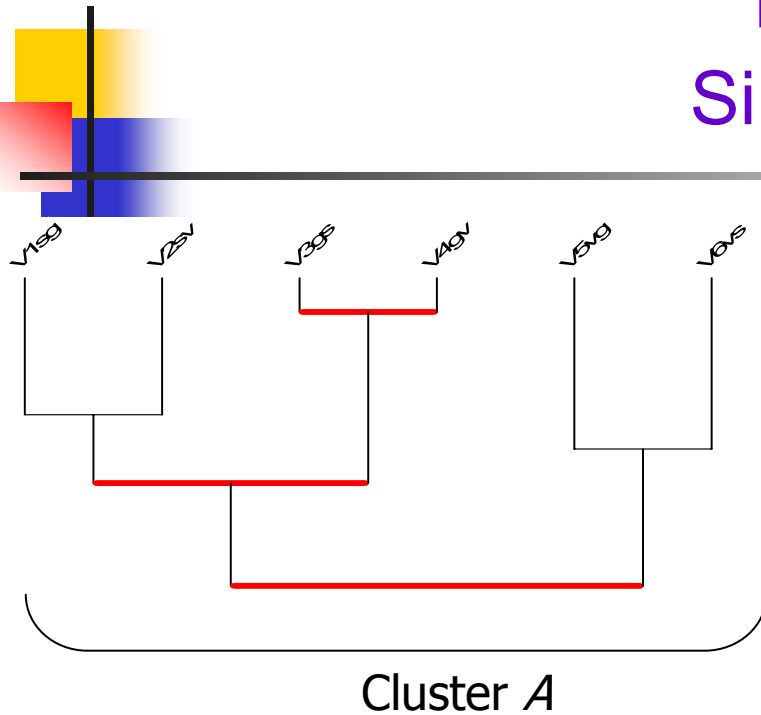
Similarity analysis



The structure of the diagram reveals a cognitive difficulty that arises from the need to accomplish flexible and competent conversion back and forth between different types of probabilistic representations. Thus, this particular structure of the diagram indicates a compartmentalization of the tasks of the tests.

RESULTS

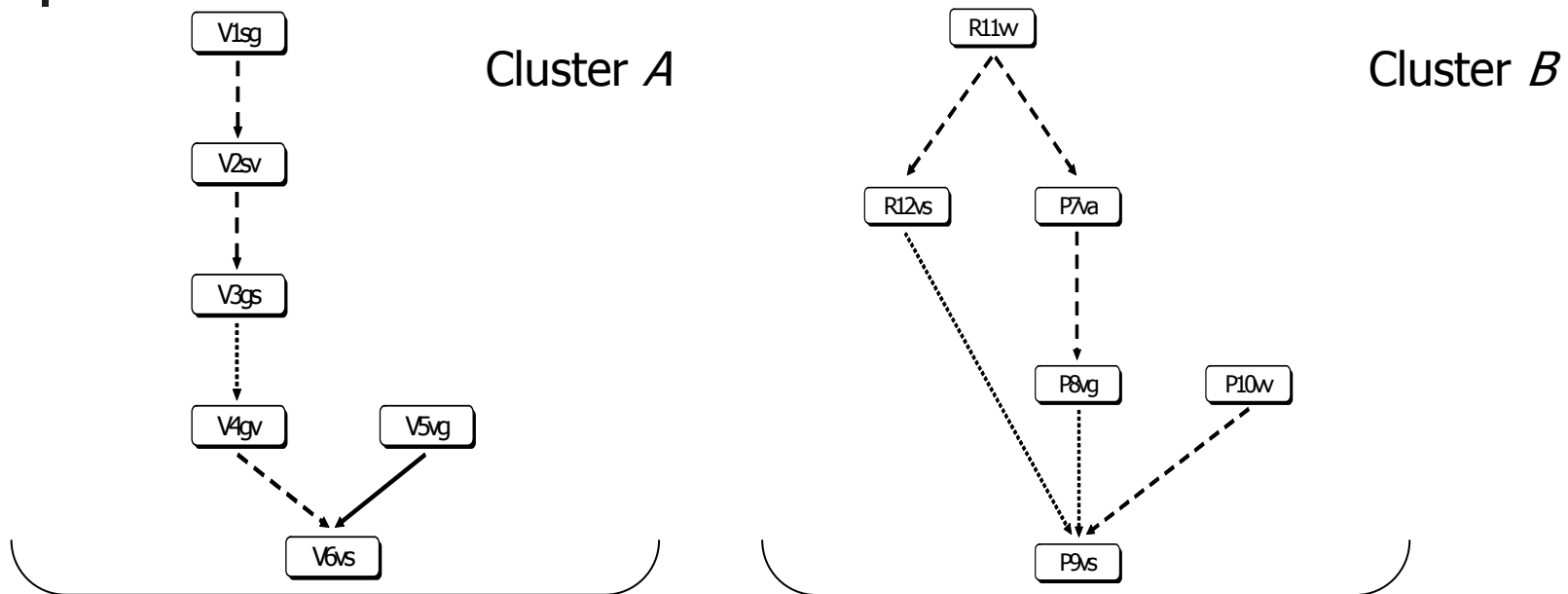
Similarity analysis



On the one hand there are the tasks associated to Venn diagrams and on the other those tasks, which involve probability problems. Students approached the two groups of tasks in a completely distinct way, Therefore, possible instructive activities would focus on the identification of the two different groups. The strongest similarity (almost 1) occurs between variables (V3gs-V4gv). Furthermore the similarity (V1sg-V2sv, V3gs-V4gv) is also important (0.923).

RESULTS

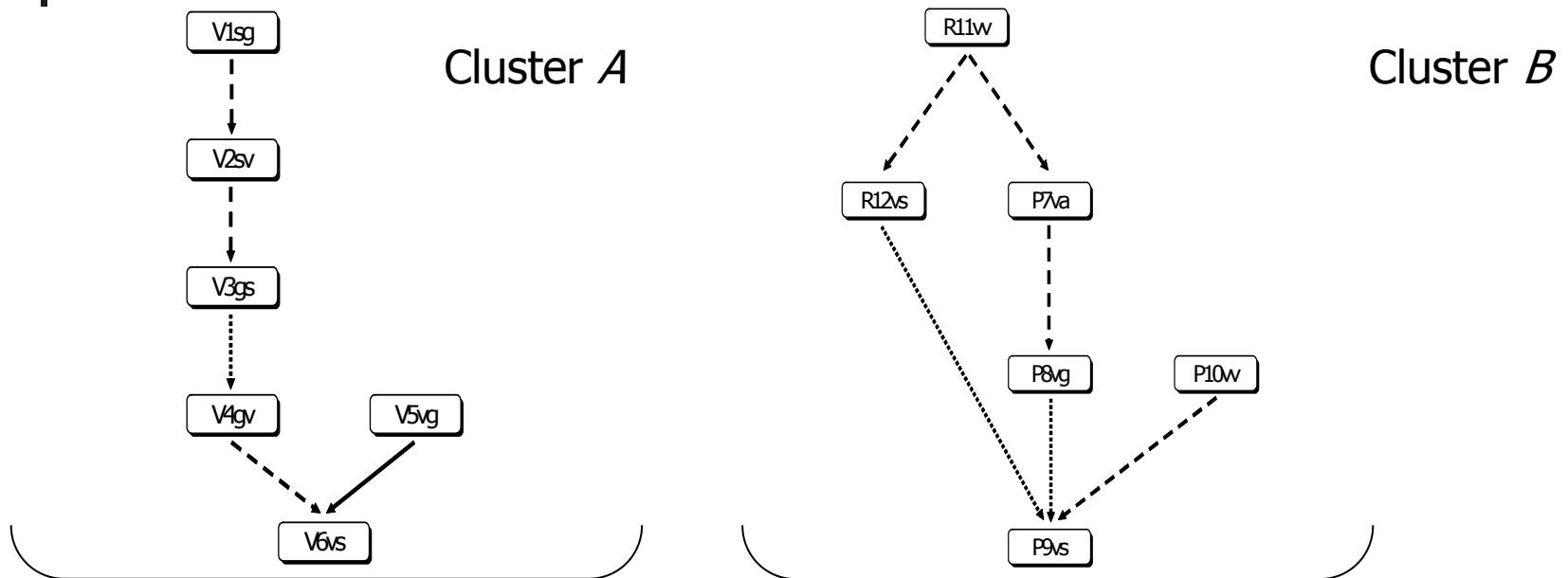
Implicative diagram



The implicative diagram shows the implicative relations between the variables. According to this diagram, not all the tasks of the test are connected by implicative relations. The implications represented by straight, dashed, or dotted lines represent relations significant at levels of 99%, 95%, or 90% respectively.

RESULTS

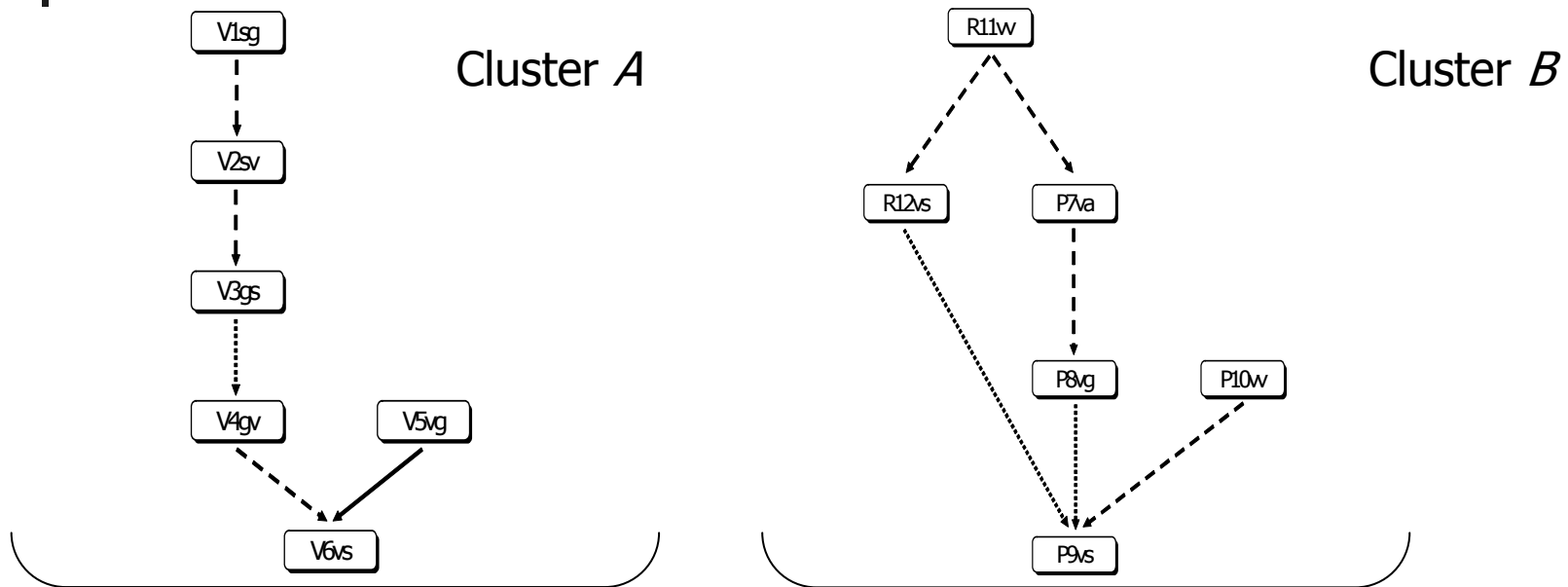
Implicative analysis



Two distinct chains of variables maybe seen from the implicative diagram:
(V1sg → V2sv → V3gs → V4gv, V4gv, V5vg → V6vs) and
(R11w → R12vs → P7va → P8vg → P9vs, R12vs, P8vg, P10w → P9vs)

RESULTS

Implicative analysis



The first one in cluster *A* concerns representation of events with the aid of Venn diagrams. The second one in cluster *B* involves variables related to probability problem solving. This grouping suggests that students dealt similarly with the conversions involving probability problems. The implicative diagram of the pre-service students' responses is exactly in accordance to the similarity diagram.



CONCLUSIONS

Representations enable students to interpret situations and to comprehend relevant relations embedded in probabilistic problems.

Thus, we consider representations to be extremely important with respect to cognitive processes in developing probabilistic concepts.

The main contribution of the present study is the identification of pre-service teachers' abilities to handle various representations and to translate among representations related to the same probabilistic relationship.



CONCLUSIONS

Our findings provide a strong case for the role of different modes of representation on pre-service teachers' performance to tasks.

At the same time they enable a developmental interpretation of students' difficulties in relation to representations of Venn diagrams.

Lack of connections among different modes of representations in the similarity diagram indicates the difficulty in handling two or more representations in probabilistic tasks.



CONCLUSIONS

This incompetence is the main feature of the phenomenon of compartmentalization in representations, which is detected in this study.

This inconsistent behaviour may be seen as an indication of students' conception that different representations of the same concept are completely distinct like if they were autonomous mathematical objects and not just different ways of expressing the meaning of the same notion.



CONCLUSIONS

Probability instruction needs to engage students in activities including translations between different modes of representation.

As a result, students will be able to overcome the compartmentalization difficulties and develop their flexibility in understanding and using a concept within various contexts or modes of representation and in moving from one mode of representation to another.



CONCLUSIONS

It seems that there is a need for further investigation into the subject with the inclusion of a more extended qualitative and quantitative analysis.

In the future, it is interesting to compare strategies and modes of representations students use in order to solve the problems.

Besides, longitudinal investigations might reveal new insights how the flexibility in using the multiple representations grows.



**THANK YOU
FOR YOUR ATTENTION**