The Queensland Responsible Gambling Strategy

Responsible Gambling Education Unit: Mathematics A & B

Teachers' Guide

Outline of the Unit

This document is a guide for teachers to the *Responsible Gambling Education Unit: Mathematics A* and the *Responsible Gambling Education Unit: Mathematics B* student workbooks.

The Teachers' Guide is divided into eight sections.

There are seven sections for both Mathematics A and Mathematics B and one section for Mathematics B alone. Additional material for Mathematics B is also provided in the first seven sections. All Mathematics B specific material is on pages shaded light blue.

There are four sets of exercises.

These exercises are followed by answers on pages shaded light blue. Students will not have the exercise answers in their booklets.

Teachers notes.

Teachers' notes are in **blue boxes with blue text**. The teachers notes and exercise answers are provided in this document and on the supplied CD ROM (see note below).

Information for students.

The information that will appear in the student booklets is in black text.
Student activities are in black boxes with black text.
The complete student booklets are incorporated in this document and are available as separate documents on the supplied CD ROM (see note below).
These separate documents are designed for student use.

CD ROM

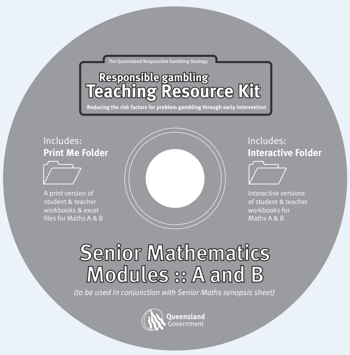
The Senior Mathematics Modules :: A and B CD ROM encorporates two methods for accessing data. On opening the CD you will find two folders available. One is a version for traditional paper based work and the other an interactive program that can be downloaded and made available to individual computers. Refer to the CD synopsis sheet for more information.

Linked Microsoft Excel files.

The Excel files are provided separately on the supplied CD ROM.
Instructions for use of the Excel files are in the appendices of this guide and the Student workbooks.

Includes:
Print Me Folder

Print versions of Teachers' Guide and Student Workbooks for Maths A & B including Microsoft Excel files.



Includes:
Interactive Folder

Interactive versions of Teachers' Guide and Student Workbooks for Maths A & B including linked Microsoft Excel files.

The Maths A syllabus

The subject matter for this unit relates to sections of the Maths A senior syllabus subject matter contained in the QSA syllabus documents 'Data collection and presentation', 'Exploring and understanding data', and the elective topic 'Introduction to models for data'.

It would be expected that any school planning to use this unit in Maths A would include this elective topic in their work program. In addition it would be expected that the unit would provide a venue for 'Maintaining basic knowledge and procedures' (Syllabus Appendix 1).

The Maths B syllabus

The Maths B senior syllabus subject matter relates mostly to the QSA syllabus documents 'Applied statistical analysis' with applications and examples from 'Introduction to functions', 'Exponential and logarithmic functions and applications'. In addition it would be expected that the unit would provide a venue for 'Maintaining basic knowledge and procedures' (Syllabus Appendix 1).

References to specific syllabus subject matter are given throughout.

A note on Excel

Most of the mathematical content can be done without Excel, however access to and use of Excel will greatly facilitate the conceptual development. No prior experience with Excel is assumed and use is explained in each section. No advanced or sophisticated features are employed, such as macros.

Several spreadsheets incorporate the use of scrollbars to enter data. For those computers that will not operate with scrollbars, data may be entered directly into the cells indicated. When generating random numbers, the rand() is used rather than randbetween(a,b) as the latter will not work on all computers.

Teachers with experience in the use of Excel may care to make their own adaptations or extensions.

Web references

There are many websites on the topic of responsible gambling, although most do not address the mathematical ideas involved. If you require more background information, consult the Queensland Government site at the following address.

http://www.responsiblegambling.qld.gov.au/

One site that does include examples of probabilities associated with a variety of gambling situations is the Powerhouse Museum in Sydney.

http://www.powerhousemuseum.com/gambling/

The Powerhouse Museum also has a teaching package that is particularly suited to Maths A.

http://www.powerhousemuseum.com/pdf/education/gambling_education_kit.pdf

The package provided with this document includes a CD that has copies of all the Excel files for use with this unit and links to the above websites.

References

Bernstein, P. L. (1998), *Against the gods. The remarkable story of chance*. Wiley. New York.

Peard, R. (1998), Ethnomathematics and gambling in the Australian social context. In N. Ellerton & A McIntosh (eds.) *Research in mathematics education: a contemporary perspective. (pp. 66-90)* Perth: Edith Cowan University.

Peard, R. (2003), Mathematical expectation: An analysis of the operation of a professional gambling syndicate. *Fourth Southern Hemisphere symposium on Undergraduate Mathematics and Statistics Teaching and Learning,* New Zealand Statistical Association, Queenstown, New Zealand.

Peard, R. *(1998),* The understanding of mathematical expectation in gambling situations. In W. Bloom, G. Booker, I. Huntley & P. Galbraith. *Mathematical Modelling: teaching and assessment in a technology rich world. (pp. 149-159)*. Horwood. Sussex, England.

Introduction

One of the keys to responsible gambling is an understanding of the mathematics governing the operation of all gambling contexts. This allows for an understanding that in the short-term, there will be both winners and losers, but that in the long-run all gamblers must lose.Mathematically, there can be no systems or methods to overcome this.

The key to this requires first an understanding of the concept of randomness and independence and secondly the ability to compute the mathematical expectation of any gambling situation. This is the product of the probability of a gambling outcome and the return associated with that outcome. This is generally expressed as an amount per bet or more commonly as a per cent (%).

In all gambling contexts such as casino betting, poker machines, bookmaker and agency betting, totalisator betting, lotto, keno etc., the mathematical expectation varies from 60% (lotto) to 97.3% (roulette) but is always less than 100%. In all situations, the long-term return to the gambler equals the expectation.

The mathematics required to fully understand this is outlined in the sequence of objectives and activities listed in this guide. The responsible gambler should first be aware of the random nature of the outcomes, be able to compute or estimate the mathematical expectation of any situation, be aware of the concept of mathematical independence that the probabilities will not change as a result of any outcomes, and that the long-term return cannot be affected by any short-term wins or losses.

To achieve this, certain basic mathematical skills and knowledge are required. Research has shown that these skills can be acquired by students with limited previous success in mathematics and are clearly within the capabilities of Mathematics A students.

Maths A and B

1. Basic probability, random generators

This section contains much conventional probability theory with most examples in a gambling context. The treatment of this could be either to introduce the study of probability and then extend the study to other applications or to use this unit after a traditional introduction. Whilst the former is recommended, if choosing the latter treatment, many sections (for example tree diagrams) may be skipped over. Simple probability distributions for Maths B students are introduced.

2. Mathematical expectation

This topic will be met elsewhere in Maths B, but not necessarily in Maths A since it occurs in the elective topics (see above). The applications and student activities here are confined to gambling contexts. Other applications, for example in business, may be encountered elsewhere.

3. Expected mathematical return and casino betting

In all forms of casino gambling, the gambler's expectation is less than 100%. This means that the gambler cannot, in the long run, return a profit from casino betting. In this section the mathematics of computing the expected return for common forms of casino betting are examined. The computation of probabilities, odds, expected return and casino long-term profit from several forms of gambling are explained.

4. Bookmakers and betting agencies operations: betting at fixed odds

With sports betting the true probabilities for most situations are not known. Therefore a bookmaker or betting agency will start taking bets on an event with estimated odds, generally set high and then adjusted down as bets are placed and the bookmaker gets an idea of how bets will proceed. In this section the student learns how to compute bookmaker or agency profit margins and player expectation for various betting situations.

5. The mathematics of totalisation betting

The mathematics of this is quite simple. In all totalisation situations, in the absence of other information, your expected return can be calculated from the profit margin. If the profit margin is 15%, the system returns 85% of bets. This is the expected return.

6. Wagers and "fair" bets

The concept of mathematical fairness differs from the everyday meaning of the term. Mathematical fairness does not necessarily imply equal likelihood but equal expectation. In this sense all gambling situations are not fair but are biased towards the house, bookmaker or agency. Individual wagers against each other may or may not be fair. The criteria and mathematics for determining mathematical fairness are developed here.

7. More complex probabilities and probability distributions.

Again, these topics will be met elsewhere in Maths B. Teachers may choose to introduce these separately prior to using the Responsible Gambling unit. However, should you choose to introduce these concepts via the unit the activities outlined are appropriate. The applications and student activities here are confined to gambling contexts. Other applications, for example in business, could then be studied separately. Further probability distributions for Maths B students are illustrated.

Maths B

8. Hypothesis testing

Again, this topic will be met elsewhere in Maths B. Teachers may choose to introduce it separately prior to using the responsible gambling unit. However, should you choose to introduce it via the unit the activities outlined are appropriate. The applications and student activities here are confined to gambling contexts. Other applications, for example in science, could then be studied separately.

Contents

Maths A & B

1. Basic probability

Subject matter, Maths A:

Maintaining basic knowledge and procedures (Syllabus Appendix 1)
- *Numbers in various notations including fraction, decimal, scientific*
- *Rates, percentages, ratio and proportion*
- *Simple algebraic manipulations*

Data collection and presentation (p. 16)
- *Identification of continuous and discrete data*
- *Practical aspects of collecting data*
- *Graphical and tabular displays*

Exploring and understanding data (pp. 21-22)
- *What a sample represents*
- *Relative frequencies to estimate probability*
- *Interpretation and use of probability as a measure of chance*
- *Misuses of probability*

Introduction to models for data (p. 26)
- *Basic probability of complements and unions*
- *Odds as an application of probability*

Subject matter, Maths B:

Maintaining basic knowledge and procedures (Syllabus Appendix 1)
- *Rates, percentages, ratio and proportion*
- *Basic algebraic manipulations*
- *Tree diagrams*

Introduction to functions
- *Concepts of function, domain and range*

Applied statistical analysis (p. 23)
- *Identification of continuous and discrete variables*
- *Practical aspects of collecting data*
- *Graphical and tabular displays*
- *Relative frequencies to estimate probability*

Exponential and logarithmic functions and applications
- *Use logarithms to solve equations involving indices*

1.1 Some historical background and the meaning of some terms

Teachers

Orienting students to the topic

Before moving into the mathematics of gambling it is useful to ensure that all students have an understanding of what gambling is and some of the historical context behind gambling as an activity and the mathematical study of probabilities.

As a class work through the following orienting activities with your students:

- Refer to the Resource Sheet #1 from the *Responsible Gambling Teaching Resource Kit* – 'Well, is it gambling or not?' and complete the activity.
- As a class group discuss your answers. What made you think some activities were gambling and others weren't? What things did gambling/non-gambling have in common?
- What do we mean by the terms - games of chance and games of skill? Can you give examples? Refer to OHT #1.
- From this activity try to write a definition of the term gambling.
- Find a definition of gambling in the dictionary. Does the definition help you decide what activities in the list were gambling activities? Refer to OHT #2.
- What are some other gambling activities?

Some historical background

As a class group read through the historical background provided:

The study of probability is the study of how chance events can be quantified and measured mathematically. The study of probability is historically a late arrival to the field of mathematics having its origin in the analysis of gambling situations in the 17th century. Although the history of gambling goes back thousands of years, people gambled without any theoretical basis or understanding of probability.

Initial opposition to the study of probability came largely from religious bodies. For them chance did not exist; everything was determined by God.

It is difficult to imagine that in the 17th century when the study of algebra and calculus was well advanced, there was no mathematical structure to answer even simple questions in probability. (*For a good account of the history of the development of probability see Bernstein, 1998.*)

It is recognised that gambling is not confined to the race track or the roulette wheel. Many decisions in life are a 'gamble':

- when to buy a house
- what term of mortgage to take
- will interest rates go up or down and what interest rate to take
- where to invest
- what stocks to buy, what to insure, etc., etc.

Insurance is in fact a type of gambling. When you insure your car against accident, for example, you are betting the insurance company that you will have an accident. If you do, you win enough to pay for the repairs. If you don't, you've lost your premium. Of course, it's a bit more complex than this, but this is the correct mathematical analysis and the mathematical principles that are discussed here apply to all forms of gambling.

Teachers

Class Activity

Refer to Resource Sheet #3 from the *Responsible Gambling Teaching Resource Kit*– 'Some History'. Discuss with the class:

- What were some of the earliest forms of gambling?
- When were the first lottery and horse race held in England?
- How could dice have been used to predict the future?
- Where was the first slot machine invented?

Refer to the historical timeline 'Gambling in Queensland' (OHT #3) and/or 'Gambling – changes and continuities' (Resource Sheet #4).

In small groups ask the students to find the answers to these questions by looking at the timeline:

- When was the first lottery held?
- What was the name of the lottery company? What was the money raised for?
- When was the first horse race held in Australia/Queensland/Brisbane/your local area?
- When and where did the first casino open?
- When were gaming machines introduced to Queensland?
- How many gaming machines are there now in Queensland/in your local area (refer to www.qogr.qld.gov.au for information on gaming machine numbers)?

Refer to OHT 25 – Read the article and discuss as a class group:

- What did you learn?
- What was new to you?
- Did anything surprise you? Why/Why not?

Common forms of gambling include raffles and lotteries, casino gaming, track and sports betting and wagers between individuals. To understand the mathematics of any gambling situation we need to develop an understanding of the chance factors operating, and the random nature of the outcomes.

Consider the *experiment* of rolling a single unbiased die. In one *trial*, there are six different *outcomes*. Since the die is unbiased (and is symmetrical in shape), it is reasonable to assume that each of these is **equally likely**. Thus the outcomes occur **at random** and are **independent** of any other outcomes.

The term **experiment** is used to describe any situation which can result in a number of different **outcomes**. A single performance of the experiment is sometimes called a **trial**.

The set of all possible outcomes of an event is called the **sample space**, though this terminology is no longer widely used and will not be employed here.

The term **event** is sometimes used instead of *experiment*, but strictly speaking *event* refers to the results of the experiment and can mean all the outcomes or a subset of them. For example if the experiment is the tossing of a die, there are six outcomes. We talk of the *event* of getting a six, or the *event* of getting a number less than three. In this sense the language is the same as the everyday language "In the event of …"

1.2 How to express probability

If a coin is tossed, there are just two possible outcomes. Since the coin is symmetrical, it is reasonable to assume that heads and tails are equally likely and the two outcomes occur at random. We can then express the probability of getting heads in any of the following ways:

one chance in two

1/2

50%

0.5

or 1:1 odds or even chances

All probabilities can be expressed in any of these ways. We generally use the one that is most convenient. For example, if we want to express the probability of a lotto draw, we can say:

'One chance in a million' - This is more convenient than 0.000 001 or 0.0001%, though mathematically all three are the same.

Or we might say that there is "an 80% chance of rain". Again, this is more convenient than 4 chances in 5 or 4/5.

1/6 is an easier way of expressing the probability of getting any number from 1 to 6 on a single throw of a fair die, although you could say one chance in six or 16.7%.

Teachers

Prerequisite knowledge

Probability is one of the few situations in which we often prefer to work with fractions rather than decimals.

Mathematics required: the ability to recognise that numbers can be expressed as fractions, percentages and ratios and how to convert one to the other. Also that 4 chances in 5 is the same ratio as 8 chances in 10 or 80 chances in 100. These are equivalent ratios.

1. 3 Views of probability and randomness

1.3.1 Axiomatic, symmetrical, or theoretical

When, for example, we assume that the two sides of the coin are equally likely we have no real proof of this. It simply makes sense to do so and we have no real reason to believe otherwise. Such an assumption in mathematics is called an axiom and this way of looking at probability is called axiomatic. Our reason for treating each side of the coin or face of the die as the same is based on the symmetry of the coin or die and the approach is sometimes called symmetrical. Similarly, drawing names from a hat or other ways of selecting at random are considered symmetrical and therefore equally likely.

Determining symmetrical probabilities: The formula

Consider the roll of a single fair die. In one *trial*, there are six different *outcomes*. The probability that on the throw of a single die, the outcome will be any of the six numbers is 1/6. If we consider the outcome of getting a three or greater (3, 4, 5, or 6) the probability will be 4/6 or 2/3. Another example might be the rolling of a pair of dice, one red and one green. Each *trial* can be thought of as having 36 *outcomes* since each of the two dice can land in six different ways (a 1 to 6 on the red can be accompanied with a 1 to 6 on the green).

Activity 1.1

The number of outcomes

Roll a pair of differently coloured dice and convince yourself that there are in fact 36 outcomes, noting for example, that a red 4 and a green 3 is a different outcome to a green 4 and a red 3.

Each of these 36 outcomes is *equally likely* to happen, so each has an equal *probability* of 1/36, occurring *at random*. We can also consider other *outcomes* of the event such as the sum of the numbers being greater than seven which, as we will see, have different probabilities associated.

In general, if an *experiment* (or one *trial*) has **t** equally likely outcomes and the event we want to consider includes **n** of these, the common formula is used:

p = n/t

This is a very simple formula that is easy to use.

Example 1. What is the probability of a number › 2 on the throw of a single die?

Solution: There are 4 ways this can happen: 3, 4, 5, or 6 so n = 4 and the total number of outcomes t = 6. (1, 2, 3, 4, 5, 6). So p =4/6 or 2/3

Example 2. What is the probability that you will get an odd number on a roll of a roulette wheel? All we need to know is:

Experiment: roll roulette wheel. One single *trial*.

Event: an odd number shows.

t: total number of outcomes, 37 (18 odd, 18 even, 1 zero)

n: the number of ways an odd number can occur, 18

so

p = n/t =18/37 or 48.64%

However, the formula is limited to situations in which there are:

1. A finite (countable) number of outcomes, and
2. These outcomes are all equally likely.

The formula does not apply if we wish to find say, the probability that Snowball will win the next race or the probability that you will die before the end of the year since these events do not meet the criteria. In these situations we must use other means to estimate the probability.

Nor does the formula apply to outcomes that are not equally likely. If we toss a thumb-tack for example there are two possible outcomes: point up or point down. We do not know whether or not these are equally likely and so we cannot assume that they are. It is a well documented misconception among people to assume equal likelihood in situations where none exists.

The sum of probabilities of all the outcomes of any experiment must always equal one or 100%.

Activity 1.2

A card game to introduce axiomatic probability

See Appendix 1: Card game

Note: The mathematically "optimal" strategy in this game, as in all such games, is to bet the minimum (here 1) when your probability of winning is less than 0.5. If your probability of winning is greater than 0.5, bet the maximum (here 3) as long as you can afford to lose.

Maths B

Random variable

If we roll a single die, there are six possible outcomes. If we denote the score as X, clearly X is a variable that can take values from 1 to 6 at random. We say that X is a random variable.

If we toss a coin there are two outcomes, head or tail. Thus we have two values that the random variable can take H and T with probabilities $p(X = H)$ and $p(X = T)$. If the outcomes are equally likely each must = 1/2, since the sum of all of the values of $p(X)$ for any experiment must equal 1.

Since X can take only a fixed number of values, we call it a **discrete random variable**.

The **probability** of X being any particular value, $p(X = x)$ is generally written simply as **p(x)**.

Once a discrete random variable has been identified there are two properties to consider:

1. What values it can take (its domain) and
2. What probability is associated with each value (its range)

These together enable us to describe and graph the distribution of the probabilities. For any variable, the sum of all probability values must equal 1.

For further examples of the distribution of discrete random variables, see Exercises Set 1, Q18.

Maths A & B

1.3.2 Frequentist probability (sometimes called "experimental" probability)

If we toss a thumbtack, there are just two ways it can land: point up or point down. However, there is no reason to assume that these are equally likely, and so we cannot use the formula to determine the probability that it will land either up or down. In situations like this we determine the experimental relative frequency of each outcome and equate this to the probability.

Activity 1.3

Toss a large number of thumbtacks and observe how often the point lands up or down. We say that the probability of point up is equal to frequency of point up, that is the number of times point up to the total tosses.

What value did you get for this?

Compare your result to that of others. Why might they differ?

What was the average result of the class?

Why might this be a better measure of the probability?

Note that we do not know how close to the true probability this frequentist probability is. For example, if a fair coin was tossed 100 times and landed heads 55 times, we would assign an experimental probability of 0.55 when, in fact, the true probability is 0.5.

We may use the same approach for symmetrical situations when we don't have the theoretical knowledge to solve the problem or when it may be easier to simulate it than solve theoretically. For example if we wanted to know the probability of getting a pair of aces in a five card poker hand, it may be easier to get an experimental probability than to calculate a theoretical one.

It is a general principle of probability theory that as the number of experiments or trials of an event increases, the experimental probability (frequentist) approaches the true or theoretical probability. (See Activity 1.4)

p = RF

1.3.3 Random generators

Random generators include any device that produces events that are random. These include coins, dice, roulette wheels, spinners, poker machines, lotto and keno number generators etc. and may be used to produce frequentist probabilities.

Independence

When using any random generators the events of each trial are independent of previous trials and what has happened in the past cannot change the probability.

For example, if a fair coin is tossed a large number of times, we expect an equal number of heads and tails. As the number of tosses increases the *relative frequency* approaches the theoretical probability of 50% for heads and tails. There is a **misconception** that the actual number of heads and tails gets closer to being equal as the tosses continue. The exact opposite is true. The difference between the number of heads and tails increases with the number of tosses, but the relative frequency approaches 0.5. The following table indicates what might happen:

# Tosses	Heads	Tails	Difference	Relative f (heads).
10	6	4	2	0.6
50	28	22	6	0.56
100	55	45	10	0.55
1,000	512	488	24	0.512

Activity 1.4

To show that the frequentist probability approaches the theoretical when the number of trials is large. This is sometimes called "the law of large numbers".

Random generators: Coins, cards, dice, wheels, spinners etc.

In the use all of these we assume that all outcomes are equally likely and apply the above formula to the situation: p = n/t

We can simulate many real life gambling situations using these random generators and obtain frequentist probabilities.

Alternatively, we can simulate the generator itself in a number of ways:

- Coin toss;
- Table of random numbers, phone book numbers;
- Odd: heads, even: tails;
- Use of Excel to generate random numbers and simulate random generators; and
- Use the Excel files *random generators* and *spinnersim* to show that the frequentist probability approaches the theoretical in all of the gambling situations described.

Note that the Spinners file shows clearly that this happens while the probabilities remain fixed.

See Appendix 2 for more information on the use of these files.

1.3.4 The gambler's fallacy

The **misconception** leads to what has been called the gambler's fallacy. An example is if a roulette player observes the ball lands on red five times in a row. Knowing that red and black are equally likely in the long run, he incorrectly concludes that black is more likely on the next roll so as to "balance up" the probabilities. The relative frequencies do become equal but this happens without any "balancing" and, as with the coins, the absolute difference continues to increase.

The following spreadsheet illustrates this by simulation.

Activity 1.5

The gamblers' fallacy

The Excel file *Gamblersfallacy* simulates this misconception.

We suppose that an unlikely sequence of events has happened.

For example: The roulette wheel has landed on High (> 18) 10 times in a row.

The common gambler's fallacy is to reason that it is now more likely to be Low (< 18) next time because in the long run the two are equally likely.

The simulation shows that in the long run, the occurrences of both do approach equality, but that this happens without any change in the probability of each trial.

See: Appendix 3, Use of the gambler's fallacy simulation

1.3.5 Intuitive probability

Finally, there are situations in which we use neither the axiomatic nor the frequentist approach to make an estimation of probability. For example, when the weather bureau estimates the probability of rain tomorrow they are making an estimation based on their interpretation of certain information available to them. In these situations we say that the approach is intuitive, even though the estimation may involve a good deal of mathematical analysis, not just intuition.

Activity 1.6

Intuitive probability

Two equally matched teams play a six game series (no draws).

What is the most likely outcome: 6-0, 5-1, 4-2 or 3-3?

Most people will answer 3-3 intuitively.

Simulate this situation either with the Excel file "*6 games*" or using a set of random numbers.

What is the most likely outcome?

See Appendix 4, Use of the 6 games spreadsheet.

Teachers

The role of intuition in the estimation of probability.

Most people are surprised to learn that in a group of as few as 23 people, the probability that at least two of them have the same birthday (i.e., same day and month) exceeds 0.5, and that in a group of 30 people this probability is more than 70%. (See exercises Set 1 (Maths B). These results are counter-intuitive and illustrate how our intuition is unreliable when it comes to estimating probability.

Activity 1.7

A card game with a **counter-intuitive probability**:

Shuffle a deck of cards. Name any three cards without reference to suit e.g. Jack, Queen, King. I will bet you that in a maximum of three draws of a card from the deck I can draw one of the cards nominated.

Will you bet me?

Most people think that intuitively the odds are against me.

Do the activity using a deck of cards a number of times to obtain a frequentist probability, or simulate the play with the Excel file *3cardcut*.

An analysis of the situation using symmetrical (equally likely) probability shows that the probability I will win is approximately 0.55 or 55%. On the average I will win 55% of the time and you will lose 45% of the time. This is drawing one card and replacing it if it is not a match, reshuffling and drawing again each time. If I simply draw three cards the probability of my win rises to 56%.

See Appendix 5, Use of the 3cardcut spreadsheet.

1.4 More about probability

1.4.1 Complementary events

We have seen that for any event the sum of the probabilities of all outcomes always adds to one or 100%. For example if the probability of getting a six on the roll of a die is 1/5, then the probability of not getting a six must be 5/6. These two probabilities are called *complementary*.

1.4.2 Compound events

A compound event is one in which one set of outcomes is followed by another one or more. A compound event may be a succession of two or more simple events. For example, the "Daily Double" consists of selecting the winners of two horse races. The event of the two races can be considered as one compound event.

We have seen that the total number of ways in which a pair of dice can land is 6 x 6 = 36. Three dice could land in 6 x 6 x 6 = 216 ways, and each of these would be equally likely.

The probability that all three would be sixes is then 1/216 since there is only one way in which this can happen.

The probability of two sixes and a five (score of 17) would be 3/216 since there are three ways this could happen (665, 656, 566).

The total number of ways a compound event can occur is the product of the ways of each of the simple events. It follows from this that the probability of a compound event is the product of the probabilities of each component event. This is known as the product rule or **multiplication principle** in probability.

Activity 1.8

Horse race game with a pair of dice. *Appendix 6*

Teachers

See Appendix 6

The theoretical probabilities show: most likely to win 7, then 6 & 8, 5 & 9, 4 & 10, 3 & 11, 2 & 12. The frequentist results will show that 7 wins about 50% of the times with 6 & 8, 5 & 9 most of the other times. 4 & 10 or 3 & 11 very rarely win, 2 & 12 almost never. The theoretical probabilities for these are very complex and well beyond high school mathematics. However, frequentist probabilities can be obtained and used later to set the odds. (Section 4.1, Activity 4.2)

The multiplication principle and trifectas/quinellas.

To find the number of ways in which the trifecta (first three horses selected in correct order) can occur in an eight horse race, we need to consider the event of finishing first followed by second followed by third, to find the total number of all possible ways in which the event can happen. There are eight ways in which the first place can be filled, and with *each* of these there are seven ways in which the second place can be filled and six for the third. Thus the total is the *product* 8 x 7 x 6 = 336.

The number of ways the first two horses can finish is 8 x 7 = 56. In selecting the quinella these do not have to be in the correct order. If you choose horses A and B, AB or BA will result in a win, so the number of combinations is 8 x 7/2 = 28.

Of course these are not all equally likely, so the formula for probability does not apply, but the method of finding the product of the individual outcomes is quite general.

1.4.3 The multiplication principle and probability

$p(ABC) = p(A) \times p(B) \times p(C)$

Which simply means that the probability of A then B then C is the product of the individual probabilities.

For example if you estimated your chance of selecting the winner on the first race of the Daily Double was 50% or .5, and on the second race was 25% or 0.25, then your chance of winning both is found by multiplying the two probabilities:

$p = 0.5 \times 0.25 = 0.125$ or 12.5%

or

$p = 1/2 \times 1/4 = 1/8$ (one chance in 8)

We need to be aware whether the events are independent. For example the probability of getting an ace on the cut of a deck of cards is 4/52 or 1/3. To find the probability of drawing an ace twice in a row from a deck of cards:

- If the first card is replaced after the first cut, then the outcomes of the second cut are independent of the first and we calculate:

  $p(AB) = p(A) \times p(B) = 1/13 \times 1/13 = 1/169 = 0.0059$

- However, if the first card is not replaced after the first cut, then the probabilities of the outcomes of the second cut will depend on what card has been removed, and the calculation becomes more complicated.

  Now $p(A) = 1/13$

to find p(B): If an ace is removed after the first cut there are 51 cards left of which three are aces,

so p(B) = 3/51

so p(AB) = 1/13 x 3/51 = 0.00452

Use of exponential notation when writing probabilities

If the probability of getting a head on the toss of a coin is 1/2, the multiplication principle tells us that the probability of four heads in a row will be 1/2 x 1/2 x 1/2 x 1/2.

We write this as $(1/2)^4$

Evaluate this on your calculator or using Excel.

Note: If we evaluate say, the probability of getting six sixes in a row when rolling a single die $(1/6)^6$ using a calculator or Excel we will get an answer like 2.14335E-05

Or 2.14335×10^{-5}

This the same as 0.0000214335

We can interpret this as about two chances in 10^5

Or two in 100,000 or one chance in 50,000

See Exercises Set 1

1.4.4 Complementary probabilities and the multiplication principle

In a certain dice game, I need to throw a six to get started. I have rolled the dice 10 times and have not yet got a six. What is the probability of this happening?

To answer this we first note that the probability of getting a six, p(X = 6), is 1/6. We then compute the *complement* of this event, not getting a six.

In general, if *p* is the probability of any event, X, the probability of its *complement*, X, is *1 – p*.

So p(X) = 5/6

And now using the multiplication principle we see that the required probability is

$(5/6)^{10}$ = 0.1615 or about 16%

Another example: If the probability of getting a broken windscreen on a particular drive was 10%, then the probability of making the drive with windscreen intact is 90%.

What is the probability that you will break a windscreen on the drive there and back?

A common mistake is to add the two 10% and get 20%. The correct procedure is to multiply the two complementary probabilities to find the probability of not breaking the windscreen:

p(AB) = p(A) x p(B) = .9 x .9 = 0.81 = 81%

So the probability of a broken windscreen is 100% – 81% = 19%

You might think that the answer of 20% is close enough. However, this is not so. Suppose, instead of a 10% chance there was a 30% chance of a broken windscreen on each drive. The combined probability is not 60%, but is 1 – 0.7 x 0.7 = 51%, which is considerably different. Or suppose you were going to make the return journey twice. If you add the probabilities you would conclude that p = 1.2 which is clearly impossible. In fact p = 1 – (0.7^4) = 0.76 which is greatly different.

The use of the complement of an event and the multiplication principle is an important strategy in computing probabilities in many gambling contexts.

Maths B

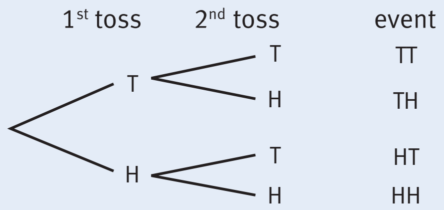
Activity 1.9
Analysis of the 3 Card Cut
Excel file *cuttingthedeck*

Use of file, see Appendix 7

1.4.5 Compound probabilities, Union and tree diagrams

Teachers
This will parallel most conventional textbook treatments of the topic and may be supplemented with any such material.

The tree diagram for the tossing of two coins is shown below. There are 4 equally likely events but as the middle two (TH, HT) are indistinguishable, the probability of tossing a head and a tail is 0.5, while the probability of tossing two heads is 0.25 and the probability of tossing two tails is also 0.25.



Many of the exercises in Set 1 can be solved by using a tree diagram.

Maths A & B

1.4.6 Some simple but useful probabilities

Teachers
The following results in bold are given without proof or explanation although it is not difficult to see the reasons. These results are very useful in the analysis of gambling situations and are often not included in introductory probability.

1. If the probability of any particular number showing on a die is 1/6, the average number of throws required for this to occur will be six. If the probability of zero showing on the roulette wheel is 1/37, the average number of rolls required for this to occur will be 37. If the probability of an outcome is 2/5, the average number of trials required for it to occur is 5/2 or 2.5.

In general: **If the probability of an outcome is p, the average number of trials required for it to occur is 1/p**

2. Take two well-shuffled decks of cards and turn the top card over on each one at a time. Continue until you have been through the 52 card deck. What is the average number of matching cards you expect? (intuitive probability). Try it a number of times and see what happens (frequentist probability).

Repeat the procedure using just the hearts from the decks.

Repeat the procedure using just the aces from the decks.

Teachers
This can be generalised to any situation. The familiar envelope problem *(Exercises Set 1 Q14)* is an example.

The theoretical average number of matches in ALL cases is 1.

1.4.7 Probability expressed as odds

Most people are familiar with the term odds. Odds may be expressed as odds for or odds against. For example, if an event has six equally likely outcomes (roll a die), of which one is favourable (score a 6), then we can see the relationship between probability and odds:

p = number of favourable outcomes/total = 1/6

Odds for = number of favourable outcomes: number of unfavourable = 1:5

Odds against = number of unfavourable outcomes: number of favourable = 5:1

Unless otherwise specified bookmaker odds are always odds against.

When p is the probability of an outcome and the odds for this are expressed as x:1

Odds for; $x = p/(1 - p)$

Odds against; $x = (1 - p)/p$

e.g. $p = 1/2$, odds for or against are 1:1

If $p = 1/4$

$1 - p = 3/4$

odds against = $(3/4)/(1/4) = 3:1$

If $p = 0.05$

$1 - p = 0.95$

$x = 0.95/0.05 = 19$

odds against 19:1

The reverse procedure is to calculate the probability if we know the odds. For example, if the odds against are 2:1 then:

Number of favourable outcomes = 1

Number of unfavourable outcomes = 2

Total outcomes = 3

p = 1/3

Another term often used in gambling is odds on. For example 2:1 on. This simply means odds in favour rather than against. Thus 2:1 on is the same as 1:2 against.

We have noted that regardless of how we calculate the probabilities of an event the total of all the probabilities of the various outcomes must always add to one. (If your answer to any probability computation is more than one you know that you have made a mistake).

For example, if there are seven other horses in the race with Snowball, and we estimate that Snowball has a 25% chance of winning, then the sum of the other probabilities must be 75%. If the probability of rain today is estimated at 40%, then the probability of no rain must be 60%. If a cricket match has three possible outcomes – a win for either team, or a draw – the sum of the three associated probabilities must add to one.

Comparison of odds. Which is better 4:7 or 2:3?

There are a number of ways of doing this:

(i) *Using equivalence 4:7 is 12:21*

2:3 is 14:21 which is better

(ii) *Find the win per $ bet:*

4:7 Bet 7 win 4

4/7 or 0.57

2:3 Bet 3 win 2

2/3 or 0.66

Bookmaker and casino odds are not the same as true or fair odds, and when we compute probabilities from bookmaker odds we do not arrive at the correct probability. For example, the odds against red on roulette are given as 1:1.

p= 1/2 or 0.5

The "true" probability however is 18/37 or 0.4865

We will see later that this is how they make their profit.

1.5 Further misconceptions about gambling: fallacies

1.5.1 Failure to recognise independent events

We have already seen how the gambler's fallacy arises. This, in part, is a failure to recognise independence. The heart of the gambler's fallacy is the misconception of the fairness of the laws of chance. Naive gamblers expect that any deviation in one direction will soon be cancelled by a corresponding deviation in the other.

Teachers

It is very important for students to be able to distinguish between situations in which probabilities stay the same, and those in which they change as a result of other events. For example, if cards are drawn from a deck without replacement (such as in blackjack), the probabilities for the remaining cards will depend on what has been already drawn i.e. probabilities that are dependent on past events. However, the probability that the roulette ball will land on an odd number cannot be influenced by what has happened on previous rolls. It is independent of past events. A lack of understanding of the concept of independence leads to the systematic misconceptions previously mentioned as the gambler's fallacy.

1.5.1.1 The gambler's fallacy and lotto.

The selection of lotto numbers by the drawing of numbered balls or any other mechanism is very carefully constructed to ensure that all numbers are equally likely to be drawn. Further, the draws are independent of each other. The balls have no memory and what has happened in the past cannot possibly have any influence on what will happen on the next draw. Thus the publication in the newspaper of the frequency of occurrence of each of the numbers and how long it has been since each was drawn last are of absolutely no value whatsoever. It is true that in the long run each lotto number will be drawn approximately the same *proportion* of times. If there are 40 numbers in a lotto, after a very large number of draws each will have been drawn about 1/40 or 2.5% of the total. However, at no time in this sequence of events do past events have any effect on what will happen in the future. All numbers remain equally likely at all times.

To help understand the gambler's fallacy in lotto consider the following situation.

In 200 lotto draws (from 40 numbers) we expect that each number will show about 1/40 of the time, that is five times. However, random variations will result in some numbers showing more often than others. Suppose the "least frequent" number was 17, which showed only twice. After 400 draws we expect each number (including 17) to have shown about 10 times. Thus the fallacy goes that 17 is now more likely to occur than any other number since it has to "catch up" and show eight times in the next 200 draws. This is simply not true. After the next 200 draws (total now 400), we expect the 17 to have shown seven times , that is to say, the two times that it has shown in the first 200 and the five times we expect it to show in the next 200 draws. Thus we have:

Number of draws	Frequency of the number 17 occurring
200	2/200 = 1%
400	7/400 = 1.75% (expected)
....	
4,000	97/4,000 = 2.425% (expected)

As the draws proceed, the relative frequencies approach the expected 2.5% *without* any change in the probabilities. The law of large numbers holds true without affecting the probability in any way. What has happened in the past cannot possibly affect what will happen in the future. The balls have no memory. It is quite amazing that so many people are unaware of this simple yet fundamental principle of probability when it comes to the selection of lotto numbers.

The following site explores myths and misconceptions associated with the playing of lottos.

http://www.solidsoftware.com.au/Products/LottoCheck/LottoMyths.html

1.5.1.2 The gambler's fallacy and poker machines

Teachers
To improve students' understanding of how poker machines work, refer to the orienting activities in Idea Sheet 6 - Gaming Machines are big business from the *Responsible Gambling Teaching Resource Kit.*

Again, the heart of **misconceptions about poker machines** is the lack of understanding of the concept of "randomness". Each play is a random event.

There are two important principles relating to this.

Firstly, the machines are constructed so that every "outcome" is independent of previous outcomes. If a machine has not paid out for a long time the probability of a payout next go is no different to that of a machine that has just paid out a jackpot.

Second is that the machines are programmed to pay out a fixed proportion of the money taken. (See section 2, Mathematical Expectation).

Other misconceptions about gambling include:

1.5.2 Lucky and unlucky streaks

This belief follows from the gambler's fallacy. Instead of viewing random sequences as such, they are viewed as lucky or unlucky streaks. A sequence of losses may be viewed as an unlucky streak and the gambler mistakenly believes that it must end and keeps gambling, often increasing bets in the belief that this will even out. A sequence of wins may be viewed as a lucky streak and the gambler keeps gambling, often increasing bets in the belief that he/she is on a roll.

1.5.3 The illusion of control

In all gambling situations in which the random generator results in random independent events the gambler can have no control. In selecting numbers for roulette, for example, your probability of winning remains the same whether you choose the numbers or place the bets at random. Gamblers will often attribute a random sequence of wins to something that they have done. Again, the misconception is a lack of understanding of randomness.

1.5.4 Not understanding the situation: extraordinary events, coincidences

A recent Ripley's *Believe it or not* cited the case of a baby girl born in the USA on 7 December, the same date as her mother and grandmother, quoting the incredible odds of less than 1 in 48 million (mathematically correct). Clearly many would think this an incredible occurrence. However, given the population of the USA, if we consider the probability that somewhere some child will be born on the same date as a parent and a grandparent of the same sex, we find that even over a relatively short period of time, this is nearly certain to happen. The only believe it or not situation would be if it didn't happen. The misconception here is in not understanding what the problem is.

1.5.5 Counter-intuitive results: the birthday problem

Most people are surprised to learn that in a group of 30 people the probability of two people having the same birthday is over 70%, since this result is counter-intuitive.

Teachers

Again the analysis of this is within the realm of elementary probability and can be understood by students of Maths A. Maths B students can consider the probability as a *function* of the number of people and explore the range and domain of the function.

Starting with two people, the probability that they share a birthday is 1/365.

The probability that they do not is then 364/365.

If the group increases to three, the probability that the third person does not match either of the other two is 363/365 and hence the probability of no match at all is 364/365 x 363/365. See *Exercises Set 1*, *Qs 19* and *20*.

Maths B

Teachers

Introduction to functions

**Concepts of function, domain and range*

With the use of modern computers the analysis of such situations as these is within the scope of Maths B students. We simply set up an Excel program as shown below and make the necessary estimations and assumptions. We see that P (the probability of at least one occurrence) is a function of each variable when the others are held constant. Teachers can discuss the possible domain and range of these functions.

Activity 1.10

Maths B: Excel spreadsheet *coincidence*

Consider n is the number of births per year in the country

Y is the number of years of observation

p is the probability that any child born has the same birthday as both parent and grandparent of the same sex = $1/365^2$.

Then (1 – p) is the probability that they do not

And (1 – p) nY is the probability of no match for n^Y births

Hence $P = 1 - (1 - p)^{nY}$ is the probability of at least one match

We see that in even a relatively small country with 30,000 births per year, the probability of this believe it or not occurrence is about 20% in any one year. In a large country such as the USA it is virtually certain to happen every few years.

Use the Excel spreadsheet of Activity 1.10 to explore this.

Teachers

We should note that these calculations are only approximate. This is because if two people are born with the same grandmother, their outcomes in regard to the events of interest here will be correlated. Also we have ignored the minor effect of leap years as we will do in the following problem.

Another example of the ease with which we can be fooled by probability is illustrated by the following: At a European casino some years ago, the same number showed six times in a row on one of the roulette wheels. It was reported, correctly, that the probability of this happening ($1/37^6 = 3.9 \times 10^{-10}$) was less than one in a hundred million (10^{-9}). Surely, this is an extraordinary event?

Yes and no. The probability that the particular number would come up six times in a row on that particular roulette wheel at that particular time was, in fact $(1/37)^6$ (the multiplication rule) or 3.89753E – 10, about 4 in 10^{10} or less than one in a billion (10^9)! This is truly extraordinary. However, if we calculate the probability that at some time in your life, at some roulette table somewhere in the world, some number will come up six times in a row we find that this probability is well over 90%. Not only is the outcome not extraordinary, it is much more likely to happen than not.

Activity 1.11

Excel file *howextraordinary* allows the user to enter several variables, such as the number of roulette wheels in the world, the number of spins per hour and the time in years.

The spreadsheet computes the probability of the event (six in a row) not happening and thus the probability of at least one occurrence.

Maths A & B

Teachers

The misconception that the probabilities change has been referred to earlier as a type of misuse of the representativeness heuristic. Representativeness is a process by which those who estimate the likelihood of an event do so on the basis of how similar the event is to the population from which it is drawn. This results in a fallacy by which equally likely sequences are viewed as unequally likely. This can occur in two ways. For example, a subject observes a fair coin lands heads on eight out of ten tosses, and then incorrectly reasons that the likelihood of a head on the next toss is less than one half. The subject is incorrectly reasoning that a balance is expected. This is commonly known as the belief in the gambler's fallacy or the law of averages. In the second instance, the subject erroneously reasons that a head is more likely using the preceding short run of outcomes as being a representative base rate. That is to say, the person thinks that the coin is biased.

There is ample evidence that lacking an understanding of independence, people will use the representativeness and availability heuristics in the selection of lotto numbers. Newspapers in Australia publish a table each week showing the lotto number frequencies for the last 200 draws, and the number of weeks since each number has shown. This information presumably enables prospective bettors to use representativeness to select numbers that have not shown for a long time. The availability heuristic may be also demonstrated in the selection of lotto numbers by the reported avoidance of patterns. Lotteries go to great lengths to ensure that the draw of numbers is independent.

Availability is a term used to explain the process that occurs when people tend to make predictions about the likelihood of an event based on the ease with which instances of that event can be constructed or called to mind. This results in combinational naivety. One event may be judged more likely than another if instances of it are easily recalled. One may assess the risk of heart attack among middle-aged people by recalling such occurrences among one's acquaintances or the probability that a given business venture will fail, by imagining various difficulties it could encounter.

Availability is a useful clue for assessing frequency or probability, because instances of large classes are usually reached better and faster than instances of less frequent classes. However, availability is affected by factors other than frequency and probability, and the reliance on availability leads to predictable biases. Other instances where such biases occur include the example that the impact of seeing a house burning on the subjective probability of such accidents is probably greater than the impact of reading about a fire in the local paper. Recent occurrences are likely to be relatively more available than earlier occurrences, and that it is a common experience that the subjective probability of traffic accidents rises temporarily when one sees a car overturned by the side of the road. In these cases, the estimation of the frequency of a class of the probability of an event is mediated by an assessment of availability.

The perceived difficulty of throwing a six with the throw of a single die results from the ease of recall by the child of instances in which a six was wanted and not obtained. This misconception is therefore an example of the misuse of the availability heuristic.

The counter-intuitive nature of many results in probability theory is attributable to violations of representativeness that occur when people view a sample randomly drawn from a population as highly representative, that is, similar to the population in all essential characteristics.

1.5.6 The equally likely fallacy

Teachers

The assumption of assigning equal likelihood to the outcomes of an event is one that is not always justified. We see this in its simplest form when, for example, young children are asked: In a class there are 12 girls and 16 boys. The teacher puts the name of each child in a hat and draws one out at random. What is the probability of the name being a boy? Many children (and adults) will answer 1/2, arguing that it can be either a boy or girl and that these are equally likely. More subtle is the situation where we ask a group of people to select a number from 0 to 9 "at random". There is a tendency to assume that each of the 10 digits is equally likely and that about 1/10 of the group will choose each. People are often surprised to learn that the numbers 7 and 3 are much more likely than any of the others. Unlike drawing numbers from a hat, people do not select at random. Even when we assign probabilities to tossing coins or rolling dice, we make assumptions of equal likelihood based on the symmetry of the materials used, which may not be correct. However, in the absence of other information, in symmetrical situations, it is reasonable to accept equal likelihood.

Activity 1.12

Make three cards: one is black on both sides, one red on both sides, and one is red on one side and black on the other. Draw one of these from a hat at random and place so that only one side can be seen. If the side showing is red, estimate the probability, p, that the unseen side is also red. If the side showing is black, estimate the probability, p, that the unseen side is also black. Turn the card over and see.

Perform the activity enough times to get a frequentist value of p.

What value did you get? Can you explain this?

Teachers

Students will reason that there are two equally likely possibilities, red or black, and answer 50%. But this is incorrect as the activity will demonstrate. Red and black are not equally likely and this is another example of the equally likely misconception. Two of the three cards are the same colour on both sides, so if the red is showing the probability that the other side is also red is 2/3. Similarly, if black is showing, the probability that the other side is also black is 2/3.

1.5.7 System fallacies

The bet and double fallacy

In this system the gambler bets $5 on the red. If it shows black, next bet is $10 on the red. If black again, then bet $20 on the red and keep doubling until the red eventually shows, at which time you will be ahead. This looks like a good system, but it is in fact a very bad one with which you stand to lose a good deal.

Suppose the black shows eight times in a row. Your next bet must be $1,280 on the red on the next throw. This is probably in excess of the house limit and you will have lost $1,275 already! The probability of black showing eight times in a row is one in 256.

Using this system, sooner or later, this is bound to happen, and all your small winnings of previous bets will disappear along with a good deal more! The overall effect of this system is that you have a high probability of winning a very small amount, and a small probability of losing a very large amount. Overall, your expected return remains at 97.3%. (See Section 2)

Activity 1.13

Simulate the bet and double fallacy with Excel spreadsheet *doublef*

See Appendix 8 for use of this spreadsheet.

Belief in systems. Suppose a person goes to the casino armed with the bet and double system. Even though the probability of winning is no different to betting at random, it is still possible that they will come home winning. It is quite likely that the person will then attribute the random win to the system and may continue to do so until the inevitable loss occurs.

Exercises Set 1 - Maths A & B

1. If there is a 20% chance of rain tomorrow what is the probability that it will not rain? Express the chance of rain as a decimal, as a fraction, as "one chance in", as "the odds against rain are".

2. A roulette wheel has 37 numbers (0 to 36): 18 of these are red, 18 black and 1 green. On a single roll, what is your probability of (a) a red (b) a red or a black (c) a green (d) an odd number (e) a red number?

3. (a) A well shuffled deck of cards is cut. The Pr (Club›three) [Ace high] is ?
   (b) The numbers 1 to 100 are drawn at random from a hat.
   Pr (odd and ›5) is ?
   (c) The probability of a score of 7 or more on the roll of a pair of dice is ?

4. A four wheeled poker machine has 10 different symbols on each wheel.
   What is the total number of possible outcomes for the machine?

5. A three wheeled poker machine has 10 symbols, three of which are bells and all the others different, on each wheel. What is the total number of possible outcomes for the machine?

6. The "daily double" consists of selecting the winning horse in each of two races. If the first race has 12 horses and the second 18, how many possible "doubles" are there? If you choose two horses at random, what is the probability of selecting the winning double?

7. The Melbourne Cup has 24 horses.
   (a) In how many ways can the trifecta (places 1, 2, 3) occur?
   (b) The quinella consists of selecting the first two horses not necessarily in the correct order. How many possible quinellas are there?

8. Draw a tree diagram for the tossing of 3 coins and calculate the probabilities of the following events.
   (a) three tails
   (b) two tails and one head
   (c) one tail and two heads
   (d) three heads

9. In a certain town 60% of the cabs are yellow and 40% are black. 50% of the drivers are male and 50% are female. If I hail a cab at random, what is the probability that it will be a yellow cab with a female driver?

10. If I drive from A to B there is 50% chance that I will see a kangaroo on the road. What is the probability that I will see at least one kangaroo on the road if I drive there and back? What is this probability if I make the return journey twice?

11. If I drive from A to B there is 5% chance that I will get a broken windscreen.
    (a) What is the probability that I get a broken windscreen if I drive there and back?
    (b) What is this probability if I make the return journey
    - twice?
    - 100 times?

    (c) About how many times would I need to make the return trip to have a more than 50% chance?

12. Which of the following in each pair is the better odds for the punter?
    (a) 2:1, 3:1
    (b) 5:2, 9:4
    (c) 7:4, 2:1
    (d) 3:2, 8:5

13. What is the probability of HH on the toss of two coins? On the average how many times will I need to toss a pair of coins to get two heads?

14. I have 10 party invitations addressed to 10 different people. In my usual disorganised manner I forget that I have personalised each invitation and put them in the addressed envelopes at random. On the average, how many people will get their own invitation?

Exercises Set 1 - Maths B

15. If two cards are dealt from a well-shuffled pack, what is the probability of getting:
    (a) Both suits the same?
    (b) Both of different suits?

16. If three cards are dealt from a well-shuffled pack, what is the probability of :
    (a) All suits the same?
    (b) All different suits?
    (c) Two of one and one of the other?

17. A machine dispenses lucky number tickets for $1 each. One in every ten tickets made has a winning number but the tickets are dispensed by the machine in random order. A ticket with a lucky number earns a prize of $5. If a person buys five tickets, what is the probability of
    (a) not getting a prize?
    (b) not losing any money?

18. In each of the following experiments, the random variable is defined.
    In each case state:
    (i) What are the values that the variable can take?
    (ii) Are these values equally likely?
    (a) Experiment: A single roll of a roulette wheel. X = number showing
    (b) Experiment: A pair of dice are rolled. X = sum of numbers showing
    (c) Experiment: A family has 4 children. X = the number of girls
    (d) Experiment: A plays B in tennis for 6 games. X = the number of games A wins.
    (e) Experiment: A five card poker hand is dealt. X = the number of aces in the hand.
    (f) In the game of bridge, the deck of 52 cards is dealt to four players. The High Card Points (HCP) are computed by scoring Ace = 4, King = 3, Queen = 2, Jack = 1 and all other cards zero.
    Experiment: A player evaluates the HCP of his hand. X = score

19. (a) What is the probability that a person selected at random from the population has a birthday in the same month as yours?
    (b) What is the probability that they do not?
    (c) What is the probability that a third person does not have a birthday in the same month as either of you?
    (d) Thus, what is the probability that none of the three of you have the same birth month?
    (e) What is the probability that at least one of you share a birth month with one of the others?
    (f) Extend (e) to a group of four, five or six people.
    (g) How many people do we need in a group to have at least 50% chance of a match in birth month?
    (h) How many people must I take to be 100% certain that two have a birthday in the same month?

20. Repeat Q19 using “on the same day” instead of “in the same month”.

Answers Set 1 - Maths A & B

Q1. 80%, 0.2, 1/5, one chance in 5, 4:1 against

Q2. 18/37, (b) 36/37, (c) 1/37, (d) 18/37, (e) 18/37

Q3. (a) 11/13, (b) 47/100, (c) 21/36 or 7/12

Q4. 10000

Q5. 8 x 8 x 8 = 512

Q6. 12 x 18 = 216, 1/216

Q7. 24 x 23 x 22 = 12144
(a) 24 x 23/2 = 276

Q9. 0.6 x 0.5 = 0.3 or 30%

Q10. p(not) = 0.5
p(not and not) = 0.5 x 0.5 = 0.25
p(at least one) = 1 – 0.25 = 0.75

Q11. p(not broken) = 0.95
(a) 1 – 0.952 = 0.0975 or 9.75%
(b) 1 – 0.954 = 0.186 or 18.6%
(c) 1 – 0.95200 › 99.9%
(d) We need to find n, so that 1 – 0.95n › 0.5
When n = 14, p = 51.2% so 7 return trips

Q12. (a) 2:1
(b) 5:2
(c) 2:1
(d) 8:5

Q13. ¼, 4

Q14. One (This is an example of the probabilities of section 1.4.6)

Answers Set 1 - Maths B

Q15. (a) 12/51.
(b) 39/51

Q16. (a) 12/51 x 11/50 = 0.0518
(b) 39/51 x 26/50 = 0.3977
(c) 0.551 (1 – 0.0518 – 0.3977)

Q17. (a) 9/10 5 = 0.59
(b) 1 – 0.59 = 0.41

Q18. (a) 0 ‹ = X ‹ = 37, yes.

(b) 2 ‹= X ‹ = 12. No

Distribution

X	#ways/36	p(X)
2	1	0.027778
3	2	0.055556
4	3	0.083333
5	4	0.111111
6	5	0.138889
7	6	0.166667
8	5	0.138889
9	4	0.111111
10	3	0.083333
11	2	0.055556
12	1	0.027778

(c) 0 ‹ = X ‹ = 4. No

Distribution

#girls	p
0	0.0625
1	0.25
2	0.375
3	0.25
4	0.0625

(d) 0 ‹ = X ‹ = 6. No

Distribution

X =#Games	p(X)
0	0.015625
1	0.09375
2	0.234375
3	0.3125
4	0.234375
5	0.09375
6	0.015625

(e) 0 ‹ = X ‹ = 4. No

Distribution
Intuitively, p(X=0) is the greatest with p(X=1) › p(X=2) › p(X=3)› p(X=4)

(f) 0 ‹ = X ‹ = 37 (4A, 4K, 4Q, 1J). No
Distribution
Mean = 10 (there are 40 points in deck and 4 players)
Hence distribution is skewed with p(X=0)›› p(X=37)

Qs 19 & 20 see Excel file *answersset1*

Maths A & B

2. Mathematical expectation

Subject matter, Maths A:

Maintaining basic knowledge and procedures (Appendix 1).
- *Simple algebraic manipulations*
- *Graphs and tables*

Introduction to models for data (p.26)
- *Probability distributions and expected values for a discrete variable*
- *Uniform discrete distributions; random numbers*

Subject matter, Maths B:

Maintaining basic knowledge and procedures (Appendix 1).
- *Summation notation*
- *Basic algebraic manipulations*

Introduction to functions (p. 12)
- *The reciprocal function and inverse variation*

Applied statistical analysis (p. 23)
- *Probability distribution and expected value for a discrete variable*

Exponential and logarithmic functions and applications (p. 17)
- *Solution of equations involving indices*

Maths A & B

2.1 Expected value and the mean

The mean or expected value of a random variable.

If 10 fair coins are tossed together (or one coin 10 times), there are 11 possible outcomes of the compound event: zero to 10 heads. If the random variable, X, is the number of heads, then the expected number of heads is:

$E(X) = n \times p = 10 \times 1/2 = 5$

If we repeat the experiment many times, we will find that the average or mean number of heads is 5.

If we roll a single die a large number of times, we will get the scores of 1, 2, 3, 4, 5, or 6 with equal probability. Thus the mean or average score will be $(1 + 2 + 3 + 4 + 5 + 6)/6 = 3.5$.

Maths B

In general, if there are i outcomes of an experiment (X_i) and each outcome has a probability p_i then the Mean or Expected Value is the sum of the product p_i x X_i all of these.

We write this as $E(X) = \sum p(X_i) X_i$

In the example of the die each p_i = 1/6 so using this formula we get

E(X) = 1/6 x 1 + 1/6 x 2 + 1/6 x 3 + 1/6 x 4 + 1/6 x 5 + 1/6 x 6

= 1/6 (1 + 2 + 3 + 4 + 5 + 6)

= 3.5

Note that it is not possible to ever get 3.5 on any one roll but this is the mean or average score from a large number of rolls.

Maths A & B

2.2 Expected gambling return

2.2.1 Odds, return and probability

While casino betting is at given "odds", many bookmakers and betting agencies have replaced odds with $ Return per $ bet.

Instead of quoting odds of 1:1, they will quote the return of $2 (per $ bet)

Odds		Return (per $1 bet)	Bookmaker Probability
2:1	(bet 1, win 2)	$3	1/3
10:1	(bet 1, win 10)	$11	1/11
3:2	(bet 2, win 3, total 5)	$2.50 ($5 for a $2 bet)	2/5
5:4	(bet 4, win 5, total 9)	$2.25 ($9 for a $4 bet)	4/9

In each case the Return x probability = 1

In any gambling situation, the less likely the outcome, the higher the return will be should that outcome eventuate.

Conversely, the more likely the outcome, the lower the return.

Maths B

In mathematics, this type of relationship is called an inverse proportion

If $y = f(x)$ is the return and x the probability, $xy = 1$ or $f(x) = 1/x$

2.2.2 Expected return

If you bet $10 on the red on a roulette wheel, the probability of winning is 18/37. If you win you will receive $20 (your $10 bet plus your $10 win).

Your expected return (ER) is the product of the probability of winning and the return from such a win.

$$ER(X) = p(X) \times R$$

where p(X) is the probability of winning, and

R is the total amount of payment you will receive if you win.

In this case ER = 18/37 x $20 = $9.73

In the long run you can expect to get back $9.73 for every $10 bet. Although on any one particular play you will get either $20 or nothing, the figure of $9.73 is what you expect to get on the average. Sometimes we refer to this as the mean or average return. This figure is better expressed as a % of the outlay. In this case:

$$\%ER = 9.73/10 \times 100\% = 97.3\%$$

Exercises Set 2 and Section 3.1 will show that all bets on roulette (in Australia and Europe) have an expected return (ER) of 97.3%

This means that for every $100 bet, the house pays out $97.30 and keeps $2.70.

(In the USA roulette wheels have 38 outcomes including 00, so the ER is 94.6%).

Maths A & B

2.3 Computing bookmaker/betting agency profit margin

Consider the following situation:

A game with two equally matched teams and one must win.

e.g. Tennis match between A and B.

Player	Return	p
A	$2.00	0.50
B	$2.00	0.50
		1.00

If there is no margin or profit each player will be at odds of 1:1 (true odds, for true probability of 0.5) and the return for each will be $2.00.

To convert these to probabilities we simply take the reciprocal of each. (See 2.2.1).

As expected the sum of the probabilities is 1.00.

Note: The inverse proportion between return, R and probability, p

$R \times p = 1$

$R = 1/p$

$p = 1/R$

For a bookmaker or betting agency to show a profit, they must pay out less than $2.00 for either win. Suppose we replace the $2.00 with $1.90.

Player	Return	p
A	$1.90	0.5263
B	$1.90	0.5263
		1.0526

In the above table p=1/R

The bookmaker's probabilities now rise to a sum of 1.05. Thus they are not true probabilities but include a margin for profit of approximately 5%.

Suppose that there is $1,000 bet on each of A and B. The agency has collected $2,000 and will pay out $1,900 to those who have bet on the winner. Thus their profit is $100 or 100/2000 = 5% of the bets taken or 100/1900 = 5.26% of the payout.

If player A is more likely to win, then the return on A must be less than the return on B. Suppose, for example, that we think A is twice as likely to win:

$p(A) = 2/3$ and $p(B) = 1/3$

Now, our table becomes

Player	Return	p
A	$1.50	0.666667
B	$3.00	0.333333
		1

In the above table p=1/R

In order to maintain a profit margin we enter lower returns for A and B. For example, if we reduce each by 10% we get $1.35 for A and $2.70 for B. This gives the following probabilities and profit margin of 10% of the take or approximately 11% of the payout.

Player	Return	p
A	$1.35	0.7407
B	$2.70	0.3704
		1.1111

Activity 2.1

To set the returns for any two player situation so as to give a profit margin for the bookmaker or agency, go to the Excel file: *returns*.

See Appendix 9 for use.

If there are more than two outcomes the procedure is the same. Each probability is the reciprocal of the return and the returns are set so that the sum of the probabilities exceeds 1.00 by the desired profit margin.

If there are three equally likely outcomes, “fair” odds would be 2:1 for each and the returns would be $3 for each.

Team	Return	p
A	$3.00	0.333333
B	$3.00	0.333333
C	$3.00	0.333333
		1

Now adjusting returns by 10% to allow for a profit:

Team	Return	p
A	$2.70	0.37
B	$2.70	0.37
C	$2.70	0.37
		1.11

If $1,000 is bet on each of the three outcomes, the agency will take in $3,000 and pay out $2,700 for a profit of $300, 10% of the take or 11.1% of the payout.

If there are several possible outcomes, the procedure is the same.

Take the reciprocal of the returns and sum them.

For example: AFL Premiership winners

Premiership Odds

Essendon		$1.55	0.645161
Brisbane		$6.00	0.166667
Carlton		$9.00	0.111111
Hawthorne		$14.00	0.071429
Pt Adel		$26.00	0.038462
Collingwood		$51.00	0.019608
Roos		$34.00	0.029412
Richmond		$26.00	0.038462
Adelaide		$101.00	0.009901
Sydney		$36.00	0.027778
Dogs		$34.00	0.029412
Geelong		$67.00	0.014925
Melb		$501.00	0.001996
			1.204322
Profit margin			20%

Further examples, see Exercises Set 2

Activity 2.2

Go to websites for Centrebet, TABs and compute profit margins on various types of betting situations with several outcomes. Set up your own spreadsheet to do this.

2.4 The mathematical expected return (ER) of gambling situations with several outcomes

Suppose a raffle has 100 tickets at $2 each.

First prize is $100 and second prize is $50.

What is the expected return (ER)?

If you have one $2 ticket, your probability of winning either 1st or 2nd prize is 0.01

Maths B

$E(X) = \sum p(X_i) X_i$

$= 0.01 \times \$100 + .01 \times \$50$

$= 0.01 \times \$150$

$= \$1.50$

Thus for each $2 ticket, your expectation is $1.50

Of course you will never get $1.50. You will get either $100, $50 or nothing with probabilities of 0.01, 0.01 and 0.98 respectively.

The $1.50 is the expected value or *average*.

Expressing this as a percentage per $ bet

%ER = $0.75 per $ or 75%

Another way of computing mathematical expectation in situations like this, where we know the total amounts taken and paid out is known, is simply to consider the $ amount paid out and the $ amount taken in, then

ER = $ out/$ in

In this case Ex = 150/200 = 0.75 = 75%

In situations where these two amounts are known, this is the easier way.

For example, if lotto has several prizes, we simply consider:

ER = $ out/$ in

Maths A & B

2.5 The mathematics of lotto, lotteries, raffles and keno

Most lottos pay out about 60% of what they take in and thus have an ER of 60%.

This figure represents the product of a very large amount and a very small probability.

Consider the following situation. A charity advertised fantastic odds on their raffle. Only 10,000 tickets would be sold at $50. The prize was a $200,000 car. How fantastic were the odds?

ER = pA = 1/10,000 x 200,000 = $20

= 20/50 (per $)

= 40%

or

ER = $out/$in = 200,000/500,000 = 40%

2.5.1 Calculating lotto probabilities

The play of lotto involves the random selection from a set of 45 numbered balls. In each game, six balls are selected and these are called the winning numbers. A further two balls are selected as the supplementaries. Prizes are awarded in five "divisions", division one being the most valuable (and least likely).

Division 1 All 6 winning numbers

Division 2 Any 5 winning numbers and either supplementary

Division 3 Any 5 winning numbers

Division 4 Any 4 winning numbers

Division 5 Any 3 winning numbers and either supplementary

The Golden Casket Gold Lotto site has information about playing the game.

http://www.goldencasket.com/gold_lotto/default.asp

Students must be closely monitored when accessing the Golden Casket website and teachers must ensure students do not access any of the gambling opportunities available.

See Exercises Set 2

2.5.2 Why people gamble on lotto

Teachers
Refer to OHT26 for classroom discussion material

Betting on lotto and lotteries is generally motivated by the very small chance of winning a very large amount of money. Your expected return on a $10 lottery ticket may be only $6, but you have bought the dream of winning the million and it is for this hope that you are expecting to lose your $10.

For example, if first prize requires you to pick six numbers from 45, the probability of doing this is 6/45 x 5/44 x 4/43 x 3/42 x 2/41 x 1/40 = 1.22774E-07or about one chance in 10 million. To appreciate how small this is, consider that if you bought your ticket a week before the draw the probability of your accidental death before the draw is considerably higher than your chance of winning. Furthermore, if other people also share your numbers, the prize is shared equally among you.

2.5.3 Increasing expectation in lotto

You cannot improve the *probability* of winning lotto by any method other than buying more tickets. However, you can improve *your expectation* by avoiding numbers and combinations that are popular with other people. The effect of this is that if you select lotto combinations of numbers that others avoid, your expectation will increase not because of any increased probability, but because of a greater payout should you win. Recall

$$ER = p\,R$$

The overall expectation of the lotto is unchanged, but your expectation is greater than those choosing "shared" combinations. The Canadian lotto which is a 6 from 49 selection records and publishes the most popular choices. These are 7, 11, 3, 9, 5, 27, 31, 8, 17.

2.5.4 Professional gamblers and lotto

The identifying characteristic of the professional gambler is that of motivation. The professional gambler is motivated by an expectation of winning, and relies on gambling as a major source of income.

An examination of the operation of professional gamblers (including syndicates), shows that they operate from two mathematical axioms:

1. A positive mathematical expectation of return - a belief that the odds are in their favour, and thus, that in the long run, they will come out ahead; and
2. The ability to sustain short-term losses.

Professional gamblers are well aware of situations in which the odds are not in their favour and do not gamble in these situations such as casino betting. Do not confuse professional gamblers with casino high rollers or heavy bettors at the track.

Lotto expectation increases when the payout jackpots. By betting only when the payout has jackpotted your probability of selecting the lucky combination does not change, but as the ratio of payout to amount paid increases, so does your mathematical expectation. There are many large gambling syndicates that are well aware of this. They employ mathematicians to compute when their expectation exceeds 100%, and use psychology in selecting combinations noted above. They also have large resources and can afford to lose. Betting when your expectation is greater than 100% does not mean that you will always win, but it does mean that you will win in the long run. Professional gamblers do this and are able to cover short-term losses.

2.5.5 Raffles

Most raffles have very low expectation and are not really considered as a serious form of gambling, more a donation to a cause or charity e.g. surf lifeclub raffle meat tray or seafood platter.

2.5.6 Keno

Keno is similar to lotto in that the bettor selects a set of numbers from a larger group. Certain combinations result in payouts. In general, at clubs and casinos, Keno returns are a higher percentage than for lotto and thus your expectation is greater. Keno payouts can jackpot in the same manner as lotto and professional gamblers and syndicates have been known to bet heavily on Keno under these conditions.

2.6 Return from poker machines

Of all forms of gambling, the one that seems to be associated with most gambling related problems is betting on electronic gaming machines.

Refer to the Excel files *spinner*, and *gamblers fallacy*

Teachers

Stress: The operation of the poker machine is the same as any independent generator. Just as the spinner frequencies approach the theoretical with repetition, so too will the poker machines. As with the spinner or roulette wheel, what has happened in the past events cannot affect this. Whether the machine has just paid a jackpot or has never paid a jackpot has no bearing on the probability of it paying on the next play.

The expected return of all machines (product of probability and return) is fixed at well less than 100%, generally at about 80% (currently in Queensland 85% or 92%).

Note: The expected return on gaming machines is over the life of the machine, not for each individual bet.

Teachers

Refer to OHTs 15 and 16 from the *Responsible Gambling Teaching Resource Kit* as teaching aids in this discussion.

Maths A & B

3. Expected mathematical return and casino betting: calculating casino profit

Subject matter, Maths A:

Maintaining basic knowledge and procedures (Appendix 1).
- *Simple algebraic manipulations*
- *Graphs and tables*

Introduction to models for data (p.26)
- *Probability distributions and expected values for a discrete variable*
- *Uniform discrete distributions; random numbers*

Subject matter, Maths B:

Maintaining basic knowledge and procedures (Appendix 1).
- *Summation notation*
- *Basic algebraic manipulations*

Introduction to functions (p. 12)
- *The reciprocal function and inverse variation*

Applied statistical analysis (p. 23)
- *Probability distribution and expected value for a discrete variable*

Exponential and logarithmic functions and applications (p. 17)
- *Solution of equations involving indices*

Casinos in Australia return about 95% of all money wagered. The remaining 5% goes to operating costs, government costs, taxes, and finally profit. Thus casinos require a high turnover for a small profit margin. However, it must be noted that the casino cannot lose in the long run. While some casinos may lose money as a result of high operating costs or lower than anticipated turnover, they never lose from the betting in the long run. This is simply because in all forms of casino gambling, the gambler's expectation is less than 100%. As we have seen, this means that you cannot, in the long run, return a profit from casino betting. With the exception of blackjack, professional gamblers do not bet on casino games.

Teachers
Refer to OHT27 for discussion of professional gamblers and poker

If this is the case, why gamble at casinos in the first place? Casino gambling should be viewed only as entertainment without any long-term expectation of winning.

The return on all casino bets are shown as Odds:1

35:1 means bet $1 win $35, receive $36 back.

In each of the following casino situations we will calculate the **expected return** (ER) per $1 bet as a percentage and the long-term **casino profit**.

3.1 Roulette

The ER for roulette is greater than most other casino games. If you bet on either red or black, odd or even, your expected return is 97.3%. This figure is constant for *any* bet on roulette (Exercise Set 2). Hence, the **casino profit** for roulette is **2.7%** of the total bet. While the casino might lose on any particular wheel in the short term, on the average and in the long term the casino profit will always average 2.7% of the total bet.

The options for betting on roulette are:

1. Straight up, any number or zero: Odds 35:1
2. Split, any two numbers: Odds 17:1
3. Street, any three numbers: Odds 11:1
4. Corner, any four numbers: Odds 8:1
5. Sixline, any six number: Odds 5:1
6. Dozen, first, second or third twelve: Odds 2:1
7. Column: Odds 2:1
8. Low (1 to 18) or High (19 to 36): Even money.
9. Red or Black: Even money
10. Odd or Even: Even money

Casinos go to great lengths to ensure that the wheels are fair in the sense that each number is equally likely and that the wheels stay fair. Thus, each roll is independent of what has happened before and cannot be affected in any way by past outcomes.

The mathematical consequence of this is that ***there is no system*** of winning at roulette nor can there ever be any system. The probabilities remain unchanged in the same way as shown with the numbers in the lotto example. It is interesting to note that the casino will hand out cards for you to keep track of the numbers that have shown. They rely on the gambler to use the gamblers fallacy (Section 1) in the selection of numbers. Of course, it makes no difference to the casino *what* numbers the gamblers choose. The gamblers fallacy means the *amount* the gamblers will wager increases, in the mistaken belief that the probabilities have changed. For example, if red shows six times in a row, many will now put large amounts on the black, thinking that this is now more likely, when in fact the *expectation* for both the gambler and the casino remains totally unchanged.

Why play roulette if you cannot increase your expectation? The only answer to this can be for the social enjoyment. If you enjoy social gambling, roulette can be a way for you to do so. However, although your ER is relatively high at 97.3%, the minimum bet is generally either $1 or $5 and you will lose money fairly quickly. (See Exercises Set 2)

3.2 Other casino games

There are many other casino games most of which give a lower return than roulette.

These include:

3.2.1 Under 7/Over 7

In this game, a pair of dice are rolled. The player can bet on either under or over seven at even money. If a seven shows, the house wins all.

There are 15 ways a pair of dice can show either more or less than seven and six ways in which a seven can show. Thus for either under or over seven p = 15/36 .

ER = 15/36 x 2 = 83.3%

This is a much lower figure than that of roulette and the gambler will lose money much more quickly at this game.

(See Exercises Set 2)

Casino Profit = 16.7%

3.2.2 Two-up

This game has a colourful history in Australia. In its simplest form, a pair of coins are tossed. The players can bet on either heads (two heads) or tails (two tails) at even money. If HT or TH show, the coins are tossed again. As such this is a fair game in the sense that ER = 100%. However most casino rules are that if a HT or TH show five times in a row, the house takes all bets. All bets are frozen after the first HT/TH.

Now, the probability of either HT or TH is 1/2 so the probability of this happening five times in a row (the multiplication principle) is $1/2^5$ = 1/32

This will happen on the average one in 32 times. Thus for any player betting on either HH or TT, p = 31/64 (one half of 31/32)

So, ER = 31/64 x 2 = 96.875%

Again, this is less than for roulette, though only slightly.

Casino profit = 3.125%

3.2.3 Sic-Bo

In this game, bets are placed on the outcomes of the roll of three dice.

For example, betting on a score of four returns 62:1. This might sound good until you do your calculations:

There are three ways a score of four can occur – 1,1,2; 1,2,1; 2,1,1

There is a total of 6 x 6 x 6 = 216 ways three dice can land.

So p(4) = 3/216

Return = 63

ER = 3/216 x 63 = 87.5%

A specific triple pays 180:1

p(triple) = 1/216

Return = 181

ER = 181/216 = 83.8%

A similar analysis of all other bets on Sic-Bo shows that the expected return is between 83% and 88%. This means that the casino profit is about 15% of all monies bet. Compare this with the 2.7% for roulette and you will see that you are more than five times as likely to lose at this game than at roulette!

3.2.4 Wheels

See Exercise Set 2, Question 8

3.2.5 Blackjack

Blackjack is the world's most popular casino card game. There are many versions of this game that go by different names: pontoon, 21, etc, all with slightly different rules. When you place a wager on blackjack you are betting that you will have a hand higher in value than the croupier without going over 21. The players place their bets then the croupier deals two cards to each player and one to the croupier all face up. The croupier then plays each player separately.

Blackjack is a special category of game since an element of decision-making is involved. Furthermore, the probabilities change slightly as the game progresses since the cards are drawn from the deck without replacement. However, most casinos use a six deck of cards and reshuffle when two decks remain. This means that it is virtually impossible to keep track of all the cards that have gone and even if you could, the changes in the probabilities of the remaining cards are only slight. Nevertheless there are methods of keeping track of the high and low cards that have gone and altering your strategy when a disproportionate number of either high or low cards remain. The mathematical expectation (ER) from blackjack will vary with the game strategy employed by the player. Using a perfect probability game strategy, the player's expected return is about 96-97% and the casino profit is about 3 to 4%.

Exercises Set 2 - Maths A & B

1. A raffle has 200 tickets at $2 each. There are three prizes: $150, $75 and $50.
   Use both methods of computation to find the ER.

2. Three dice are rolled. What is the mean or expected score?

3. If the probability of a person selected at random from the population being left-handed is 0.15, how many left handed people would you expect there to be in a group of 60 people selected at random from the population?

4. A lotto consists of selecting six numbers from 40. How many possible selections are there? What is your probability of selecting first prize?

5. A lotto consists of selecting six numbers from 49. What is your probability of winning 1st prize with one selection? (Write your answer as a decimal and as "one chance in …"). If first prize is $5,000,000 what is your expected return (assuming no one else shares the prize)?

6. Suppose you bet straight up on roulette. The odds are 35:1, so your return, should you win, is $36 per $1 bet. The probability of winning is 1/37, so

   ER = pA = 1/37 x 36 = 97.3%

   The options for betting on roulette are:
   1. Straight up, any number or zero – Odds 35:1
   2. Split, any two numbers – Odds 17:1
   3. Street, any three numbers – Odds 11:1
   4. Corner, any four numbers – Odds 8:1
   5. Sixline, any six number – Odds 5:1
   6. Dozen, first second or third twelve – Odds 2:1
   7. Column – Odds 2:1
   8. Low (1 to 18) or High (19 to 36) – Even money.
   9. Red or Black – Even money
   10. Odd or Even – Even money

   Show that the ER in roulette is the same for ANY selection.

7. If on a typical casino night $500,000 is bet on the roulette tables, what is the approximate casino profit for the night?

8. A wheel at the casino has 52 numbers: two 47s (One casino, one win), 24 ones, 12 threes, 8 fives, 4 elevens, and 2 twenty-threes.

The following odds are offered:

Complete the table

Outcome	number	Odds	pr	E
47 C	1	47:1	1/52	48/52= 92.3%
47W	1	47:1		
23	2	23:1		
11	4	11:1		
5	8	5:1		
3	12	3:1		
1	24	1:1		

9. A casino game involves the throw of a pair of dice. For each of the following outcomes, the casino pays odds as shown. Compute the gambler's ER and % casino profit for each.

(a) Any pair	4:1
(b) 2 or 12	15:1
(c) 10, 11 or 12	4:1
(d) < 6	2:1

10. Compute the mathematical expectation as a % for the following Sic-Bo outcome at the odds shown:

Any Triple pays 31:1

11. (a) If you bet $1,000 on roulette, what is your expected $ return?

(b) How much do you expect to have if you bet your whole return again?

(c) How much after 10 such bets?

(d) After how many bets will you have:

- $500 left?
- $5 left?

Exercises Set 2 - Maths B

12. (a) Express the amount left in Q13, y, after a number of bets, n, as y = f(n)
    (b) State the range and domain of f(n).
    (c) Use this function to answer Q.11 (c) without the spreadsheet.
    (d) If you start with $A and continue until only $5 is left, write the number of bets, n, as a function of A.
    (e) If $A = 2000, what is n?

13. Track betting generally has an expectation of 85%. A punter starts with $1,000 and bets it all randomly on several outcomes. The amount remaining is then bet again in the same manner.
    (a) How much remains?
    (b) How much after five such bets?
    (c) After how many bets does $100 remain?
    (d) Express the amount left (y) after a number of bets, n, as y = f(n).
    (e) If you start with $A and continue until only $5 is left, write the number of bets, n, as a function of A.

14. A Sic-Bo outcome consists of the total value on the three dice of 10. What odds should the casino offer to have a mathematical expectation for the player between 85 and 90%?

Answers Set 2 - Maths A & B

Q1. ER = amount paid out/amount taken = $(150 + 75 + 50)/$200 x 2
= 275/400
= 68.75%
or
ER = p A = 1/200 ($150 + $75 + $50) = $1.375 per $2
= $0.6875 per $1
= 68.75%

Q2. Mean = 3 x 3.5 = 10.5

Q3. X = p x n = 0.15 x 60 = 9

Q5. p = 6/49 x 5/48 x 4/47 x 3/42 x 2/41 x 1/40
= 7.15112E-08
about 7 chances in 100,000,000 or
less than 1 in 10,000,000
ER = pA = 7.15112E-08 x 5,000,000 = 0.357556192 = about 36%

Q6. Split, any two numbers. Odds 17:1, p = 2/37, A = 18, ER = 36/37
Street, any three numbers. Odds 11:1, p = 3/37, A = 12, ER = 36/37
Similarly for all other options, p x A = 36/37

Q7. Casino profit on roulette = 1/37
So 1/37 x $500,000 = $13 515

Q9. (a) 83.3% 16.7%
(b) 88.9% 11.1%
(c) 83.3% 16.7%
(d) 83.3% 16.7%

Q10. p (3 of a kind) = 1/36
A = 32
ER = pA = 32/36 = 88.9%

Q11. See Excel file Answers Set 2
After one bet ER = 36/37* amount bet = $973
Betting this again, ER = 36/37 x $973 =
We see that after two bets ER = $1000 x (36/37) 2
After n bets ER = $1000 x (36/37)n

From the Excel file
After two bets $947 remains
After 10 bets $760 remains
After 26 bets $490 remains
After 197 bets $5 remains

Answers Set 2 - Maths B

Q12. (a) $y = 1{,}000\ (36/37)n$ where \$1,000 is the amount at start (n=0)
In general, if you start with \$A, $y = A\ (36/37)n$

(b) Domain $0 <= n <= 198$ (after 197 bets only \$5 remains, assuming a \$5 minimum bet), Range $5 <= y <= 1{,}000$

(c) If $f(n) = 5$
$5 = 1000\ (36/37)\ n$
$0.005 = (36/37)\ n$
$\log (0.005) = n \log (36/37)$
$n = \log (0.005)/\log (36/37) = 197$

(d) $n = f(A) = \log(A/5)/\log(36/37)$

(e) 216

Q13 See Excel file Answers set 2
(a) After one bet ER = 0.85 x \$1,000 = \$722
(b) After five bets, ER = (0.85) 5 x \$1,000 = \$443
(c) After 14 bets \$102, 15 bets \$87
(d) After n bets $y = 1{,}000 \times (0.85)n$
(e) $n = f(A) = \log(A)/ \log 0.85$

Q14. See Excel file Answers set 2
$p(X = 10) = 27/216 = 1/8$
Fair odds would be 7:1, so casino must offer less
At 6:1, ER = 87.5%

Maths A & B

4. Bookmakers and betting agencies: betting at fixed odds; calculating bookmaker profits

Subject matter, Maths A:

Maintaining basic knowledge and procedures (Appendix 1).
- *Simple algebraic manipulations*
- *Graphs and tables*

Introduction to models for data (p.26)
- *Probability distributions and expected values for a discrete variable*
- *Uniform discrete distributions; random numbers*

Subject matter, Maths B:

Maintaining basic knowledge and procedures (Appendix 1).
- *Summation notation*
- *Basic algebraic manipulations*

Introduction to functions (p. 12)
- *The reciprocal function and inverse variation*

Applied statistical analysis (p. 23)
- *Probability distribution and expected value for a discrete variable*

Exponential and logarithmic functions and applications (p. 17)
- *Solution of equations involving indices*

4.1 Setting bookmaker and agency odds

Unlike casino betting, in sports betting the true probabilities for most situations are not known. Therefore a bookmaker or betting agency will start taking bets on an event with odds estimated by expert advisers. See Section 2.2

At the start of betting, this profit margin is generally set high and then adjusted down as bets are placed and the bookmaker gets an idea of how bets will proceed.

Bets taken at these odds are fixed for the placer of the bet. However, as betting proceeds these odds will change according to the amounts bet. If more money starts to be placed on one player, the return will be decreased on that one and increased on the other. Setting odds or returns on horse races follows the same procedure. Initial odds are set to give the bookmaker a fairly large profit margin. For example, in an eight horse race the odds and/or returns might be:

Activity 4.1

Setting the Odds

See Excel file: *trackodds*

Use See Appendix 10

Horse	Odds	p	Return
1	5:2	0.2857	$3.50
2	7:2	0.2222	$4.50
3	8:1	0.1111	$9.00
4	4:1	0.2000	$5.00
5	5:1	0.1667	$6.00
6	8:1	0.1111	$9.00
7	10:1	0.0909	$11.00
8	20:1	0.0476	$21.00

1.2354 total

23.5% profit

This represents a more than 20% profit margin for the bookmaker. As bets are taken, the odds are constantly adjusted maintaining an overall profit margin of about 15 - 20%. *(Do this using the file)*.

Activity 4.2

Use the *randomgenerators* spreadsheet to generate a large number of rolls of a pair of dice. Use these numbers to play the Horserace game of Activity 1.8.

Record the number of wins for each of the 11 horses to get a relative frequency for each.

Use this RF to estimate the probability of each horse winning.

Go to the Excel file *trackodds* (part D) and set a book of odds using your probabilities.

For use, see Appendix 10

4.2 Mathematical expected return of bookmaker and betting agencies

Since with track and sports betting, unlike casino betting, the true probabilities are never known for sure, ER = p x A must only be an estimate for any bet. However, since we can calculate the profit margin from the returns shown, we can calculate the overall ER for all bets.

Profit margin 16.7%

Now using ER = amount paid out/amount taken in

ER = 83.3%

Maths A & B

5. The mathematics of totalisation betting

Subject matter, Maths A:

Maintaining basic knowledge and procedures (Appendix 1)

- *Numbers in various notations including fraction, decimal, scientific*
- *Rates, percentages, ratio and proportion*
- *Simple algebraic manipulations*

Subject matter, Maths B:

Maintaining basic knowledge and procedures (Appendix 1).

- *Rates, percentages, ratio and proportion*
- *Basic algebraic manipulations*

5.1 Totalisation computations

Consider a simple two-outcome event. If a tie is not possible, there are only two outcomes, e.g. Qld vs NSW rugby.

Totalisation

Outcome	$ bet	$ Return
New South Wales	1,000	1.53
Queensland	800	1.91
Total	1,800	
Profit margin	15%	
Return to punters	1,530	

For most two or three outcome events, the odds will be fixed as discussed in Section 4 rather than totalised, but all TAB track bets are totalised with a profit margin of about 15% and rounding the returns (which the TAB calls dividends) down to the next five or ten cents. The procedure is very simple. (see Excel file *totalisation1*)

1. All the bets are totalled
2. The $ profit is calculated (15%)
3. The win pool, the amount to be paid back to the winners, is calculated
4. The win return (dividend) for each possible outcome is calculated by dividing the win pool by the amount bet on that outcome (This may be rounded down to the nearest five or 10 cents)

5.1.1 Place bets

The returns for win bets are computed as in Section 5.1, place bets are computed in exactly the same manner for each horse, but the total return is divided by three before sharing. (See Excel file *totalisation1*)

Activitity 5.1

Excel spreadsheet totalisation of bets and computation of returns

File: *totalisation1*

Use, See Appendix 11

5.1.2 Mathematical expectation and totalisation betting

In all totalisation situations, in the absence of other information, your expected return can be calculated from the profit margin. If the profit margin is 15%, the system returns 85% of bets and this is the ER.

5.2 Track betting

Australians spend more per head of population on betting than any other nation and much of this is on track betting. In addition, a greater proportion of Australians attend track events than in any other country.

Track bets may be placed with a licensed bookmaker at the track (with fixed odds set as described in Section 4), on the TAB at the track, or off the track at a TAB office, PUBTAB, or by telephone account or Internet TAB betting. Different TAB organisations (such as Tabcorp, UniTAB), do their own totalisation so the payouts for any particular event may vary (generally only slightly) between them.

At the moment only State TABs are allowed to offer track betting. This is because part of their revenue must be paid to the racing industry. We will see that the player's ER for track betting (about 85%) is considerably less than for other events (about 90%) for this reason.

Track and sports betting differ from casino gambling in that the probabilities of the outcomes are never known exactly.

Computations on track betting are now all done instantly by computer as bets come in from all over the country and the returns are adjusted and displayed instantly on the TV monitors. The punter must understand that it doesn't matter what the return is showing on the screen when the bet is placed. The only return will be that showing once betting is finished.

It is important to understand that when you place a bet with a bookmaker at fixed odds, you are betting against the bookmaker. When you place a bet with a totalisation system you are really betting against all the other punters. The system cannot lose and for you to win, other punters must lose.

If the returns are rounded down, the agency profit increases and the punter's expected return decreases. (See Excel file *totalisation2* for illustration of how this works).

It is possible when totalising for the return to drop below $1 if, for example, a very large amount is placed on one outcome. This is more likely to happen with a place bet than for a win. In this case the return is adjusted back up to $1 and the profit reduces. (See Excel file *totalisation3*).

Use of *totalisation2* and *totalisation3* see *Appendix 11*.

5.2.2 Quinellas and trifectas

The returns are computed and totalised in exactly the same manner by computer. There are of course many more outcomes to consider.

In an eight horse race there are 8 x 7 = 56 ways in which the first two horses can finish. However the selection of AB is the same as BA in the quinella.

Therefore there are 8 x 7/2 = 28 quinella outcomes.

There will be 8 x 7 x 6 = 336 trifecta outcomes. Since the procedure is more complex only TABs using computer systems can offer quinella and trifecta bets, and then their profit margin is generally slightly higher, 17%.

See *Exercises Set 1*, Q7

Maths A & B

6. Wagers and "fair" bets

Subject matter, Maths A:

Maintaining basic knowledge and procedures (Appendix 1)

- *Numbers in various notations including fraction, decimal, scientific*
- *Rates, percentages, ratio and proportion*
- *Simple algebraic manipulations*

Subject matter, Maths B:

Maintaining basic knowledge and procedures (Appendix 1).

- *Rates, percentages, ratio and proportion*
- *Basic algebraic manipulations*

6.1 Fair games

We need to define what is meant by this term before continuing. In everyday or colloquial use, fair can have different meanings. A football game is fair if there is no foul play. A roulette wheel is fair if each number has an equal chance of showing. A teacher is fair if all students are treated equally. A horse race is fair if there is no outside interference even though we know that each horse does not have an equal chance of winning. Fairness does not necessarily imply equally likely although when we use the term with reference to coins and dice, it does. In these situations it is better to speak of unbiased coins rather than fair coins. We need a mathematical definition of fairness:

> *Mathematical fairness*. A game is fair if each player has the same mathematical expectation of 100%.

That is to say, the probability of winning times the return for a win is the same for all players. In this sense none of the casino games we examined are fair, since the player's expected return is always less than 100%. On the other hand, the play of the games is always fair in the non-mathematical sense in that it is free from interference and the roulette wheels, coins, dice etc. are unbiased.

Games of only chance, such as two-up, when played without any house profit, generally fit the definition of fairness. Games that involve both chance and skill, such as poker or bridge, are also fair in the sense that the total paid out by the losers is the same as the total collected by the winners (there is no house take), although individual player expectations could vary (for example if a novice is playing against an expert).

6.2 Wagers between individuals

Wagers between individuals are fair if each player has the same mathematical expectation.

Example: A pair of dice is rolled. You bet on a showing of 10, 11, or 12. What odds should you get to make a fair game?

Solution: Computation of "fair" odds

First we must calculate the probabilities.

In this case there is a total of six ways in which you can win (6,4; 4,6; 5,5; 6,5; 5,6; 6,6) so $p = 6/36 = 1/6$

Next we convert this to true odds (See Section 1)

In this case $p = 1/6$, odds 5:1

Example: If you cut a deck of cards and win if you cut a heart, what are the fair odds?

Solution:

Pr (win) = 1/4, odds are 3:1

Example: If you cut a deck of cards and win if you cut a heart greater than 10, what are the fair odds?

Pr (win) = 4/52 = 1/13, fair odds are 12:1

Example: Roll a pair of dice, win if total > 8, how much should I receive as return for a $1 fair bet?

Solution:

With play at "fair" odds $ER = p \times A = 100\%$

So if $p(X>8) = 10/36$

$ER = 10/36 \times A = 1$

$A = 36/10 = \$3.60$

Example: Roll three dice, win if total < 6, how much should I receive as return for a $1 fair bet?

Solution:

How many ways can three dice result in a score of three, four or five?

Total no. of ways three dice can land = $6 \times 6 \times 6 = 216$

No. way $< 6 = 10$

So $p(X<6) = 10/216$

$ER = p \times A = 1$

$A = 216/10 = 21.6$

For $1 bet, Return = $21.60

Exercises Set 3 - Maths A & B

1. On a particular race the winning horse paid $2.65 on the TAB. Bookmaker odds were 3:2.

   Which was the better to bet with? Why would they be different?

2. In each of the following, calculate the bookmaker profit margin to the nearest whole %.

(a)	Rugby Union		
		South Africa	$2.55
		Australia	$1.45
(b)	Cricket	England	$2.45
		Draw	$2.40
		New Zealand	$3.15
(c)	AFL	Carlton	$2.40
		Western Bulldogs	$1.50

(d) Horse race with 5 horses showing returns of $3.50, $4.00, $6.00, $2.75, $11

3. A two outcome event is bet on using a totalisation system in which a margin of 10% is set.

   If the final takings are

   A: $9,000

   B: $6,500

   How much is paid per $ bet for each outcome?

4. If you cut a deck of cards and win, what are the fair odds for:

   (a) a red card

   (b) a club

   (c) an Ace

   (d) a heart › 10

   (e) a black card › 10

5. If you roll a pair of dice, how much should you receive as return for a $1 fair bet if you bet on:

   (a) any pair

   (b) an even number total

   (c) an odd number total

   (d) seven

   (e) › 7

   (f) ‹ 7

6. If you roll three dice, how much should you receive as return for a $1 fair bet if you bet on:

   (a) any triple

   (b) X › 16

   (c) X › 10

7. You and I play a game in which a coin is tossed. Heads I win, tails we toss again. On the second toss, heads I win, tails you win. I will pay you odds of 2:1 if you win.

   Is this a fair game?

   How would you change the odds to make it a fair game?

8. You and I play a game in which a single die is rolled.

   (a) I take the numbers 1, 2, 3 and you take 4, 5, 6. We bet at even money. Is this a fair game?

   (b) I take the numbers 1, and 2, and you take 3, 4, 5, 6. What odds should you pay me to make it a fair game?

   (c) You call any number from 1 to 6 and win if it shows. What odds should I pay you to make it a fair game?

9. Three people, A, B and C play a game in which a pair of dice are rolled.

   A wins if it's a 2, 3, 4 or 5; B wins on 6, 7, or 8; C wins on 9, 10, 11, 12.

   Who has the best chance of winning.

10. You and I play a game in which a pair of dice is rolled. You take the product of the numbers if it is odd and I take the even product. Is this a fair game? What odds could make it a fair game?

11. Your risk index

    Suppose you win a cash prize in a quiz and the host then offers to toss a coin for double or nothing. Would you accept if the prize was $20? $50? $100?

    For what amount would you decline? Clearly a double or nothing bet at $p = 1/2$ (unbiased coin) has an expected return of 100% but whether or not you take the risk depends on other factors.

    Suppose instead that the host offers to toss a coin but will pay you at odds of 3:2. That is if you have won $20 you will get either $50 or nothing. Now your expected return is:

    0.5 x 50 = $25 or 125%

    as a mathematical gambler you will bet when

    1. ER › 100% and

    2. You can afford to lose

    So what now is the amount of the prize at which you decline the bet?

    What if the host offers odds of 2:1?

Exercises Set 2 - Maths B

12. On a two outcome event the Return/$ for player A is $1.60

What return should be offered on B (to the nearest 10 cents) for

(1) a “fair” return,

(2) a 10% bookmaker profit (keeping A at $1.60)

13. Explain how you could place bets so as not to lose if returns were offered on a two outcome event at:

A $3.00

B $1.60

14. Consider the famous gambling problems of the 17th century.

(a) Two equally matched players play a game of tennis (first to 6 wins).

Each puts in $50 and the winner takes the $100. When the score is 5-3 rain stops the play and the game is not resumed. How should the $100 be divided fairly among the two players?

(b) A player plays a game against a bank with the following rules.

A coin is to be tossed until it shows heads. On the first show of heads, the game finishes and the bank pays the player:

$2 for a head on the first toss

$4 for a head on the second toss

$8 on third etc.

How much to play the game if the game is fair? That is to say equal expectation for player and bank.

Answers Set 3 - Maths A & B

Q1. 3:2 returns $2.50, so $2.65 is better

Q2. (a) 8%
(b) 14%
(c) 8%
(d) 16%

Q3. A pays $1.50 and B pays $2.25

Q4.

(a)	a red card	1:1
(b)	a club	3:1
(c)	an Ace	12:1
(d)	a heart > 10	12:1
(e)	a black card > 10	11:2

Q5.

(a)	any pair	$6
(b)	an even number total	$2
(c)	an odd number total	$2
(d)	seven	$6
(e)	> 7	$2.40
(f)	< 7	$2.40

Q6.

(a)	any triple	$36
(b)	X > 16	$54
(c)	p(X > 10) = p(X < = 10) = so A =	$2

Q7. p(you win) = (draw a tree diagram)
So fair odds are 3:1 NOT 2:1

Q8. (a) Yes
(b) 2:1
(c) 5:1

Q9. B

Q10. 3:1 if ODD

Answers Set 3 - Maths B

Q12. (1) $2.70
(2) $2.10

Q13. Bet, for example
A $333
B $625
You have outlaid $958 and will receive $1000 for either outcome

Q14. (a) For B to win he must win the next three games in a row
p(B win) = 1/8, so B should take 1/8 of the $100 = $12.50 and A $87.50

(b) $2 for a head on the first toss, so — R = 2 x _ =1
$4 for a head on the second toss — R = 4 x _ = 1
$8 on third etc. — R = 8 x 1/8 =1

Hence $E(X) = \sum p(X_i) X_i = 1 + 1 + 1 + 1$ ....
It would appear that E is infinite and no amount can make the game fair.
However, this applies only if the bank has infinite funds. If there is a house limit, E for a fair game can be computed. The mathematics of this is beyond Maths B.

Maths A & B

7. More complex probabilities: probability distributions

Subject matter, Maths A:

Introduction to models for data (p. 26)

* *Discrete data that could be uniform or binomial*
* *Identification of binomial situations, binomial expected values, and binomial probabilities*

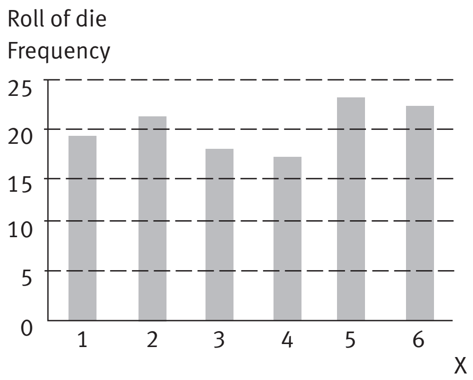
Subject matter, Maths B:

Applied statistical analysis (p. 23).

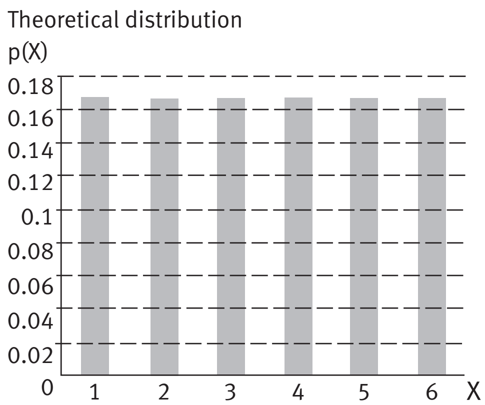
* *Probability distribution and expected value for a discrete variable*
* *Identification of binomial situations and the use of technology*
* *Normal distribution, continuous distributions*

7.1 Uniform distributions

All gambling situations in which the outcomes are equally likely result in uniform distributions. For example with the roll of a single die all 6 outcomes are equally likely. If it is rolled say 120 times, the graph of the distribution will be similar to the graph below.



The graph of the theoretical probabilities and distribution will be perfectly rectangular.



7.2 Binomial probabilities and distributions

Teachers
You may wish to cross-reference this section with the binomial theorem and binomial distribution for Maths B students. Maths A students will not know the theory behind binomial probabilities. However, this will not stop them from using the Excel function BINOMDIST.

So far we have had to compute only fairly simple probabilities. How could we calculate something like: what is the probability of getting exactly seven heads in 10 tosses of a coin? One way would be to consider all 2^{10} outcomes (1024) and count how many of these resulted in seven heads. Fortunately these days this is not necessary. We have readily available computer software or statistical calculators to do this type of computation.

This type of situation is called a **Binomial** experiment since there are two possibilities for each (H or T).

All we need to know is

(1) p = the probability of a success on any one trial (p = 0.5)

(2) the trials are independent (yes)

(3) n = the number of trials (10)

and (4) X = the number of successes (7)

Using Microsoft Excel to do this we use the statistical function and enter:

Successes (X), trials (n), probability (p), cumulative (True or False):

The Excel function statement for this is =BINOMDIST (7,10,0.5,FALSE) = 0.117118

The statement FALSE is to give the probability of exactly seven heads, entering TRUE will give the probability of zero to seven heads (0.945313) and thus the probability of eight, nine, or 10 heads would be 1- 0.945313 = 0.054687

The binomial method is very powerful in computing complex probabilities and we will see how to use it in hypothesis testing later.

Compare the following probabilities:

(1.) Exactly five heads in 10 tosses

(2.) Five or less heads in 10 tosses

(3.) Exactly 50 heads in 100 tosses

(4.) 50 or less heads in 100 tosses

Using the binomial function, we see these are:

(1.) 0.24609

(2.) 0.62304

(3.) 0.07958

(4.) 0.53979

Although the binomial method applies only to situations where there are exactly two outcomes, many experiments can be made binomial.

For example:

Roll a die: Six outcomes. Binomial outcomes: 6 or not 6, › 3 or ‹ 4 etc.

Roulette: 37 outcomes. Binomial outcomes: odd/even, › 12 not › 12, etc.

Recall the question: In a six game series between two evenly matched teams, what is the most likely outcome?

To do this we can use binomial probabilities.

n = 6; p = 0.5; x = 0, 1, 2; cumulative FALSE (we want individual probabilities) then x2 (5,1 or 1,5)

n = 6; p = 0.5; x= 3; cumulative FALSE

Outcomes of six game series

	p
6,0	0.0313
5,1	0.1875
4,2	0.4688
3,3	0.3125
	1

We see 4-2 is the most likely result.

Compare these results with the Excel file “6 games”.

We have seen how to calculate probabilities like getting a pair of fours on the throw of a pair of dice using a number of methods e.g. p = n/t = 1/36 OR p = 1/6 x 1/6 = 1/36

We can also do this using binomial probabilities, using

p = 1/6

n = 2

X = 2

The Excel function statement is =BINOMDIST(2,2,1/6,FALSE) = 0.027778

We have also seen how to calculate probabilities like getting exactly one four on the throw of a pair of dice using p= n/t (10/36).

We can also do this using binomial probabilities, using

$p = 1/6$

$n = 2$

$X = 1$

The Excel function statement is =BINOMDIST(1,2,1/6,FALSE) = 0.277778

Activity 7.1

Using Excel to generate binomial distributions

Graphical representation of binomial probabilities

Excel file *Binomialdists*

Use, see Appendix 12

Maths B

7.3 The normal distribution

Teachers
You may wish to cross reference this section with the normal distribution and standard scores for Maths B students. Mean = np and Standard deviation = sqrt npq

The normal distribution can then be used to compute probabilities. In the past, this was generally much quicker and easier than using binomial tables. However, if you can use the BINOMDIST Excel function there is nothing to be gained from using the normal approximation. One example is given.

The normal approximation to binomial can be seen from the graphs in the file ***normal***

Example of its use:

A fair coin is tossed 50 times. What is the probability of getting less than 20 heads?

Using the *binomialdists* file, we see p(0‹ = X ‹ = 19) = 0.05946 or about 6%

If we look at the graph we see that it looks like a normal curve with:

Mean = np = 50 x 0.5 = 25

Std. devn. = (npq) 1/2 = 3.54

We need to find p(X ‹ 20).

To do this we could use the NORMDIST Excel function =NORMDIST(19,25,3.45,true) = 0.045 or about 4.5%.

Note: The normal distribution is continuous whereas the actual distribution is discrete.

If we compute =NORMDIST(20,25,3.45,true) = 0.078 or about 8%

Taking the middle of 4.5% and 8% we get 6.25% which is a reasonable approximation to the true probability of 5.95%

Alternatively we could convert to a standard score where we need to find

p(0 ‹ = X ‹ = -1.4142) = 0.078

Activity 7.2

To show how the normal distribution approximates the binomial distribution.

Excel file *normal*

Use, see Appendix 13

Maths B

8. Hypothesis testing

Subject matter, Maths B:

Applied statistical analysis (p. 23).

* *Introduction to the concept of hypothesis testing*

8.1 Purpose

Teachers
The main reason for including the study of probability in the curriculum is to be able to understand its use in real world decision-making situations as such the one below. In this section the binomial distribution probabilities are used. You may care to show how normal distribution probabilities could also be used.

Suppose a new drug to cure a disease is tested by administering it to a sample of 20 patients. Without the drug, it is known that only 40% will recover. In the trial 12 patients recover. The problem for the scientists is to determine the effectiveness of the drug.

To do this we formulate the "Null hypothesis": The drug has no effect.

If this is true, then we expect 40% of 20 = 8 patients to recover.

But 12 patients recovered.

What we need to determine is whether or not this has happened just by chance or whether the drug was effective. To do this we need to find the probability that *more than* 11 of the 20 would recover by chance if the probability of recovering by chance for each one is 0.4.

(We are not interested in the probability that exactly 12 recovered, but in any number greater than 11.)

8.2 Using binomial probabilities

For this we can use the BINOMDIST function

N = 20

X = 11

P = 0.4

Cumulative TRUE will give p from 0 to 11 inclusive

Then

n	X	p	P(0 to 11)	P(12 or more)
20	11	0.4	0.9435	0.0565

We see that the probability of 12 or more recovering under the probability of the hypothesis is only 0.056 and we now decide whether or not to reject the null hypothesis. In this case, there is some doubt since a probability of 0.056 means that about 1 in 20 of such trials could produce results like this just by chance.

If however 14 patients recovered, we would have:

n	X	p	P(0 to 13)	P(14 or more)
20	13	0.4	0.9935	0.0065

Now the probability of this happening by chance is $<$ 1%, so it would be more reasonable to reject the null hypothesis and assume the effectiveness of the drug.

In scientific or medical research, the researcher might want to know whether or not a new medicine or treatment is really effective.

Suppose researchers know from observation of cases that 30% of sufferers of a certain type of cancer go into remission and recover without any treatment. (p [frequentist] = 0.3)

A new but very expensive treatment is to be trialled on a sample of 10 patients. How many need to recover to believe the treatment can be considered to be effective?

To answer this we test the H^0: The treatment has no effect.

Under H^0, p = 0.3

Using the binomial distribution with n = 10, p = 0.3 we get:

X	p(X<=)	1-p
0	0.028248	0.971752
1	0.149308	0.850692
2	0.382783	0.617217
3	0.649611	0.350389
4	0.849732	0.150268
5	0.952651	0.047349
6	0.989408	0.010592
7	0.99841	0.00159
8	0.999856	0.000144
9	0.999994	5.9E-06
10	1	0

If more than six recover, we can reject H^0 at about the 1% probability level.

If more than seven recover, we can reject H^0 at about the 0.1% probability level.

Since the treatment is expensive, we need to minimise the chance of the error of rejecting H^0 when it is true (using an expensive treatment that has no effect), so we need eight or more to recover.

8.3 Hypothesis testing in gambling situations

Suppose we toss a coin 20 times and it comes up heads 15 times. We suspect it of being biased. To test this we do the following *hypothesis test*:

1. We assume the coin is NOT biased (the NULL hypothesis, H_0)

2. Under this assumption, we calculate p. In this case p = 0.5

3. Now we calculate the probability of what we have observed happening by chance.

   In this case we want to know how likely it is that a fair coin will give such a high number of heads just by chance. We need to find the probability of getting MORE THAN 14 heads by chance. To do this we use the binomial distribution with n = 20, x = 14, cumulative TRUE to get the probability of 0 to 14 heads, p = 0.98.

   Then p $(X > 14) = 0.02$ or 2%.

   This means that with a fair coin, we will get more than 14 heads in 20 tosses about once in every 50 times.

   If the number of heads was LESS than the mean (what we expect) then we will compute the probability of this number or LESS.

   e.g if 5 heads showed, we need to find p$(X <= 5)$

   =BINOMDIST(5,20,0.5,TRUE) = 0.02

4. We now have to decide whether to accept or reject the Null hypothesis. There are no fixed rules for doing this. The consequences of being wrong must be considered and a common sense approach adopted. In this case one chance in 50 or p = 0.02 would not be strong enough evidence to reject H_0. Thus we conclude that there is insufficient evidence to assume the coin is biased.

   There are many other statistical techniques for hypothesis testing that we will not go into here. Essentially all hypothesis testing proceeds in this way.

   How many heads in 20 tosses would you need to reject H_0?

   Again, using the binomial distribution we find:

		p	1-p
=	14	0.979305	0.020695
=	15	0.994091	0.005909
=	16	0.998712	0.001288
=	17	0.999799	0.000201

p $(X > 16)$ = 0.1% or one in a thousand, and 17 or more heads would be a safe place to reject H^o.

Now the alternative hypothesis H_1 can be formulated: “The coin is biased in favour of heads.”

NOTE: Hypothesis testing always tests H_0 under the assumed probability.

8.4 Errors of judgement

We can be wrong in one of two ways.

1. At 15 heads, we have accepted H_0, but if the coin really is biased we have made a mistake in doing so.
2. At 17 heads, we rejected H_0, but the coin could be unbiased.

Sometimes we report the probability level at which our decision is made. This will help indicate the possibility of either type of error.

In determining bias in coins or dice we need to ask:

What value of p is needed to suspect bias?

There is no simple answer to this. In all statistical hypothesis testing, the value of p required to reject the Null hypothesis depends on the circumstances and the consequences of being wrong. That is:

- we might reject the hypothesis when it is fact true – in this case say that the die is biased when it isn't; or
- accept the hypothesis when it's false – again, in this case, say that the die is fair when it is really biased.

Clearly, a casino with large amounts of money bet on dice will need to be more reliant on the dice being unbiased than if we are using a dice to play snakes and ladders.

Statisticians sometimes use "levels of p" when reporting results of tests. Commonly used levels are:

$P < 0.05$ this means that there is a less than 1 in 20 chance that the observed results could have happened by chance. Or, put another way, if the observations are made many times, the observed results will happen less than once for every 20 times.

Similarly, for $p < 0.01$ (one in a hundred)

and $p < 0.005$ (one in 200)

$p < 0.001$ (one in 1,000)

The Null hypothesis is rarely if ever rejected for $p > 0.05$

If, for example, $p = 0.1035$ and we would say there is not enough evidence to assume any bias in the dice.

With $p < 0.005$, there is less than 1 chance in 200 that the observed results were in fact due to random variations and we are wrong. This is simply reported as: reject H^0 at the 0.005 level.

Suppose a casino tests a roulette wheel and gets a value of $p = 0.0092$

Should they replace the wheel? Roulette wheels are expensive pieces of machinery, and to replace a good one would be making one type of error (rejecting a true hypothesis), whereas retaining a faulty one (accepting the hypothesis when it's false) is another type of expensive error. In this case, $p < 0.01$and there is about a one in 100 chance that the wheel is fair. This is probably not low enough to warrant replacing the wheel and further testing would proceed.

8.5 A final note on hypothesis testing

We use hypothesis testing only in situations in which we do not know whether or not random factors are involved. It is inappropriate to apply the procedure if we already know that a situation is random. For example, if the probability of getting 1st prize in a lottery is one in a million, we know that any individual has a 0.000001 chance of winning. If someone wins, there is no point then in testing H^0: not a random win, and rejecting it at $p < 0.000001$ since we already know that the lottery is random.

Exercises Set 4 - Maths B

Using the Excel file binomialdists, find the following binomial probabilities.

Q1. On the toss of a fair coin 10 times

(a) Exactly seven heads

(b) Seven or less heads

(c) More than seven heads

Q2. Repeat Q1 for a biased coin for which p(head) = 0.55

Q3. If the proportion of left-handed people in the population is 0.14, in a sample of 50 people selected at random

(a) What is the probability of exactly seven left-handed people?

(b) What is the probability of more than seven left-handed people?

(c) What is the probability of more than 10 left-handed people?

(d) What is the probability of less than five left-handed people?

Q4. On the roll of 10 dice, what is the probability of

(a) No “ones”?

(b) Exactly three sixes?

(c) More than two fours?

(d) Less than three fives?

Q5. On 20 rolls of a roulette wheel, the black shows 14 times. Test the hypothesis that the wheel is biased

Q6. Johnny needs to throw a “six “ to get started in the game. After seven throws he still hasn’t got a six and thinks the die is biased. What do you think?

After how many rolls without a six would you start to suspect bias?

Q7. A roulette wheel is spun 100 times and no zero is observed. Test the hypothesis that the wheel is biased.

Q8. Do the numbers in Gold Lotto follow a uniform distribution model?

(Math B, p. 25, SLE #17)

Use the numbers given in the newspaper about previous draws to test the Null hypothesis: there is no bias (the numbers are random)

Answers Set 4 - Maths B

Q1. On the toss of a fair coin
(a) Exactly seven heads = 0.1171875
(b) Seven or less heads = 0.9453125
(c) More than seven heads = 0.0546875

Q2. 0.166478293, 0.900440348, 0.099559652

Q3. (a) What is the probability of exactly seven left-handed people? = 0.160630244
(b) What is the probability of more than seven left-handed people?= 0.401003582
(c) What is the probability of more than 10 left-handed people? = 0.08241013
(d) What is the probability of less than five left-handed people? = 0.152812886

Q4. 0.161506, 0.155045, 0.224773, 0.775227

Q5. On 20 rolls of a roulette wheel, the black shows 14 times. Test the hypothesis that the wheel is biased.
H^0: The wheel is not biased, p=18/37
Under H^0 we expect on average 10 black and we observe more than this
We need to find the probability of 15 OR MORE from an unbiased wheel.
To do this we first find the cumulative probabilities from 0 to 14
(Binomdist (14,20,18/37, true), then subtract this from 1.
p = 0.015385
Since this is $>$ 1% (0.015385), it would happen more than 1 in 100 times by chance and we would need further before we could reject H^0.

Q6. See Excel answers
p(no six) = 5/6
After n throws p(no six) = 5/6 n
When n = 7, p = 0.279082

We see that this is not at all unusual and will happen by chance about one in four times, or to about one of every four players.
To suspect bias increase n until p is low enough to reject no bias.
Using Excel we see that we need 26 throws to get $p < 1\%$
OR
solve the equation using logs
5/6 n $< 1\%$
$n > 25$

Q7. A roulette wheel is spun 100 times and no zero is observed. Test the hypothesis that the wheel is biased.
p = 0.064577 cannot reject H^0

Responsible Gambling Education Unit: Mathematics A & B

Appendices

Appendix 1

A card game to introduce axiomatic probability

Play in groups of four to six

Shuffle a deck of cards and select one person from the group to act as dealer. The dealer deals four cards to each person and only one to her/himself. Each person looks at their four cards and, in turn, 'bets' against the dealer according to the following conditions:

1. The player will win if he/she holds a card (any card) that is in the same suit as the dealer's, and is higher than the dealer's (ace is high).
2. The player must bet one, two or three (points or chips), that is, he or she cannot 'pass'.
3. The dealer pays even money. [For the event of a player winning, the dealer pays the amount bet. That is, if you win a 3-chip bet at even money, you get 6 chips from the dealer – 3 chips (your original bet) + 3 chips from the dealer (winnings).]
4. The dealer pays all winning players and collects from losing players.

After dealing, each player bets in turn. Once each player has bet, the dealer turns over her or his card and either pays or collects from each player.

Some examples will help: (In each case assume that the player 'bet' 2 chips.)

1. The player holds: diamond 10, spade jack, heart 5, club king. The dealer holds: spade 9 – the player gets 4 chips (the 2–chip bet and 2 chips won from the dealer) because his/her spade jack is higher than the dealer's spade 9.
2. The player holds: diamond 10, spade jack, heart 5, club king. The dealer holds: spade queen. Dealer wins because the dealer's spade queen is higher than the player's spade jack.
3. The player holds: diamond 4 and ace, spade king, heart 10. The dealer holds: club 2. Dealer wins. (The player does not have a club.)
4. The player holds: diamond 4 and ace, spade king, heart 10. The dealer holds: diamond king. The player gets 4 chips (the 2-chip bet and 2 chips won from the dealer) because the player's diamond ace is higher than the dealer's diamond king.

Look at your four cards. The dealer holds one card that is dealt at random from the 48 remaining cards. Calculate how many of the 48 remaining cards will beat your hand. Thus calculate the probability that your hand will beat the dealer's card. (Ignore the effect of other players holding cards.)

After a few hands, rotate the dealer. At the conclusion of play answer the following questions:

What constitutes a 'good' hand and what constitutes a 'bad' hand?

- How can the formula p = n/t be used in this game?

Calculate the probability that each of the following hands will win:

(i)	diamond ace,	spade 6,	heart 10,	club jack
(ii)	diamond 2,	diamond ace,	heart 10,	club jack
(iii)	diamond 5,	heart 8,	spade 9,	club 10
(iv)	diamond 7,	heart 3,	heart 6,	club jack
(v)	club 2,	club 5,	club 8,	club queen

Describe a hand that has a probability of winning of 1.00.

Describe some hands that have a probability of winning of 0.

- What informal probability language did the group use in playing the game?
- What is the meaning of equally likely events? What are some equally likely events you noted in this game?
- What other game strategies were used by the group?

Excel file: Random Generators

The Excel function = rand() returns a random number between 0 and 1

To use this to simulate a coin toss, we first generate a random number that is either 0 or 1 and arbitrarily assign H and T to either value.

= rand()*2 returns a random number > 0 and < 2

The function–trunc–truncates the number by removing the decimal part and leaving the whole number.

Thus = trunc(rand()*2) returns a random number that is either 1 or 0

(Alternatively randbetween (0,1) will do the same if this function is available.)

Similarly:

- To simulate the roll of a **single die** we can use
  = trunc(rand()*6+1) returns 1 to 6 at random
- To simulate the roll of a **roulette wheel** =trunc(rand()*36) 0 to 36 at random
  The spreadsheet also shows the colour (red, black, green) according to the true colours of a wheel.
- To simulate the roll of a **pair of dice** we can use
  = trunc(rand()*6+1) + trunc(rand()*6+1)

Using the copy and paste operations, any number of trials can be simulated.

Excel file: Spinnersim

This file simulates 100 spins of a three coloured spinner.

The spreadsheet used sets the probability of blue and red and the file calculates p(yellow) and displays the "theoretical" probability accordingly.

This can be done using the scrollbars or by entering the probabilities directly in cells D4 and D5 as %. The third probability is automatically computed in cell D6.

The relative frequencies of the 3 colours are computed after 10, 20, 50 and 100 spins (see sheet 2, spinner) and displayed visually.

Generally, after 10 or 20 throws the relative frequencies will differ considerably from the theoretical. However after 50 or 100 spins the two are generally very close.

The simulation can be repeated by using the scrollbar or by entering any number into cell B8.

It is important to stress that this convergence occurs without any change to the set probabilities (see further gambler's fallacy).

Note: any "enter" will automatically result in the generation of a new set of random numbers simulating 100 spins and the corresponding frequencies. Thus changing the probabilities automatically does this whether using the scroll bars or entering the probabilities directly.

Simulation using Excel spreadsheet

The attached Excel file simulates the fallacy for any two equally likely outcomes
e.g. heads/tails
Red/Black, Odd/Even Roulette
Equally matched players etc.

We start by assuming that one of the two equally likely outcomes has happened a number of consecutive times. Probabilities of this are:

# of times	Probability
5	3.125%
6	1.563%
7	0.781%
8	0.391%
10	0.098%
15	0.003%

So, starting with a rare occurrence we enter the number in cell E3 (red).
This is automatically recorded to cell B15.

Next go to Sheet 2 "20 rolls".
The program simulates the 20 rolls of the wheel (or 20 coin tosses etc.) and records the number of the outcomes (cell G13, sheet 2).

Go back to the original sheet, "Gambler's fallacy" and manually enter this number into the next cell B16.
(Note: You can program Excel to do this, but it is better for the student to do this manually to see what is happening).
Now go back to sheet 2. In cell G13 there will be another result for 20 rolls. Go back to sheet 1 and enter this number into cell B17.
Continue this process until cell B25 is complete.

The program will now compute the total # of Black and the proportion of Black as a % of the total (Cells C16 and D16), the total number of Red and the absolute difference between Red and Black.

Irrespective of what number you start with in E5, the final % will approach 50%.

This shows that the "frequentist" probability approaches the "theoretical" probability as the number of trials increases. Furthermore this happens without any change to the probability. The gambler's fallacy is that the probability changes for this to happen (i.e. the probability of red increases).

Note also, that while the difference between red and black initially may drop, it will fluctuate and eventually start to increase and continues to do so with the number of trials. The final simulation to over 1000 trials will show this.

To repeat the simulation delete the Cells B16 to B25 (in yellow).

The 6 Games Question

Two equally matched teams play a six game series (no draws).

What is the most likely outcome: 6-0, 5-1, 4-2 or 3-3?

Most people will answer 3-3 intuitively.

Simulation of problem

The Excel program generates 0 or 1 at random in blocks of 6.

(If you don't have access to Excel, you can do the same thing by looking at the last digit of a table of random numbers such as the last digit of telephone numbers and scoring for odd and even).

For each block record the score as: 6-0, 5-1, 4-2, or 3-3.

The spreadsheet does this for a "block" of 24 trials and displays the results graphically.

We will see later that the axiomatic (theoretical) probabilities are:

3,3	31.3%
4,2	46.9%
5,1	18.8%
6,0	3.1%

To repeat the simulation either use the scrollbar or enter any number in cell D2.

It will be necessary to repeat the simulation several times to see that 4-2 is more likely than 3-3.

Appendix 5

Three card cut

You may select any three cards and enter them into cells D10, D11 and D12 (Jack, 11; Queen, 12; King, 13) or you may click the scrollbar in cell B5 and three cards are selected at random.

Cells C23, 24 and 25 show whether or not there is a match.

You will need to repeat the simulation a large number of times to see that there will be more times with at least one match than no match.

Appendix 6

Work in groups of 4 or 5.

To start: Each player throws a pair of dice. Highest score gets first choice of horse, next highest second choice etc. Continue until each player has two horses. Players now take turns to throw a pair of dice. After each throw that horse number is moved forward one square on the sheet. Use counters. First to cross the finish line wins.

Play the game a few times then ask:

If you have a choice which horse would you choose?

Which is most likely, least likely etc. Why?

From the Excel file ***randomgenerators*** generate a page of numbers simulating the rolling of a pair of dice.

Using these numbers play the game several times for each group to gather enough data to get an idea of the frequentist probabilities of each horse.

Horse game chart

Race Game

Horse number and name	1	2	3	4	5	6	7	8	9	10
2 Champion										
3 Bright Eyes										
4 Olympic										
5 Bright Forrest										
6 Stewball										
7 Ugly Duckling										
8 Snowflake										
9 Ancient Runner										
10 Ghostbuster										
11 Strawberry Way										
12 Bold Warrior										

Appendix 7

Cutting the deck

Maths B

Part 1. The deck is cut into n piles and n cards are named at random (without regard to suit).
The spreadsheet computes the probability of no match, then at least one match.

Part 2. The deck is cut into 3 piles and n cards are named at random.
The spreadsheet computes the probability of no match, then at least one match.
(Of course if 13 cards are named p must equal 1)

Maths B students:
In each case write p = f(n) and state the range and domain of the function.

Appendix 8

Simulation of the bet and double fallacy

The spreadsheet simulates the following situation:
For each trial

$10 is bet on the high (> 18)

If the roll is high a win is recorded and added to the win total.
Stop (Cells E23 to E27) and Play (Cell E30) are displayed.

If the roll is low (< 19) the bet is doubled and another roll is done.
This continues up to a maximum of 6 rolls. If 6 lows turn up in a row, the player busts as the next double ($720) would exceed the house limit ($500).

The trials are repeated (Cell B18, either direct entry or scrollbar) and the spreadsheet computes the total wins until a bust occurs then gives the net loss or win.

By using the spreadsheet we see that although sometimes the player will be ahead when a bust happens, more often than not, they will be behind and incur a net loss.

Appendix 9

Enter the returns either directly into cells E9 and E10 in $ or use the scrollbars to change amounts in 5 cent increments.

The spreadsheet computes the reciprocal of each return to show the corresponding bookmaker probability and add these.

For the bookmaker to make a profit, these must add to more than 1.

Hence the bookmaker profit is computed and shown.

Appendix 10

Trackodds

The spreadsheet allows the user to enter the odds for 8 horses in a race either:

A. Directly into Cells C3, E3 to C10, E10 or

B. Directly as $ returns in cells J3 to J10, or

C. Through the use of the scrollbars.

In A and C, the odds are converted to returns and bookmaker probabilities. In B, the returns are converted to bookmaker probabilities. In each, the probabilities are then added to show the bookmaker profit.

The relative frequencies from Activity 4.2 can now be used to estimate the returns for the horse race game. These can be entered into cells C30, C40 and E30, E40 so as to give a "realistic" book for the game and show the bookmaker profit.

The spreadsheet also allows the user to enter the odds for 12 horses in a race (Activity 1.8).

Appendix 11

Totalisation spreadsheets

Enter the amounts bet on each of the outcomes and the desired profit margin.

Spreadsheet *totalisation1* computes the return on win and place bets of each outcome.

Spreadsheet *totalisation2* shows the effect on profit margin of rounding down returns to 10 cents.

Spreadsheet *totalisation3* shows the effect on the profit margin of rounding and computation that results in a return of less than $1 back up to $1.

Amounts bet and profit margins can be entered directly into cells or via the scrollbars.

Appendix 12

Excel file: binomialdists

Two distributions are shown, one for n = 10 and one for n = 50

Probabilities are entered either directly into the cells or using the scrollbars.

The spreadsheet computes each probability using the binomdist function.

Excel file: normal

An example for n = 50 is shown.
Probability is entered in Cell B6 or though the use of the scrollbar as a %

The spreadsheet computes probabilities using the binomdist function and the normdist function.

Start with p = 10% and note how the two graphs differ. Increase p until the two are indistinguishable.