Teaching the Mathematics of Gambling to Reinforce Responsible Attitudes towards Gambling

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Summary

The general acceptance afforded the national image of Australians as gamblers has given gambling a legitimacy rare in other countries. Concerns with the social effects of this have led many State governments to implement programs to counteract negative social effects. The Queensland Treasury has allocated funds for the development of teaching resources for this purpose including the development of the Unit presented here. In 2006 the author constructed a Unit of work for Queensland Senior Secondary (Years 11 and 12) Mathematics classes entitled "The mathematics of responsible gambling" as a consultancy to the Queensland State Government. Towards the end of 2007 the "Secondary Mathematics Teaching Resources Kit" was distributed to all secondary government schools. This paper describes the activities of the Unit, their relationship to the Queensland Syllabus objectives, the research upon which the Unit is based, and the current research into the effectiveness of its implementation which began in November 2008 and will continue in February of 2009.

Introduction

It has been estimated that 85% of Australian adults participate in some form of gambling at some time (Peard, 1998). The influence and extent of gambling within Australian society can be investigated by an examination of the relevant statistics relating to expenditure on various types of gambling. These provide evidence that Australia has the highest level of gambling expenditure per head of population of any country (Peard, 1998). In addition to statistics relating to expenditure, we find evidence for the widespread occurrence and acceptance of gambling as a "respectable" pastime that is not to be found in many other societies**.** The general acceptance afforded the national image of Australians as gamblers has given gambling a legitimacy rare in other countries (Peard, 1995). State governments' revenue by way of fees and taxes totals millions of dollars annually.

As noted by Jacobs (2000, p.119) "today's juveniles are the first generation to be raised in an environment where legalised gambling is so pervasive, readily accessible and socially acceptable". Jacob's comments are as relevant to Australia as to North America, perhaps more so. Recently, concerns with the social effects of extensive gambling have led many State governments to implement programs to counteract negative outcomes associated with gambling. The Queensland Treasury Department has allocated funds for research into this concern and methods of counteracting the negative social effects. In the past most of this research money had been allocated to the development of social science courses. See, for example
http://www.qsa.qld.edu.au/downloads/syllabus/kla_sose_sbm_312.pdf

At the urging from mathematics educators, some money was also allocated to the development of teaching resources specifically related to mathematics outlined in this paper.

Background to the Paper

In 2006 the author constructed a Unit of work for Queensland Senior Secondary (Years 11 and 12) mathematics classes entitled "The Mathematics of Responsible Gambling" as a consultancy to the Queensland Government. This document was published by Education Queensland and distributed to all secondary schools in the State (Queensland Government, 2007). At the end of 2007 the author conducted a number of teacher in-service workshops regarding the implementation of the Unit and in the second semester of 2008 research into the effectiveness of the Unit began. The Unit is available to classroom teachers of Senior Secondary Mathematics (Years 11 and 12) as a resource for both Mathematics A (a general non-academic subject) and Mathematics B (a more academic subject). It can be administered alone as section of work covering a period of 2 to 3 weeks or can be integrated throughout the school's two year work program for the subject. As such it is to be viewed as a supplement to existing work programs and objectives. (Note that the Queensland syllabus contains both "core" and "elective" objectives).

The Queensland syllabi in Senior Mathematics can be accessed
http://www.qsa.qld.edu.au/syllabus/1885.html
Unfortunately many resources including this Unit are secured and not generally available.

This paper will outline the development of the Responsible Gambling Unit for senior mathematics including the research upon which it was based and outline the current research being conducted by the author.

The Mathematics of Gambling

It is contended that one must be able to understand the fundamental mathematics governing the operation of gambling situations if one is to be considered as "a responsible gambler". The responsible gambler recognises that you can't win in the long run and doesn't try to chase wins (Queensland Government, 2007). It is necessary to understand that in the short term, there will be both winners and losers but that in long run all gamblers must lose whenever the mathematical expectation of the situation is less than 100%. Mathematically, there can be no "systems" or "methods" to overcome this.

Whether such understanding will in itself result in responsible gambling will require further research. The first component of such research will be to determine the effectiveness of the Unit of work in meeting its mathematical objectives. The mathematical prerequisites to this are an understanding of the concepts of "randomness" and "independence". Next is the ability to compute the "mathematical expectation" of any gambling situation defined as the product of the probability of a gambling outcome and the return associated with that outcome.

This is generally expressed as an amount per $1 bet or more commonly as a percent. In all gambling contexts such a Casino betting, poker machines, bookmaker and agency betting, totalisation betting, lottos, Keyno etc. the mathematical expectation varies from 60% (Lottos) to 97.3 % (roulette) but is always less than 100%. In all situations, the long term return to the gambler equals the expectation. The mathematics required to fully understand this is outlined in the sequence of objectives and activities of the unit. The responsible gambler should first be aware of the random nature of the outcomes, be able to compute or estimate the mathematical expectation of any situation, be aware of the concept of mathematical independence that the probabilities will not change as a result of any outcomes, and that the long term return cannot be affected by any short term wins or losses. In order to achieve this certain basic mathematical skills and knowledge are required.

It is not the intention of this unit or of the objectives of the associated research to try to address the problem of compulsive gamblers who are already addicted. It is recognised that education as to the resulting losses will not be effective; the compulsive gambler already knows that long term losses will occur in the same way that there is little point in educating addicted cigarette smokers that it is bad for their health: they already know this. Rather the objective is to educate the public via public education of the consequences of gambling in the same way that the consequences of cigarette addiction are now made known.

This paper describes the activities of the Unit and their relationship to the Queensland Senior Mathematics Syllabus objectives. Implementation of the unit is voluntary and up to the school's head of department as in Queensland all schools write their own work program based on the objective and suggested learning experiences of the Queensland Studies Authority. In December 2007 the author conducted a number of in-service workshops for practicing classroom teachers of Mathematics A and B. A few schools began implementation of the Unit in 2008. This was repeated in 2008 and one school was selected to implement the Unit in 2009 during which the proposed research would be carried out to evaluate the effectiveness of the Unit in both meeting the Mathematical syllabus objectives as well as the social objectives.

Previous Research

Earlier research by the author (Peard, 1995) has shown that these required skills can be acquired by students with limited previous success in mathematics and are clearly within the capabilities of secondary mathematics students. This research showed that low achieving students who came from a background in which track betting was a common occurrence had knowledge and skills not found in other students. They were able to perform computations in gambling contexts that other low achieving students were not and had constructed their own meaningful and mathematically correct concept of what we call expectation. That is that in any gambling situation, the less likely the outcome, the higher the return will be should that outcome eventuate and that conversely, the more likely the outcome, the lower the return; that "expectation" is the product of the probability of a gambling outcome and the return associated with that outcome.

Following recent revisions of the Queensland Senior Mathematics syllabi, for which the present author has served as an advisor, probability, including mathematical expectation, features as a large component of all syllabi (Queensland Studies Authority, 2008). Earlier research has also shown that probabilistic knowledge among school students, tertiary students and the general public is poor (See for example, Peard, 2001; Shaughnessy, 1992; Truran, 1997; Watson & Kelly, 2004). It is well documented that probability is the one field in which our intuition is often unreliable and that school students demonstrate this same lack of intuition (See, for example, Borovcnik & Peard, 1997; Fischbein, Nello & Marino, 1991; Peard, 2001; Shaughnessy, 1992).

There is also abundant research evidence that probability is a difficult topic to teach (See for example; Garfield & Ahlgren, 1998; Peard, 1996 ; Watson & Kelly, 2004). Garfield & Ahlgren (1998) noted that "… the teaching of a conceptual grasp of probability appears to be a very difficult task, fraught with ambiguity and illusion" (p.57). Teacher unfamiliarity in Queensland with both content and pedagogy in the field of probability and statistics has also been well documented by the present author (Peard, 1987, 1991). It is envisaged that the teaching probability in the gambling contexts of the Unit will help both teachers and students in this difficult field. Research into this will be conducted.

However, elsewhere more recent developments have indicated a decrease in the role of probability within the curricula. Borovcnik has reported on several occasions that the international trend in stochastics education has focused more on data handling (See, for example, Borovcnik, 2006). Chernoff (2007) goes so far as to claim that the state of probability measurement is in crisis. An investigation among researchers within the didactics of stochastics (Nemetz, 1997, cited in Borovcnik, 2006) has confirmed that probability is diminishing in the curricula internationally. The reasons he cites are;

(i) Probability is orientated too much towards mathematics;

(ii) Probability is too tightly connected to games of fortune; and

(iii) Probability is only required to justify the methods of inferential statistics.

Borovcnik, commenting on these statements in a paper to the Probability Study group at ICME 11 has commented that:

> …reason (ii) shows a public puritanism neglecting the huge business behind games of fortune including state lotteries. It implies that morally, games of fortunes are undesirable and therefore we should keep our young students apart from it instead of helping them to clear up the situation by teaching the mathematics behind it to them (ignoring the historical connection between games of chance and probability theory). The counter argument is that a sound understanding has to refer to these roots in order to understand the peculiarity of the concepts.

However Australian society does not demonstrate the public puritanism that Nemetz cites as a reason and it is unlikely that these comments are relevant to the situations here. Furthermore, Borovcnik continued:

> Reason (iii) reveals a basic misconception of probability, or at least a basic ignorance about its relevant character as a tool to investigate and or structure reality.

However the Queensland syllabus documents make clear that the role of probability in everyday life and decision making is important. These objectives are in line with current research conclusions such as those made by Borovcnik (2006) who has maintained that only a sound notion of conditional probability enables learners to grasp any method of inferential statistics; probability concept are necessary to reveal the peculiarity of stochastic thinking in contrast to logical, causal, or mystic thinking; and that only a sound understanding of probability will help to clarify the abundance of intuitive and related misconceptions.

Misconceptions Dealt with in the Unit

There is abundant evidence that misconceptions in probability are widespread. Furthermore, it is recognized that in order to correct misconceptions teachers must understand their natures. As Stohl (2005, p. 351) comments “The success of any program … depends on teacher understanding of concepts … and deep understanding of misconceptions”. As del Mar & Bart (1989) reported, these misconceptions are not confined to naïve subjects. del Mar & Bart (1989) observe that many people are surprised to learn that there is no real decision making involved in the play of roulette. The extensive teacher notes in the unit of work address these misunderstandings.

One of the major objectives of the Unit is to ensure that the students are free of the misconceptions about probability that are reported in the literature as common. These misconceptions, including the “gamblers’ fallacy", are not confined to naive subjects and are prevalent among both secondary and tertiary students (Peard, 1996; Shaughnessy, 1992). Prerequisite to the remediation of these misconceptions and included in the Queensland syllabus are the concepts of independence and mathematical expectation.

Simulation using the Excel technology is used extensively in an attempt to remedy misconceptions. The author is aware that simulation in itself is inadequate and that there are problems associated with its use. Borovcnik and Peard (1996) note that while simulation may lead to the solution of a problem it does not necessarily do so by improving conceptual understanding. For this reason simulation and theory are combined wherever possible in the unit. It is also recognized that further research such as that outlined here is needed to evaluate the effectiveness of the approach. For example:

The Gambler's Fallacy

The well documented "gamblers' fallacy" arises in part from a failure to recognise independence of events. The heart of the gambler's fallacy is the misconception of the fairness of the laws of chance. Naive gamblers expect that any deviation in one direction will soon be cancelled by a corresponding deviation in the other and that the probabilities change in order to accommodate this. This has been referred to by some as a type of misuse of the "representativeness" heuristic (Shaughnessy, 1992). Cognitive psychologists Tversky & Kahneman (1982) reported on this misconception in depth. Earlier studies by them (Tversky & Kahneman, 1973) had noted that risk proneness of individuals increases after repeated losses. The gamblers' fallacy is common in many forms; roulette and other Casino games, lottery number selection and poker machine play.

The Unit addresses this misconception by simulating the rolling of a roulette wheel that has already landed a large number of times on the black (green is ignored in this simulation). Students are able to enter this number as a variable and then simulate a number of further rolls to show that the proportion of red and black does in fact approach 50% when p is set at ½. (Appendix, Figure 1 and 2). Furthermore they see that the difference between the number of red and black initially fluctuates up to 200 rolls (Appendix, Figure 3). but eventually starts to increase up to 1000 rolls (Appendix, Figure 4).

The Gambler's Fallacy and Poker Machines

Of problem gamblers in Australia, those causing the greatest concern are players of Poker machines. Directions from Queensland Treasury to the author included the specific inclusion of poker machine operation. Again, the heart of misconceptions about poker machines is the lack of understanding of the concepts of "randomness" and "independence". Each play of the machine is a random event. The machines are constructed so that every "outcome" is independent of previous outcomes. If a machine has not payed out for a long time the probability of a payout next go is no different to that of a machine that has just payed out a jackpot. Secondly the machines are programmed to pay out a fixed proportion of the money taken and this expectation remains constant. The unit addresses this misconception from a theoretical standpoint relying on an improved understanding of randomness and independence.

The Misuse of Heuristics

Misuse of the heuristics of representativeness and availability are well documented in the literature. The counter intuitive nature of many results in probability theory is attributable to violations of representativeness that occur when people view a sample randomly drawn from a population as highly representative, that is, similar to the population in all essential characteristics. For example, most people are surprised to learn that in a group of as few as 23 people, the probability that at least two of them have the same birthday (i.e., same day and month) exceeds 0.5, and that in a group of 30 people this probability is more than 70%. The Unit addresses these issues in detail by discussing the underlying theory, and using Excel to perform the necessary computations. For example the probabilities for a group of people of any number are easily computed once the underlying theory is established (See Appendix, Figure 5 and 6).

The Equally Likely Fallacy

The assumption of assigning equal likelihood to the outcomes of an event is one that is not always justified. We see this in its simplest form when, for example, young children are asked: "In a class there are 12 girls and 16 boys. The teacher puts the name of each child in a hat and draws one out at random. What is the probability of the name being a boy"? Many children (and adults) will answer ½, arguing that it can be either a boy or girl and that these are equally likely. Again, the Unit addresses this misconception by simulating situations.

Not Understanding the Situation: Extraordinary Events, Coincidences

As an example we include a recent newspaper Ripley's *Believe it or not* which cited the case of a baby girl born in the USA on 7 December, the same date as her mother and grandmother, quoting the incredible odds of less than 1 in 48 million (mathematically correct). Clearly many would think this an incredible occurrence. However, given the population of the USA, if we consider the probability that somewhere some child will be born on the same date as a parent and a grandparent of the same sex, we find that even over a relatively short period of time, this is nearly certain to happen.

The misconception here is in not understanding what the problem is, failing to distinguish between the specific and the general. Using modern technology (graphical calculator or spreadsheet) the analysis of problems of this type are now possible by Senior students and are included in the Unit. (See Appendix Figure 7 and further analysis later in Use of Technology).

Lucky and Unlucky Streaks

The belief in these follows from the "gambler's fallacy". Instead of viewing random sequences as such they are viewed as "lucky" or "unlucky" streaks. A sequence of losses may be viewed as an "unlucky streak" and the gambler mistakenly believes that it "must end" and keeps gambling often increasing bets in the belief that this will "even out". A sequence of wins may be viewed as a "lucky streak" and the gambler keeps gambling often increasing bet in the belief that he is "on a roll".

The Illusion of "Control"

In all gambling situations in which the random generator results in random independent events the gambler can have no control. In selecting numbers for roulette for example your probability of winning remains the same whether you "choose" the numbers or place the bets "at random". Gamblers will often attribute a random sequence of wins to something that they have done. Again, the misconception is a lack of understanding of randomness.

The following site explores myths and misconceptions associated with the playing of lottos.

http://www.solidsoftware.com.au/Products/LottoCheck/LottoMyths.html

The Mathematics in the Unit

The Mathematics objectives for the QSA syllabus documents (Queensland Studies Authority, 2006) include:

- Maintaining basic knowledge and procedures
- Numbers in various notations including fraction, decimal, scientific
- Rates, percentages, ratio and proportion
- Simple algebraic manipulations
- Data collection and presentation
- Identification of continuous and discrete data
- Practical aspects of collecting data
- Graphical and tabular displays
- Exploring and understanding data
- Interpretation and use of probability as a measure of chance
- Probability of compliments and unions
- Relative frequencies to estimate probability
- Compound and conditional probabilities
- Tree diagrams
- Misuses of probability and misconceptions
- Odds as an application of probability
- Decision making and hypothesis testing
- Introduction to functions
- Concepts of function, domain and range
- Exponential and logarithmic functions and applications
- Use logarithms to solve equations involving indices

An examination of those objectives relating to probability shows that they are consistent with ideas from current research. For example, the probability chapter in the *Second Handbook of Research on Mathematics Teaching and Learning* (Jones, Langrall, and Mooney, 2007) states: "With respect to probability content, the big ideas that have emerged…are *the nature of chance and randomness*, *sample space*, *probability measurement* (classical, frequentist, and subjective), and *probability distributions*" (p. 915).

The syllabus objectives above are all integrated into the Unit which is structured as follows. The Unit is provided both as hard copy and CD ROM with interactive Word and Excel files.

Teachers' Guide

This elaborates much of the probability theory and information about misconceptions. It has been extensively reported that the misconceptions common in probability are "not confined do naïve subjects" (del Mar & Bart, 1989, p. 43) and are often held by teachers themselves. This is particular true in Queensland where many Mathematics A classes are taught by marginally qualified teachers (Peard, 1987, 1995, 2001). The Unit of work Teachers' Guide are divided into eight sections. There are six sections for both Mathematics A (non-academic general mathematics) and Mathematics B (academic) and two sections for Mathematics B alone. Additional material for Mathematics B is also provided in the first six sections. There are four sets of exercises. The teachers' notes and exercise answers are provided in this document and on the supplied CD ROM. The complete student booklets are incorporated in this document and are available as separate documents on the supplied CD ROM.

Students' Material

Separate documents are designed for student use. The Senior Mathematics Modules A and B CD ROM incorporates two methods for accessing data. On opening the CD two folders are available. One is a version for traditional paper based work and the other an interactive program that can be downloaded and made available to individual computers.

Linked Microsoft Excel Files

The Excel files are provided separately on the supplied CD ROM. Instructions for use of the Excel files are in the appendices of this guide and the Student workbooks. The subject matter for this unit relates to sections of the Maths A senior syllabus subject matter contained in the QSA syllabus documents 'Data collection and presentation', 'Exploring and understanding data', and the elective topic 'Introduction to models for data'. It would be expected that any school planning to use this unit in Maths A would include this elective topic in their work program. In addition it would be expected that the unit would provide a venue for 'Maintaining basic knowledge and procedures'

The Maths B senior syllabus subject matter relates mostly to the QSA syllabus documents 'Applied statistical analysis' with applications and examples from 'Introduction to functions', 'Exponential and logarithmic functions and applications'. In addition it would be expected that the unit would provide a venue for 'Maintaining basic knowledge and procedures'.

References to specific syllabus subject matter are given throughout. Throughout the Unit links are made to conventional syllabus content and objectives.

Example 1. For example, the link between expectation and "inverse proportion" is stressed. If $y = f(x)$ is the Return and x the probability, $x \cdot y = 1$ or $f(x) = 1/x$

Example 2. In the birthday problem discussed under misconceptions, once established the probability can be viewed as a function of the number of people in the group and linked to the syllabus objective relating to functions, range, domain, graphing etc.

$$P = f(n) = 1 - \frac{365 \cdot 364 \cdot \ldots \cdot (366 - n)}{365^n}$$

The Eight Sections are:

1 Basic Probability, Random Generators

This section contains much “conventional” probability theory with most examples in a gambling context. The treatment of this could be either to “introduce” the study of probability and then extend the study to other applications or to use this unit after a “traditional” introduction. Several Excel files simulate random generators.
For example, one file simulates 100 spins of a 3 coloured spinner. The spreadsheet user sets the probability of blue and red and the file calculates p(yellow) and displays the “theoretical” probability accordingly. This can be done using the scrollbars or by entering the probabilities directly in cells. The relative frequencies of the 3 colours are computed after 10, 20, 50 and 100 spins and displayed visually as a pie graph (See Appendix, Figure 8). Generally, after 10 or 20 throws the relative frequencies will differ considerably from the theoretical. However after 50 or 100 spins the two are generally very close.

2 Mathematical Expectation

This topic is met elsewhere in the probability section of the Senior secondary mathematics. The applications and student activities here are confined to gambling contexts. Other applications, for example in business, may be encountered elsewhere. The operation of professional gamblers and gambling syndicates operating with an expectation of more than 100% has been studied by the author (Peard, 2003); details of this are included in the Unit.

3 Expected Mathematical Return and Casino Betting

In all forms of casino gambling, the gamblers’ expectation is less than 100%. This means that the gambler cannot, in the long run, return a profit from casino betting. This section develops the mathematics of computing the expected return for common forms of casino betting. The computation of probabilities, odds, expected return and Casino long term profit from several forms of gambling are explained.

4 Bookmakers and Betting Agencies Operations: Betting at Fixed Odds

With sports betting the true probabilities for most situations are not known. Therefore a bookmaker or betting agency will start taking bets on an event with estimated odds, generally set high and then adjusted down as bets are placed and the bookmaker gets an idea of how bets will proceed. In this section the student learns how to compute bookmaker or agency profit margins and player expectation for various betting situations. Figure 9 in the Appendix shows an example of a two outcome betting event.

5 The Mathematics of Totalisation Betting

The mathematics of this is quite simple. In all totalisation situations, in the absence of other information, your expected return can be calculated from the profit margin. If the profit margin is 15%, the system returns 85% of bets and this is the expectation. Figure 10 in the Appendix shows an example of an eight outcome totalised betting event.

6 Wagers and "Fair" Bets

The concept of mathematical fairness differs from the everyday meaning of the term. Mathematical fairness does not necessarily imply equal likelihood but equal "expectation". In this sense all gambling situations are not "fair" but are biased towards the house, bookmaker or agency. Individual wagers against each other may or may not be fair. The criteria and mathematics for determining mathematical fairness are developed here.

7 More Complex Probabilities and Probability Distributions

Again, these topics will be met elsewhere in Senior Secondary Mathematics. Teachers may choose to introduce these separately prior to using the Responsible Gambling Unit. The applications and student activities here are confined to gambling contexts. Other applications, for example in business, could then be studied separately. Further probability distributions are illustrated.

8 Hypothesis Testing

Again, this topic will be met elsewhere in Senior Secondary Mathematics. Teachers may choose to introduce it separately prior to using the Responsible Gambling Unit, however should they choose to introduce it via the unit the activities outlined are appropriate. The applications and student activities here are confined to gambling contexts. Other applications, for example in science, could then be studied separately. The main reason for including the study of probability in the curriculum is to be able to understand its use in real word decision making situations. In this section the binomial and normal distribution probabilities are used to do this.

The Use of Technology

The Australian Syllabus on the Use of Technology

The QSA syllabus documents for Mathematics A specify that throughout:

> A range of technological tools must be used in the leaning experiences and the corresponding assessment. These range from pen and paper, compasses, measuring instruments and tables through to technologies such as scientific calculators, global positioning systems, graphing calculators and computer programs.

http://www.qsa.qld.edu.au/downloads/syllabus/snr_maths_a_08_syll.doc (p. 5)

For Mathematics B:

> The minimum level of higher technology appropriate for the teaching of this course is a graphing calculator. Although student ownership of graphing calculators is not a requirement, *regular and frequent student access* to appropriate technology *is necessary* to enable students to develop the full range of skills required for successful problem solving during their course of study. Use of graphing calculators or computers will significantly enhance the learning outcomes of this syllabus.
> To meet the requirements of this syllabus schools should consider the use of:

- general purpose computer software that can be used for mathematics teaching and learning, e.g. spreadsheeting software
- computer software designed for mathematics teaching and learning, e.g. dynamic graphing software, dynamic geometry software
- hand-held (calculator) technologies designed for mathematics teaching and learning, e.g. graphics calculators with and without algebraic manipulation or dynamic geometry facilities.

http://www.qsa.qld.edu.au/downloads/syllabus/snr_maths_b_08_syll.pdf (p. 12)

Use of Technology in the Unit

In this Unit, the minimum requirement is the use of a graphing calculator and it is strongly recommended that students have access to and are able to use an Excel spreadsheet. Excel is used to analyze various problems in probability and well as for simulation of random generators: coins, dice, roulette, spinners; the Gamblers' fallacy and other fallacies. Students participating in the research component all have access to and are experienced in the use of Excel. Borovcnik (2007) has noted that the new technologies have revolutionised the teaching of probability and statistics.

The author is aware of the various criticisms of Excel and its limitations. However the decision to use Excel was largely pragmatic: there is no cost; all students have ready access to it on their home and school computers; many students are already familiar with its use. Nevertheless, no prior experience with Excel is assumed and use is explained in each section. No advanced or sophisticated features are employed, such as macros. Several spreadsheets incorporate the use of scrollbars to enter data. For those computers that will not operate with scrollbars, data may be entered directly into the cells indicated. When generating random numbers, the rand() is used rather than randbetween (a, b) as the latter will not work on all computers. Teachers with experience in the use of Excel may care to make their own adaptations or changes.

Examples of how technology has revolutionised the teaching of probability are illustrated throughout the Unit. Computations are now possible that were not previously so. See for example the Excel file Figure 7, Appendix for the previously mentioned misconception. The Excel computation allows the variables: number of births, time period to be entered and computes the probability.

Examples to Illustrate the Power of Technology, Links to other Topics, and Potential Insights

This section is also linked to *Introduction to functions: Concepts of function, domain and range.*

Example 3. What is the probability that any child born has the same birthday as both parent and grandparent of the same sex?

Consider n is the number of births per year in the country
Y is the number of years of observation
p is the probability that any child born has the same birthday as both parent and grandparent of the same sex $=1/365^2$.
Then
$(1-p)$ is the probability that they not match is found for one birth and
$(1-p)^{nY}$ is the probability of no match for nY births.
Hence
$1-(1-p)^{nY}$ is the probability of at least one match.

Even with only 50,000 births per year over a 10 year period the probability of the occurrence is over 98%. Entering 400,000 births (the average for the US over the last few years) we see that in only 2 years the probability is over 99.7%. The only believe it or not situation would be if it didn't happen. The misconception here is in not understanding what the problem is and we see here an example of how technology can be used to illustrate this.

Another example of the ease with which we can be fooled by probability is illustrated by the following.

Example 4. At a European casino some years ago, the same number showed six times in a row on one of the roulette wheels. It was reported, that the probability of this happening (for a specified number)

$(1/37)^6 = 3.9 \times 10^{-10}$

was less than one in a hundred million (10^{-9}). Surely, this is an extraordinary event?

The probability that the particular number would come up six times in a row on that particular roulette wheel at that particular time was, in fact $(1/37)^6$ or about 4 in 10^{10} or less than one in a billion (10^9)! This is truly extraordinary. However, if we calculate the probability that at some time in your life, at some roulette table somewhere in the world, some number will come up six times in a row we find that this probability is well over 90%. Not only is the outcome not extraordinary, it is much more likely to happen than not. Again this type of computation would not be possible without the use of modern technology. The Excel file "6 in a row" (Appendix, Figure 11) allows the user to enter several variables, such as the number of roulette wheels in the world, the number of spins per hour and the time in years. The spreadsheet computes the probability of the event (six in a row) not happening and thus the probability of at least one occurrence.

Other Sources for "Responsible" Gambling

There are many websites on the topic of responsible gambling, although most do not address the mathematical ideas involved. More background information is available at the Queensland Government site at the following address.

http://www.responsiblegambling.qld.gov.au/

One site that does include examples of probabilities associated with a variety of gambling situations is the Powerhouse Museum in Sydney.

http://www.powerhousemuseum.com/gambling/

The Powerhouse Museum also has a teaching package that is particularly suited to Maths A.

http://www.powerhousemuseum.com/pdf/education/gambling_education_kit.pdf

The package provided with this document includes a CD that has copies of all the Excel files for use with this unit and links to the above websites.

Current Research

In November, 2008 a Brisbane metropolitan schools implementing the Unit was selected. The classroom teacher had done the one day in-service workshop on the implementation of the Unit and the author and the teacher have together designed the research instrument. In February of 2009, working with the classroom teacher who coordinates the Mathematics A work program for the school the author has integrated the Unit of Work into the schools Maths A program. Although all classes will follow this program for the purpose of the research only one class of approximately 25 students will be used. All students have access and are able to use excel. In March of 2009 these students will be pre-tested with a questionnaire and clinical interview constructed and conducted by the author and based on previous research by the author (Peard, 1995). This will be followed by a written multiple choice pre-test.

This data will determine:

- Basic mathematical knowledge and procedures in a probability context
- Numbers in various notations including fraction, decimal, scientific
- Rates, percentages, ratio and proportion
- Understanding of randomness and independence
- Misuse of heuristics of representativeness and availability
- Presence of "The gamblers' fallacy"
- Ability to compute mathematical expectation
- Understanding of mathematical "fairness"

Following the implementation of the parts of the school work program that integrate the Unit from April to June a parallel form post test and interview will be administered. Data analysis will be both qualitative and quantitative. Qualitative data will include attitudes towards and expectations of gambling while qualitative data will involve comparisons of mathematical performance. Other qualitative data will relate to attitudes to gambling and possible difficulties encountered in the use of Excel.

Conclusion

Successful implementation of the Unit and achievement of its objectives would suggest that the Unit be given widespread use throughout the State of Queensland and could have similar implications for other Australian States and possibly other countries. The research should also show if and what modifications to the program may be required for future use.

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Appendix

Gamblers Fallacy Simulation

# of rolls	# of black	total black	% black	B	R	Difference
10	10	10	100.0	10	0	10
30	12	22	73.3	22	8	14
50	8	30	60.0	30	20	10
70	8	38	54.3	38	32	6
90	10	48	53.3	48	42	6
110	11	59	53.6	59	51	8
130	9	68	52.3	68	62	6
150	8	76	50.7	76	74	2
170	13	89	52.4	89	81	8
190	4	93	48.9	93	97	4
210	10	103	49.0	103	107	4

Figure 1. Excel Simulation of Gamblers' Fallacy.

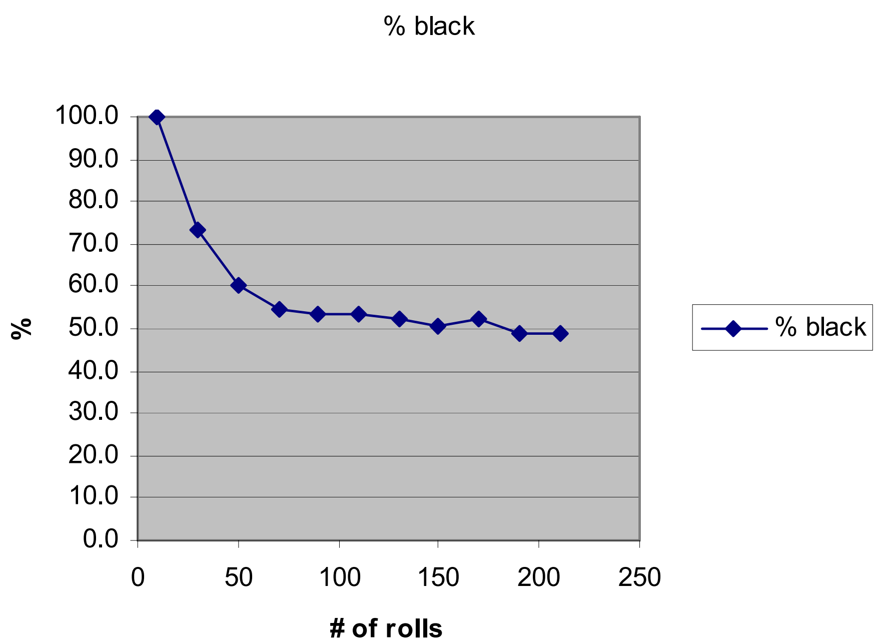

Figure 2. Relative Frequencies 200 rolls.

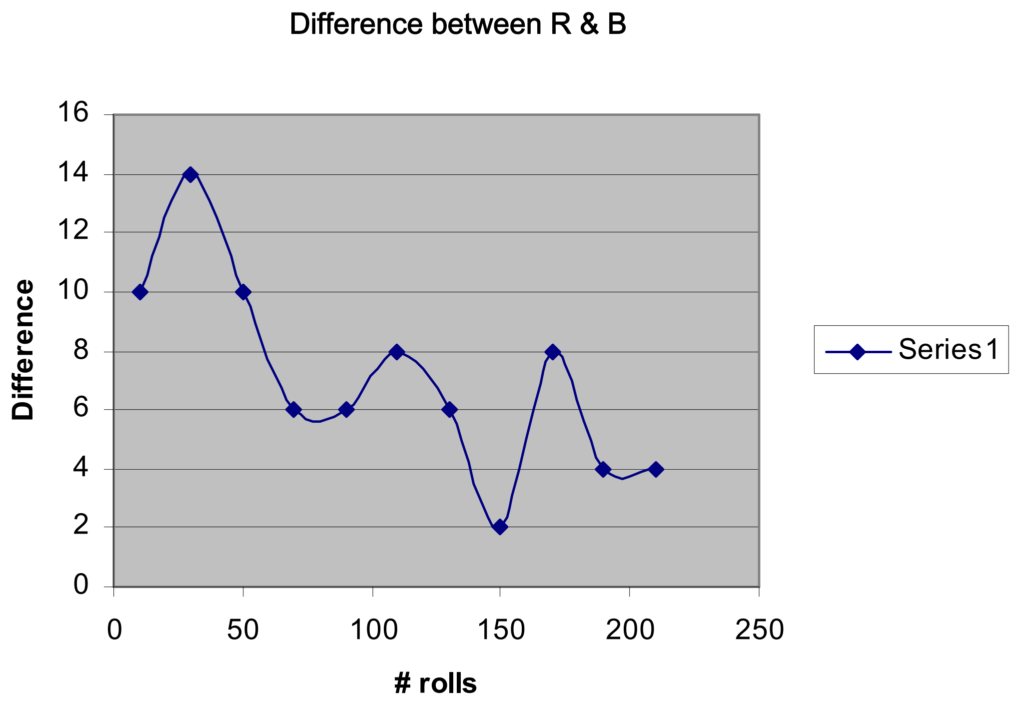

Figure 3. Differences 200 rolls.

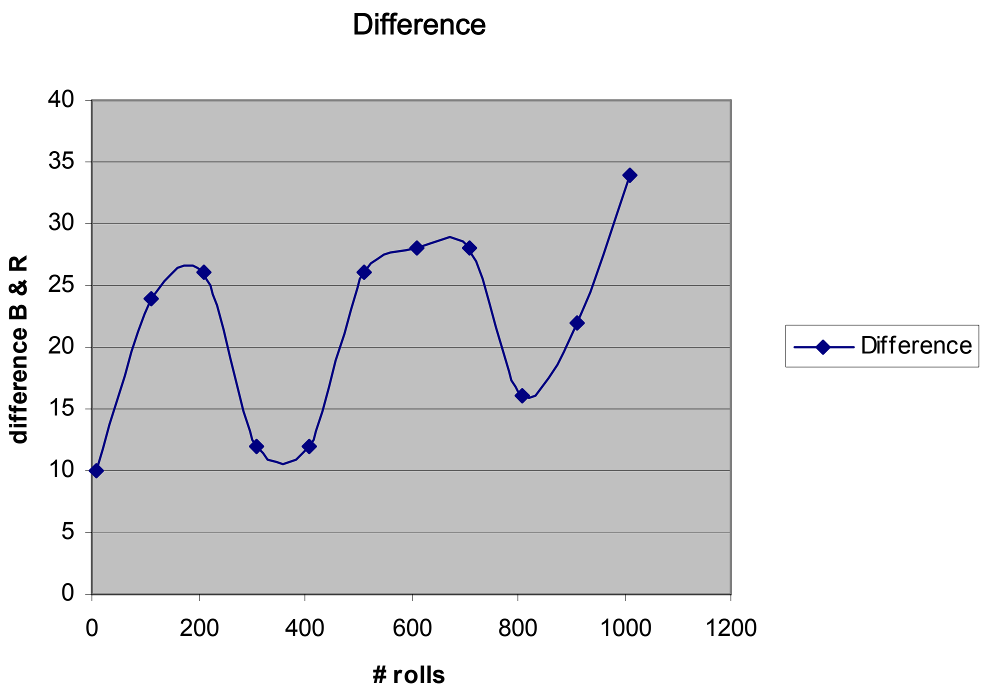

Figure 4. Differences 1000 rolls.

Matching Birthdays

# people		p(none)	p(at least 1)
2	365	0.997260	0.002740
3	364	0.991796	0.008204
4	363	0.983644	0.016356
5	362	0.972864	0.027136
6	361	0.959538	0.040462
29	338	0.319031	0.680969
30	337	0.293684	0.706316
31	336	0.269545	0.730455
32	335	0.246652	0.753348
33	334	0.225028	0.774972
34	333	0.204683	0.795317
35	332	0.185617	0.814383
47	320	0.045226	0.954774
48	319	0.039402	0.960598
49	318	0.03422	0.965780

Figure 5. Birthday Problem Excel Computations.

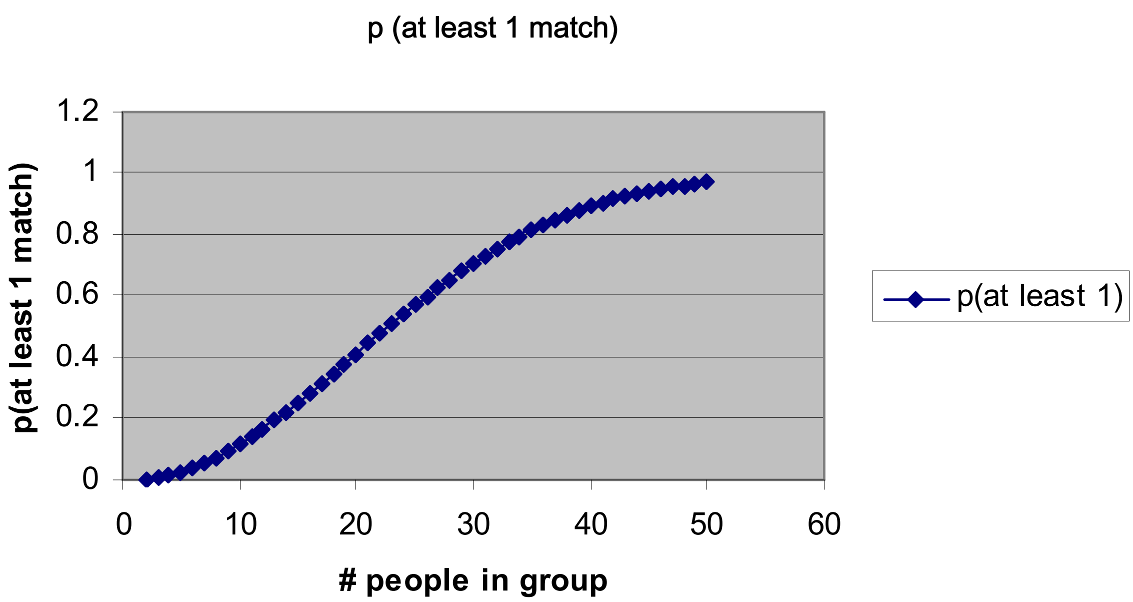

Figure 6. Birthday Problem.

Coincidences

		p	p(not happening)
Probability of same birthdays (1/365 ^2)		7.5061E-06	0.999992494
Variables			
No. of births per year	50000		
Number of years	10		
Total births	500000	**0.98**	0.02

Figure 7. Excel file Coincidences.

Spinner simulation

Colour	**p**	**%**
blue	0.33	33
red	0.33	33
yellow	0.34	34

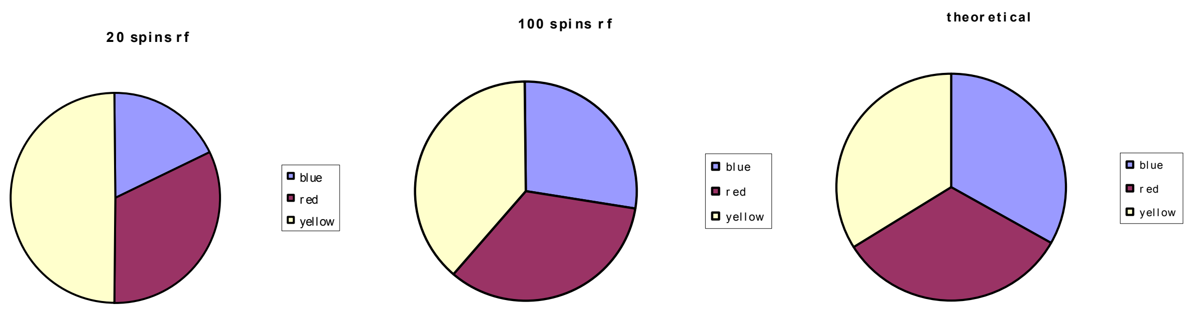

Figure 8. Excel simulation of spinner.

Schemes for Calculating the Stakes

Fixed odds

	Player	Return	p
140	A	$1.40	0.7143
260	B	$2.60	0.3846
			1.0989
	Profit		9.9%

Figure 9. Excel file: Bookmaker betting at fixed odds.

Totalisation

Outcome	$ Bet	Win Return/$
1	**$500**	$3.57
2	**$320**	$5.58
3	**$240**	$7.44
4	**$250**	$7.14
5	**$240**	$7.44
6	**$150**	$11.90
7	**$150**	$11.90
8	**$250**	$7.14
	$2,100	

Nominal profit margin 15.0%

Win pool $1,785

Figure 10. Excel file: Totalisation betting on an eight outcome event.

"How Extraordinary" Computations

		p	p(not happening)
Probability of *any* # landing six times in a row 1/37 ^ 5:		1.442E-08	0.999999986
Variables			
No. of roulette wheels	93		
Spins per hour	60		
Time (years)	3		
Total # of spins	146642400	**0.88**	0.12

Figure 11. Excel file: 6 in a row.