Concrete to Abstract in a Grade 5/6 Class

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Summary

This paper addresses the need identified by Graham Jones for more research related to the connections made by students in the middle years between classical and frequentist orientations to probability. It does so based on two extended lessons with a class of Grade 5/6 students and in-depth interviews with five students from the class. The Model 1 version of the software TinkerPlots was used in both settings to simulate increasingly large samples of random events. The aim was to document the students' understanding of probability on a continuum from Experimental to Theoretical, including consideration of the interaction of Manipulatives, the Simulator, and the Law of Large Numbers. A cognitive developmental model was used to assess students' understanding and recommendations are made for classroom interventions.

Introduction

The motivation for this study grew from the first author's interest in the contribution of technology to learning in the classroom. The narrowing of the focus resulted from the availability of the *TinkerPlots* (Konold & Miller, 2005) Sampler, which was under development, and a reading of *Exploring Probability in School* (Jones, 2005). Jones, reflecting on the current teaching of probability in classrooms, noted,

> In spite of the apparent robustness of research on elementary school children's probabilistic reasoning, it is evident that there is a void in the research associated with the frequentist approach to probability; that is, research dealing with children's cognitions on experimental probability. In fact, there is almost no research on whether children can make connections between classical and frequentist orientations to probability even though teachers are encouraged to use these connections in the classroom. … such research needs to document effective classroom practices including those that use the technology and software that is becoming available for young children. (p. 368)

Only recently has research commenced in this area. Stohl, Rider, and Tarr (2004) investigated how sixth grade students used computer simulated experimental data to make inferences about unknown probabilities of a loaded die. They examined "the ways in which students' understanding and use of sample size, independence, fairness, and variability interacted with their use of external resources such as the task context, multiple representations of data and social negotiations with a partner" (p. 1). Stohl et al. (2004) concluded that students who used larger sample sizes and multiple representations of data made appropriate inferences regarding fairness.

Although there has been no published research on student outcomes associated with the *TinkerPlots* Sampler, its predecessor, *ProbSim* (Konold & Miller, 1993) was seen to assist students in appreciating the contribution of sample size in decision making in relation to a version of Kahneman and Tversky's (1972) famous hospital problem (Watson, 2000; 2007). As well, *TinkerPlots* itself has been recognized as making a valuable contribution to classroom practice and student understanding of other data handling topics (Ben-Zvi, Gil, & Apel, 2007; Watson, Fitzallen, Wilson, & Creed, in press).

The Sampler provides two ways of simulating a natural phenomenon: The Mixer and the Spinner. The Mixer in Figure 1 is similar to a bag of marbles, where the balls are the same size and shape and sample space is altered by adding additional balls to the Mixer. The Mixer simulates randomness visually by bouncing the balls around the bag and releasing an outcome out of the bottom of the bag. The Spinner in Figure 1 is an area model. The size and proportions of the sample space are altered by the percentage of area allocated to each outcome on the Spinner. The Spinner simulates randomness visually by spinning a pointer around, which eventually stops, giving the result of the trial.

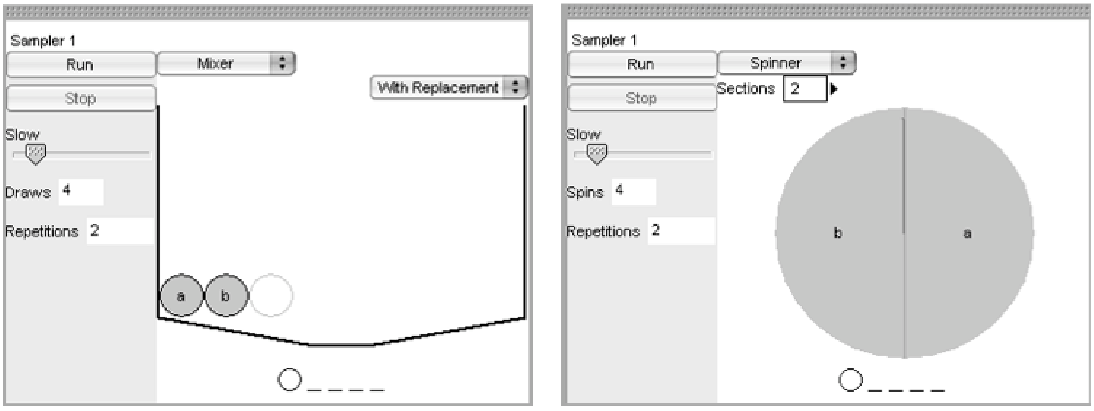

Fig. 1: The Mixer and the Spinner in *TinkerPlots* Sampler

Computer simulations imitate the process of "real-world" phenomena on a computer (Malhotra, Hall, Shaw, & Crisp, 1996) and "should imitate the internal processes and not merely the results of the thing being simulated" (Wordnet, n.d., para. 1). Students, need not only to make conceptual connections between the three-dimensional concrete manipulatives, such as coins, and the two dimensional abstract representation on the simulator, but also to make the connection between the internal processes of the simulation and the process of obtaining outcomes using the coin. Many questions arise. For example, what is the students' understanding of how the computer chooses outcomes randomly? Do they believe that the computer actually randomly chooses the results? Do the students' make the connections between the computer simulation and the concrete manipulatives? What understandings are required for this transition? Much research has been carried out into the use of concrete models and computer simulations to develop other abstract mathematical concepts for students (Clements, 1999; Mills, 2002; Rider & Stohl Lee, 2006; Sarama & Clements, 1998). Except for the work of Pratt (e.g., Pratt & Noss, 2002) on randomness based on the notion of situated abstraction, there appears to be limited research concerning what is required to facilitate an effective transition from concrete models to abstract computer simulations in the field of probability. This realisation has provided further impetus for the study.

Konold (2006), discussing the design of the *TinkerPlots* software, gives some insight into how the transition between concrete manipulatives and the simulation might be accomplished.

> We then worked to implement these operations in the software in a way that would allow students to see the computer operations as akin to what they do when physically arranging real-world objects. This sense–that one already knows what the primary software operators will do–becomes important in building up expectations about how the various operators will interact when they are combined. (p. 4)

It is the connection suggested by Konold, which links the concepts of concrete and abstract, that underlies this study. The literature reveals many definitions of both concepts and their interaction (Erickson, 2006; James & James, 1959; Steen, 2007; Stein & Bovalino, 2001). The ideas, that the concrete leavens the abstract (Erickson, 2006) and that abstract concepts can become concrete (Basson, Krantz, & Thornton, 2006), suggest a continuum linking the two extremes with the two ideas reinforcing each other depending on context. In the area of probability the identification of experimentation using actual objects with the concrete end of the continuum and theoretical probability with the abstract end, appears feasible. With manipulatives associated with experimental probability and the simulator providing one link from the manipulatives to the theoretical, the law of large numbers fits also in a proposed model along the continuum (Stohl & Tarr, 2002) as shown in the framework in Figure 2. A further development of the model is presented in Ireland (2007).

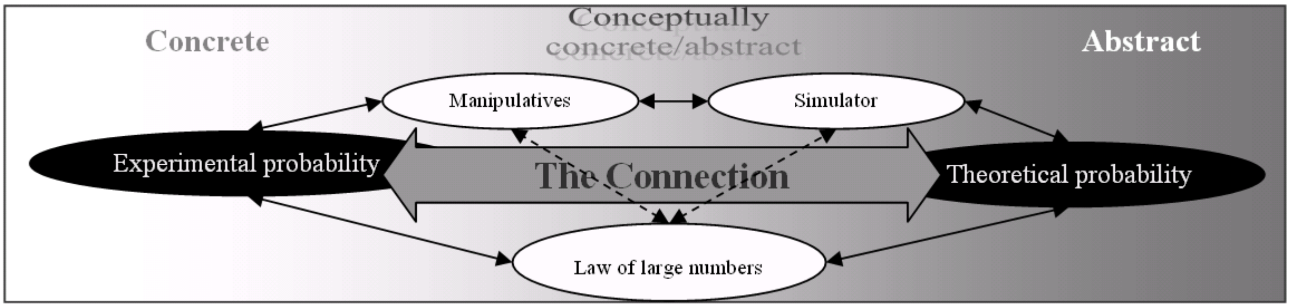

Fig. 2: Concrete to Abstract conceptual framework

The key research question for this study is then, “What connections do students make between experimental and theoretical probability with the aid of *TinkerPlots*?”

Methodology

The study took place during the first term of 2007 in a grade 5/6 classroom (ages 10-12) in a government primary school in Hobart, Tasmania. The class consisted of 27 students, 11 grade fives (5 males and 6 females) and 16 grade sixes (7 male and 9 female), with varying degrees of mathematical competence. This class was chosen based on their age in relation to Piaget’s theory of cognitive development (1955), which places the students’ understanding on the cusp of the transition from the concrete operations stage to the formal (abstract) operations stage.

A survey was administered to the class prior to the lessons on probability. The section relevant to this paper included questions on basic probability chosen from the survey items used by Watson and Callingham (2003), an assortment of open and closed questions, often including a request for explanation or justification. The responses were used to judge the level at which to begin the lessons and later to assist in selecting students to be interviewed.

The two extended lessons were taught by the first author in conjunction with the classroom teacher as part of the normal curriculum. The first lesson began with a structured discussion of Probability including (i) its meaning in terms of events like the sun setting, (ii) the ordering of chance phrases, and (iii) meanings of specific terms like “variation” and “random.” Then a coin was introduced and students asked to predict outcomes as it was tossed and outcomes recorded. After 10 trials and discussion, the idea of “theoretical probability” was introduced, including favourable and total possible outcomes, the fraction 1/2, decimal 0.5, and percent 50%. These ideas were then linked back to the earlier trials and the term “experimental probability” introduced. The relationship between the two was mooted and the question asked about how many trials would be expected for the experimental probability to confirm the theoretical probability.

The lesson then turned to the coin tossing activity with students being given recording sheets, asked to record their estimates about how many heads they would get in 10 tosses, and allowed to carry out the 10 tosses and record the fraction of outcomes that were heads. Sticky labels were used by students to record their outcomes on a class chart; these were compared to the theoretical value of 5/10 to illustrate the variation involved. Students then combined their results with neighbours to create samples of size 20 and then 40. These were also recorded on the class chart (see Figure 3). The discussion at the end of the first lesson focused on the closeness of the results to the theoretical probability and the effect of sample size.

The following lesson continued with the same format and involved the use of the *TinkerPlots* Sampler to investigate the connection between the theoretical probability and the observed experimental outcomes in larger trials of 100, 1000, and 10,000 (see Figure 4). In the whole class lesson, the Mixer was used to simulate a coin. Figure 5 is a screen dump of the actual activity. The students were able to see: the Mixer, a running tally of both heads and tails in the form of a bar graph, a dot graph tracking the variance between heads and tails, and a stacked dot plot graph tracking the number of heads in a row.

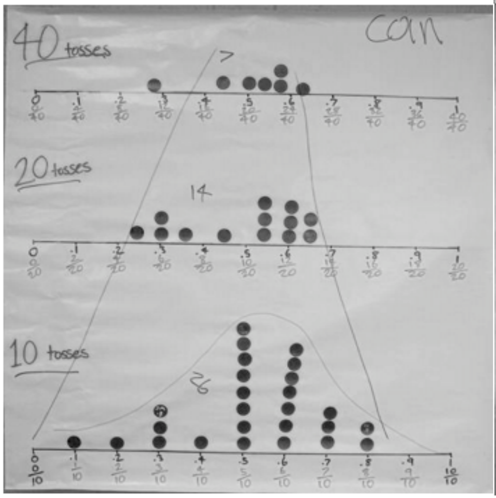

Fig. 3: Tracking the tossing of a coin

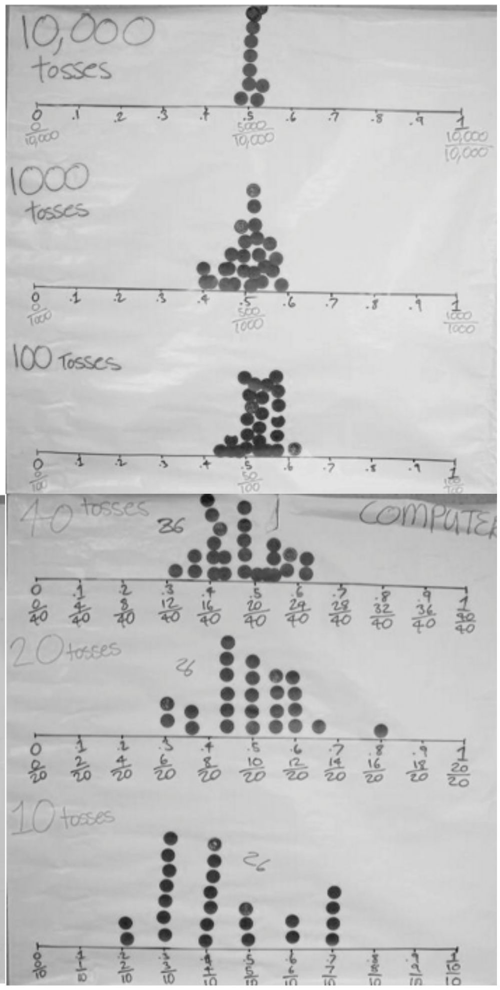

Fig. 4: Tracking the simulated tossing of a coin

Work samples were collected from the students in the form of a recording sheet and a reflective summary at the end of the second lesson. The reflective summary asked students to answer three questions: What is experimental probability? What is theoretical probability? How are they connected? Responses to these questions helped to confirm the choice of students for interview.

The students chosen for interview, in consultation with the classroom teacher, were those who performed well in the pre-test and the reflection sheets. Five of the students are discussed here. The interview protocol used a die and the *TinkerPlots* Sampler. The introduction of a die in the interview protocol, instead of coins, provided an opportunity for students to demonstrate their understanding in a new context (Blythe, 1998). During the interview, students were asked to explain, generalize, find evidence, and apply their current probabilistic understanding of dice. As part of the interview protocol, students were exposed to three scenarios involving the simulator and loaded dice. The Spinner was chosen to simulate the die as there was more flexibility in the loading of the outcomes. Three scenarios were developed: one without loading, one with a large loading, and one with a slight loading. These loadings were hidden from the student. Figure 6 is a screen dump of one of the actual activities. The students could see the hidden Spinner, the table of results, and a stacked dot plot with the experimental outcomes above each column.

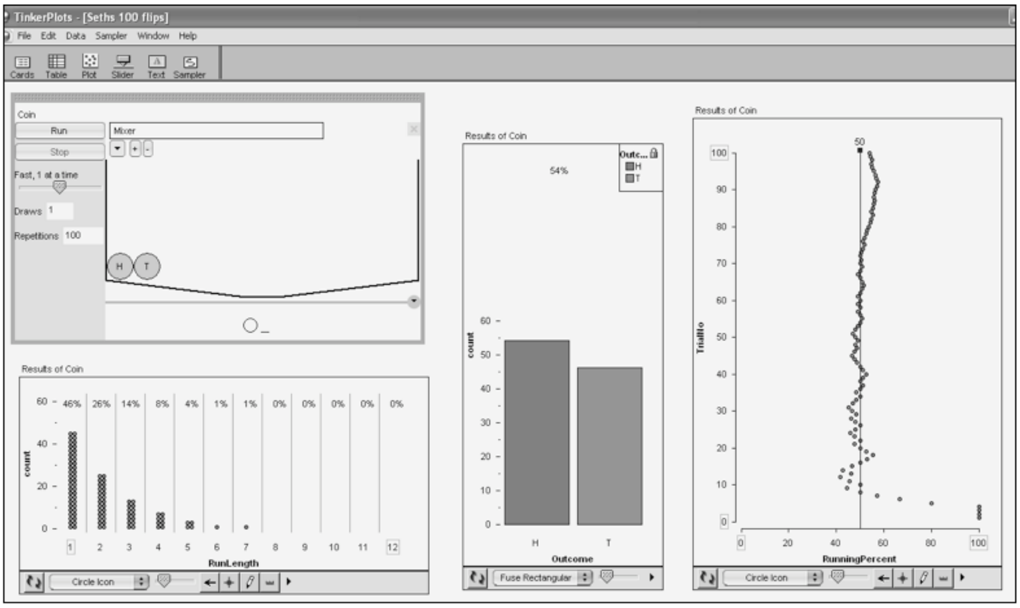

Fig. 5: Screen dump of the Sampler whole class coin activity including three graphs

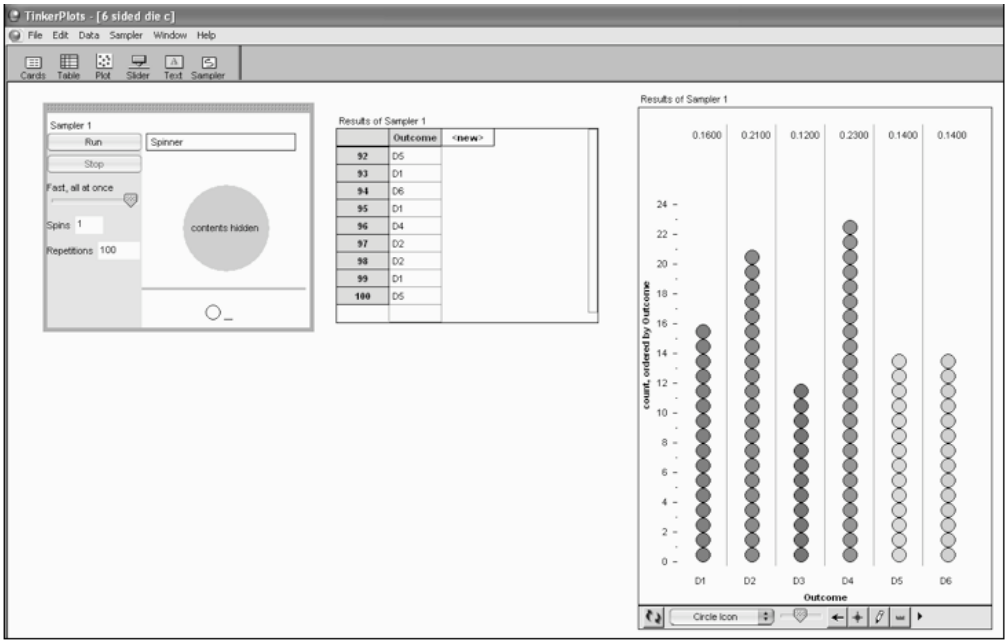

Fig. 6: Screen dump of the Sampler personal interview dice activity including a table and graph

The specific questions in the protocol were designed to cover the concept framework in Figure 1. The interview began with a discussion of a tangible die and the associated theoretical probability. Then the student was asked to draw graphs estimating the outcomes for tossing the die 10, 20, 100, and 1000 times and to discuss how different they would be. Students were also asked what the outcomes from a loaded die would look like. Then the Sampler was introduced with discussion of its relationship to the real die on the table and three cases with loaded or unloaded dice considered. The interview ended with a discussion of the relationship between experimental and theoretical probability as experienced during the interview.

The determination of the level of student response was based on mappings of the ideas expected to be included in the students' responses in relation to the five elements in Figure 1. These are shown in Figure 7. The connections among the five elements themselves are shown in Figure 8. The way the components of the figures were combined, for both Figures 7 and 8, was assessed using the SOLO model of Biggs and Collis (1982, 1991) (cf. Lidster, Watson, Collis, & Pereira-Mendoza, 1996; Watson & Moritz, 2003). Three levels of the model were employed in this study. Unistructural responses (U) were characterized by single aspects of the element and a lack of recognition of contradictions when they occurred. Multistructural responses (M) contained a series of aspects of the element with contradictions likely to be recognized but unresolved. Relational responses (R) demonstrated a linking together of aspects of the element, resolving to a large extent conflict that arose.

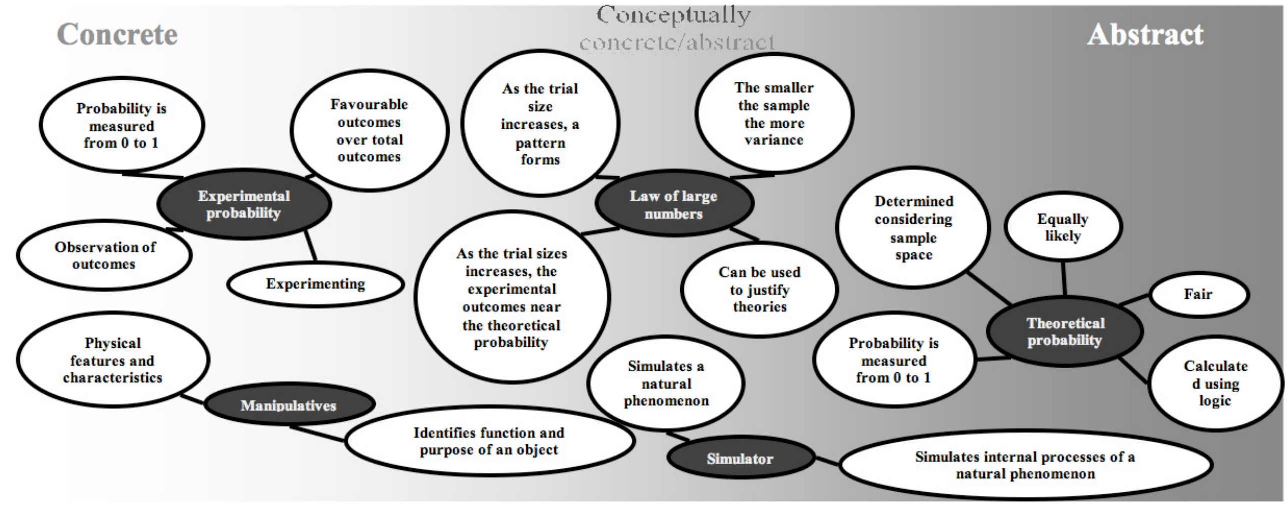

Fig. 7: Mapping in relation to each of the five elements in Figure 1

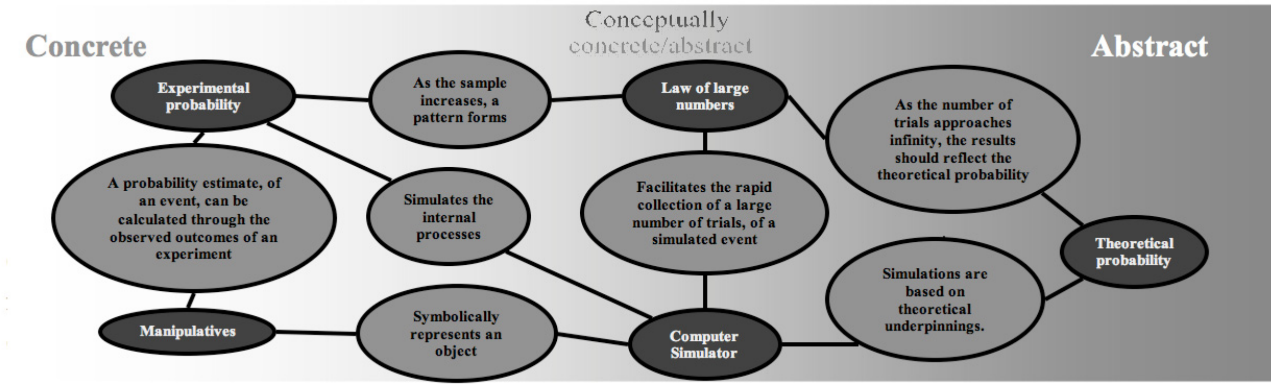

Fig. 8: Connections among five elements in Figure 1

Results

The survey administered to 26 students before the lessons was marked out of a possible total score of 30. Of the students, 10 achieved a score greater than or equal to 20, 13 achieved between 10 and 19, and 3 received a score of 9 or less. The results indicated that a third of the students had a good basic appreciation of calculating probabilities and some intuitions on the variation involved in an experimental setting.

During the classroom lessons, it was observed that some students had difficulty using decimals to the hundredths and thousandths places and converting between fractions and decimals. Students' sketches on work sheets however indicated that 70% appreciated the levelling out of the experimental outcomes as the sample size increased as seen in Figures 3 and 4. The responses to the three reflection questions were coded at four levels, with 0 being idiosyncratic, and 1 to 3 representing a greater degree of understanding of the topic in the question. Summaries of the results for the 23 students who completed the reflection are shown in Table 1. Finding the language to answer the questions in Table 1 was not easy for students. Most had a general idea of what was involved but tended not to hit on the essential elements.

Table 1. Number of Students Achieving Each Level of Reflection for Three Questions about Probability

Code	What is experimental probability?		What is theoretical probability?		How are they connected?	
0	Idiosyncratic	2	Idiosyncratic	7	Idiosyncratic	2
1	Experimenting with chance, probability	8	50/50 chance, contextual	7	They are similar or same, 50/50 chance	10
2	Experimenting with coins, dice, etc, try something, do it	10	Using logic, thinking about it	5	"Prove" each other	4
3	Looking for patterns confirm ideas	3	Calculation based on sample space	4	Eventually come together and join. In long run they converge	7

Because the interview questions were specifically related to the five elements of the model in Figure 1, it was possible to assign SOLO levels to the students' observed understanding of each, in terms of the connection of ideas to the elements in Figure 7. Considering the entire framework together related to the connections among the elements in Figure 8, it was also possible to assign a SOLO level to the observed understanding of the connection between experimental and theoretical probability. The levels assigned for the five students are given in Table 2. It can be seen that the five students interviewed displayed an integrated, relational understanding of the Manipulatives and the Simulator in the context they encountered. Only three performed at this level in relation to Experimental Probability and only one for Theoretical Probability. Displaying an appreciation of the Law of Large Numbers was the most difficult expectation for the students with three able only to describe a single attribute. In terms of the overall connection between Experimental and Theoretical Probability, a wide range of understanding was observed.

Table 2. SOLO Levels for the Five Elements and Connections for Five Interviewed Students

	Manipulatives	Simulator	Experimental prob.	Theoretical prob.	Law of large No.	Connection
S1	R	R	M	M	R	R
S2	R	R	R	M	U	U
S3	R	R	R	R	R	R
S4	R	R	R	M	U	M
S5	R	R	M	M	U	U

In relation to the connection between experimental and theoretical probability, only S1 and S3 were able to articulate and demonstrate this understanding clearly. Although S1's initial intuitions regarding the distribution of numbers were incorrect, believing that she hardly ever rolled a 1 or a 6, she spoke of a smoothing out of the results and the images reflected this in relation to her conceived model (Figure 9). S3's initial graphs (Figure 9) did not reflect his explanation that followed. Both students discussed a levelling out of the results in the larger trials and used the larger trials to determine fairness, and were therefore assessed to have a relational understanding. S3 also showed a more sophisticated understanding of the relation between the theoretical and experimental probabilities by using decimals. It was common among the other students to rely heavily on the visual representation of the levelling out of the data. S3, however, tracked the experimental outcomes of the trials on a piece of paper. After using a trial of 100,000 he explicitly demonstrated a connection between his experimental outcomes and the theoretical probability; he stated, "they are all .16 [referring to the experimental outcomes], just as we found last time, the theoretical probability was .167, here they are all [.167 roughly]." The use of the decimals was considered to show a more sophisticated understanding.

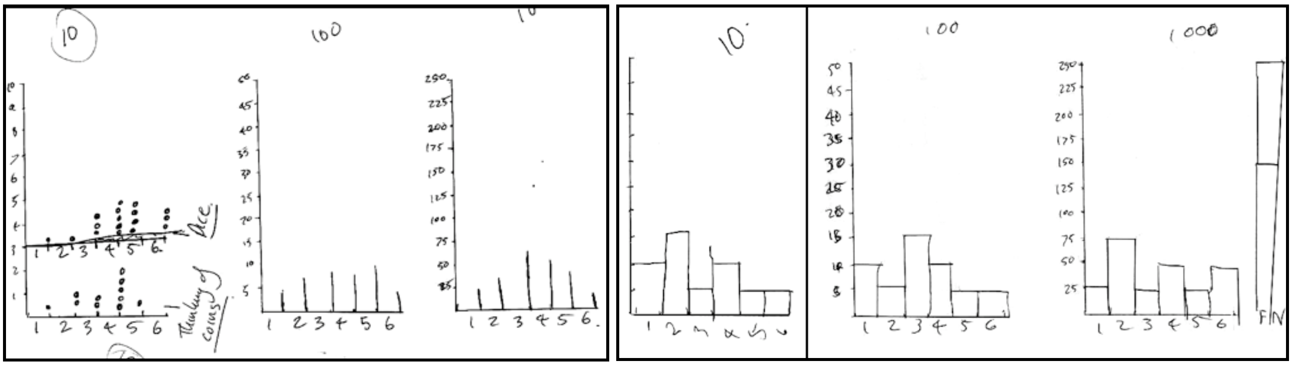

Fig 9: S1's and S3's estimated rolls of a dice 10, 100, and 1000 times

The other students, although able to articulate a basic understanding, were unable to apply this understanding in the hypothetical and simulated scenarios. S2 and S5 both demonstrated a developing understanding of the connection. Both at times articulated an understanding of a levelling out of the data in the larger number of trials; however a belief that fairness was demonstrated by short-term variance in the outcomes hindered the development of an understanding of the connection. It appears that these students saw the connection as: theoretically, the outcomes are equally likely and as the results are random, short term variance in experimental outcomes demonstrates fairness. As there was a variety of contradictory unresolved issues seen for example in their initial sketches (Figure 10), a unistructural understanding was demonstrated.

S4, during the hypothetical estimated trials of 10, 100, and 1000 die rolls, demonstrated an understanding that experimental outcomes should reflect the theoretical probability, dividing each hypothetical trial size into roughly equal portions of .167. This was reflected in his initial graphs (Figure 11). Concern was raised, however, when he estimated the levelling out of data with small trial sizes of less than 100. This concern was reinforced later when he expressed surprise at the variation in outcomes in a trial size of 24, expecting a levelling out of the data. Even though S4 was able to identify correctly the loaded die in each hypothetical scenario, he was not judged to have a relational understanding. As S4 used multiple samples of less then 100 to determine fairness, and did not recognize the significance of "large" trials, he was unable to use the law of large numbers to determine fairness and demonstrate the link between the theoretical and experimental probabilities. Therefore a multistructural understanding was assigned.

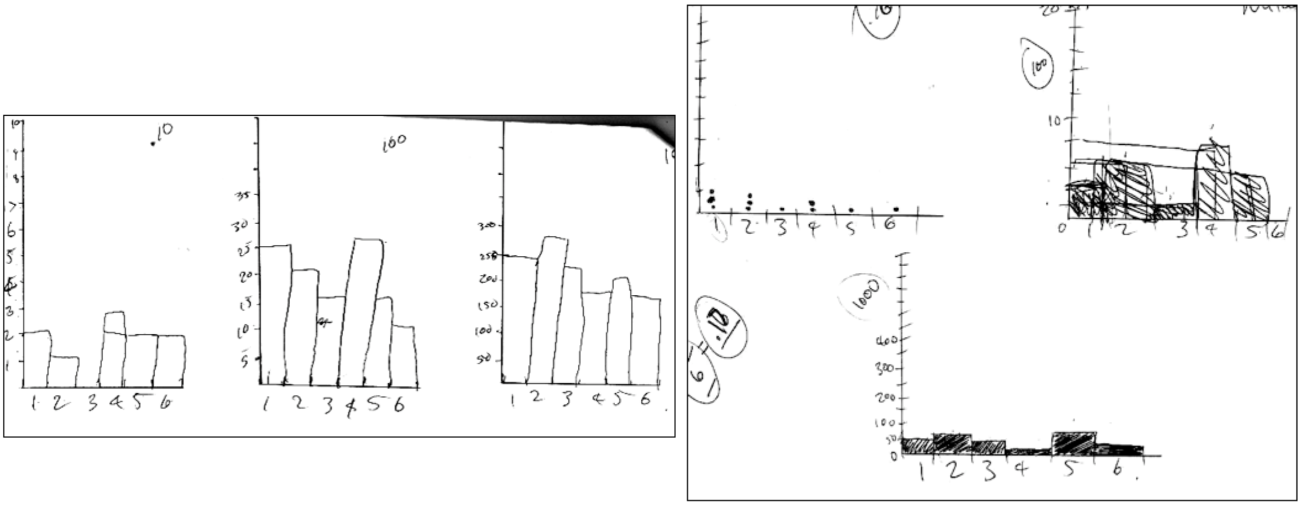

Fig 10: S2's and S5's estimated experimental outcomes for 10, 100 and 1000 rolls of a die

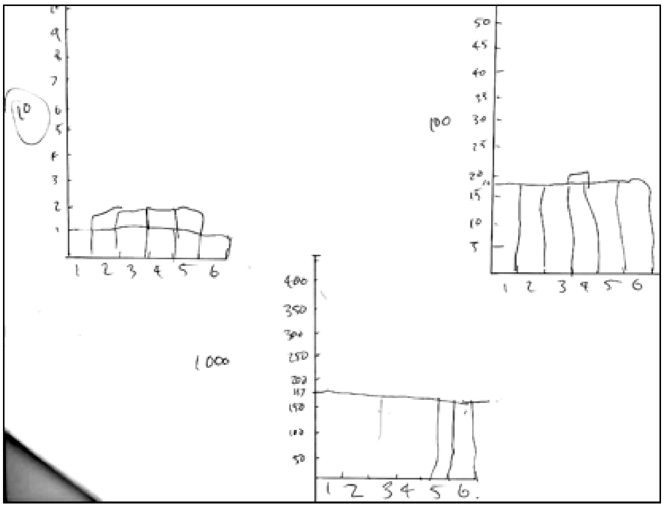

Fig 11: S4's estimated outcomes for 10, 100 and 1000 rolls of a die

Discussion

All of the students interviewed were able to articulate, at some stage during the research, a basic level of understanding of the relationship between experimental and theoretical probability, mentioning a levelling out of results or "proving" of one another. S3 provided the richest example of a student making the connections between experimental and theoretical probability. He was able to connect the experimental outcomes on the simulator with the appropriate theoretical probability. He demonstrated an understanding of the law of large numbers, proportional reasoning, and the underpinning theoretical concepts of fairness and equally likely outcomes. S1, also deemed to have a relational understanding, demonstrated similar understandings. These findings support the conclusions of Stohl, Rider, and Tarr (2004) that students who used larger sample sizes made appropriate inferences regarding fairness. In addition to these outcomes, it is apparent that a linked network of concepts, proportional reasoning, and an understanding of the implications of the underpinning theoretical concepts of fairness and equally likely outcomes are key to developing students' understanding between the experimental outcomes and the associated theoretical probability.

In relation to the other interviewed students, the lack of understanding of the implications of the underpinning theoretical concept of fairness was a barrier to connecting theoretical probability to the experimental outcomes. The determination of fairness and the testing of the theoretical probability appeared conceptually different to the students' minds. The students understood fairness to mean short-term randomness, so to test this required a small trial. When testing the theoretical probability some students appeared more comfortable with larger samples. S3 demonstrated this responding "6 rolls" to a question about determining fairness but then proceeding to use trials of 10,000 to test the theoretical probability. S1 demonstrated the clearest connection, referring back to the whole class lesson and the large number of flips it required to test the fairness of the coin. Linking these two concepts will help students understand better the connection between the theoretical probability and the experimental outcomes. Providing broader contexts may also assist the students to develop a better understanding of the connection. In common board games that involve 10 to 20 rolls of a die, fairness could be considered in short term randomness. However, suggesting to the students the real world context of a casino, where a large number of rolls would occur, may provide a greater appreciation of the importance of determining fairness in the long-run.

When engaging in the simulated trials the students demonstrated a reliance on the visual representations to determine fairness. In the continuum from concrete to abstract the reliance on the graphs from simulations is conceptually in the middle (concrete-abstract), the link to understanding more abstract concepts. An over reliance on the graphs raised concern in this study, as scale plays a large roll in providing the perceived levelling out of the data. The Sampler automatically adjusts the scale of the graph from the smaller number of trials to the larger number of trials, keeping the overall graph the same size. Appropriate interpretation relies on a high level understanding of proportional reasoning. Van de Walle (2004) identified proportional reasoning as an important curriculum content connection to the understanding of probability. If the Sampler did not alter the scale, then the graph would not visually show a levelling out of the results. This highlights the importance of focusing the students' attention on the decimal summary of outcomes, as it gives results in proportion to the total and is easily comparable to the theoretical probability.^

The use of a probability simulator such as the *TinkerPlots* Sampler can provide a meaningful link to concrete manipulatives and can make large samples, and potentially the law of large numbers, easily accessible. Students in the classroom and in the interviews were able to follow the generation of outcomes in relation to the corresponding tangible experimental outcomes, which in turn assisted in developing the overall connection between experimental and theoretical probability. On occasions however, the students did express doubt in the computer's ability to produce random and fair outcomes, identifying that a computer is bound by its rules and calculated responses. If students do not believe the computer can truly simulate randomness, then will they trust the outcomes sufficiently to develop probability intuitions? At times, the first author found that even he questioned the computer's outcomes and its ability to be random. This question provides an opportunity for further research and was beyond the scope of this study. Such doubts, however, did not appear to influence the final levels of performance assigned to students in this study.

Conclusion

In connecting students' understandings of the connections between theoretical and experimental probability many students demonstrated the building blocks necessary to bridge understandings of the two concepts (see Figures 7 and 8). It is evident from this study that it is insufficient for educators to assume that students understand theoretical probability if they can calculate it. Instead, there is need to investigate and develop students' understanding of the underpinning concepts of fairness, equally likely, and random in large and small numbers of trials. It is also apparent from this study that it is insufficient for educators to focus on the calculation of the theoretical probability and the observation of experimental outcomes to develop students' understanding of the connection between these two concepts; this connection needs to be taught explicitly.

Acknowledgement

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