Primary School Children's and College Students' Recency Effects in a Gaming Situation

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Summary

We investigated the evolution with age of probabilistic reasoning and some related biases, such as the negative/positive recency effects. Primary school children and college students were presented probability tasks in which they were asked to estimate the likelihood of the next occurring event after a sequence of independent outcomes. Results indicate that older children perform better than younger children and college students. Concerning biases, the positive recency effect decreases with age whereas no age-related differences are found for the negative recency effect. Theoretical and educational implications of results are discussed.

Introduction

Probabilistic thinking has been found to be related to cognitive heuristics. The application of heuristics leads sometimes to reasonable outcomes, but their activation sometimes produces systematic errors (Tversky & Kahneman 1974; Kahneman, Slovic, & Tversky 1982). One well-known heuristic that affects probabilistic reasoning is the so called *representativeness heuristic* (Kahneman & Tversky 1972; Kahneman, et al. 1982): people estimate the likelihood of an event by taking into account how well it represents its parent population, and the application of this heuristic can cause some predictable errors (biases) in certain situations.

The so called *negative recency effect* (or *gambler's fallacy*) is an example. For instance, if a coin is flipped four times and there is a consecutive sequence of four *tails*, people commonly believe that *heads* is more likely to occur next. That is, the equal probability of getting either *heads* or *tails* is not considered. Rather, by applying the representativeness heuristic, the consecutive sequence of four *tails* is not representative of the expected 50:50 distribution. That is why, according to this heuristic, *heads* must follow so as to equilibrate the proportion. In other words, people become victims of the fallacy because they believe the likelihood of an occurring event is related to the outcomes of previous tosses, i.e. the fact that single tosses are independent events is not taken into account.

Several studies have investigated negative recency effects in college students and children (Kahneman & Tversky 1972; Green 1982; Konold, Pollatsek, Well, Lohmeier, & Lipson 1993; Batanero, Godino, Vallecillos, Green, & Homes 1994; Konold 1995; Williams & Amir 1995; Afantiti-Lamprianou & Williams 2003). However, the effect of age has not been completely clarified. Fischbein and Schnarch's study (1997) have found that the negative recency effect decreases with age. In contrast, a recent study (Chiesi, Primi, & Gronchi 2007) reveals negative recency effects in the performance of young children and adults, whereas older children are found to be almost immune.

Another fallacious approach used in gambling situations is the *positive recency effect*. For example, if a coin is flipped four times and a consecutive sequence of four *tails* is obtained, a person might ignore the independence of events, and instead might believe that the *coin* is not fair. In other words, past outcomes of random events are used as indicators of future events, and, learning from experience, a particular outcome is considered to be more likely. It has been found that from age ten the positive recency is almost absent (Fischbein, & Schnarch 1997), whereas younger children show this bias (Chiesi, et al. 2007).

Several criticisms have been reported to the notions of heuristics and biases. In the main, the adaptive value of heuristic modes of reasoning has been largely documented (Gigerenzer & Todd 1999; Evans & Over 1996). Concerning probabilistic reasoning, Gigerenzer (1991, 1996) argues that certain biases are not biases considering that it is not possible to establish one correct answer. If we aim to describe the functioning of human reasoning, the statement that probabilistic problems have only one correct answer - the one claimed by the normative rules - has to be rejected.

Whereas the relevance of these claims concerning human decision making can not be discarded, actually statistics and mathematics teachers need to transmit the normative rules, and they have to deal with the following matter: students have intuitions on probability that have a cognitive foundation (i.e., intuitions are not irrational). Sometimes intuitions coincide with scientifically accepted statements, but sometimes may contradict them (Fischbein & Gazit 1984). Concerning probability, some normative rules are *counter-intuitive*, and consequently students are likely to have intuitions which are at variance with the commonly accepted reasoning, and as a result probability is a hard subject to learn and teach (Fischbein 1975; Kapadia & Borovcnik 1991; Shaughnessy 1992).

Starting from the assumption that intuitions on probability could impede learning probability, it becomes relevant to investigate students' heuristic reasoning and biases. Italian national curricular reform[1] includes probability at primary school level. For this reason our study is designed to describe primary school children's ways of handling probability concepts. Specifically, we are interested in two issues that have been introduced in mathematics curricula at the primary level: the base-rate computation and the independence notion.

In line with previous studies (Jacobs & Potenza 1991; Davidson 1995; Fischbein & Schnarch 1997; Chiesi, et al., 2007; Morsany & Handley 2007; Chiesi, Gronchi, & Primi 2008), these topics are investigated comparing children and adults' performance. In the pilot study, reported in this paper, primary school and college students were asked to estimate the likelihood of the next occurring event after a sequence of independent outcomes. Combining base-rates and sequences of previous outcomes, different trials were created in order to highlight normative (i.e. correct responding) and heuristic reasoning (i.e. recency effects), and to explore age-related differences.

Method

The sample consists of 23 third graders (mean age = 8.7), 25 fifth graders (mean age = 10.9) and 35 college students (mean age = 25.3). The young students come from a primary school located in Livorno, Tuscany. The university students come from different faculties of the University of Florence. University students were volunteers and they did not receive course credits for their participation in this study.

The three age groups included children before they were formally taught proportional reasoning (third graders), children who were taught proportional reasoning (fifth graders), and college students who encountered mathematical issues related to probability during their high school years. We used a game with marble bags that is suitable for both children and college students, which, compared to the classic test involving the tossing of a coin, allowed to generate base-rates different from equiprobability (Klaczynski

2001).

The task was presented individually to each participant. It measured the ability to estimate the probability of a single event after a sequence of independent outcomes. The experimenter showed the participants a bag with some marbles inside and read out the following story:

"*Simon and John are playing together with a bag in which there are 15 Green and 15 Blue marbles. Simon drew marbles from the bag four times. Each time the drawn marble is put back into the bag. One after the other, Simon drew four Green marbles. What do you think is more likely Simon to draw next, a Blue or a Green marble, or is each colour marble just as likely?"*

Outcomes are independent because the number of marbles remains always the same since the marbles were replaced after each draw. In this way the likelihood of the next occurring event has to be estimated upon the initial base-rate. The task was performed again, and that time the sequence of outcomes was four Blue marbles. The two sequences of outcomes (four Blue and four Green marbles) were randomly presented. Different trials of the same task were obtained with two bags containing different proportions of marbles. One contained a prevalence of Green (21 Green & 9 Blue), the other a greater prevalence of Blue marbles (3 Green & 27 Blue). Each bag was randomly presented. The experimenter filled a record sheet for each participant.

Results

One point was given for each correct answer (i.e., answers consistent with the normative rule: the answer "Green" (G) when the bag had a prevalence of green, the answer "Blue" (B) when the bag had a prevalence of blue marbles, the answer "Equally likely" (G/B) when the bag had the same number of blue and green marbles.

To examine the participants' level of *normative reasoning*, a composite score was formed based on the sum of correct answers (Klaczynski 2001). The scores ranged from 0 to 6, with higher scores indicating a higher level of normative reasoning. The one-way ANOVA (Age Groups: Grade 3, Grade 5, College) reveals a significant effect of age, $F(2, 76) = 12.71$, $p < 0.001$, with a large effect size, $\eta^2 = 0.25$. In particular, post-hoc comparisons (Tamhane test) yield that all comparisons are significant: older children ($M = 4.30$, $SD = 1.49$) perform significantly better than younger ($M = 2.39$, $SD = 0.66$, $p < 0.001$) and college students ($M = 3.24$, $SD = 1.45$, $p < 0.05$); adults perform better than younger children ($p < 0.05$) (see Figure 1).

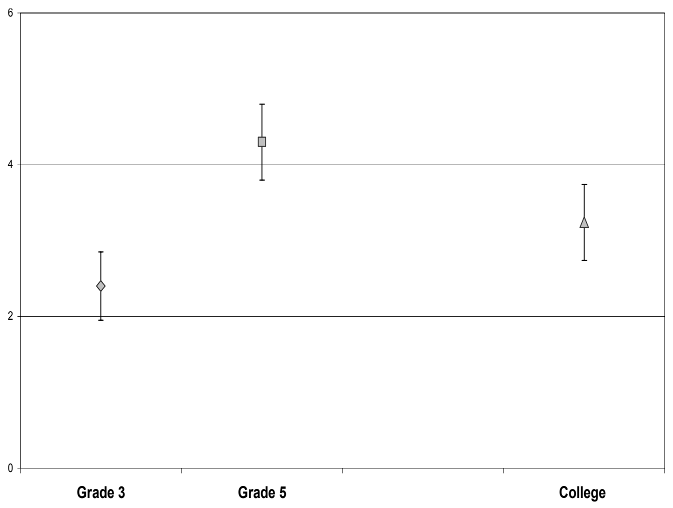

Figure 1. Means for the *Normative Reasoning Scores* per Age.

Concerning *heuristic reasoning*, to examine the participants' *positive recency bias*, one point was given to the participants for each wrong answer coherent with the positive recency effect. In the same way, to measure the participants' *negative recency bias*, one point was given to the participants for each wrong answer coherent with the negative recency effect (see Table 1 for the scoring rationale). Composite scores of heuristic reasoning (Klaczynski 2001) were obtained ranging from 0 to 4. Higher scores indicate more biased responding.

		Positive Recency (sequence + answer)	***Negative Recency*** (sequence + answer)
Bag	**15B & 15G**	GGGG + G BBBB + B	GGGG + B BBBB + G
	21G & 9B	BBBB + B	GGGG + B
	3G & 27B	GGGG + G	BBBB + G

Table 1. *Heuristic Reasoning* Scoring Rationale.

The one-way ANOVA (Age Groups: Grade 3, Grade 5, College) reveals a significant effect of age ($F(2, 76) = 13.95$, $p < 0.001$, $\eta^2 = 0.27$) on positive recency. More in detail, Tamhane post-hoc comparisons indicate that younger children show this bias to a higher degree than older children ($M = 0.95$, $SD = 1.30$, $p < 0.01$) and college students ($M = 0.58$, $SD = 1.20$, $p < 0.001$) (see Figure 2).

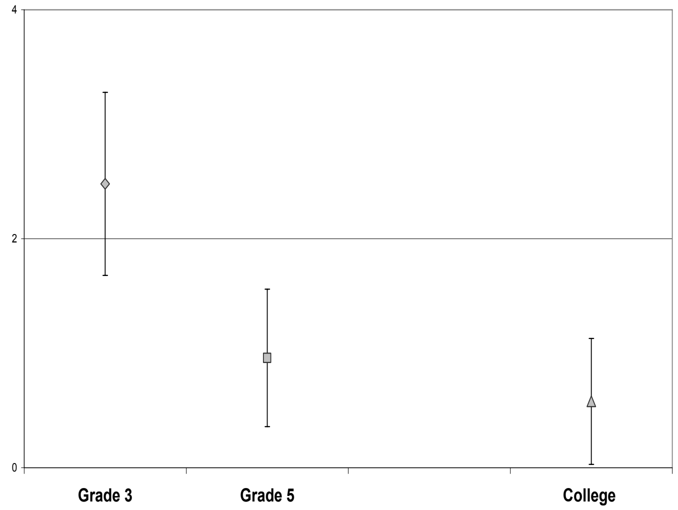

Figure 2. Means for the Positive Recency Bias Scores per Age Groups.

Regarding negative recency, no significant effect of age is found ($F(2, 76) = 1.22$, $p = 0.30$, $\eta^2 = 0.03$). In more detail, the gambler's fallacy appears to have the same effect on younger ($M = 1.65$, $SD = 1.11$), and older children ($M = 0.74$, $SD = 0.81$), and college students ($M = 1.30$, $SD = 0.97$) (see Figure 3).

It is remarkable that some college students considered the two possible outcomes as equally likely, regardless of the actual green/blue proportion in the bags. Accordingly, all responses "Equally likely" were coded as *equiprobability bias*. A composite score ranging from 0 to 4 was obtained with higher scores indicating higher bias levels. College students' mean score on this scale is found to be 0.88 ($SD = 1.47$).

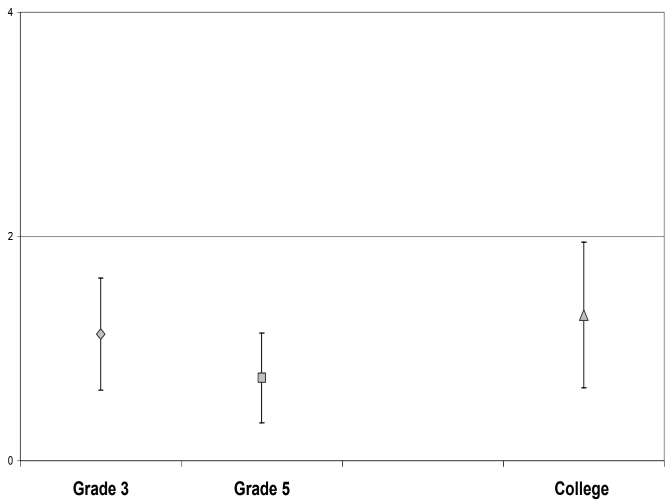

Figure 3. Means for the Negative Recency Bias Scores per Age.

Different typologies of heuristic respondents were created considering the prevalence (at least 3 over the maximum 4) of responses in one direction across tasks. With this respect, younger children tend to be *Positive Recency respondents* (about the 50% of them show this bias across tasks), *Negative Recency respondents* are found among young children and college students in similar proportions (about 20%), whereas *Equiprobability respondents* are found among college students (see Figure 4).

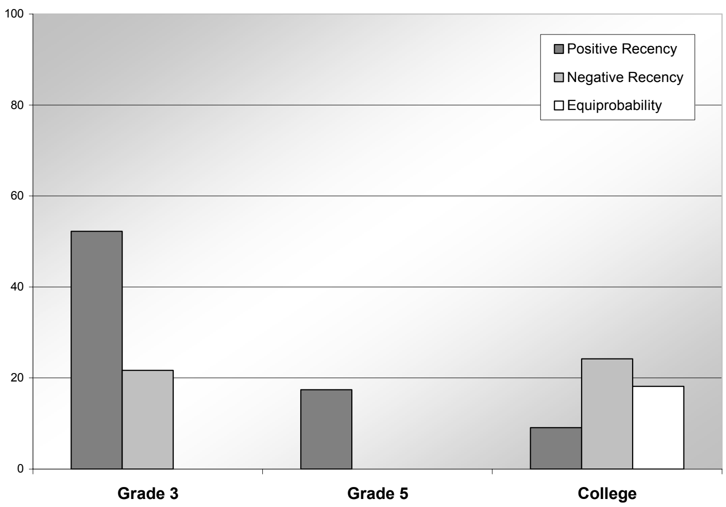

Figure 4. Different Typologies of Heuristic Respondents per Age.

Discussion and Conclusions

Primary school and college students show both normative and heuristic reasoning when asked to estimate the likelihood of the next occurring event after a sequence of independent outcomes.

Younger children, that have not formally been taught proportional reasoning, obtain a poor performance. They rarely take base-rates into account, instead, they largely use the sequence of previous independent events as a cue to estimate the likelihood of the future outcome, i.e. their answers are consistent with the positive recency bias. It can be claimed that their heuristic mode of reasoning is probably due to a lack in numerical computational ability.

Older children, who have already been taught proportional reasoning, perform well. They take the value of base-rates into account, and use them to make normatively correct decisions. As a consequence, older children use less often heuristic modes of reasoning than the other two groups. One possible explanation might be the fact that they dispose of the arithmetic proficiency needed to solve base-rates problems. As documented for different arithmetic skills (i.e., performance on equivalence problems) (McNeil 2007), children seem to apply rules inflexibly while they are learning them and acquiring expertise. In this case, older children's level of proficiency leads them to the correct answer, and the numerical information on base-rates overrides any other information (e.g., the sequence of independent previous outcomes). Nonetheless, sometimes they show the positive recency effect, as younger children, and the negative recency bias - i.e., the next event likelihood is estimated on the basis of previous outcomes in order to equilibrate the proportion in the sample according to the population characteristics.

College students' performance is poor. They show not only the negative recency bias, but also the equiprobability bias - i.e., dichotomous events are assumed to be equiprobable regardless of the base-rates. This bias has been documented using different types of tasks (e.g., the probability of outcomes obtained rolling two dices) (Green 1982; Fischbein et al. 1991; Lecoutre 1992; William & Amir 1995; Canizares & Batanero 1998), and refers to the application of the notion of equal probability (each of the possible outcomes of an experiment has the same probability of occurring) in situations where the outcomes are not equally likely.

According to previous studies (Jacobs & Potenza 1991; Davidson 1995; Fischbein & Schnarch 1997; Chiesi, et al. 2007; Morsany & Handley 2007; Chiesi, Gronchi, & Primi 2008), our results present a view of development of normative reasoning that runs contrary to what is described by traditional developmental theories: Whereas an increase of the ability is found from younger to older children, college students' performance reasoning seems to get worse with age.

The present investigation supports the idea that the connection between age and probabilistic reasoning performance is mediated by the progressive acquisition of both numerical competencies and heuristic issues linked to the notion of probability. We can argue that young children and college students' performances are wrong for different reasons. Young children's heuristic reasoning seems to be causes by the lack of normative competence. College students adopt heuristic reasoning because they have been misguided by task cues (i.e., the combination bag *plus* the sequences of previous outcomes) eliciting private conceptions originating from experiences other than instruction (e.g., gambling), but also from probability notions they learned by instruction (e.g., random sample distribution). In this way, incorrect responses (i.e., in contrast with the normative rule) are not irrational or derived from an illogical analysis. According to Gigerenzer & Todd (1999), this can be an example of the adaptive value of heuristics and biases in human decision making. From an educational point of view, it is to be noted that sometimes these processes lead to conceptions and conclusions, which are at odds with the formal rules.

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Footnote:

[1] Riforma della scuola (XV legislatura) DL del 19.02.2004 (in S.O. n. 31, G.U. del 2.03.2004, n. 51).