INTERNATIONAL STATISTICAL INSTITUTE

Newsletter

This issue of the Newsletter features two articles which are very differently focused. The first gives a brief report on three current or recently completed statistical education projects at Appalachian State University; *STAT-MAPS*, *STAT-LINK* and *SIM-PAC*. The second article, originating from Klagenfurt, explores some of the more philosophical aspects of cognition in stochastics, suggesting ways in which the particular characteristics of the discipline may hamper conventional approaches to its teaching and its learning.

STATISTICAL EDUCATION PROJECTS

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The goals of the statistical education projects at the Appalachian State University are to effect change in the secondary mathematics curriculum and in the styles and methods of statistical instruction. Three projects have been funded since 1987, covering curriculum design, research and development of teaching materials and methods, and teacher-training. They have been built on three fundamental assumptions:

(1) The prevailing style of presentation in the secondary mathematics curriculum is too narrow and formalised. We must broaden the curriculum and encourage active learning styles which stress experimental and exploratory approaches to learning. Mathematics instruction needs to get out of the textbook and into the laboratory.

(2) The role of the teacher is absolutely crucial to effect change in the curriculum and styles of instruction. The teacher's attitude about what is important and how it should be taught is the most important factor in the change equation.

(3) Statistics should be presented in a coherent fashion and must be considered as a "whole problem" solving process consisting of question formulation, data collection, analysis, interpretation and inference. Statistics should be approached through problems, not just techniques. The role of statistics in society and statistics across the curriculum are important objectives of statistical education.

STAT-MAPS, "Statistics Materials and Activities for Problem Solving" (1991-94), is a project developing and testing a comprehensive secondary curriculum in statistical education and producing supporting materials. The curriculum will address the needs of students in grades 9-12, both those who do not intend to go to college and also those who do. The curriculum is based on active learning through applications of statistics. It stresses exploratory and experimental approaches to learning, and emphasises statistics in society. STAT-MAPS is developing categories of statistical problem solving based on eight types of statistical interpretations; (1) Fundamentals - interpreting tables and graphs, (2) Interpreting Univariate Data, (3) Interpreting Bivariate Data, (4) Interpreting Designed Experiments, (5) Interpreting Times Series, (6) Estimating Population Values from Samples, (7) Model Fitting, (8) Statistical Decision Making/Assessing Statistical Significance. STAT-MAPS intends to produce a student text, an activities manual, and a teacher's manual. John Wasik is a project associate for evaluation. STAT-MAPS is seeking additional teachers who are interested in class testing materials. Only English versions of the materials have been produced, but others might want to adapt the ideas and produce them in another language.

STAT-LINC, "Statistics Leaders in North Carolina" (1990-93), is a teacher education project, using a cascade model of training. It is built around 24 Lead Teachers (secondary mathematics) who were selected during the spring of 1990. The Lead Teachers attended an intensive four week institute during each of the summers of 1990 and 1991. The institutes emphasised statistical concepts and problem solving, teaching statistics, curriculum, and the role of the Lead Teacher. Participants practised learning through problem solving activities and less than fifteen percent of institute sessions were in a lecture format. STAT-LINC held two conferences for the Lead Teachers and biology teaches from their respective schools in the fall of 1992 to consider the role of statistics in the biological sciences and ways that the statistics teacher and biology teacher can collaborate. Workshops for middle and secondary school teachers have been held at which Lead Teachers served as workshop staff. STAT-LINC also hosted a conference on statistical education and the curriculum for administrators and co-ordinators during 1992-93.

SIM-PAC, "Simulations in Mathematics-Probability and Computing" (1987-91), developed instructional strategies for learning probability concepts through the study of computer simulation models. The materials produced include simulation models implemented for both IBM-PC and Macintosh computers, student activities for each model and a teacher manual. The materials are designed for the secondary student in grades 9-12. The models are 1) DICE, 2) BINOMIAL, 3) GEOMETRIC, 4) SPINNER, 5) TOKENS, 6) RUNS, 7) RANDOM WALK, 8) ESTIMATE. Each SIM-PAC program provides simulations of a random experiment and statistical summaries of at least one random variable. Summaries of simulated trials include bar graphs, serial mean plots, and box plots; frequency, relative frequency, and cumulative relative frequency tables.

WHY TEACHING PROBABILITY AND STATISTICS IS SO DIFFICULT

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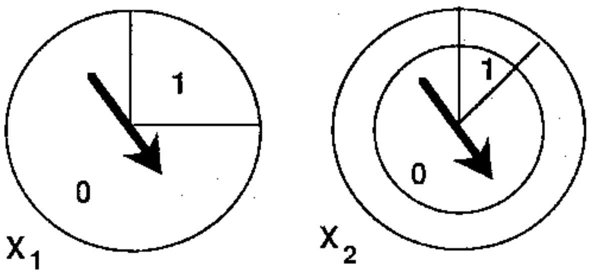
It is the gap between individual ideas and theoretical concepts which makes it difficult to teach probability and statistics successfully. It is argued here that there are some deep-seated reasons for that gap. Stochastics, as a discipline, shows some peculiarities which underlie the acquisition of the correct concepts and which make it truly different from other fields of mathematics.

1. No learning by trial and error

Trial and error is a basic type of learning which accompanies us from our earliest age and is also applicable to mathematical concepts, at least at the basic levels. Starting from concrete operations upon a material representation of a concept, the individual gets the required feedback leading to the revision of existing but vague primary ideas. For example, concrete counting leads to the structure of the natural numbers and establishes clear ideas about them. In mathematics, counter-intuitive results are not met until after the student has reached high levels of abstraction. The same cannot be said about the process of acquiring stochastic concepts.

In other fields of mathematics, modelling involves producing a first representation of the real problem, getting more data as feedback of its fit, and then refining the model to make it fit better to reality. This implies that the feedback from reality has a regularity about it which makes its interpretation relatively straightforward. However, even the very basic concepts of probability reveal characteristics which defy such direct feedback. In a sense, probability is much more of a heuristic, a strategy, to explore reality in something like a phase space yielding only *possible* scenarios of the real world.



To demonstrate this, the reader should consider the two wheels of fortune in the figure; if the pointer stops in the sector marked by a '1', the player will win. For a series of spinning the chosen wheel there is, after some instruction, no problem in persuading students to take the wheel with the larger winning sector. The problem arises with a single, one-off, decision in which probability plays but one role among several approaches and gives only an abstract weight for the possible results.

In fact, one can 'win' using a subsidiary strategy. The choice of the worse wheel X_2 may lead to success, and likewise the choice of the better X_1 may lead to failure. To convince someone who has chosen the 'wrong' wheel but who has nevertheless won may not be such a simple matter. It can be counter-productive if the teacher tries to prompt students to rely on a trial and error approach, because this can lead to wild speculations which hinder the acquisition of adequate concepts. Children get lost in deliberations about the "special" margin of the wheel X_2. Furthermore, the popularity of astrological predictions suggests that this kind of behaviour is not only the domain of children. It is not before one has experience with *series* of trials that some feedback and its utilisation become possible. However, probability is also an important strategy to apply to a *single* situation, and there are many reasons why this single situation differs from a series of uncertain events.

2. The gap between actions or operations and reflections

There are various approaches in the didactics of mathematics which are based on the interplay between actions and reflections, starting with Piaget. *Actions* upon a material representation of the concept form the ingredients of a process of *reflection*. During that process new objects will emerge from the concrete operations which allow a higher level of action. By stepwise hierarchical stages of actions with new objects and ensuing reflection, the individual establishes the concepts at higher levels. With probability, it is suggested that the concrete operations which should provide the basis of this process as well as the feedback from operations are missing, thus hindering the reflection phase. This author argues that the interplay between intuitions and theoretical concepts proposed by Fischbein provides a much more promising approach for teachers of probability. According to this perspective, raw primary intuitions are revised by partial theoretical inputs into secondary intuitions whereby intuitions generally form the key to understanding and acceptance of the theory.

Operations and reflections use different cognitive strategies.

An analysis of empirical research on concept understanding reveals that the communication between an individual and a researcher might completely break down if the individual answers questions in terms of concrete operations and the researcher expects an answer at the level of reflections. This breakdown of communication applies also to teaching and individual concept formation. This can be illustrated by a simple coin tossing situation. The *operation* is 'predicting the outcome of the next toss', whereas the *reflection* means 'evaluating the weight of Heads'. The concrete operation may be based on the physical symmetry or on an indifference argument. This usually leads to allocating abstract weights of the uncertain events such as 1:1, or assigning the probability of heads as 1/2.

With coin tossing, the individual witnesses a conflict right from the beginning. Intuitive ideas, on the one hand, indicate that it is impossible to predict the next outcome with absolute certainty, and yet the individual feels the need to master the chaos in the environment. At this point, mathematics enters the stage and promises to calculate randomness. However, while *reflection* yields only abstract weights which are not intuitively accessible, *action* would necessitate a procedure for accurate prediction. This primary intuitive conflict is exacerbated by the lack of a direct feedback from reality. leading to a re-interpretation of any statement from theory into a recipe to solve the prediction problem. Consequently, mathematical statements are either over-interpreted (e.g. the law of large numbers being misused to derive the prediction of Tails after five consecutive Heads) or abandoned, e.g. re-establishing causal schemes for the prediction.

3. Mathematics offers a justification but yields no insight
Classical and Bayesian approaches to theory justify different intuitions.

The basic idea of the classical approach is the tendency of a repeatable experiment to produce relative frequencies, whereas the Bayesian approach centres its theory around the idea of probabilistic judgements in the form of weight of evidence. The axiomatic theory of either position is thought to justify solely its own basic idea. In fact, may of the frequently quoted probability paradoxes are characterised by an uncomfortable inter-mixing of classical and Bayesian ideas. Among them are puzzles related to the Bayes formula, which highlight the suggestion that the concept of conditional probability has a poor representation within the classical approach. *Conceptual thinking is not reducible to the logic of mathematics.*

Besides the controversy between Bayesian and classical ideas, there are many other ideas which are important to the individual, but which are not covered at all by the development of the theory. This might not be a problem as long as one remains within pure mathematics, but things are different when one applies or teaches mathematics. Then, the fact that a logically consistent argument to justify a concept cannot mirror all of its features has t be brought out into the open. The concept of independence provides a good example that the cognitive processes of justifying and understanding are different. Independence may be mathematically reduced to the multiplication formula and become the basic ingredient of theorems about repeated random experiments, like the law of large numbers or the central limit theorem. Independence thus gains an important role within theory, but its mathematical definition by no means affects the individual's causal interpretations. *More about the author's ideas can be found in Kapadia R. Borovcnik M (eds) 1991 "Chance Encounters: Probability in Education", Kluwer which is reviewed in this issue of Teaching Statistics.*

NEWS ITEMS

ICOTS-4: The 4th International Conference on Teaching Statistics will take place 25-30th July, 1994, in Marrakech, Morocco. Full details of the programme, with details of session leaders for those wishing to contribute, may be obtained from Professor Yves Escoufier, Université Montpellier II, Science et Technique du Languedoc, Place E Bataillon-34095, Montpellier Cedex 5, France (Tel: +67-14-35-69, Fax: +67-14-35-58, E-Mail: yes@montpellier.inra.fr).

Accreditation: The RSS (Royal Statistical Society) now has accreditation of statisticians in the form of the award of CStat (Chartered Statistician). In recent months, the ASA (American Statistical Association) has been deliberating on the subject of accreditation. The ISI has formed a committee to consider the issues, which include professional conduct, and the implications for the provision of initial and continuing in-service statistical education.

What is Research in Mathematics Education and What are Its Results: This interesting paper by Balacheff *et al* recently appeared in the 14th issue of the Newsletter of The Royal Society, Mathematics Instruction Subcommittee (MIS). Although not specifically dealing with Statistical Education, the article nevertheless covers, by analogy, a number of issues of considerable importance for statistical educators and researchers. The Newsletter also carries relevant conference announcements and calls for papers. (For further MIS information or to join the UK mailing list contact: Robert Rees, The Royal Society, 6 Carlton House Terrace, London, SW1Y 5AG. Tel: 071-839-5561 ext. 311).

First Scientific Meeting of IASE and *49th Session of the ISI.* These took place August/September 1993. A report on the proceedings will appear in subsequent issues of the *IASE Newsletter.*

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