DESCRIBING CRITICAL STATISTICAL LITERACY HABITS OF MIND

by

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ABSTRACT

NINA G. BAILEY. Describing Critical Statistical Literacy Habits of Mind. (Under the direction of DR. ALLISON W. MCCULLOCH)

This dissertation adopted the three-article format. The entire dissertation aims to explore Describing Critical Statistical Literacy Habits of Mind (CSLHM). The first article describes the creation of the CSLHM framework. The second describes how secondary preservice mathematics teachers enact CSLHM. The third article describes a common and an unusual case of secondary preservice mathematics teachers' CSLHM enactment. Below is an abstract for each of the three articles.

The first article presents the CSLHM framework. Given the vast number of data representations that people encounter daily, it is imperative that people become critical consumers of the representations they will face. This requires the development of particular habits of mind. The purpose of this paper is to share the critical statistical literacy habits of mind framework. We articulate how we drew on the literature related to statistical literacy, critical mathematics, and critical statistical literacy to identify the habits of mind needed to enact critical statistical literacy in the context of consuming data representations. We describe the refinement process using qualitative interview data. To illustrate what each of the critical statistical literacy habits of mind looks like when enacted, we share examples from statistics teachers making sense of a data representation.

The second article aims to explore how preservice secondary (middle and high school) mathematics teachers (PSTs) enact critical statistical literacy habits of mind when engaging with a statistical message from the media. Standards documents (e.g., National

Governors Association Center for Best Practice & Council of Chief State School Officers, 2010; North Carolina Department of Public Instruction, 2020) emphasize statistical literacy from the consumer orientation (e.g., making sense of data representations from the real world). Making sense of real world statistical messages requires the adoption of a critical lens (e.g., focus on power and equity). How statistics are wielded and presented in the real world cannot be separated from the fact that social issues operate within systems of marginalization, privilege, and power. Findings reveal that preservice teachers emergently enacted the CSLHM, and some enacted particular CSLHM robustly.

The third article describes a common and an unusual case of secondary preservice mathematics teachers' CSLHM enactment. When making sense of data representations from the media, preservice secondary mathematics teachers typically enact CSLHM emergently. The goal of this study is to examine the ways that PSTs enact CSLHM when making sense of data representations from the media using an instrumental multiple case study design. Two cases were selected from the broader study, one typical CSLHM enactment and the other unusual. The study examined how each enact the CSLHM across two tasks and examined the differences between the two cases. Findings revealed several important differences between the cases. The PST who more robustly enacted the CSLHM integrated context into her enactment, directly discussed the social issue, and evidenced elevation of some CSLHM from emergent to robust on a single task. These findings are important for the field to consider how to support preservice teachers CSLHM development so that they can help their students develop the same habits of mind.

Keywords: critical statistical literacy, habits of mind, preservice mathematics teachers

DEDICATION

This work is dedicated to my 5th grade mathematics and science teacher Mrs. Jean Sam. You believed in me when I did not believe in myself, and I wish I could tell you thank you.

This work is also dedicated to the academics and lovers of learning that left our world much too soon; this is for Dr. Matthew Edward Fiss and Mrs. Gayle Olenik.

Finally, this work is for all those afraid to advocate for themselves. Finding our voice takes courage and time; I have faith in you.

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LIST OF ABBREVIATIONS

CSL	Critical Statistical Literacy
CSLHM	Critical Statistical Literacy Habits of Mind
GAISE	Guidelines for Assessment and Instruction in Statistics Education
JMB	*The Journal of Mathematical Behavior*
JMTE	*Journal of Mathematics Teacher Education*
LOCUS	Levels of Conceptual Understanding in Statistics (Jacobbe et al., 2014)
PST	Preservice Secondary Mathematics Teacher
SERJ	*Statistics Education Research Journal*
SL	Statistical Literacy

CHAPTER 1: INTRODUCTION

> I believe that we owe it to our children to prepare them for the world that they will encounter—a world driven by data. Basic data fluency is a requirement not just for most good jobs, but also for navigating life more generally, whether it is in terms of financial literacy, making good choices about our own health, or knowing who or what to believe. (Levitt, 2019, 42:34)

We live in a world that buzzes with data. As Bersin and Zao-Sanders (2020) stated in a *Harvard Business Review* article: "We've entered a golden era of data. You don't have to be Walmart or IBM to build a data lake in your company — that opportunity is now available to every company through cloud-based systems at modest cost" (para. 2). But data is not just reserved for businesses; it permeates our daily lives. Most people are familiar with the phrase "fake news," but is everyone equipped to spot fake news? Are we setting up our students and citizens to be able to make sound decisions based on the data they encounter? Economist Tim Harford (2021) warns against developing a purely cynical view of statistics and advocates for a balance between appreciation for statistics as a tool to improve society and healthy skepticism for how it can be misused:

> If we give in to a sense that we no longer have the power to figure out what's true, then we've abandoned a vital tool. It's a tool that showed us that cigarettes are deadly. It's our only real chance of finding a way through the coronavirus crisis—

> or, more broadly, understanding the complex world in which we live. But the tool is useless if we lapse into reflexive dismissal of any unwelcome statistical claim. Of course, we shouldn't be credulous—but the antidote to credulity isn't to believe nothing, but to have the confidence to assess information with curiosity and a healthy skepticism. (p. 9)

Harford's admonishment is salient, particularly given the shift to emphasize statistics and data science in K-12 education (e.g., National Governors Association Center for Best Practice & Council of Chief State School Officers, 2010; North Carolina Department of Public Instruction, 2020). Now that we have this emphasis, it is vital to consider how individuals can develop habits that lead to an improved balance between skepticism and appreciation as Harford described.

Historically statistics have been incorporated into the K-12 and undergraduate curriculum from a production perspective (i.e., creating statistical messages thus calculation driven) not a consumer perspective (i.e., sense making with statistics). As national organizations (e.g., American Statistical Association) articulate calls for statistical literacy and mathematics standards are revised to place more emphasis on statistics, it is likely that we are on the cusp of change. Consider the newly adopted standards for Integrated Math 4 in North Carolina. Not only do the Math 4 standards include 50% of the course devoted to inferential statistics, but the standards reflect a combination of the production and consumer orientations. For example, SP.1.4 reads: "Interpret non-standard data visualizations from the media or scientific papers to make sense of real-world phenomena" (North Carolina Department of Public Instruction, 2020, p. 20). Similar emphasis on both orientations is evidenced in standard and/or curriculum

revisions nationwide and is particularly evident in California's new push for the inclusion of data science and big data (e.g., Boaler & Levitt, 2019; Gewertz, 2020).

While it appears that many researchers, economists, and educators are taking up the position that we need to help the public digest statistical information, there tends to be a distinct gap between the curriculum in K-16 schools and the types of knowledge needed to effectively consume statistical messages (Gal, 2002; Nicholson et al., 2019). When describing civic statistics (i.e., making sense of statistical messages concerned with civic life), Nicholson et al. (2019) noted that the skills involved "often [require] understanding of topics and issues that are different from or go beyond the knowledge gained from regular statistics curricula" (p. 2). Despite this growing emphasis on statistical literacy and data science literacy, there is a disconnect between the skills needed to effectively consume real world statistical messages and what is taught in schools (Nicholson et al., 2019). Furthermore, literature suggests that preservice secondary mathematics teachers (PSTs) are not prepared to teach statistics (Lovett & Lee, 2018), thus illuminating the need for articulating and understanding how PSTs teachers enact Critical Statistical Literacy Habits of Mind (CSLHM) for making sense of such messages, particularly using a critical lens. The CSLHM are the thinking behaviors called upon to make sense of statistical messages with a focus on how the statistical message is used to uphold or dismantle structures of inequity. Students and adults alike need to be able to navigate through the abundance of quantitative messages they will undoubtedly encounter. Additionally, students will need to be able to create and communicate their own messages supported by statistical investigation or the credible statistical investigation of others. These actions are often referred to as statistical literacy or SL (Gal, 2002; Rumsey, 2002).

While there are varying conceptions of statistical literacy, the consumer orientation is widely adopted (Gal, 2003; Kaplan & Thorpe, 2010; Wallman, 1993). From this orientation, SL requires a set of skills needed to consume statistical messages effectively in the real world (e.g., the media). The importance of developing SL was articulated in the *Guidelines for Assessment and Instruction in Statistics Education* (GAISE) reports (Bargagliotti et al., 2020; Carver et al., 2016; Franklin et al., 2007) with particular emphasis on statistical thinking and thinking critically about statistical issues. Some scholars have criticized such efforts as superficial. For example, Rubel et al. (2021) called attention to the perfunctory inclusion of a critical lens in the *PreK-12 Guidelines for Assessment and Instruction in Statistics Education II* (Bargagliotti et al., 2020) stating that the skepticism promoted in the report "can expose unfairness in a dataset but falls short of seeking accountability for justice" (p. 219). Rubel et al. (2021) posited that CSL should specifically aim to challenge and dismantle the systems of inequity and consider how data can further marginalize and/or privilege. Moving forward my use of the word critical aligns with Rubel et al. (2021); I use the word critical to refer to how it is generally used in critical literacies to emphasize the relationship between literacy and power (Lankshear & McLaren, 1993).

Weiland (2017) referred to SL as reading and writing the world and emphasized using a true critical lens to do so. Such a critical lens includes attending to different perspectives sometimes from a sociopolitical standpoint. When SL is approached from this perspective it is described as *critical statistical literacy* (Weiland, 2017). For the

purposes of this study[1], I define critical statistical literacy (CSL) as the practice of interrogating statistical content to inform action or change with a specific focus on power and equity. To help bridge the gap between curriculum and forms of statistical literacy, specifically CSL, I approached this study from the perspective of habits of mind. By habits of mind, I am referring to the collection of thinking behaviors that when used appropriately lead to sense-making (Costa & Kallick, 2000b). In the words of Harford (2021), I aim to better understand what makes someone a good data detective to begin to think about how we can nurture such behavior in school and beyond. Thus, in my dissertation study, I aim to describe the habits of mind needed to enact CSL and understand how PSTs enact CSLHM.

Significance of this Study and Research Questions

Research is rich with work that legitimizes SL as worthy of study (Watson, 1997; Watson & Callingham 2003; Weiland, 2017) and describes the components of SL (Gal, 2002; Kaplan & Thorpe, 2010). Other researchers have detailed how to assess statistical literacy (e.g., Budgett & Pfannkuch, 2010), developed hierarchical frameworks (e.g., Watson & Callingham, 2003), and described curriculum design or reform (e.g., Schield, 2004; Tishkovskaya & Lancaster, 2010). While SL has gained momentum in the field, there are fewer examples of research that provide rich examples of how students express *critical* statistical literacy. In the most recent GAISE college report (Carver et al., 2016), the first goal listed states that "students should become critical consumers of statistically-

[1] "Study" refers to the collection of three articles that comprise this dissertation.

based results reported in popular media, recognizing whether reported results reasonably follow from the study and analysis conducted" (p. 8), thus substantiating the need for CSL from the consumer orientation. While there is ample research validating SL as worthy of study and identifying what SL entails, there is still a need for research that focuses on taking a critical lens.

As consumers we need to move beyond critical thinking and adopt a lens focused on sociopolitical inequity and issues of power. Weiland (2017) has synthesized the differences between critical literacies and SL to develop a critical perspective within statistical literacy. His framework for CSL focused on reading and writing the world, arguing that statistical literacy should include consumption of data as well as inquiry. Weiland (2017) described reading the world as making sense of and evaluating statistical messages with a lens focused on sociopolitical inequity. In contrast, writing the world emphasizes the actions taken to investigate and combat such social inequities. Weiland (2017) has both described and established the importance of CSL, however we need to know what CSL looks like in action. While explicit research on CSL is still emergent, anecdotal work supports the development of statistical literacy from the critical perspective (Harford, 2020), and also implies that the habits of individuals who are able to enact such skills are important to consider.

Like any area of mathematics or statistics, engaging in CSL requires particular habits of mind. Habits of mind refer to the collection of thinking behaviors that when used appropriately lead to sense-making (Costa & Kallick, 2000a, 2000b, 2008). While there are habits of mind for statistical thinking which highlight the importance of critical thinking (Lee & Tran, 2015), they do not explicitly address the sociopolitical perspective

that Weiland points out is an essential aspect of making sense of data. Thus, the broad aim of my work is to describe CSLHM.

I adopted the three-article model for this dissertation study. The three articles represent a cohesive progression for describing CSLHM specifically among the PST population. The first article (Chapter 2) presents the CSLHM framework, describes the process of creating and refining the framework, and provides examples of both emergent and robust enactment. Building off the development of the framework, the second article (Chapter 3) describes how PSTs enact the CSLHM. Finally, the third article (Chapter 4) examines PST enactment more deeply by describing and contrasting two cases: one common and one unusual. Across these three articles, I aimed to answer the following research questions:

- What are the habits of mind that individuals use to enact CSL when presented with statistical messages from the media?
- How do PSTs enact CSLHM when presented with statistical messages from the media?

Methodological Overview

While the methods for each article (Chapters 2-4) are different and will be described within each article, I would like to broadly address the overlap of participants and consistent interview structure. All three articles employed qualitative research methods using subsets of interview data from preservice and inservice K-16 teachers. I will briefly address the consistency in interviews, differences in participants, and overall methodological approaches across the three articles.

The interview protocol and procedures across the three articles was identical. All participants took part in semi-structure task-based interviews (Goldin, 2000). The participants were presented with statistical messages from the media and asked to think aloud as they made sense of the data representations. Once they finished making sense of the initial task, they were asked what a conversation about the task would sound like with a confidant with similar beliefs. Then, they were asked the same thing but with a confidant who has dissimilar beliefs. This process repeated for all six tasks. Questioning was limited to clarification (e.g., I'm not sure I understand what you mean, can you explain that one more time or in a different way?) or elaboration (e.g., Can you tell me more about that?). After they cycled through all six tasks, they were asked if there were any which she would like to revisit.

The participants in this dissertation study were all preservice or inservice K-16 teachers with varying experience. In total there were 30 teachers. The first article (Chapter 2) used the interview data from all 30 teachers (inservice and PSTs). I intentionally used all of the teachers to refine the CSLHM framework as I did not want to only use "experts". The CSLHM framework includes guiding questions for both emergent and robust CSLHM enactment, which required a broad variety of teachers (with respect to their experience teaching statistics and experience teaching statistics from a critical perspective) to refine the framework and be informed by actual enactment of CSLHM. The second article (Chapter 3) specifically studies the CSLHM enactment of PSTs, thus only the 17 teachers that are PSTs were used and the 13 inservice teachers were excluded. The final article (Chapter 4) focuses on the CSLHM enactment of 2 of the 17 PSTs.

While the methods in each article were different, they all utilized the qualitative semi-structured task-based interview data. The first article (Chapter 2) used the interview data to refine the CSLHM framework (including descriptions and guiding questions). The second article (Chapter 3) followed a multiple case study design (Yin, 2018) that described PSTs' enactment of CSLHM when presented with statistical messages from the media. The third article (Chapter 4) followed an instrumental multiple case study design (Bloomberg & Volpe, 2018). Two cases were selected from the broader study in Chapter 3, one who exhibited typical CSLHM enactment (i.e., few CSLHM and emergently) and the other unusual (i.e., many CSLHM robustly).

This brief methodological overview is intended to provide the needed context of how the three articles draw from the same pol of participants and interviews yet employ different methods and use different subsets of participants.

Statement of Subjectivity

I find it essential to provide my readers with the knowledge they need about me to make sense of how I view CSL and how my experiences and beliefs have shaped the lens I employed for approaching this dissertation. When I began my teaching career in 2005, I infused teaching mathematics for social justice, although I never knew or used that term back then, into my teaching. This infusion is partly because I have always fought for myself and the people in my life in various ways. It is not surprising to me, or my loved ones, that I have decided to use my privilege as a PhD candidate to work towards framing CSLHM and encouraging educators to consider how to nurture CSLHM in their students.

My experiences have shaped who I am, what I believe in, and how I teach. My sister and I both have learning disabilities. Our experiences in school were vastly different. I was fortunate to have a teacher who helped me understand the types of strategies I needed to employ to learn in a system of education that privileges students who are able to capture what they need from traditional schooling. My sister's story is quite different. I can vividly remember the tears my sister shed after publicly being called "lazy" and "stupid" by an elementary school teacher, who I will refer to as Mrs. Neil. Mrs. Neil told my mother that my sister was lazy and stupid because she "passed" the test. Mrs. Neil was referring to the fact that my sister had the aptitude to score right above the threshold on an intelligence test that would have qualified her for special education services. I often think about Mrs. Neil and ponder if opportunities to develop her critical consciousness would have changed how she treated the children in her classes. My sister's educational experience undoubtedly shaped my lens. And, if I am being completely transparent, my sister's horrible experiences in school are exactly why I became an educator in the first place and why I majored in special education and psychology for my undergraduate studies. I believe that every child is intelligent. The experiences we educators and adults provide to our children can spark curiosity and demonstrate the ability of *all* children to learn.

In high school and college, I developed a love for statistics because of its usefulness. When I started teaching after my undergraduate degree, my role was split between running the special education program and teaching mathematics (mostly AP Statistics). My passion for advocating for others and the passion of my students to understand their world led me to focus on real data and issues that unearthed the

inequities. My students and I were more interested in data that centered on important issues like the gender wage gap in the United States than we were by the contrived datasets in the textbook. It appeared that their desires matched my own appreciation for the utility of statistics. I also felt a responsibility to help my students develop the skills they would need to question statistical content and avoid being duped by anyone with ulterior motives. While I did not realize that what I wanted to develop in my students was critical consciousness, this was the beginning of wanting to situate my work in and build upon the work of critical mathematics scholars. I viewed developing my students' critical consciousness (although I did not use that term years ago) on par with developing their statistical literacy and statistical content knowledge. For what good is knowing about statistics if you are not taught to use it with a critical lens, or to consider how your own worldview influences your interpretation of and use of statistics? There are few scholars who have explicitly melded the worlds of statistical literacy and critical mathematics. The work of Travis Weiland (2017) is the one exception I encountered. In thinking about how I could take Weiland's (2017) theoretical work and operationalize it in a way that it would be helpful to teachers, I intentionally decided to explore and focus upon habits of mind since the knowledge valued by society changes over time. Currently there is an explosion of data and an emphasis on statistical literacy as essential. While this is wonderful and unlikely to change, I also think that the dynamic nature of valuing particular types of knowledge implies that education should value habits of mind. We are preparing students to enter jobs which have yet to be realized and this means we need to develop students' skills to tackle the unknown. Focusing on CSLHM works toward this

goal as the CSLHM will persist even if the content knowledge has shifted in value. I share this to illuminate the reasons and moments that crystallized my research agenda.

Given that my research is situated within equity research, I must share with you my relationship to and with equity research. To contextualize how I make sense of equity research, critical mathematics, and statistics, and how I position my dissertation work, I must share my privilege and (un)privilege. Like many, I dance between the intersectionality of my privilege and my societally defined handicaps. I am a white, cisgender, straight passing, seemingly able-bodied, educated, native English-speaking individual. Society deems me (un)privileged as I am a womxn, I am pansexual, I do not identify as a Christian, I have a learning disability, and I have a hidden physical disability (I say hidden as you cannot see the damage to the inside of my dominant hand, but you can observe the pain I experience if you watch me carefully). While I experience many benefits from my privilege, it is important to note that the ways in which I am often deemed (un)privileged by society are mostly things that I can hide. I choose not to hide them anymore, but for years society taught me that it was easy to temporarily suppress or cover up parts of me to navigate the world more comfortably. Being a womxn in a world that defaults to men is my most pronounced (un)privilege. As a survivor of sexual harassment, sexual assault, and gender discrimination, I feel a sense of urgency to disrupt the male default. These experiences, privileges, and societally deemed handicaps have shaped who I am, how I view the world, and most importantly why I think this dissertation work is needed.

With my privilege and perceived handicaps in mind and with my deeply rooted beliefs in teaching mathematics for social justice and developing critical consciousness, I

recognize that knowledge is not neutral. Mathematics and Statistics are not neutral. I firmly believe we are doing a disservice to students if we ignore the inherently biased nature of knowledge. I have grounded my work in the work of scholars who have articulated this non-neutrality clearly (e.g., Frankenstein, 1983; Skovmose, 1994; Weiland, 2017). Given the biased nature of knowledge, it is appropriate and prudent to adopt a critical lens. It is impossible to learn without attending to injustice and equity. How we use statistics and mathematics in the real world (to make decisions and predictions) is inextricably tied to the social context, and social context is inextricably tied to issues of equity and power. By the very nature of making decisions, there are issues of power and equity at play with who is making the decisions, for whom, and why. We attend to injustice, whether consciously or not, and we should help students navigate this viewpoint so that they can more deliberately challenge systems that marginalize and their own implicit biases. I genuinely believe we should be more careful to consciously attend to injustice and equity as researchers. So that is the stance I am taking in this work.

CHAPTER 2: DESCRIBING CRITICAL STATISTICAL LITERACY HABITS OF MIND

Journal

This theoretical article was written as a full report and was submitted to the Special Issue of *The Journal of Mathematical Behavior (JMB)*. Please note that a revised version of this article has been accepted as part of the Special Issue of *JMB.* This Special Issue focused on the topic of *Mathematics in Society: Exploring the mathematics that underpins social issues*. The introduction of the CSLHM and examples from statistics teachers that demonstrate both emergent and robust enactment of each CSLHM would benefit the audience of JMB of researchers and mathematics teacher educators. Specifically, this article provides a potential conceptual and analytical framework that can be used to guide the design of lessons/activities that support the development of CSL or as an analytical framework as the CSLHM and guiding questions teases out the important aspects of CSL related to making sense of statistical messages. Please note that Allison W. McCulloch is listed as second author on this article.

Abstract

Given the vast number of data representations that people encounter daily, it is imperative that people become critical consumers of the representations they will face. This requires the development of particular habits of mind. The purpose of this paper is to share the critical statistical literacy habits of mind framework. We articulate how we drew on the literature related to statistical literacy, critical mathematics, and critical

statistical literacy to identify the habits of mind needed to enact critical statistical literacy in the context of consuming data representations. We describe the refinement process using qualitative interview data. To illustrate what each of the critical statistical literacy habits of mind looks like when enacted, we share examples from statistics teachers making sense of a data representation.

Keywords: critical statistical literacy, habits of mind, learning, framework

Describing Critical Statistical Literacy Habits of Mind

Statistical literacy (SL) refers to both making sense of statistical messages in the real world (consumer orientation) as well as creating and communicating messages supported by statistical investigation (production orientation, e.g., Gal, 2002). Given the numerous data representations that people encounter daily, it is imperative to support people in becoming truly *critical* consumers of such representations. When these actions entail interrogating statistical content to inform action or change with a specific focus on power and equity, it is referred to as critical statistical literacy (CSL; Weiland, 2017).

Interpreting data representations effectively requires developing particular habits of mind, which are the collection of thinking behaviors that when used appropriately lead to sense-making (Costa & Kallick, 2000a, 2000b, 2008). For example, Lee and Tran (2015) described seven statistics habits of mind which included thinking behaviors like "always consider the context of the data," "anticipate, look for, and describe variation," and "be a skeptic throughout an investigation." These habits of mind are helpful for engaging in and describing work related to the statistics investigation cycle (Wild & Pfannkuch, 1999) but are less helpful when consuming data representations created by

others, especially those that center issues of equity or power. To date, the habits of mind research within statistics does not include a critical lens[2]. This missing lens suggests the need for describing habits of mind from the perspective of CSL.

Critical Statistical Literacy Habits of Mind (CSLHM) are the thinking behaviors called upon to make sense of statistical messages with a specific focus on how the statistics and/or statistical message are used to uphold or dismantle structures of inequity. In line with this special issue, the development of such habits of mind provides the field a framework with which to describe the questioning disposition associated with considering how statistics underpins social issues.

The purpose of this paper is to present the CSLHM framework and to provide examples of what each of the CSLHM looks like when enacted. First, we synthesize the literature we drew upon to inform the initial draft of the CSLHM (including literature related to SL, critical mathematics, and CSL). We then describe the refinement process of the CSLHM based on interview data. Next, to illustrate what each of the CSLHM looks like when enacted in context, we share examples of statistics teachers making sense of a dynamic data representation featured in a *New York Times* article on racism (Badger et al., 2018). Finally, we will discuss the analytical and conceptual potential of the CSLHM.

Background Literature

[2] When we state "a critical lens," we are referring to any lens or collection of lenses that center issues of equity or power, thus acknowledge the plurality of critical lenses.

Before moving forward, it is important to clarify what we mean by *critical* as this word has many meanings generally and within scholarship. SL models typically use the word to describe critical thinking (i.e., analysis needed to arrive at conclusions and decisions based on statistical information). Sometimes scholars are explicit in their reference to critical thinking, such as Schield (1999), who defined SL as "a basic skill: the ability to *think critically* about arguments using statistics as evidence" (p. 1; italics added for emphasis). While others use synonymous phrases such as "critically evaluate" (Gal, 2019, p. 2). Consistent across SL models is the use of the word *critical* to indicate analysis to inform one's conclusions drawn or decisions to be made. However, this use of the word *critical* may or may not involve a focus on power.

In contrast, the use of the word *critical* when describing Critical Mathematics or CSL draws from critical literacies and critical theory. While there is variation in the use of the word *critical,* even among critical literacies and critical theories, what is consistent is the emphasis on the relationship between literacy and power (Lankshear & McLaren, 1993). Of the previously discussed SL models, only Gal (2002) drew attention to such use of the word within his dispositional element of *critical stance*. Gal (2002) described how our sociopolitical context shapes the lens through which we view and interpret statistical information. Moving forward, the word critical will refer to how it is generally conceptualized in critical literacies.

Many scholars noted the importance of developing SL (e.g., delMas, 2004; Rumsey, 2002; Utts, 2003). Gal (2002) argued that adults need SL to be "informed citizens and employees" (p. 1). Similarly, Watson (1997) insisted on moving people away from "automatically [believing] everything they read in the media" and beginning to

"intelligently question data and claims" (p. 110). Other researchers have noted that SL entails more than what typically is taught. In describing making sense of statistical messages concerned with civic life (e.g., poverty), Nicholson et al. (2019) pointed out that the skills involved "often [require] understanding of topics and issues that are different from or go beyond the knowledge gained from regular statistics curricula" (p. 2). Statistical messages are ubiquitous, and as consumers we need to be able to navigate such messages.

While there is consensus on the importance of SL, there is not around what is meant by the phrase *statistical literacy.* Yet the literature does reveal a consistent emphasis on understanding basic statistical terminology and concepts, understanding the need for and generation of data, interpreting different representations and conclusions, and acknowledging the potential for data to generate conflicting interpretations (e.g., delMas, 2004; Gal, 2002; Garfield & Ben-Zvi, 2007). The absence of a critical lens in this consensus serves to uphold the notion that statistics is a neutral tool. However, as a human construct, statistics is not neutral (e.g., Frankenstein, 1983; Freire, 1970), and SL should be broadened to consider how the use of statistics can serve to privilege or marginalize. Furthermore, one's personal lens (i.e., their own perspectives, beliefs, and experiences) undoubtedly influences how one interprets statistical messages (e.g., Weiland, 2017).

Critical perspectives within the world of mathematics are more established than that of statistics. Frankenstein (1983) established the construct of critical mathematics by outlining how Paulo Freire's *Pedagogy of the Oppressed* (1970) can be applied within mathematics education in the United States. She suggested that most U.S. mathematics

curriculum positions knowledge as neutral and instead advocated for an understanding of knowledge as a human construct that is both inherently biased and non-neutral. According to Frankenstein (2009), the goal of critical mathematics is to "[understand] how to use mathematical ideas in struggles to make the world better" (p. 111). She, like Freire, focused on letting students dictate what ideas to investigate based on their own interests and values (often referred to as generation of themes). When juxtaposed with goals described by other researchers, a common goal of confronting injustice emerges. For example, Skovsmose (1994) detailed how critical mathematics involves grappling with "social problems, inequalities, [and] suppression" (p. 37) with the goal of making "education an active progressive social force" (pp. 37-38). He stated that mathematical literacy is the true goal of mathematics education since education prepares students for the workforce. Skovsmose wondered if mathematical literacy "[can] be used for the purpose of empowerment because it can be a means to organize and reorganize interpretations of social institutions, traditions and proposals for political reforms?" (p. 39). Ultimately, he argued that mathematics education entails traditional mathematical skills, the application of those skills, and the evaluation of or reflection upon the application. He stressed reflection which includes assessing the "political and social function of applying mathematics to a certain situation" (1998, p. 199). Skovsmose emphasized the role of empowerment and the notion of who wields power, whereas Frankenstein (1983) focused on the role of mathematics to overcome and combat inequity. Consistent in both is that critical mathematics aims to help students learn how to use mathematics as a tool to improve our society.

Teaching mathematics for social justice (e.g., Gutstein, 2003) similarly is driven by investigating inequity and is often used synonymously with critical mathematics. Teaching mathematics for social justice focuses on the development of "sociopolitical consciousness, a sense of agency, and positive social and cultural identities" (p. 40). Like Frankenstein (1983), Gutstein (2003, 2006) emphasized the Freirean use of student direction or generative themes. He advocated for moving away from the traditional passivity of school mathematics in favor of wielding mathematics as a tool for interrogating injustice. Similarly, Gutiérrez (2002) described critical mathematics, or teaching mathematics for social justice, as: "mathematics that squarely acknowledges students are members of a society rife with issues of power and domination" (p. 151). Critical mathematics (including teaching mathematics for social justice) focuses on the relationship between mathematical understanding and challenging the inequity in our world. Thus, critical mathematics asserts that students should learn to use mathematics as a tool to investigate and challenge inequity and consider issues of power and privilege.

Drawing on the work of critical mathematics scholars, Weiland (2017) synthesized the differences between critical literacies and statistical literacy to formulate a CSL framework focused on reading and writing the world. His framework integrates both the consumer (making sense of the statistics of others) and production orientations (when the individual themselves creates the statistical message). He described reading the world as making sense of and evaluating statistical messages that focus on sociopolitical inequity and writing the world as the actions taken to investigate and combat such social inequity. This framework helps us think broadly about the importance of taking a critical lens to SL, something that has been largely missing from previous descriptions.

There are several examples of the adoption of a critical lens within data science literacy (e.g., D'Ignazio & Klein, 2020; Engel, 2017). However, these examples approach data science from a predominantly production orientation. Moreover, there are nuanced differences between SL and data science literacy that are beyond the scope of this paper (for more on this see Hoerl et al., 2014 as an example). With respect to SL, and from an explicitly consumer orientation, the critical lens is largely absent. Though Weiland (2017) has both described and established the importance of CSL, we need to know what CSL looks like when enacted. Like any area of mathematics/statistics, engaging in CSL requires particular habits of mind.

Cuoco et al. (1996) described two classes of habits of mind: general habits of mind that permeate disciplines and content-specific habits of mind which are specific to one particular domain (e.g., mathematics). Here we are focused on one domain: statistics. The scholarship on habits of mind within statistics is limited. Lee and Tran (2015) described habits of mind for statistical thinking detailing the routinized ways an expert would make sense of a statistical investigation. Their statistical habits of mind emphasize critical thinking from the production orientation but do not include the critical lens that is needed for making sense of statistical information in the real world (e.g., informing action or change). Thus, CSLHM aims to fill the void in the literature by describing habits of mind within statistics that have an explicit critical lens. The CSLHM are content-specific habits of mind as they focus explicitly on CSL from the consumer orientation. We define CSLHM as the thinking behaviors called upon to make sense of statistical messages with a specific focus on how the statistics and/or statistical messages are used to uphold or dismantle structures of inequity.

Describing Critical Statistical Literacy Habits of Mind

In this section, we will describe the process used to create the CSLHM framework. Our process was two-fold. First, we share how we drew on literature to create a draft of the CSLHM. Then, we describe how we refined the CSLHM using interview data. This process is detailed in subsequent sections.

Drawing on the Literature to Describe the CSLHM

To describe CSLHM, we began with Gal's SL model (2002) which includes both a knowledge and a dispositional component. While most of his model does not explicitly draw attention to sociopolitical inequities, the dispositional element of critical stance is grounded in the actions people take and the knowledge needed to navigate statistical messages. He acknowledged that "messages aimed at citizens in general may be shaped by political, commercial, or other agendas which may be absent in statistics classrooms or in empirical enquiry contexts" (p. 15), which established the need for consumers to employ a critical stance. Gal described critical stance as playing a vital role in moving from passive interpretation to informed action, specifically by adopting a questioning nature needed to critically assess and examine statistical information. He included a list of "worry questions" designed to elicit and develop a critical stance when interpreting statistical messages.

To describe CSLHM, we began with Gal's list of worry questions. We looked for themes across his worry questions and captured those broad ideas. For example, consider the following two worry questions from Gal's (2002) model:

> (4) What is the shape of the underlying distribution of raw data (on which this summary statistic is based)? Does it matter how it is shaped? (5) Are the reported

> statistics appropriate for this kind of data, e.g., was an average used to summarize ordinal data; is a mode a reasonable summary? Could outliers cause a summary statistic to misrepresent the true picture? (p. 16)

Question five explicitly considers if the statistical measures are aligned with the type of data. Question four considers the shape of the data and how to proceed with analysis given that shape. Both questions connect to whether the statistical measures, procedures, and/or tests align with the data. We identified this theme as *recognizing appropriate statistics*. We continued this process with all of Gal's worry questions, which resulted in the extraction of six themes: noticing potential bias, questioning the sample size and sampling methods, desiring additional information, acknowledging alternate conclusions, recognizing appropriate statistics, and acknowledging ethical considerations. The descriptions of each were informed by Gal's model and refined using Weiland's (2017) framework. The latter was used to ensure that a critical lens was captured within each description.

Gal's (2002) work provided a solid starting point for describing the habits of mind needed to enact CSL, and though we drew on Weiland's work to inform their descriptions, a true critical lens was still missing. As CSL is situated within the broader literature of critical mathematics, the focus on grappling with social inequity is integral. Weiland's (2017) theoretical framework for CSL stressed the importance of placing emphasis on sociopolitical inequity and the actions needed to disrupt and dismantle such inequity. There are two main components to his discussion: (1) wielding statistical messages and statistics to interrogate and actively work towards disrupting and dismantling injustice, and (2) communicating and understanding the influence of one's

own social and political position. Using Weiland's framing, we then described the habits of mind that would be necessary to enact CSL. His two main components informed the development of *recognition of one's own sociopolitical or critical consciousness* and *employing active citizenry*. This process resulted in eight CSLHM (noticing potential bias; questioning the sample size and sampling methods; desiring additional information; acknowledging alternate conclusions; recognizing appropriate statistics; recognition of one's own sociopolitical consciousness; employing active citizenry; and acknowledging ethical considerations) and included identifying broad habits, descriptions, and guiding questions one might ask as they enact each habit.

Using Interview Data to Refine the CSLHM

After we developed our first draft of the CSLHM based on the literature, it was refined through qualitative analysis of interview data in which individuals made sense of a variety of data representations. What distinguishes our process from other habits of mind work was the intentional inclusion of a range of participant backgrounds, not just "experts," in the refinement process. We made this choice because we wanted the final descriptions of the habits of mind to be informed by actual enactment of CSLHM by people with different backgrounds. In the sections that follow we describe our refinement process including the participants, data collection, and data analysis.

Participants

The 30 participants were all secondary or post-secondary statistics teachers. We focused on statistics teachers because they are responsible for teaching such content and thus should have ways of making sense of data representations in different contexts. The 30 statistics teachers had varied experience. Most were either new to teaching statistics

content or new to teaching it from a critical lens (N=26). However, some had considerable CSL experience as they had been teaching statistics from a critical standpoint for at least three years and had previously engaged in some form of scholarship related to issues of equity and social justice (N=4). The teachers were predominantly female identifying (N=20). There were several male identifying teachers (N=10), and none identified as non-binary. The teachers were predominantly white (N=25). There were three Black teachers, one Asian teacher, and one multiracial teacher. None of the teachers involved had experience with the CSLHM prior to the interviews.

Data Collection

The participants took part in a semi-structured task-based interview (Goldin, 2000). The interviews took place on Zoom and were recorded. Participants were asked to make sense of a series of data representations that featured either a static or dynamic graph. Prior to starting the interview, participants were asked to identify two people with whom they honestly discuss the media or news, one with whom they share similar beliefs on social issues and one with whom they do not. This information was used to tailor questioning later in the interview. The interviewer (first author) presented participants with a graph and directed them to think aloud: "As you are making sense of this graph, please share what you notice." After the participants made sense of the graph, they were asked to share what they would discuss with a confidant who held similar beliefs (that they identified earlier) if they were talking about this data representation: "Now imagine that you saw this graph and you are having a conversation with [name]. What would you and [name] likely discuss?" Then they were asked to imagine discussing it with a confidant holding dissimilar beliefs. This process was repeated for all six graphs. As to

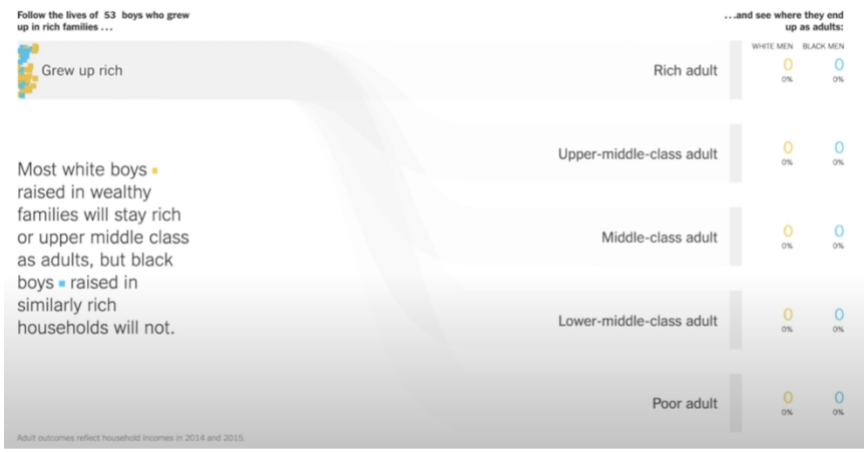
not elicit particular CSLHM, questioning during the interview was limited to clarification or elaboration questions such as “I’m not sure I understand what you mean. Can you explain that one more time or in a different way?” or “Can you tell me more about that?”

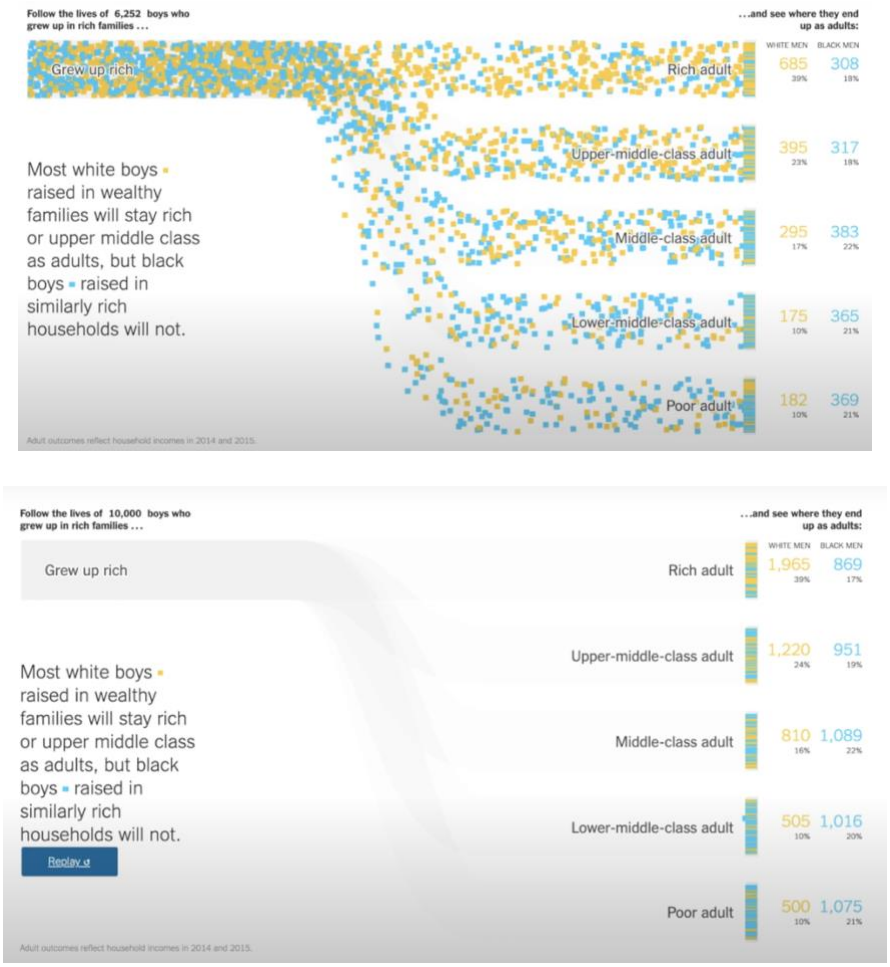
Example Data Representation

As an example, we describe one of the data representations used in the task-based interviews. The representation is from a *New York Times* article on the reach of racism for Black boys (Badger et al., 2018). This dynamic data representation animated 10,000 colored squares, each representing a boy (50% Black boys represented by blue squares and 50% white boys represented by yellow squares) who grew up in a rich household, to their resulting income categories as an adult (see Figure 2.1). The dynamic representation takes approximately two minutes to complete. The teachers were asked to watch the dynamic data representation and talk aloud as they made sense of it, watching as many times as needed. We then proceeded with the rest of the interview protocol (as described above).

Figure 2.1

Snapshot at the beginning, middle, and end of the NY Times data visualization (Badger et al., 2018)





Analysis for Refinement of the CSLHM

We used DeCuir-Gunby et al.'s (2011) framework for the development of an interview codebook as a process for refining the CSLHM based on the interview data. The first stage of our refinement included coding a set of randomly selected interview transcripts using the draft CSLHM as our theory driven a priori codes. We compared our coding and discussed any discrepancies. If discrepancies were due to unclear descriptions, we refined them and then used the revised codebook to code another set of randomly selected transcripts.

Throughout this process, we also watched for the enactment of CSLHM that was not captured by the existing codes. We repeated this process until the codebook was

applied consistently. The first author then coded the remaining interviews independently and employed random spot checks with the second author to ensure consistent coding was upheld. Throughout this process the first author watched for the emergence of additional CSLHM.

Through our analysis of the interview data, no new CSLHM were identified, but two were collapsed and the descriptions (including guiding questions) for others were refined. For example, we came to realize that two of our original habits of mind (noting potential bias and acknowledging ethical considerations) were related to all the others which was why we were having difficulty using those codes consistently. As a result, we incorporated noticing potential bias (i.e., skepticism) and ethical considerations into the existing descriptions of each of the other habits of mind (see Table 2.1 for an example).

Table 2.1

Example of Refinement of Guiding Questions to Integrate Skepticism and Ethical Considerations

CSLHM	Original Guiding Questions	Refined Guiding Questions
Questioning Sample Size and Methods	Is the sample representative of the population?	Is the sample representative of the population? Was the sample intentionally selected to create a statistical message that misleads or deceives?
Desiring Additional Information	N/A	How transparent was the author about the statistical message and/or methods?

In another example, a discrepancy arose when a participant questioned the source, and one of us coded *Questioning Sample Size and Methods* and the other *Desiring Additional Information*. In the original codebook, the source was not mentioned in the guiding questions for *Questioning Sample Size and Methods*, and it was only briefly

mentioned for *Desiring Additional Information* (Table 2.2). We refined the codebook to indicate that *Questioning Sample Size and Methods* included when someone was wondering about the source by wondering about who the people in the study are and how they came to be in the study, whereas *Desiring Additional Information* included if they were wondering about the source in a way that questioned whether they trusted the person or organization sharing the information.

Table 2.2

Example of Refinement of Guiding Questions to Integrate Source Considerations

CSLHM	Original Guiding Questions	Refined Guiding Questions
Questioning Sample Size and Methods	N/A	Where are the people in the sample from? Where is the data from? Who is the source, and do I trust them? *Note: questioning the data and source in these questions refers to the people/sample being studied.*
Desiring Additional Information	Who is the source of the message and/or data?	Who is the source of the message and/or data? Do I trust them? *Note: questioning the data and source in these questions refers to who created/collected the data.*

In refining the CSLHM, it became evident that the level of enactment of a habit varied greatly from vague to very thoughtful. We realized that having a way to describe these differences would be helpful. We deviate from the typical habits of mind literature that employs the use of novices and experts to illuminate the similarities and differences in the ways that habits of mind are enacted (e.g., Goldenberg, 1996), to emphasize the dynamic nature of the ways individuals might enact the CSLHM. We know that enactment can vary across different data representations and contexts based on an individual's prior knowledge and personal experiences (Weiland, 2017). Yet, it is helpful to use contrasting examples to illustrate the robust enactment of a habit of mind. To that

end, we set out to categorize examples of enactment (not individuals) as either emergent or robust.

Using both the literature and the interview data, we created guiding questions to describe the differences between emergent and robust enactment of each of the CSLHM. Emergent enactment of a particular CSLHM is characterized by vague wonderings (i.e., lack of depth and criticality). Robust enactment of a particular CSLHM is characterized by depth and criticality (for the full codebook with guiding questions for emergent and robust enactment see the Appendix A). For example, emergent enactment of the CSLHM *Questioning Sample Size and Methods* would include consideration of who was sampled but may fail to consider who is missing from the sample and why that matters, whereas robust enactment would necessarily include the latter. Quotations for a single CSLHM were extracted and then coded for emergent or robust enactment. Again, we discussed and reconciled any discrepancies. It is important to note that participants often enacted multiple habits of mind at once, thus more than one code might be applied to a quotation. In addition, since we were coding at the quotation level, a participant might have enacted the same CSLHM in different ways at different times. The final product is a set of six CSLHM (Table 2.3; see the Appendix A for the full CSLHM with guiding questions to demonstrate how one might enact each CSLHM).

Table 2.3

CSLHM Descriptions

CSLHM	Description
Questioning Sample Size and Methods	Individual demonstrates healthy skepticism regarding the sample, sample size, sampling technique, sampling bias, or lack of information regarding sampling that may lead to invalid inference on a target population. This includes considering who is missing, why, and how that influences the statistical message and the generalizability of the results, and the potential power of the message.

Recognizing Appropriate Statistics & Appropriate Representations	Individual questions whether the type of statistics and/or the way it is represented is the most appropriate for the data. This includes considering if the data representation employs techniques to mislead or deceive, thus questioning the motivation behind presenting the data in the way it was shared. Individual questions the role of outliers in the given representation. Individual questions whether the conclusions align with the selection of statistical test/procedure.
Desiring Additional Information	Individual demonstrates a need for additional information to draw a reasonable conclusion. Individual demonstrates healthy skepticism of the information, including the type of study, context of the study, source (who collected the data and how), author's motivation for sharing the statistical message (if not connected to appropriateness of the representation), and credentials of the person/people sharing the statistical message. (Note: this does not include desiring additional information of ideas particular to the other habits of mind).
Acknowledging Alternate Explanations	Individual acknowledges the potential for alternative interpretations for the meaning of findings or different explanations for what caused them, e.g., Was there an intervening moderator variable that affected the results? Are there additional or different implications that are not mentioned?
Recognition of One's *Own* Sociopolitical/ Critical Consciousness	Individual recognizes how one is integrating *their own* social, political, economic, etc. understandings to make sense of injustice within the statistical message. Individual recognizes the degree to which one is engaged in critical reflection and critical action/active citizenry (see below). Individual recognizes the gaps in one's knowledge needed to interpret the statistical message.
Employing Active Citizenry	Individual is aware of inequities within the statical message. Individual expresses a desire to disrupt and dismantle inequities. Individual is motivated to act and describes next steps (action includes wanting to research the context, as education is an important part of being an active citizen).

There are two important aspects of the CSLHM framework to discuss. First, the theme of skepticism permeates all the habits of mind. Part of effectively consuming statistical messages is having a questioning disposition (e.g., Gal, 2002). For example, when enacting *recognizing appropriate statistics and appropriate representations* one may question whether the measure (e.g., mean or median) was appropriate, when enacting *questioning sample size and methods* one may question the transparency of the sampling method (e.g., who was surveyed and how were they selected?), or when enacting *acknowledging ethical considerations* one might question if the sample was selected in a manner to intentionally deceive readers. As D'Ignazio and Klein (2020) discussed in *Data Feminism*, healthy skepticism is not sufficient for truly considering

power and inequity, thus the guiding questions were worded to emphasize systemic connections, the potential of statistical messages to further marginalize or privilege and consideration of who the statistical messages serve and who benefits or profits from them. Second, similar to how the development of reasoning about data may not be linear (Konold et al., 2015), how we make sense of statistical messages is not necessarily linear. Therefore, some individuals may enact multiple habits of mind simultaneously, and others may do so one at a time.

Examples of Critical Statistical Literacy Habits of Mind

In the sections that follow, we will provide examples of what enacting CSLHM looks like in practice. It is natural that different data representations might elicit particular CSLHM more than others. For this reason, we discuss the enactment of each CSLHM more generally across a variety of data representations and then provide more detail using the *NY Times* representation (Figure 2.1) as an example. We selected quotations that demonstrated emergent and robust enactment to illustrate the differences in enactment. Given the ways in which individuals intertwine ideas as they speak about a representation, at times more than one CSLHM code was applied to a quotation. To make our illustrations here clear, we chose examples in which only one CSLHM code was applied. When there were multiple examples of a particular habit, we chose those quotations that demonstrated typical enactment most clearly.

Questioning Sample Size and Methods

Questioning the sample size and methods entails skepticism regarding who was sampled, the sample size, the sampling method, and potential sample bias. This includes

considering who is missing from the sample, why, and how that influences the statistical message. Quotations evidencing emergent enactment of this habit of mind typically focused generally on who was sampled and why. Regardless of the data representation, quotations demonstrating emergent enactment of questioning the sample size and methods rarely explicitly focused on measures aimed to reduce bias or the specific sampling methods. These quotations often exhibited wonderment about aspects of the sample that are answered through analysis of the representation. In contrast, quotations evidencing robust enactment of this habit of mind typically exhibited specific skepticism about who was sampled, who was missing from the sample, and why. These quotations also typically employed questioning whether the methodology used would make the sample representative of the population (or not).

With the *NY Times* representation (Figure 2.1), examples of emergent enactment of questioning the sample size and methods had two main themes: (1) general questioning of where the people are from and (2) desire to know if there were an equal number of Black and white boys in the sample (Table 2.4). For example, quotations 9:23 and 20:1 included questions about whether the sample was from a specific geographical location revealing a desire to know more about who was sampled, indicating that the geographical location was an important variable to consider implied a consideration of why those individuals were sampled. Quotations 14:3 and 20:1 included wonderment about the breakdown of Black and white men within the sample; given that there were 10,000 boys represented and the counts were listed for each adult category, it could be determined that there was an equal racial split. Quotation 14:3 included a comment about expecting the split to be equal, demonstrating a common misconception about having

equal numbers in each group instead of a representative sample. Quotations with robust enactment of questioning the sample size and methods, such as quotation 29:8, explicitly tied sampling wonderment to the sourcing and indicated a belief that the *NY Times* has reputable sourcing. However, it is noteworthy that despite this acknowledged faith in the source, quotation 29:8 showed a desire to verify the primary data source to ascertain whether the sample was representative of the population. Representativeness of a sample ties back to the methodological choices that were employed to reduce bias in the sample.

Table 2.4

Examples of Questioning Sample Size and Methods Enactment

Emergent Enactment	Robust Enactment
9:23 Um, okay. Um, where's this data being collected from? Is this, I wonder if this is national or if this is like in a certain city? **14:3** So we don't know if the same number of Black men and white men are being looked at, or actually maybe if we added these up, I just can't. I mean, maybe they are, I would have to, like, maybe add those up to be sure though. Cause you would expect about 5,000 each. **20:1** I wonder where the data comes from. I mean, are they pulling it from the same city or are they, do they have the same sample, same population? Were they all throughout the country or world, or were they just picked in certain regions of the area? I mean, the different variables have got to make a difference here. Were half of them Black, half of them white? Was that a statistic or a, or a variable? I mean, I'd like to know that, uh, that little bit of information too.	**29:8** So one thing I haven't done yet is source any of this. I'm actually kind of just trusting it. Partially because it's the *New York Times*, and generally their editorial board does a decent job of sourcing. But I probably would want to check this because I'm not actually entirely clear where this data comes from. It does make me curious; I'm guessing it's ACS data, American Community Survey, because I'm not sure what other representative sample they would have for this sort of study.

Recognizing Appropriate Statistics and Appropriate Representations

Recognizing appropriate statistics and appropriate representations entails thinking about whether the statistical measures/tests align with the data and if the data aligns with the choice of graphical representation. Quotations evidencing emergent enactment of this habit of mind tended to consider whether the graph was easy to read or not, sometimes including consideration of the choice of statistical measure in a questioning manner (e.g., would a rate be better here?). Quotations evidencing robust enactment typically emphasized the connection between the statistical message and the choice of measure, test, and/or representation. Often this meant assessing if there was a better choice of statistical measure/test to demonstrate the point (e.g., to make a fair comparison, a rate should have been used) while also attending to issues of correlation versus causation.

Quotations with emergent enactment of this habit of mind, such as 3:5 (Table 2.5), focused on the appropriateness of the color choice and the appeal of the movement. Some, such as 14:4, included a comment on the choice of representation and expressed confusion or dislike for how the bars collecting the blue and yellow squares for each income category were muddled, but seemed to trust the creator's choice. This contrasted with quotations demonstrating robust enactment of this habit of mind. Consider quotation 29:2, which pointed to the muddled bars as a poor choice and indicated that a stacked bar graph would be more ideal for comparison. Furthermore, 29:2 included a discussion of how the counts were visually larger than the percents (i.e., font size), but that the percents were the more appropriate measure for comparison. This analysis led to wariness about drawing conclusions given the uncertainty surrounding how well the representation reflects the proportionality and rigor of the calculations. Quotation 29:2 also noted the

power of the representation in demonstrating the literal fall in status from growing up rich to becoming an adult of a lower income class that is demonstrated as the blue and yellow squares fall yet expressed legitimate concern on how this representation was created.

Table 2.5

Examples of Recognizing Appropriate Statistics and Appropriate Representations

Emergent Enactment	Robust Enactment
3:5 Yeah. Um, this, I dunno, I kind of like that they chose yellow and blue, um, for the, the dots, because it shows like you have a pretty yellow bar up here with rich adult and then a little bit more blue. So, it's getting to green, um, down here in the upper-middle-class adult, and then it gets bluer as you go down. Um, so I think any other colors might've been, uh, harder to differentiate, um, cause like I can see that it's yellow and the top one for rich adult and like really blue for poor adult. Um, but it's also, it's really cool to see like the dots move along the screen.	**29:2** I think the visualization is powerful in the sense that you can see it. I struggle with it sometimes though, because there is a lot of black box magic behind the scenes, so to speak, and any time that's happening in something, it makes me somewhat hesitant just because I am guessing. It was probably more of a computer science type person that built this thing, and they may have taken creative license to visualize it in certain ways. And so, I don't know how accurately it matches the data. I do think it has this dramatic effect of the Black boys being the blue dots falling. So, you've fallen in stature and in SES and you've fallen visually speaking. Um, so they're certainly trying to highlight that. I also don't like how they emphasize the counts, but then shrunk the percentage. Because to me what's more important to look at are the percentages. That's what really tells me what's going on. And the percentages are constant. Those aren't changing. Um, at least I didn't notice them changing drastically.
14:4 Okay. We'll go all the way to where it stops. Yeah. So, I also am curious about these bars and the color-codedness of those, I guess it just stacks whenever to kind of show the distribute, but it probably doesn't matter when the colors come just the overall share of who goes where.	This whole color bar nonsense here [referring to bars filled with blue and yellow], um, to me is just useless visualization because I don't know what's, I don't know if it's actually proportional. Why didn't they just turn this into like a stacked bar where there's a yellow segment and a blue segment? I want to intuitively go look there's evidence of systemic racism, but I don't think that there actually is per se, because I don't know enough about how this was calculated to really say that it was done rigorously to say that I could actually conclude something that powerful from it.

Desiring Additional Information

The CSLHM desiring additional information refers to wanting or needing more information regarding the statistics or methodology to be able to make sense of or draw conclusions about the statistical message being presented (that is not captured by other CSLHM). When expressing a need for more information, quotations evidencing emergent enactment of this habit of mind typically included general wonderment, such as asking questions like "why did they do this?" or "when did this happen?" Certainly, such questions are worthy of consideration, but what makes the quotations showing robust enactment of this habit of mind stand out is that these questions were often explicitly connected to why such information was desired. Furthermore, when expressing a need for more information, quotations demonstrating robust enactment tended to include a desire for more information regarding specific aspects of the methodology that were not transparent in the representation. This desire usually included suggesting a search for a source to dig for answers. Often such quotations attended to wanting to know how the variables were defined, what the creators of the data representation meant by specific terms, what the categories represented, or whether the intervals were consistent.

With the *NY Times* representation, quotations evidencing emergent enactment of this habit of mind, such as quotation 19:9 (Table 2.6), included general wonderment about the timeline. This contrasts with quotations showing robust enactment of this habit of mind, such as 29:9, which included wonderment about the timeline with respect to how far back the data went since the representation lacked specific information about the time points (i.e., it only states household income in 2014/2015). Many quotations

evidencing robust enactment included requests to know more about how the variables are operationalized. For example, 14:1 communicated a desire to know more about how racism was being measured. Quotation 29:9 included a desire to know how the income categories were defined and connected this to hesitation about drawing any conclusions since there was a lack of transparency about the variables in this data representation.

Table 2.6

Examples of Desiring Additional Information

Emergent Enactment	Robust Enactment
19:9 What time period was this? And so, before I can, we can have a real conversation. You would have to know certain things.	**14:1** My, the first thing I think about is I wonder how they measure racism and the way that is measurable. **29:9** I'm not sure what timespan it's for, because it would have to be some sort of time series data to go from, grew up rich to what they are as an adult. And all I see down here is "adult outcomes reflect household incomes in 2014 and 15". I would imagine they would have had to have traced back to, to make sense of that. So, I'm not sure like how far back they went. Um, so that starts to raise questions of methodology of where this came from. I'm still really hesitant to make any conclusions here because for one, I don't know what "grew up rich" meant, nor do I know what these categories are or where they came from. I'm assuming there's some sort of income intervals that they're using for this, but I don't know what they are. So that makes it all those questions of like, where did these numbers come from? Make me really hesitant to actually conclude anything from this.

Acknowledging Alternate Explanations

Acknowledging alternate explanations refers to discussing the possibility of other conclusions, explanations, and/or causes for the phenomena in the statistical message. All quotations evidenced curiosity about other variables playing a role and often connected

this to personal experience with the world (thus sometimes simultaneously enacting recognition of one's own critical/sociopolitical consciousness). However, quotations evidencing emergent enactment of this habit of mind tended to detail potential reasons from one perspective, whereas quotations evidencing robust enactment detailed potential explanations from both sides of an issue or argument. Quotations showing robust enactment typically included connections to prior reading, research, or attempts to try to understand the argument from an opposing viewpoint. Furthermore, such quotations also exhibited questioning if those variables were considered and/or controlled for, if information was omitted or not clear, and how transparent the creator of the visualization/message was about their goal in sharing the information (sometimes simultaneously enacting acknowledging ethical considerations).

Quotations evidencing emergent enactment of this habit of mind, such as quotation 18:7 (Table 2.7), evidenced consideration of what could have contributed to the disparity seen in the *NY Times* representation by asking questions such as "What's the underlying cause of this?" or "What is the driving force?" Other quotations, such as 7:9, included examples of factors that may influence how the boys grow up; however, the suggested factors often aligned with one side of a narrative. For example, some quotations discussed if drugs or violence played a role, whereas others suggested education, police brutality, and discrimination as factors. Contrastingly, quotations evidencing robust enactment of this habit of mind, such as 25:7, included a focus on how the creator of this message could clearly articulate how the groups were comparable to strengthen the argument that systemic inequities exist. Specifically, 25:7 included anticipation of some other variables involved and indicated that much could happen in

one's life between childhood and adulthood that may contribute to one's economic status. Additionally, there was an implied discussion of how we can control for such things and communicate to others that such variables were considered.

Table 2.7

Examples of Acknowledging Alternate Explanations

Emergent Enactment	Robust Enactment
7:9 Um, I wonder if that has anything to do with the culture, do you, do you know what I mean? Like, um, I, I guess I'm going to make a generalization, but it's often that you see white parents, especially affluent, they kind of baby their children and I feel like black or African-American families. They often try to push their children to pursue their dreams and to learn from their failures. So, I wonder if that has anything to do with it. Um, I'm thinking like they encouraged their, their child to pursue, um, like veterinarian school or medical school or law school. **18:7** Like I see this, and I don't fully understand it. Um, I mean, I do understand like the numbers, but I don't understand why, like, why is it, um, that 12 percent more of Black men end up poor even when they grew up rich? Like, is it college education? Is it police, uh, discrimination, brutality? Like what's the, what's the underlying cause of this? I really look at what, what made that, that young man that was raised in a rich family, white or Black, what may, what situation, or what instance made that person or that young man become poor as an adult? Was it, was it drugs? Was it, uh, violence? I'm just wondering what, what is the driving force behind that?	**25:7** I'm also thinking about how can we strengthen the argument by making sure that the two groups are comparable. Perhaps we need to know more about other demographic aspects of those families and their professional lives. I know that there's a lot of literature in sociology about all the other factors that would contribute to this. I mean, there's obviously the geography aspect of it. Based on the little I have read, I know a lot of policies have driven the lives of Black persons in certain ways in the US in the last couple of decades. That's, that's sort of the best reference I have, but in terms of, uh, what happens to their lands, where the, uh, gentrification is a term I have read about, um, I don't know too much, but I think I know enough to understand that it would have affected them negatively. I want to know what are, what are the other aspects of this story? Cause there's clearly more to it, and I want to learn more about that. I think one question that I would definitely want to keep in mind is what are the confounders, because this is not enough. Where you started matters, but a lot happens to you during your life before you become an adult, that's going to drive your social economic status when you're an adult, so, yeah.

Recognition of One's Own Critical Consciousness

Recognition of one's own critical or sociopolitical consciousness occurs when an individual considers how they are integrating *their own* social, political, economic, or worldly understandings to make sense of the statistical message, specifically the injustice within the message. It was less common for quotations evidencing emergent enactment of this habit of mind to identify the influence that their own critical consciousness plays in interpreting the data. If present, it was often via a discussion of what had been witnessed in the world. It was more common for quotations evidencing robust enactment to explicitly recognize and/or stop to acknowledge beliefs and experiences and how they influence one's assessment of the statistical message being presented. Some quotations included an implicit discussion of one's stance on the issue being addressed.

Quotations evidencing emergent enactment of this habit of mind, like quotation 18:5 (Table 2.8), included connections between what is seen in the community and the way one perceived the message of the *NY Times* representation. This quotation evidenced acknowledgment that the wealth gap is more commonly discussed from the vantage point of lower-class individuals struggling to improve their income status, whereas this representation focuses on those who grew up wealthy. This starkly contrasts quotations showing robust enactment of this habit of mind, such as quotation 25:4, which identified deep care about dismantling racial injustice and that it would be hard to convince such a person that such injustice does not exist. Perhaps more importantly, it was common for quotations evidencing robust enactment, like 29:13, to intentionally note that an analysis of this particular representation should not be driven by these beliefs.

Table 2.8

Examples of Recognition of One's Own Critical Consciousness

Emergent Enactment	Robust Enactment
18:5 Um, so I guess when I think of the wealth gap, I'd always thought of, like, I had never thought of it from this angle. So, I'd always thought that, I guess the reason there was such a wealth gap, um, between like Black Americans and white Americans was that, uh, Black Americans had never like broken out of the lower class, like the poverty shell, which not all this is, but, um, like inner city, they had been in a poorer. Cause that's what I see, not with Black Americans, but with white Americans around, we don't have the percentage of Black people in our town is very small. Um, but for white Americans around me, the primary, primarily what I see is lower class families staying in that lower class, um, area and not, and I think this is the conversation I'll be having with probably, yeah, the people that I'm, that I share beliefs with around me, this would be this conversation. So, what I had mostly seen was, um, people in, white Americans, low, low to middle class or poor staying in that, in my town. And so, when I had thought of the wealth gap, I was always thinking of kind of that group of people and then Black people who were also initially in the lower class or poor class staying in that class.	**25:4** I'll start by recognizing that this is an issue I care about deeply. And in this particular moment, unless I actually have somebody who can give me an informed opposing argument, I'm not going to bother considering it, just because of this issue and how, how, important I think it is. **29:13** I want to intuitively go look there's evidence of systemic racism, but I don't think that there actually is per se, because I don't know enough about how this was calculated to really say that it was done rigorously to say that I could actually conclude something that powerful from it. And I really don't want to fall into the pitfall of making conclusions that match what my beliefs are without actually making sure that the data was telling me that's what what's happening. Um, because you know, bad data goes, knows no political affiliation. They, they both do it. Everybody does it. Um, everybody's biased. So, I'm still hesitant to make any kind of firm conclusions here.

Employing Active Citizenry

Employing active citizenry occurs when individuals consider their own actions as a result of making sense of the statistical message. While everyone enacts active citizenry in different ways, all examples of this CSLHM in this study acknowledged inequity and expressed some desire to either learn more or converse on the topic with the goal of challenging someone who does not see or acknowledge the inequity. Quotations

evidencing robust enactment of this habit of mind tended to include a desire to research in order to better understand the context with the goal of being able to better "dig into" the statistical analysis. Similarly, to supplement the desire to engage in discussion, there was frequent inclusion of discussing precisely how they will do so. Quotations evidencing robust enactment explicitly demonstrated how one could or could not use a data representation to challenge the views of others.

Quotations showing emergent enactment of active citizenry, such as quotations 18:6 and 19:17 (Table 2.9), demonstrate the different types of active citizenry enactment regarding the *NY Times* representation. First, we saw a desire to learn more through research with expressed confusion as to why this disparity exists. Second, we saw some quotations (19:17) that expressed a desire to use the data representation to start a conversation. While not explicit, there is an implied sense of using the representation to press on the views of others. Finally, we saw other quotations (18:6) include explicit details of how one could challenge the views of individuals who believe that racial injustice is attributed to individual choice. These examples demonstrate the variability typically evidenced within quotations showing emergent enactment. In contrast, quotations evidencing robust enactment of active citizenry, such as quotation 25:9, illuminated the level of detail used to describe how one might enact active citizenry that was common among quotations evidencing robust enactment. This quotation details a specific plan for how to engage in pressing on another's views to help the other acknowledge the inequity and think more deeply about the reasons for such inequity. The quotation included anticipating that such a person would draw on anecdotal evidence and a consideration of how one might combat such an approach.

Table 2.9

Examples of Employing Active Citizenry

Emergent Enactment	Robust Enactment
18:6 And I had never realized that even for Black, um, like Black men that grow up rich, there's such a spread that like less Black men that grew up rich stay rich than white men. Like I just didn't realize that spread. And I, I don't really have, I mean, I don't have an explanation or a full understanding, so I think this would just leave me like the conversation I would have just kind of, do you see this or understand this? or, um, just kinda start I would start researching, I guess. That's kind of what I do when I don't really get something, is just try to research what's going on… You know, I think a lot of people, I think with people that have more ultra conservative views than I am, there would be like, probably go back to like the personal choice narrative, um, that they've, these Black men have probably made personal choices that have kept, caused this. Um, and that's where I would say. And then I think I would say, but that doesn't like, it's not that there's something in Black men that are going to cause them to make personal choices that cause them to be poorer than white like that doesn't pan out over this [inaudible] data for one or, you know, for a small handful. Everybody's personal choice is going to affect to some point where they end up on this, but it doesn't with these staggering of percentages. You can't all be about personal choice. Like it's not the white men are just making that much better choices. Um, then Black men like that just doesn't like, when you look at these numbers that doesn't pan out. **19:17** I'm going to have to share this graphic and stir up some conversations. No, I wonder, I wonder if people are ready to have these tough conversations. And I wonder if this, if people are really bothered by things like this? You know, because I could, I could see how somebody, you know, when look at this, you	**25:9** And with mom, economics, ooh, with mom, this could be a heated discussion. Um, I think this piece, this piece of, um, causation versus association, wouldn't be obvious to her. And I think I'll have to spend some time, reminding her that the social demographics of the family you're born into don't define who you are. The parts shape your life in many, many ways. Some of them positive, some of them negative, but that's not who you are. I think just because of cultural references, the question of crime would come into the discussion. Like when we talk about rich Black families, where does it come from? And that's not the answer to that question. So, I think there'll be a lot of resistance to, resistance from her to the idea that neither the racial makeup of your parents, nor the economic situation of your parents should themselves define where you end up when you grow up. That just because somebody might be born into a Black family, and if we forget that socioeconomic status doesn't mean that yeah, sure, they're just going to be poor. I think I'll be able to get across to her that there's just a lot of questions to ask. And the question is not very simple. I think the piece that I will have a resistance to is if she tries to provide anecdotal evidence. I mean the relative discussion may not be about race. It would be

know, with, and indifferently, if, if, especially if they're not rich, it's like, oh, well, this is just a rich people.	about caste, but in a lot of ways the issues are similar, the questions are similar.

Discussion

To address the need for operationalization of CSL in the context of consuming statistical messages (e.g., Callingham & Watson, 2017; Cannon, 2020; Weiland, 2017), we set out to define CSLHM. Drawing on both the SL and CSL literature (delMas, 2004; Gal, 2002; Garfield & Ben-Zvi, 2007; Rumsey, 2002; Utts, 2003; Weiland, 2017) along with the more established field of critical mathematics (e.g., Frankenstein, 1983; Gutstein, 2003; Skovsmose, 1994) we aimed to make explicit what enacting CSLHM look like in practice. When comparing the CSLHM to the more established field of critical mathematics, there are parallels. Similar to Frankenstein's (1983, 2009) emphasis that mathematics is inherently biased, the CSLHM acknowledges the non-neutrality of making sense of statistical messages by evoking skepticism related to the motivation of the author or consideration of one's own beliefs in how they perceive the message. Skovsmose (1994) and Gutiérrez (2002) pointed to the need to grapple with social issues and issues of power. The CSLHM illuminates the need to interrogate the statistical message's connection to inequity and the need to consider what resulting actions one needs to take as a result of that knowledge. These roots in critical mathematics demonstrate how the CSLHM has integrated a true critical lens into SL.

The distinct differences among quotations that evidenced either emergent or robust enactment of a particular CSLHM was evidence of its power as an analytical

framework. As can be seen in the examples above, it is a useful analytical tool for teasing out the important aspects of CSL related to reading the world (e.g., Gutstein, 2003; Weiland, 2017), something that was missing in the literature. Additionally, the examples provided suggest that the six elements described in the CSHLM are a helpful grain size for identifying the differences in the ways that individuals enact each of the individual elements. The guiding questions for robust enactment aligned with Weiland's Freirean vision of reading and writing the world which emphasizes both sense-making and action as it relates to issues of power and equity.

It is important to note that there are several limitations to this work. First, the sample was voluntary and not random. It is possible that the teachers who participated were all motivated by the statistical nature of the call, by sharing their thinking aloud, or simply by wanting to contribute to research. It is possible that teachers who chose not to participate would have enacted the CSLHM differently. Second, we focused on statistical messages from tweets, thus we cannot extrapolate CSLHM enactment to other types of statistical messages. Future research should aim to both include a broader audience of participants and different types of statistical messages to further refine the framework.

In response to the continued call for students and teachers to engage in SL with a critical lens (Weiland, 2017) and research that suggests that this is an area in which students need support (e.g., Callingham & Watson, 2017), we envision the use of CSLHM to guide the design of instruction for both K-16 students and their teachers. Since the CSLHM describes each habit of mind and provides guiding questions, we see the potential for CSLHM to support the intentional development of activities that elicit and/or unpack each CSLHM and specifically those aspects more common to emergent

enactment. Such activities can nurture the understanding of the connections between statistics and society, specifically the notion that neutrality is impossible and that all statistical messages are human constructs tied to issues of power, privilege, equity, and bias. Future research should endeavor to reveal the potential of CSLHM to be used in this way. As one potential limitation of this work is our focus on graphs from the media, it is possible that consuming statistical messages in different mediums (e.g., news broadcasts, podcasts, or radio) warrants further research.

Nicholson et al. (2019) pointed out that the skills involved in making sense of social issues and statistical messages are often beyond the scope of the statistics curriculum. The CSLHM is a potential framework to begin closing this gap. As habits of mind, the CSLHM holds the potential to develop domain-specific content in a way that is sustainable for students. As Goldenberg (1996) emphasized, habits of mind have a cross-curricular influence. He explained that as educators prepare students for jobs that do not yet exist, rather than placing too much emphasis on content that may become extinct, educators should focus on the habits of mind needed to effectively navigate the content. Goldenberg further argued that if multiple domains focus on teaching habits of mind, students are likely to make connections across those domains. As such, it is our hope that the CSLHM can foster cross-curricular connections and nurture statistical content. Future research should endeavor to determine if the CSLHM wields similar and increased benefits of habits of minds.

We envision that the CSLHM is not only helpful from the consumer orientation of CSL but also from the production orientation of CSL. Wild and Pfannkuch's (1999) work on SL not only described the statistical investigative cycle of SL which aligns with the

production orientation, but they also introduced the interrogative cycle (i.e., generate, seek, interpret, criticize, judge), which aligns with the consumer orientation. They argued that these two cycles are synergistic, suggesting that habits of mind from one could (should) inform the other. For example, consider Cannon's (2020) study, which showed that inquiry-based lessons in middle grades mathematics that began with engaging students in interrogating a data-based argument produced by the media (i.e., consumer orientation) could serve as a segue into exploring a dataset related to that context before moving on to the investigative cycle (i.e., production orientation). Cannon's work shows the potential of connecting the two orientations of SL. Using the CSLHM to interrogate data-based arguments from the media on sociopolitical issues can provide a launch into a statistical investigation of critical and relevant topics. We speculate that the CSLHM can foster rich discussion of societal issues, support the development of statistical knowledge that underpins social issues, and provide a skill set that would translate to thinking more critically within the context of statistical investigations. Future research should consider the potential of CSLHM to be used in this way.

By describing CSLHM, we are advocating for emphasis on critical statistical analysis from a consumer perspective. Additionally, by demonstrating the potential CSLHM holds as both an analytical and conceptual framework, it is our hope that researchers will continue the conversation and conduct much needed research on how people develop CSLHM and more specifically, how we can support them to become robust enactors.

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Appendix A

CSLHM Framework - Descriptions and Guiding Questions

Questioning Sample Size/Methods		
Description	**Emergent Guiding Questions**	**Robust Guiding Questions**
Individual demonstrates healthy skepticism regarding the sample, sample size, sampling technique, sampling bias, or lack of information regarding sampling that may lead to invalid inference on a target population. This includes considering who is missing, why, and how that influences the statistical message and the generalizability of the results, and the potential power of the message.	1. Were the sampling methods discussed? 2. Who was sampled and why? 3. How many were sampled? 4. The sample feels biased. 5. Were measures taken to reduce bias? 6. The sample was too small/ /large/convenient? 7. Discuss "cherry picking" without explicitly considering representation within the sample. 8. Where are the people in the sample from?	1. Were the sampling methods discussed? AND if not, why? 2. Who was sampled and why? AND Who is missing and why? Does that influence the results? 3. Could non-response or other sampling issues influence this data or the generalizability of the results? 4. How many were sampled AND why? 5. Were measures taken to reduce bias? 6. Was the sample too small? Too large? Convenient? AND why this matters? 7. Is the sample representative of the population? AND/OR was the sample intentionally selected to create a statistical message that misleads or deceives? 8. Where are the people in the sample from? Where is the data from? Who is the source, and do I trust them? (Note: questioning the data and source in these questions refers to the people/sample being studied)
Appropriate Stats and Appropriate Representations		
Description	**Emergent Guiding Questions**	**Robust Guiding Questions**
Individual questions whether the type of statistics and/or the way it is represented is the most appropriate for the data. This includes considering if the data representation employs techniques to mislead or deceive, thus questioning the motivation behind presenting the data in	1. Are the reported statistics appropriate for this kind of data? AND does not justify why. o Why was the mean used instead of the median or vice versa? o Was a mean used to describe ordinal data? o What is the influence of outliers on the statistics used? 2. n/a 3. n/a	1. Are the reported statistics appropriate for this kind of data? AND justifies why. o Why was the mean used instead of the median or vice versa? o Was a mean used to describe ordinal data? o What is the influence of outliers on the statistics used? 2. Is correlation confused with causation?

the way it was shared. Individual questions the role of outliers in the given representation. Individual questions whether the conclusions align with the selection of statistical test/procedure.	4. n/a 5. Do the variables measure what they are intended to measure? 6. Is the visualization appropriate for this kind of data and/or statistics? AND does not justify why. o Is the type of graph appropriate for the data? o Are the comparisons being shown appropriate? (e.g., comparing counts/frequencies vs. percentages) 7. Are the scales appropriate? Are the intervals/bins appropriate? 8. n/a 9. Were any techniques employed that are used to sway readers' opinions? For example, was the graph truncated in a way that exaggerates differences between groups? Or was the scale manipulated? AND does not justify why this seems like it is misleading or deceiving.	3. Is the size of the difference described appropriately? 4. Is there evidence of Simpson's Paradox at play? 5. Do the variables measure what they are intended to measure? AND if so, who do they serve being measured in the way they are? 6. Is the visualization appropriate for this kind of data and/or statistics? AND justifies why. o Is the type of graph appropriate for the data? o Are the comparisons being shown appropriate? (e.g., comparing counts/frequencies vs. percentages) 7. Are the scales appropriate? Are the intervals/bins appropriate? 8. Why might the author of this message have chosen to display the information they did? 9. Were any techniques employed that are used to sway readers' opinions? For example, was the graph truncated in a way that exaggerates differences between groups? Or was the scale manipulated? AND justifies why this seems like it is misleading or deceiving.
Desiring Additional Information		
Description	**Emergent Guiding Questions**	**Robust Guiding Questions**
Individual demonstrates a need for additional information to draw a reasonable conclusion. Individual demonstrates healthy skepticism of the information, including the type of study, context of the study, the source (who collected the data and how), the author's motivation for sharing the statistical message (if not connected to appropriateness of the	**Expressing a need for more information without justification as to why they want or need this information to make sense of the data rep** 1. Do I need to know more about the methods used? (without explaining why) 2. n/a 3. Do I need to know more about *how* the variables are defined/operationalized? 4. Was vital information about the context of the study omitted (e.g., when was the study performed)? Without explaining why that information is needed	**Expressing a need for more information with justification as to why they want or need this information to make sense of the data rep** 1. Do I need to know more about the methods used? AND explaining why 2. Do I need to know more about how and if the assumptions were met? 3. Do I need to know more about *how* the variables are defined/operationalized AND why? AND Do I need to know more about if there are other ways to measure these variables that are more equitable?

representation), and the credentials of the person/people sharing the statistical message. **This does not include desiring additional information about the sample as that is captured in the *Questioning Sample Size/Methods* **This does not include desiring additional information about the construction of the data representation as that is captured in *Recognizing Appropriate Statistics* & Appropriate Representations. **This does not include desiring additional information related to the context as this is captured in *Employing Active Citizenry*	5. How transparent was the author about the statistical message and/or methods? 6. How transparent was the author(s) about their personal lens (e.g., political affiliation or other beliefs) and/or motivation? 7. Vaguely questioning the source (e.g., I do/don't trust them) 8. Vaguely questioning the credentials of the individual sharing the statistical message. Who is this person? Are they qualified? 9. Vaguely wondering who the message serves	4. Was vital information about the context of the study omitted (e.g., when was the study performed, does it only represent one slice in time)? AND explaining why that information is needed. 5. How transparent was the author(s) about the statistical message and/or methods? AND why does that matter? 6. How transparent was the author(s) about their personal lens (e.g., political affiliation or other beliefs) and/or motivation? AND why does that matter? 7. Who is the source of the message and/or data? Do I trust them? (Note: questioning the data and source in these questions refers to who created/collected the data) AND explaining why that matters. 8. What are the credentials of the individual sharing the statistical message? AND why does that matter? 9. Who does this message serve? AND why that is important
Acknowledging Alternate Explanations		
Description	**Emergent**	**Robust**
Individual acknowledges the potential for alternative interpretations for the meaning of findings or different explanations for what caused them, e.g., Was there an intervening moderator variable that affected the results? Are there additional or different implications that are not mentioned?	**Only includes alternative explanations from one perspective/narrative** 1. Are there alternate interpretations that can be gleaned from the message? 2. Are there other variables that play a role that should have been considered and were excluded? 3. Did the author of the message fail to include vital information (e.g., did not address lurking or confounding variables) that I would need to make an informed decision?	**Includes alternative explanations from different perspectives/narratives AND/OR explains why those alternate explanation influence how the statistical message is interpreted** 1. Are there alternate interpretations that can be gleaned from the message? 2. Are there other variables that play a role that should have been considered and were excluded? 3. Did the author of the message fail to include vital information (e.g., did not address lurking or confounding variables) that I would need to make an informed decision?
Recognitions of One's Own Sociopolitical/ Critical Consciousness		

Description	Emergent	Robust
Individual recognizes how one is integrating *their own* social, political, economic etc. understandings to make sense of injustice within the statistical message. Individual recognizes the degree to which one is engaged in critical reflection and critical action/active citizenry (see below). Individual recognizes the gaps in one's knowledge needed to interpret the statistical message. **Simply stating one's beliefs is not enacting this CSLHM	1. n/a 2. n/a 3. n/a 4. Do I recognize if I am applying a particular lens (e.g., feminism)? 5. What is my understanding of this particular context? Are there gaps?	1. Am I considering the perspective of others? 2. How does my life experience shape how I think about this message? 3. How does my identity influence how I think about this message? 4. Do I recognize if I am applying a particular lens (e.g., feminism)? If so, how does that lens shape how I think about this message? 5. What is my understanding of this particular context? Are there gaps? Why is that important in how I make sense of the statistical message?

Employing Active Citizenry

Description	Emergent	Robust
Individual is aware of inequities within the statical message. Individual expresses the desire to disrupt and dismantle inequities. Individual is motivated to act and describes next steps (action includes wanting to research the context, as education is an important part of being an active citizen).	1. What do I need to read/research about to understand the context better? 2. Does this message unearth injustice or lack of equity (explicitly or implicitly)?	1. What do I need to read/research about to understand the context to be able to appropriately make sense of this message? Do I need to consult someone/an expert to make sense of the context, or the mathematics involved in understanding this statistical message? 2. Does this message unearth injustice or lack of equity (explicitly or implicitly)? How is this statistical message being used? Is it serving to further marginalize or privilege? Who benefits and/or profits from this statistical message? And more importantly, what are my actions in response to that? ○ How can this message be used to promote the dismantling or disruption of inequity? More specifically, what are my next steps? (e.g., do I need to read more on the topic to understand it? Do I need to write to a

		legislator? Do I need to spread the word about this issue? Can I share this with friends or family to push on their views?) o And conversely, could this message serve to uphold inequity in some manner? Or serve to maintain systems that marginalize? If so, what are my next steps? 3. How will I press on others' views in conversations about this statistical message? Is it worth engaging? In what context would I decide not to engage and why?

CHAPTER 3: DESCRIBING PRESERVICE SECONDARY MATHEMATICS TEACHERS' CRITICAL STATISTICAL LITERACY HABITS OF MIND ENACTMENT

Journal

This article was written as a full report and will be submitted to the *Journal of Mathematics Teacher Education* (JMTE). The goal of JMTE is to share research aimed at improving mathematics teacher education. I believe that the results of this study would be of interest to and benefit the JMTE audience of statistics and mathematics teacher educators. With rising emphasis on CSL across national organizations and standards documents (e.g., Bargagliotti et al., 2020) and within research (e.g., Weiland, 2017), it is imperative that the field aim to better understand how PSTs enact CSL. This will enable future researchers to consider how to support PSTs in developing CSLHM and consequently how to foster such habits in their future students. Please note that Allison W. McCulloch is listed as second author on this article.

Abstract

Standards documents emphasize statistical literacy from the consumer orientation (e.g., making sense of data representations from the real world). Making sense of real world statistical messages requires the adoption of a critical lens (e.g., focus on power and equity). How statistics are wielded and presented in the real world cannot be separated from the fact that social issues operate within systems of marginalization, privilege, and power. This study aims to explore how preservice secondary (middle and high school) mathematics teachers enact critical statistical literacy habits of mind when engaging with a statistical message from the media. Findings reveal that preservice teachers typically emergently enact CSLHM, and some enact particular CSLHM robustly.

Keywords: critical statistical literacy, habits of mind, preservice mathematics teachers

Describing Preservice Secondary Mathematics Teachers' Critical Statistical Literacy Habits of Mind Enactment

The Common Core State Standards (National Governors Association Center for Best Practice & Council of Chief State School Officers, 2010) initiated an emphasis on statistics and representations of real data sets across the middle and high school standards. More recently, individual states have moved to include an even larger focus on statistics and data science in their standards, including from the consumer orientation of statistical literacy. The production orientation (i.e., the creation of statistical messages) has traditionally been emphasized in schools. As the production involves engagement in the statistical investigative cycle (Wild & Pfannkuch, 1999), it can be calculation driven.

In contrast, the consumer orientation encompasses making sense of statistical messages in the real world and aligns with Wild and Pfannkuch's (1999) interrogative cycle (i.e., generate, seek, interpret, criticize, judge) which emphasizes questioning. As an example of including the consumer orientation within standards, consider the recently adopted standards for Integrated Math 4 in North Carolina where SP.1.4 reads: "Interpret non-standard data visualizations from the media or scientific papers to make sense of real-world phenomena" (North Carolina Department of Public Instruction, 2020, p. 20). Similar emphasis is evidenced in standard and curriculum revisions nationwide and is particularly evident in California's new push for the inclusion of data science and big data (e.g., Gewertz, 2020).

During this push for statistical literacy (SL; e.g., Gal, 2002), instructional routines including *data talks* (e.g., Boaler et al., 2021), *notice and wonder* (e.g., Rumack & Huinker, 2019), and *slow reveal graphs* (e.g., Laib, n.d.) are being promoted and adopted in K-12 to develop SL, specifically from the consumer orientation. Such instructional routines and resources are imperative and provide students with meaningful ways to engage with statistics from the consumer orientation. However, we must also consider that as facilitators of such discussions, teachers must have robust statistical knowledge, SL, and critical SL to be able to effectively build upon student wonderings to make such discussions fruitful. It is important to clarify that the use of the word *critical* in this paper refers to the emphasis on the relationship between literacy and power (Lankshear & McLaren, 1993). Moving forward, the word critical will refer to how it is generally conceptualized in critical literacies. Given that research has indicated that preservice mathematics teachers (PSTs) are not prepared to teach statistics (Lovett & Lee, 2017,

2018), do not feel prepared to teach statistics (Banilower et al., 2013, 2018), and do not feel comfortable teaching or discussing issues of social justice (e.g., Simic-Muller et al., 2015), we need to better understand how PSTs are making sense of statistical messages from the real world. Particularly to support PSTs to develop the knowledge and confidence to teach statistics from the consumer orientation.

Critical mathematics scholar Skovsmose (1998) argues that "social, political, and economic interests can be pursued by means of the powerful language of mathematics" (p. 197). How the use of statistics is wielded and presented in the real world cannot be separated from the fact that social issues operate within systems of marginalization, privilege, and power (e.g., D'Ambrosio, 1994; Frankenstein, 1983). Thus, it follows that making sense of statistical messages should involve a critical lens. Therefore, the call for and inclusion of the consumer orientation entails the enactment of critical statistical literacy habits of mind (CSLHM; Bailey, Chapter 2). CSLHM are the thinking behaviors called upon to make sense of statistical messages with a focus on how the statistical message is used to uphold or dismantle structures of inequity. As students are expected to consume statistical messages from the real world, it is vital that PSTs develop robust CSLHM. Before the field can consider how to support PSTs in developing CSLHM, we must understand how they enact CSLHM. This study aims to explore how PSTs (middle and high school) enact CSLHM when engaging with statistical messages from the media.

Background Literature

To provide a background, we begin by providing an overview of what we know about research on pre and inservice teachers statistical reasoning and statistical thinking

as there is overlap with SL. We then share research on SL, CSL, habits of mind, and the CSLHM. Finally, we share what is known about teachers' SL and CSL. Many scholars consider SL and data literacy as synonymous (e.g., Gould, 2017). However, since data literacy can be viewed as a specific subset of SL in which teachers and policy makers apply SL to educational data to inform decision making (e.g., Mandinach & Gummer, 2013), it does not align with the consumer orientation that we are studying (i.e., making sense of data representations from the media), thus we did not include it.

There are numerous studies that explore in-service teachers' and PSTs' statistical reasoning and/or thinking (for more on the nuanced differences between SL, statistical reasoning, and statistical thinking see Ben-Zvi & Garfield, 2004; Britz et al., 1996; delMas, 2004; Dransfield et al., 1999; Mallows, 1998; Moore, 1997; Snee, 1990). While these studies do not explicitly focus on SL, they provide some insight into how teachers reason about statistical ideas central to SL. Research has indicated that teachers (both in-service and PSTs) have difficulties with several statistical concepts such as conceptual understanding of the mean and median (e.g., Estrada et al., 2004; Leavy & O'Loughlin, 2006), interpretation of graphical representations (e.g., Bruno & Espinel, 2009), understanding variation in data collection (e.g., Makar & Confrey, 2005), understanding variation in chance contexts (e.g., Canada, 2006), distribution (e.g., Ciancetta, 2007), understanding sampling (e.g., Watson, 2000), and interpreting statistical inference (e.g., Liu & Thompson, 2009). Based on a study of 221 PSTs, Lovett and Lee (2017, 2018) indicated that PSTs lack a robust understanding of the statistical content taught at the secondary level. While they did point to some strengths, such as identifying the appropriate measure of center for a particular context, there were more areas of need,

including conception of variability, sampling distributions, and inferential statistics. What this means is that teachers need more opportunities to develop their statistical knowledge, and while these specific studies did not focus on SL, they do provide additional support for developing SL among PSTs.

The literature on SL reveals consensus on the importance of SL and less unity on what SL entails. Despite this lack of unity, the literature does reveal a consistent emphasis on understanding the basic statistics terminology and concepts, understanding the need for and generation of data, interpreting different representations and conclusions, and acknowledging the potential for data to generate conflicting interpretations (e.g., delMas, 2004; Gal, 2002; Garfield & Ben-Zvi, 2007; Kaplan & Thorpe, 2010). This emphasis reveals a strong focus on purely statistical content and an absence of a critical lens. This absence serves to uphold the notion that statistics is a neutral tool which is problematic since statistics is a human construct and thus inherently not neutral (e.g., Frankenstein, 1983; Freire, 1970). SL must consider how the use of statistics can serve to privilege or marginalize. Furthermore, an individuals' personal lens (i.e., their own perspectives, beliefs, and experiences) undoubtedly influence how they interpret statistical messages (Weiland, 2017).

Critical Statistical Literacy (CSL) integrates SL with the importance of taking a critical stance. Weiland (2017) synthesized the differences between critical literacies and SL to formulate a CSL framework focused on reading and writing the world (drawing from the work of Freire, 1970). His framework integrates both the consumer and production orientations. Weiland described reading the world as making sense of and evaluating social justice statistical messages and writing the world as the actions taken to

investigate and combat social inequities. This framework addresses the missing critical lens from the SL literature and demonstrates the importance of taking a critical lens. Weiland explicitly discussed the need to study the development of students' CSL, and the need of the mathematics teacher educators to provide PSTs with opportunities to read and write their world using statistics in contexts connected to their politicized world.

Like any area of mathematics or statistics, engaging in SL or CSL requires particular habits of mind. Habits of mind describe the thinking behaviors that experts employ with ease and frequency to problem solve (Goldenberg, 1996). There are general habits of mind that cross disciplines and content-specific habits of mind that are distinct to a domain (Cuoco et al., 1996). Goldenburg (1996) suggested that as we prepare students for jobs that are yet to exist, developing habits of mind in conjunction with content focuses on developing the needed sensemaking to tackle novel problems. Providing students with a repertoire of thinking behaviors is more sustainable than focusing on content that may expire as technology and job demands are dynamic. While habits of mind research within mathematics has flourished, there are few examples within statistics. Lee and Tran (2015) described habits of mind for statistical thinking from a production orientation focused on how experts would make sense of a statistical investigation. To operationalize the work of Weiland and previous SL models, Bailey (Chapter 2) created a CSLHM framework. The CSLHM are content-specific habits of mind as they are focused explicitly on CSL from the consumer orientation. Thus, CSLHM are the thinking behaviors called upon to make sense of statistical messages with a focus on how the statistical message is used to uphold or dismantle structures of inequity.

While there are a larger number of studies that investigate how K-12 students or adults enact SL, there are few that specifically focus on mathematics/statistics teachers' (PSTs or in-service) SL and none that explicitly study teachers' CSL or enactment of CSLHM (e.g., Watson & Callingham, 2003, 2005). The studies on elementary preservice mathematics teachers (EPSTs) reveal concerns. Guven et al. (2021) conducted a study of EPSTs at four different universities. They indicated that overall, the EPSTs evidenced low SL. They cited a number of difficulties among EPSTs including considering context. They did not comment on many strengths, however, they did note that the EPSTs performed better on the SL component connected to sample selection. Ultimately, Guven et al. (2021) raised the concern about the ability of EPSTs to engage in statistical investigation without providing opportunities to more fully develop SL. Nahdi et al. (2021) reported mixed results regarding the SL of EPSTs. They indicated that while the EPSTs demonstrated strong understanding of data and the visualizations, they exhibited challenges when drawing a logical argument or conclusion from the data and data representations.

Similarly, at the secondary level, there are concerns regarding teachers' SL. Tak et al. (2017) examined PSTs' understanding of sampling using Watson's (1997) statistical hierarchy. They found that PSTs often disregarded the context, which is an essential component of SL (e.g., Gal 2002, 2019). Furthermore, taking university level statistics content courses did not appear to support PSTs' SL development (Tak et al., 2017). Other researchers have noted concerns with the translation of SL into teachers' lessons. Muñiz-Rodriguez et al. (2020) commented on how the secondary mathematics teachers in their study relied more on data from textbooks thus missing the opportunities afforded by

using real data, particularly the opportunities for students to engage with messy data. What the limited research on PSTs' and in-service teachers' SL reveals is that teachers experience challenges with SL, and we need more research. Furthermore, we did not find any studies that explicitly studied CSL. This study aims to contribute to the extant body of research by examining how PSTs enact CSLHM when presented with data representations from the real world.

Framework

To frame this study, we used the CSLHM framework (Bailey, Chapter 2) which resulted from the operationalization of Gal's (2002) critical stance within his SL model and Weiland's (2017) CSL framework (summarized in Table 3.1). Bailey (Chapter 2) draw attention to two key aspects of the CSLHM framework. First, since employing a questioning disposition is essential to effectively consuming statistical messages, skepticism is evidenced within each habit of mind. Second, the CSLHM are not necessarily enacted in a linear fashion. In other words, it is possible to enact multiple CSLHM at once.

Table 3.1

CSLHM Descriptions

CSLHM	Description
Questioning Sample Size and Methods	Individual demonstrates healthy skepticism regarding the sample, sample size, sampling technique, sampling bias, or lack of information regarding sampling that may lead to invalid inference on a target population. This includes considering who is missing, why, and how that influences the statistical message and the generalizability of the results, and the potential power of the message.

Recognizing Appropriate Statistics & Appropriate Representations	Individual questions whether the type of statistics and/or the way it is represented is the most appropriate for the data. This includes considering if the data representation employs techniques to mislead or deceive, thus questioning the motivation behind presenting the data in the way it was shared. Individual questions the role of outliers in the given representation. Individual questions whether the conclusions align with the selection of statistical test/procedure.
Desiring Additional Information	Individual demonstrates a need for additional information to draw a reasonable conclusion. Individual demonstrates healthy skepticism of the information, including the type of study, context of the study, the source (who collected the data and how), the author's motivation for sharing the statistical message (if not connected to appropriateness of the representation), and the credentials of the person/people sharing the statistical message.
Acknowledging Alternate Explanations	Individual acknowledges the potential for alternative interpretations for the meaning of findings or different explanations for what caused them, e.g., Was there an intervening moderator variable that affected the results? Are there additional or different implications that are not mentioned?
Recognition of One's *Own* Sociopolitical/ Critical Consciousness	Individual recognizes how one is integrating *their own* social, political, economic etc. understandings to make sense of injustice within the statistical message. Individual recognizes the degree to which one is engaged in critical reflection and critical action/active citizenry (see below). Individual recognizes the gaps in one's knowledge needed to interpret the statistical message.
Employing Active Citizenry	Individual is aware of inequities within the statistical message. Individual expresses desire to disrupt and dismantle inequities. Individual is motivated to act and describes next steps (action includes wanting to research the context, as education is an important part of being an active citizen).

We used the CSLHM framework in our study of PSTs' CSLHM enactment because it operationalized what adopting a critical lens looks like in practice when making sense of statistical messages. The framework held the analytical power we desired, with guiding questions to support analysis. But more importantly, we choose a

habits of mind lens to draw attention to the fact that there are thinking behaviors that, if used frequently and with ease, can lead to more robust and critical enactment of CSL.

Methods

This study followed a multiple case study design (Yin, 2018) that described PSTs' enactment of CSLHM when presented with statistical messages from the media. The cases are defined by the individual tweets that featured a data representation and will be described in detail shortly. Specifically, we aimed to answer the following research question: How do PSTs enact CSLHM when presented with data representations from the media?

Participants

The participants in this study are all PSTs. They were recruited from 4-year universities in the southeast by asking mathematics education professors to send out a recruitment email including a link to the consent and a preliminary survey. Twenty PSTs completed the initial recruitment survey, and of these 17 (across three different universities) consented to and took part in the study.

Of the 17 PSTs that completed interviews, all 17 also completed the demographic questions. The majority (N=14) identified as female, some identified as male (N=3), and none identified as non-binary. The majority (N=15) of the participants identify as white. There was one participant who identified as Black and one who identified as multi-racial. Sixteen of the PSTs were a typical college age (20-24 years old), and one indicated that they were 25-29 years old at the time of the study. There was also representation from all political affiliation categories. Five PSTs identified as democratic, five indicated that they

do not affiliate (not registered), four participants identified as republican, two identified as independent, and one indicated that they preferred to skip this question.

Data Collection

Each participant took part in a semi-structured task-based interview (Goldin, 2000) during the Fall 2021 semester with the first author. The interviews lasted on average for 46 minutes and included six tweets (order was randomized). The interviews took place on Zoom and were screen and audio recorded. Prior to starting the interview, participants were asked to identify two people with whom they honestly discuss the media or news, one who shares similar beliefs to themselves and one who shares dissimilar beliefs to themselves. This information was used to better tailor questioning later in the interview. Next participants were presented with a tweet and directed to think aloud: "As you are making sense of this tweet, please share what you notice." After the participants had time to respond to the initial tweet, they were asked to share what they would discuss with the person with similar beliefs (that they identified earlier) if they were talking about this tweet. For the sake of example, let's say that a participant indicates that they talk most openly with their friend Demet who shares similar beliefs, then I would have asked: "Now imagine that you saw this tweet and you are having a conversation with Demet, what would you and Demet likely discuss?" Then they were asked the same question but asked the participant to imagine they are discussing the tweet with the person they identified as having dissimilar beliefs. This process repeated for all six tweets. As to not elicit particular CSLHM, questioning during the interview was limited to clarification (e.g., I'm not sure I understand what you mean, can you explain that one more time or in a different way?) or elaboration questions (e.g., Can you tell me

more about that?). After participants cycled through all six tweets, they were asked if there were any tweets which they would like to revisit.

The Tweets

Each of the six tweets are a single tweet featuring a data representation (five were static and one was dynamic). All six tweets were related to issues of social justice in some manner as those require a critical read (Butwell, 2020; DeSantis, 2021; JordanUhl, 2021; Krugman, 2020; Mobley, 2020; Purcell, 2017). Topics included: the gender wage gap, COVID-19 and education, flow of military gear to police, hate crimes, systemic racism among boys who grew up rich, and perceptions of deadly police encounters. We will share findings across all six tweets but will use one to provide detailed examples (see Figure 3.1). All six tweets can be found in the Appendix B.

Figure 3.1

Tweet by Governor Ron DeSantis (2021)

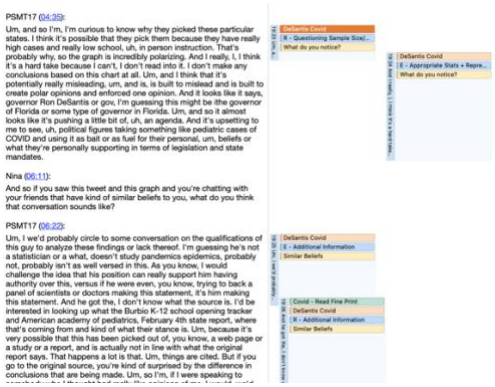
Analysis

The blinded and transcribed interviews were first coded using a priori theoretical coding based on the CSLHM descriptions (see Table 3.1) and developed according to the guidelines set forth by DeCuir-Gunby et al. (2011). To ensure that the codes were being applied consistently, two researchers independently coded a randomly selected interview. We compared our coding and discussed all discrepancies. This first comparison revealed that the coding and codebook were being used with strong consistency. The first author coded the remaining interviews independently and employed random spot checks with the second researcher to ensure consistent coding was upheld (see Figure 3.2 for a snapshot of coding). Throughout this process we watched for the emergence of additional CSLHM. While there were no new CSLHM that emerged, there was evidence of what we

will refer to as preliminary habits of mind (not yet emergent enactment). It felt important to capture instances where PSTs enacted part of a habit of mind with potential to develop it fully (more so because these instances present teachable moments for PSTs to develop CSLHM). We used the constant comparative method to look through all of the data using the new codes *Pre-Employing Active Citizenry* and *Pre-Recognition of One's Own Critical or Sociopolitical Consciousness* (DeCuir-Gunby et al., 2011).

Figure 3.2

Snapshot of ATLAS.ti Coding



After each transcript was coded for evidence of CSLHM, we coded for emergent or robust enactment of each CSLHM. Emergent enactment of a particular CSLHM is characterized by vague wonderings (i.e., lack of depth and criticality). Robust enactment of a particular CSLHM is characterized by depth and criticality. For example, emergent enactment of the CSLHM *Questioning Sample Size and Methods* would include consideration of who was sampled but may fail to consider who is missing from the sample and why that matters. We revisited each coded quotation to determine if that quotation evidenced emergent or robust enactment of the particular CSLHM. As we coded what the PSTs said aloud, the emergent and robust codes only reflect what the

PSTs stated aloud in the interview; it is possible that they had thoughts that were not verbalized. After coding was complete, we looked for emerging themes across the codes.

Findings

Different data representations naturally elicit particular CSLHM more than others. For this reason, we will look both at one specific tweet and across all six tweets. The narrow view enables us to see the enactment in context, specifically in the context of the DeSantis (2021) tweet (Figure 3.1). Then, we will present how the PSTs enacted each CSLHM across all six tweets. This broader view provides a general description of how the PSTs enacted the CSLHM.

The Narrow View: CSLHM Enactment with the DeSantis (2021) Tweet

Looking at findings related to the DeSantis (2021) tweet, PSTs commonly enacted the CSLHM *Acknowledging Alternate Explanations* (12 of 17 PSTs; see Table 3.2). *Questioning the Sample Size and Methods* (9 of 17 PSTs; 2 of which were robust) and *Desiring Additional Information* (8 of 17 PSTs; 2 of which were robust) were also commonly enacted. With respect to this tweet, it was far less common for PSTs to enact *Employing Active Citizenry* (1 of 17 PSTs) and *Recognition of One's Own Critical or Sociopolitical Consciousness* (1 of 17 PSTs).

Table 3.2

Summary of PST CSLHM Enactment on the DeSantis (2021) Tweet

	CSLHM					
PST	Questioning Sample Size/ Methods	Appropriate Stats & Representations	Additional Information	Alternate Explanations	Sociopolitical/ Critical Consciousness	Active Citizenry
1	E			E		
2	E	E	E	E		
3		E	E			
4				E		
5	R		R	E		

6			E	E		
7	E	E		E		
8	E			E		
9				E		
10	E		E	E		
11						
12						
13			E	E		E
14	E	E	E	E		
15	E					
16						
17	R	R	R	E	R	

Note: R indicates that the PST enacted the CSLHM robustly on the DeSantis (2021) tweet. E indicates that the PST did so emergently.

Questioning Sample Size and Methods

Questioning the sample size and methods entails skepticism regarding who was sampled, the sample size, the sampling method, and potential sample bias. On the DeSantis (2021) tweet, PSTs commonly enacted questioning the sample size and methods emergently by demonstrating general wonderment about the sample and oftentimes expressed distrust. Some PSTs expressed skepticism about the sample, but this was frequently something they could have discerned from further analysis of the data representation. For example, PST 2 was skeptical of the sample and stated: “We can also talk about how Florida has a high retirement population. And so that could skew it because a lot of older people live there.” PST 2 overlooked the fact that a pediatric rate per 100,000 children was used, thus the retirement population is less of a concern unless PST 2 articulated why an increased retirement population would directly impact the pediatric rate of covid. Similarly, PST 7 considered the number of children and disregarded the fact that a rate was used: “there is no information about the percentage of children in each state,” however PST 7 also began to think about how the information was gathered and who is included in the data: “and there is no information about how they got this survey in the first place.”

PSTs expressed distrust in the sample or how the author selected the sample by wondering if the sample was somehow “cherry picked” or chosen intentionally to tell a particular narrative. The PSTs that indicated potential cherry picking did so in different ways. Some PSTs wondered without implying deceit such as PST 1:

> So, I guess first off, I mean, I understand why we chose Florida because that's what we're talking about, but I wonder, um, why we chose Ohio, Illinois, and California. Um, is that a strategic choice to make Florida look better? Or are those, is there another reason behind it? Like, um, it kind of looks like maybe Ohio had the next, maybe they were, um, had the next closest person of students offered in-person instruction or I guess I'm just wondering why they chose these three. What's the reasoning behind that?

Whereas other PSTs implied that intentional manipulation could be driving the decision of which states to include. For example, PST 15 stated

> There's only four states here out of 50. Okay. There we go. I think that I hit something there there's only four states out of 50. So why don't, if you're wanting to lean this a way of looking at making Florida look bad, put them next to three states that are going to have super low, um, in person instruction opportunities, but with about the same amount of, uh, pediatric cases. So, this could definitely be fixed.

Similarly, PST 7 suggested the selection of states aligned with DeSantis’ agenda: “Um, and so I'm, I'm curious to know why they picked these particular states. I think it's possible that they pick them because they have really high cases and really low school, uh, in person instruction. That's probably why, so the graph is incredibly polarizing.” PST

7 expanded upon this by wondering how the information was obtained implying that the cities or districts within the sample could have also been intentionally selected to tell DeSantis' desired message:

> I think they're also missing a piece like about population. Cause if Florida has a bigger or denser population than California, Ohio, and [Illinois], then that kind of accounts for… I wonder if they went to every district to find this information, um, or maybe they cherry picked certain cities within each state to portray the message they want to, um, yeah.

PST 17 took a similar approach to PST 7 by wondering what the bigger story would be if the data was expanded to include all states:

> I would, you know, open up the conversation about what does this look like with all the other states? How does this information relate to other states? Is it significantly worse than other states, for example, because what if, you know, 3,794 is starkly higher than, you know, two dozen other states are those two dozen other states like doing the wrong thing. I'd like to see, like what's their percent of students offering in-person instruction.

Several PSTs continued to wonder about cherry picking in conjunction with DeSantis' (2021) language in the tweet (states are "of similar size"). For example, PST 10 stated: "I hope when they're comparing the other states of similar size, they're not talking about like size, like land and they're talking about people instead of the size part." This PST is drawing on their knowledge of the general area of the four states and reasoning that DeSantis' comment about comparing states of similar size is questionable. Instead suggesting that maybe the states are similar with respect to population. While the states

are also not similar in population size, the PST was appropriately questioning how these four states were selected for the sample when the claim is that they are similar in size.

Two PSTs enacted this CSLHM robustly. Both did so by extending the idea of who was sampled and why to wonder about who was missing and how that influences the statistical message. PST 5 discussed the idea of not taking a test: “If you haven’t gotten a test that doesn’t, it doesn’t mean you haven’t had it. It just means that it hasn’t been recorded… that affects the numbers, that affects the data, that affects what we see on the graph.” Similarly, PST 17 explicitly considered representation by thinking about who is not captured in the sample. PST 17 stated:

> Florida has the highest rate of in-person instruction, but also the lowest rate of pediatric cases. Um, but it doesn't, there's nothing on here that talks about how often people get tested, how they get tested and how it's reported, you know, if they have at home tests and I don't know if that has, if people always report that or if they just stay home. Or it also doesn't talk about, um, or there's no way for us to know if this is, um, I forgot where, what I was going to say something about schools. Like, is it just, do they count a student that is quarantined versus the student that has tested positive? So, I dunno, there's a lot of ways you could argue that this is, or is not a complete argument.

Both PST 5 and PST 17 pointed out that the pediatric cases represented on the graph can only include those people who tested, therefore we are missing the people who did not take a test but have the virus.

Recognizing Appropriate Statistics and Appropriate Representations

Recognizing appropriate statistics and appropriate representations entails thinking about whether the statistical measures/tests align with the data and if the data aligns with the choice of graphical representation. With respect to the DeSantis (2021) tweet, the PSTs emergently enacted this CSLHM in a variety of ways. Several PSTs noted that DeSantis did not state he was discussing COVID-19 (in either the tweet or the data representation). PST 14 noted this lack of transparency in connection to the labels on the graph: "pediatric cases of COVID I'm assuming it doesn't even say." Similarly, PST 3 questioned "Is this, um, COVID related?" It is possible that some PSTs felt comfortable assuming the tweet was about COVID given the timing of the tweet and their experience with education during the pandemic.

As there were two distinct scales and graphs on one data representation, several PSTs looked carefully at the two scales. Some PSTs looked for consistency of the scaling (i.e., are the jumps in value consistent and evenly spaced). Whereas, other PSTs were confused by the two scales, the inconsistent color coding, and identifying which scale aligned with the horizontal white lines on the graph (particularly given that both axes are white, and the legend is white and orange). Consider PST 3 who wondered: "why does the line not reach 30%?" PST 3 assumed that the line highlighted in pink on Figure 3.3 was connected to the scale on the left (circled 30% on Figure 3.3) and wondered why Ohio's bar was below the line. This PST did not make the connection that the horizontal lines on the graph are actually connected to the scale on the right for pediatric cases. PST 7 also questioned the scales, but noted explicitly that the horizontal lines on the graph did not match the scale on the right: "the lines for like the percentages down here [referring

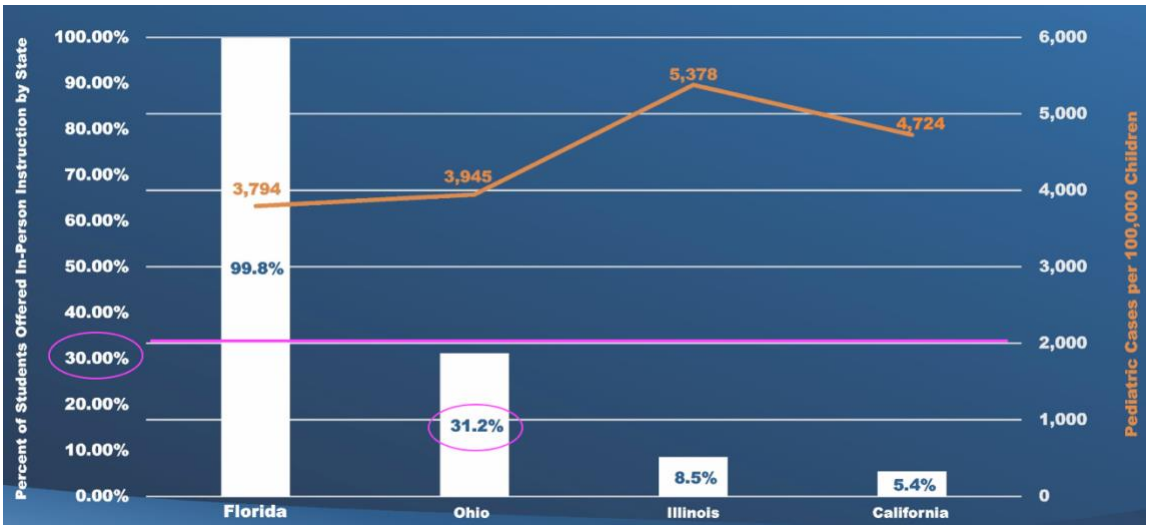
to right axis with percentage of students offered in person], they don't, um, they don't really align with the lines here [referring to the lines on the graph]. I see that the lines are supposed to align with like the pediatric cases." By questioning the scale, the PSTs do raise the important concern of why using a data representation with two graphs in one is a poor choice and difficult to understand. In fact, PST 2 comments on this explicitly:

> First thing, the fact that it's a bar graph and a line graph is very misleading [chuckles] because you can't compare the data that way…I guess they're trying to make the argument, that like the bar goes above the, where the line is at. So, you know, fewer, I don't know. That's weird. I don't like that.

While PST 2 does not clearly articulate why this is confusing, they are taking note of the fact that there is something confusing about presenting two different types of graphs in one representation. None of the PSTs extended this conversation to comment on how the use of a line graph was inappropriate given the data, particularly since such graphs typically track change over time not differences across groups.

Figure 3.3

Annotated Graph Snapshot from the DeSantis (2021) Tweet



Beyond the scales on the graph, the PSTs also seemed concerned about the choice of statistical measure, specifically the pediatric rate. Several PSTs suggested there might be something off about the choice of using a rate, but they did so implicitly. Consider how PST 5 wondered if states with more children would potentially cause an issue with this data representation: "And, um, and I think I would, like, I would want to know how the amount of, how many, like, I guess like under 18 year olds they have in Florida, like in each state, to know like, okay, this is like of how many kids, because I feel like those numbers could affect the, the like rate a bit." Only one PST explicitly discussed their confusion with respect to the units of measurement and the graph itself. PST 2 stated: "And pediatric cases per 100,000 is that pediatric cases, so I guess that is pediatric cases in the country? Per 100,000. So, that means for every 100,000 people." PST 2 questioned the unit of measurement (i.e., the rate) and whether it was per 100,000 children or per 100,000 people. The legend at the top of the graph is unclear as it only states: "Pediatric Cases per 100,000" and does not clarify what is meant by a pediatric case. However, on the right y-axis the label indicates that the rate is "Pediatric cases per 100,000 children." Even with the distinction on the y-axis, several PSTs were not sure what per 100,000 meant, and others wanted to know if a pediatric case only included school-aged children or all children.

While many PSTs questioned the labels and scale, only one PST began to consider correlation versus causation. PST 17 said: "Does this correlation mean that there's some causation?...I think it's a big generalization…I don't think this, this chart doesn't speak to any other circumstances surrounding what might be causing pediatric cases of COVID-19." By indicating that the correlation DeSantis is illuminating does not

imply any causation, PST 17 robustly enacted this CSLHM by making the connection between the representation, how the data was being portrayed, the intended message of the tweet and graph, and ultimately if the conclusions being drawn are grounded statistically.

Desiring Additional Information

The CSLHM desiring additional information refers to wanting or needing more information regarding the statistics or methodology to be able to make sense of or draw conclusions about the statistical message being presented (that is not captured by other CSLHM). Nine PSTs enacted this CSLHM while making sense of the DeSantis (2021) tweet, however there was no consistent pattern of the type of information the PSTs wanted to know. Some PSTs wanted to know more about what the percentages mean in terms of a count of children. Such as PST 1 who said: “But like, what does 99.8% of like, how many, like how many students is that? I don't like, I don't know how many students are enrolled in like Florida schools.” Some PSTs wanted to know what the trends look like in other states, such as PST 17:

> But also, I would be interested to see how these, I would, I would, you know, open up the conversation about what does this look like with all the other states? How does this information relate to other states? Is it significantly worse than other states, for example, because what if, you know, 3,794 is starkly higher than, you know, two dozen other states are those two dozen other states like doing the wrong thing. I'd like to see, like what's their percent of students offering in-person instruction.

Other PSTs wanted to know more about how the variables are defined. Such as PST 5 wanted to know more about what "offered in-person instruction" means: "And also it says like students offered, offered in person I'm not su-, I don't know if that means that they actually went in person or if it was just like offered to them, but it was also like online offer option offered as well. Cause I know that's happened." PST 5 raised an important consideration about the difference between how many students were offered and how many actually attended in person.

While most PST enacted this CSLHM emergently, there were two who did so robustly. Specifically, they connected their desire for more information to critical elements and provided justification for why they wanted more information. PST 5 wondered about the timing:

> Um, also like when was this done? Like when was this taken, you know, was this, were they all from the same like, uh, like where did these pediatric cases, numbers that we see where they all taken from the same time. I've seen some graphs where they've taken them, you know, to make it look better for, um, their data and stuff. So that's just you know kind of like more about what this is from

Later they expanded upon this argument: "all the rates are up and down, up and down and certain states really had higher, um, percentages overall in the state." PST 5 indicated that depending on when the data was taken, the story could be drastically different, and that is a vital consideration when making sense of this statistical message and the motivation for sharing such a message. PST 17 robustly enacted this CSLHM by wanting to know about the effect size:

> You know, and this is, this is hitting me like the orange line of, of pediatric cases. Yeah. They're higher in Illinois, they're higher in California. Um, but how significant is that difference? You know, for example, it's a little over a thousand more cases and in terms of California, it's right at about a thousand more cases, which is, um, you know, 1% of a hundred thousand, um, which, you know, does mean something because these are lives.

PST 17 did not explicitly use the term effect size but is clearly grappling with the idea of the magnitude of the difference. None of the PSTs explicitly wanted to know more about the specific aspects of the methodology that are not transparent in the representation.

Acknowledging Alternate Explanations

Acknowledging alternate explanations refers to discussing the possibility of other conclusions, explanations, and/or causes for the phenomena in the statistical message. This CSLHM was most commonly enacted by considering the rate at which testing occurs and several other factors associated with testing that are not captured in the data representation or DeSantis' tweet. For example, PST 9 stated: "so it could just be that Illinois is testing significantly more, um children for COVID then Florida during this time period." Similarly, PST 4 considered the testing rate: "Maybe like testing, like Florida or people who live in Florida did not go and get COVID tests as much, or go to the doctor as much to make those numbers be where they could have been, possibly." PST 5 approached this by considering those that don't take a test but could be unknowingly infected: "if you haven't gotten a test that doesn't, it doesn't mean you haven't had it. It just means it hasn't been recorded… and so that affects it. That affects the numbers that affects the data that affects what we see on the graph." PST 14 also

considered testing but in conjunction to reporting: “There's nothing on here that talks about how often people get tested, how they get tested, and how it's reported, you know, if they have at home tests and I don't know if that has, if people always report that or if they just stay home.” PST 14 continued to consider how schools count students by wondering if there is a difference in counting and reporting students who are quarantined versus tested positive. Many of the PSTs also mentioned different factors that might influence the spread of the virus including mask mandates, compliance with mask mandates, the number of adults and other people that students have contact with, vaccination rates, and several other relevant variables that are not captured in the tweet and data representation.

Another factor that the PSTs considered was the density of the population. For example, PST 17 suggested population density by considering living conditions: “Kind of how living conditions are a little bit tighter in California, potentially compared to Florida. I know there's a lot of rural Florida, potentially more than rural California.” PST 17 is suggesting that density of the population can affect the spread of the virus. Similarly, PST 7 suggested that “Florida has a bigger or denser population than California, Ohio, and [Illinois].” PST 13 drew upon their knowledge of Illinois: “Also at the same time, I also instantly go to, well, you have to think about density of population… I know for a fact that Illinois is a very, um, you know, regardless of the population, uh, it's very, uh, highly populated in very small areas.” Often PSTs suggested a factor that was not considered and used that as a springboard to consider other potential factors. Such as how PST 13 extended their argument about population density to consider the impact of poverty and resources: “Illinois also has a very low socioeconomic average, and there's a big, big

portion of people in there that live in poverty. So it kind of makes sense that they wouldn't have access to a lot of different things or access to a lot of the type of information." Overall, the majority of PSTs identified at least one variable that was not considered in the tweet and data representation that would be important to consider. While the PSTs did consider a variety of variables that potentially play a role, none explicitly wondered if the analysis attempted to control for such variables or if the data was pulled from a study that controlled for other variables. Nor did they consider alternate explanations from multiple viewpoints or narratives, thus none of the PSTs robustly enacted this CSLHM.

Recognition of One's Own Critical or Sociopolitical Consciousness

Recognition of one's own critical or sociopolitical consciousness occurs when an individual considers how they are integrating *their own* social, political, economic, or worldly understandings to make sense of the statistical message, specifically the injustice within the message. Only one PST enacted this CSLHM when making sense of the DeSantis (2021) tweet and did so robustly. PST 17 acknowledged that they rely on scientists for information regarding COVID-19 and would want to refer to such information before drawing conclusions:

> Um, but this is a tough conversation. And it, I, I, um, I think I would openly say that I only feel so qualified to share. What's been reported by credible sources, by scientists and by, you know, credible organizations. And I, I don't think I would feel in the same way I'm criticizing the governor for making some type of statement about like, this means that it's good to have in person instruction. I would want to refer to professionals in the field before I try and make some

> conclusion, um, because it's just not super in my scope of expertise. And I think sometimes that makes people uncomfortable, um, to not feel like I'm the right person to be this authority of knowledge, especially when things turn into a debate, you'd like to be the authority of knowledge. Um, but I'm actually, I've become more comfortable, especially in the last year with saying, you know, I'm not, I'm not positive right now.

PST 17 embodied this CSLHM by acknowledging the gap in their own knowledge that is needed to interpret the statistical message. More importantly, PST 17 indicated that they would like to complete that research before drawing any conclusions.

Employing Active Citizenry

Employing active citizenry occurs when individuals consider their own actions as a result of making sense of the statistical message. PST 13 was the only PST to enact the CSLHM Employing Active Citizenry on the DeSantis (2021) tweet and did so emergently. In thinking about how they would have a conversation with their cousin (the person they identified as someone they trust who holds different beliefs), PST 13 carefully considered how they would challenge their cousin's thinking:

> I think I would kind of just counter ask her like, okay, just because this many students are offered in person classes does not mean that, you know, all the students are taking it or whatever it may be. And I think here, we would just have a bit more, um, she would very much so want to back the data that is shown in the graph. And I would want to just kind of play devil's advocate and talk about other variables that would go into it.

PST 13 identified an issue and potential point of disagreement with their cousin, consider how their cousin would think about the issue, and generate ways to challenge their cousin's thinking. This element of thinking about how to press on the views of others is key from moving from passive interpretation to informed action.

Similar to how there were several PSTs that could be supported to make the leap from identifying their beliefs to enacting recognizing their own sociopolitical consciousness, the same trend emerged for employing active citizenry. While there was not much evidence of employing active citizenry on this particular tweet, I do think it is important to note that several PSTs mentioned things that with some practice could be elevated from passive interpretation to informed action. For example, some PSTs began to consider what an argument from an opposing viewpoint might entail but did not further that conversation to think about what actions they might take to press on such an interpretation of the data representation. Similarly, several PSTs indicated that they wanted to research aspects about the context but did not then indicate how that might inform their interpretation of the data representation and how they might navigate conversations about it.

The Broad View: CSLHM Enactment across All Six Tweets

Looking across all six tweets, *Acknowledging Alternate Explanations* was the most commonly enacted CSLHM (16 of 17 PSTs; 2 of which were robust; see Table 3.3). This is consistent with enactment on the DeSantis (2021) tweet. *Desiring Additional Information* was also commonly enacted (13 of 17 PSTs; 2 of which were robust). It was less common for PSTs to enact *Employing Active Citizenry* (8 of 17 PSTs; 2 of which were robust) and *Recognition of One's Own Critical or Sociopolitical Consciousness* (7

of 17 PSTs; 1 of which was robust). Please note that as we discuss general trends, we will use some examples that come from the other tweets in the study, but the examples should illustrate the point without the need of further context, however if you need a reference the tweets can be found in the Appendix B.

Table 3.3

Summary of PST CSLHM Enactment across All Six Tweets

	CSLHM					
PST	Questioning Sample Size/ Methods	Appropriate Stats & Representations	Additional Information	Alternate Explanations	Sociopolitical/ Critical Consciousness	Active Citizenry
1	E		E	R		E
2	R	R	E	E		E
3	E	E	E	E	E	E
4			E	E		R
5	R	E	R	E	E	
6		E	E	E		
7	R	R	E	E	E	
8	E			E		
9	R	E	E	E	E	
10	E	E	E	E	E	
11				E		
12						R
13	E		E	R	E	E
14	R	E	E	E		E
15	E	E	E	E		
16	E	E		E		
17	R	R	R	E	R	R

Note: R indicates that the PST enacted the CSLHM robustly on at least one tweet. E indicates that the PST enacted the CSLHM emergently on at least one tweet.

Questioning Sample Size and Methods

Across all six tweets, the most common way PSTs enacted this CSLHM was a general wonderment about who was sampled and where. Sometimes this was stated as a wondering such as when PST 9 stated “I wonder if this is national or if this is in a certain city” and sometimes asked as a question such as PST 7 who asked “Where, who did they survey? Where did they survey?” Many PSTs used the language of “cherry picking” to question the sample methods. These PSTs wanted to know if the author of the data

representation selected the sample intentionally to align with their message (many did this with the DeSantis tweet). Although none of the PSTs explicitly questioned the sampling method, the idea of cherry picking demonstrates that PSTs are implicitly considering the sample methods used.

While most enactment of *Questioning Sample Size and Methods* was emergent, five PSTs enacted this CSLHM robustly on at least one of the six tweets. Robust enactment of this CSLHM occurred most frequently when PSTs extended their general wonderment of who was sampled and why to also consider who was potentially missed (as we saw with the DeSantis tweet) or why we might want to see the data disaggregated. For example, PST 2 started with generally wondering about who was sampled: "I'd be interested to see where, who they asked." Then extended this to consider disaggregation of the data: "…depend on where you ask these questions…if you asked an officer up north… whereas in a small town." Given that they are referencing a tweet on perception of the police, knowing not just the general geography (i.e., national or state-wide), but the type of towns that were included (i.e., urban, suburban, or rural) seemed important to PST 2: "in a small town you'd be interested to see like, would that be different?" Very few PSTs addressed the idea of representative sampling, but those that did enacted this CSLHM robustly. For example, when making sense of the hate crime tweet and bar graph (Krugman, 2020), PST 9 started with the general wondering of "Is this nationally? Is this like within a specific city or state?" Then they followed that up with explicitly thinking about the demographics in the population: "And like how many, how many Black people versus how many Muslims make up this population?" PST 9 is wondering if the bars are representative of the Black and Muslim population or if there is a sampling issue where a

group is over or underrepresented. Only two PSTs discussed non-response, and only one PST considered the idea of potentially missing people based on how the variable is defined. None of the PSTs explicitly wondered about the role of randomness when they considered how the sample was collected. Several did imply they were considering if the data was biased based on the sample as we saw with PST 2 wondering about the differences in police perception across urban, suburban, and rural areas or across geographical divides (e.g., north versus the south).

Recognizing Appropriate Statistics and Appropriate Representations

Across all six tweets, emergent enactment of *Recognizing Appropriate Statistics and Appropriate Representations* was most often characterized by a focus on aesthetic aspects of the graph such as color. PST 10 commented on the color when making sense of the police perception data representation (Purcell, 2017): "but the isolated incidents catches my eye…I don't know if it's because like the color." It was also common for PSTs to focus on the construction of the graph by noting missing details regarding the axes, scaling, or labeling. For example, PST 2 questioned the labeling on the hate crime bar graph (Krugman, 2020): "I guess on the side it's people, it doesn't say, on the y-axis it's people." Similarly, PST 15 assessed the same bar graph to see if the scale was consistent: "I always try to look at the scales and math to see if they mess up… I'm kind of surprised at, that, I mean, this one's obviously good."

Some PSTs considered whether the statistical measure used in the data representation was appropriate. For example, when making sense of the hate crime bar graph (Krugman, 2020), PST 7 implicitly considers why a raw count doesn't make sense by stating: "but there's a bigger population of the Black community in many states

compared to the LGBTQ and Middle Eastern community." While others did so explicitly and robustly such as PST 14:

> I wonder if this is in frequency and not rate. I would expect the population of black people in the US to be larger than the population of Islamic people in the US… I think it would be more beneficial than this would be the rate change since before, um, 9/11, so you can see like, anti-black hate crimes, go up slightly and back down again… but for anti-Islamic hate crimes, the, the rate change is probably like 500% or something crazy because it is jumping up so high.

Not only did PST 14 question the appropriateness of the statistical measure, but they stated that percent change would more accurately capture change over time.

Only one PST considered if correlation was confused with causation. None of the PSTs considered if the variables accurately measure what they are intended to measure. While many PSTs assessed the scale on the axes, none commented on the appropriateness or inappropriateness of the groupings or intervals on the graphs.

Desiring Additional Information

PSTs very frequently wondered what the data looked like over time or wanted to see the data representation over a larger timespan. When making sense of the gender pay data representation (Butwell, 2020), PST 13 said "I'd love to see this data compared to years previous." Many PSTs also expressed a desire to know if there is a potential lurking variable or why a variable was or was not clearly defined. For example, on the hate crime tweet (Krugman, 2020), PST 5 asked "what's being defined as a hate crime?" What distinguished emergent from robust enactment of *Desiring Additional Information*, was

whether the PST included justification for why they wanted or needed the additional information to make sense of the statistical message from a critical perspective.

Only one PST demonstrated a desire to disaggregate the data presented to gain a better understanding of the issue. Only one PST demonstrated a desire to know the units on a graph. Similarly, there was only one PST who demonstrated a desire to know how the variable was measured. Only one PST wanted qualitative data or narrative accounts to accompany the quantitative data presented. Only one PST wanted to know more about the effect size of the difference being described. And only one PST commented on the absence of a source and wanted to know what the source of the information was.

Acknowledging Alternate Explanations

Many PSTs suggested potential lurking variables and explored alternate reasoning for or against the data being presented (this is different than the desire to know if there is a potential lurking variable discussed in the desiring additional information section). Some PSTs pondered any historical connections that might explain or negate the data. Often it appeared that PSTs suggested alternate explanations that aligned with their beliefs. For example, the *NY Times* data representation (Mobley, 2020) that explores systemic racism and income, generated two groups of PSTs. Some suggested that individual choice, upbringing, crime, drugs, or violence were related to what they saw, whereas others suggested that racism within education or police brutality played a role. It was rare for PSTs to suggest alternate explanations from counter viewpoints.

While many PSTs offered potential lurking variables, few of the PSTs explicitly considered if the author of the message failed to include vital information (e.g., did not address lurking or confounding variables) that would be needed to make an informed

decision or draw an appropriate conclusion from the data. And few connected their alternate explanations to equity or inequity through consideration of systemic discrimination or systemic racism.

Recognition of One's Own Critical or Sociopolitical Consciousness

While many PSTs tried to draw parallels between what they see in the world and the content of the data representation, few made the necessary connection between such observations and how they made sense of the statistical message. About a third of the PSTs acknowledged their experiences in the world, their feelings, or their political beliefs, and used them to make sense of the statistical message (thus enacting this CSLHM). Whereas, about another third partially enacted this CSLHM, which we refer to as *Pre-Recognition of One's Own Critical or Sociopolitical Consciousness*. Typically, these PSTs would make personal connections, but they did not consider how those feelings or beliefs influenced their interpretation of the data representation, thus they did not enact this CSLHM. For example, PST 5 identified that they would talk about history and events surrounding the issues but never unpacked how their interpretation of those events, feelings surrounding them, and broader political views on the issues influence how they are making sense of the data representations.

Of those that enacted this CSLHM, only two PSTs discussed how their lack of knowledge on the topic influenced their interpretation of the data representation. And only one PST enacted this CSLHM robustly (as we saw on the DeSantis tweet) and articulated that their feelings and beliefs influenced how they interpreted the graph and that it was something they should consider.

Employing Active Citizenry

Employing active citizenry occurs when individuals consider their own actions as a result of making sense of the statistical message. As the types of actions people take to be active citizens vary, there is natural variation in how this CSLHM is enacted. Most PSTs focused on how they could use the data representation or additional data to press on the views of their family and friends. While most PSTs did so generally, one PST brainstormed the types of questions they would ask their family or friends to advance their understanding of the issue. The same PST also explicitly indicated the elements that they would need to research to effectively press on other's views. Two PSTs considered how they would use the data representation to have conversations about the implications for teaching and for students. None of the PSTs detailed any actionable next steps as a result of making sense of the data representation beyond researching the topic and having conversations about it.

Approximately one fourth of the PSTs partially enacted this CSLHM. Often the PSTs would vaguely ponder why but miss the opportunity to think more deeply about issues of power or how they might press on the views of others. For example, PST 10 wondered about the military gear referenced in JordanUhl's tweet (2021): "Why are the police on track to get more military hardware, um, with Biden rather than Trump?" Wondering why is an important first step to enacting *Employing Active Citizenry*, as it sets the stage to explore who the statistical message serves and why, who benefits from this message and how it was presented, and what actions should be taken next.

Discussion and Implications for Practice

The goal of our study was to explore how PSTs enact CSLHM when engaging with statistical messages from the media. While there is variation among both the depth and frequency with which PSTs enact particular habits of mind, there is evidence that PSTs enact each CSLHM. Often the general wonderment that PSTs evidenced when enacting each CSLHM is consistent with emergent enactment as described in Chapter 2. This suggests that there is an opportunity to support PSTs in the development of CSLHM.

It is important to note that there are several limitations to this study. For example, the sample of PSTs was not random. It is possible that those that participated in the study were motivated to do so for some reason. This study focused on how PSTs enacted CSLHM when making sense of statistical messages from the media, specifically tweets that featured a data representation. It is possible that PST CSLHM enactment on other types of statistical messages is different, thus warrants further research. Even so, these 17 PSTs provide insight that can inform future studies.

With the continued call for critical statistical literacy (e.g., Bargagliotti et al., 2020) and the rise of instructional routines aimed at exploring data, such as data talks (Boaler et al., 2021), future research should aim to consider how we can support students and teachers to notice and wonder more effectively. The CSLHM is a conceptual framework that we hope can be used to advance both student, PST, and teacher thinking with respect to what they notice and wonder when presented with a data representation from the real world. Specifically, future research should investigate interventions and supports that may increase robust CSLHM enactment among teachers, PSTs, and students. It is particularly important to consider how to support pre and in-service

teachers' development of CSLHM given that research suggests that PSTs are not prepared to teach statistics (Lovett & Lee, 2017, 2018), do not feel prepared to teach statistics (Banilower et al., 2013, 2018), and that taking university level statistics content courses do not appear to support the development of statistical literacy (Tak et al., 2017).

Research suggests PSTs are not comfortable discussing or teaching social justice topics (e.g., Simic-Muller et al., 2015). We saw evidence of this in two ways. First, several PSTs did not discuss about the broader issue in their interviews. Second, PSTs seemed to be more comfortable enacting CSLHM that aligned with more traditional notions of statistics. *Questioning Sample Size and Methods* was the most common CSLHM to be enacted robustly followed by *Recognizing Appropriate Statistics and Appropriate Representations*. Both of these CSLHM provide opportunities to critically engage with the issue without sharing one's personal beliefs (*Recognition of One's Own Critical or Sociopolitical Consciousness*) or thinking about how one might challenge someone else's beliefs (*Employing Active Citizenry*) or considering alternate explanations that come from different narratives (*Acknowledging Alternate Explanations*). This finding may illuminate a potential leverage point for PSTs. It is possible that PSTs may gain confidence by learning to robustly enact these CSLHM (*Questioning Sample Size and Methods* and *Recognizing Appropriate Statistics and Appropriate Representations*) before gaining the confidence to robustly enact the CSLHM that make them feel more vulnerable.

Given that statistics education traditionally positions statistics as neutral, it is not surprising that the least commonly enacted CSLHM were *Employing Active Citizenry* and *Recognition of One's Own Critical or Sociopolitical Consciousness*. This points to the

need for statistics education to emphasize that statistics is a tool used to help drive decision making in society which means statistics is inextricably linked to societal issues and therefore is not neutral. Furthermore, there is promise that PSTs are ready to integrate a critical lens as evidenced by the emergence of *Pre-Employing Active Citizenry* and *Pre-Recognition of One's Own Critical or Sociopolitical Consciousness*. The partial enactment of these CSLHM points to potential leverage points for PSTs to think more critically. Future research should endeavor to determine if explicit instruction on the CSLHM holds the power to help PSTs unearth the non-neutral nature of statistics as well as harness their noticings into robust enactment of the CSLHM.

Additionally, Muñiz-Rodriguez et al. (2020) discussed how secondary mathematics teachers missed opportunities to engage students with real and messy data and data representations, and instead they relied on data from textbooks. It is possible that teachers will employ the use of data driven routines, such as data talks, more frequently, and perhaps more effectively, if they have a framework to support their thinking and that of their students. Using the CSLHM as a conceptual framework has the potential to help teachers facilitate discussions about data representations and pose purposeful questions to help students engage in CSLHM. Future research should endeavor to explore the conceptual power that the CSLHM holds.

We aimed to extend the field's knowledge by describing how PSTs enact CSLHM, with the specific hope that research will continue to investigate how to support PSTs in developing robust enactment of CSLHM so they can support their students in doing the same.

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Appendix B

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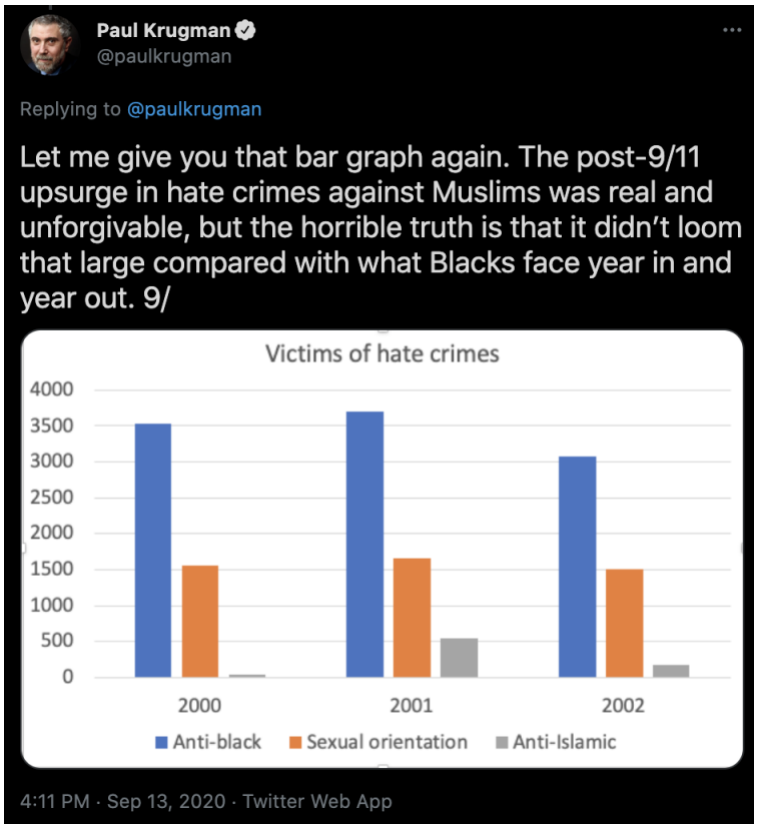
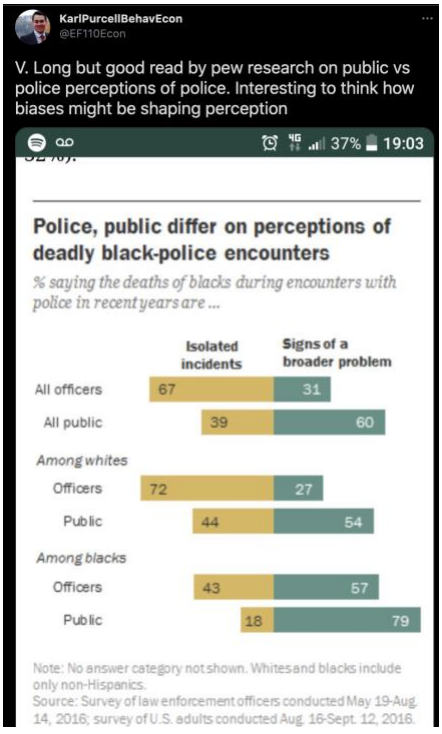


Mobley, S. [@sylvestermobley]. (2020, June 16). *If you haven't seen the @nytimes visualization that uses data to show the impact of racism on Black boys* [Thumbnail with link attached] [Tweet]. Twitter. https://twitter.com/sylvestermobley/status/1272973707565244416

Snapshot at the beginning, in the middle, and at the end of the NY Times data visualization shared in Mobley's Tweet (Badger et al., 2018)

Purcell, K. [@EF110Econ]. (2017, January 12). *V. Long but good read by Pew Research on public vs police perceptions of police. Interesting to think how biases* [Image attached] [Tweet]. Twitter. https://twitter.com/EF110Econ/status/819621499942342657



CHAPTER 4: PRESERVICE SECONDARY MATHEMATICS TEACHERS' ENACTMENT OF CRITICAL STATISTICAL LITERACY HABITS OF MIND: A MULTIPLE CASE STUDY

Journal

The third article is an instrumental multiple case study with a common and an unusual case of secondary preservice mathematics teachers' CSLHM enactment from the broader study in Chapter 3. I plan to submit this article to the *Statistics Education Research Journal* (SERJ). The goal of SERJ is to share research on the teaching, learning, and understanding of statistics and probability. I believe that the results of this study would be of interest to and benefit the SERJ audience of statisticians, statistics teacher educators, and mathematics teacher educators. With increased calls for taking a critical lens to SL, this article will provide statistics and mathematics educators with a better understanding of how a PST can robustly enact CSLHM, what that looks like, and set the stage for considering how to foster this among all PSTs. Please note that I am listed as the first and only author on this article.

Abstract

When making sense of data representations from the media, preservice secondary mathematics teachers typically enact critical statistical literacy habits of mind (CSLHM) emergently. The goal of this study is to examine the ways that preservice secondary mathematics teachers (PSTs) enact CSLHM when making sense of data representations from the media using an instrumental multiple case study design. Two cases were

selected from the broader study, one typical CSLHM enactment and the other unusual (as this PST evidenced more robust enactment). The study examined how each enact the CSLHM across two tasks and examined the differences between the two cases. Findings revealed several important differences between the cases. The PST who more robustly enacted the CSLHM (unusual case) integrated context into her enactment, directly discussed the social issue, and evidenced elevation of some CSLHM from emergent to robust on a single task. These findings are important for the field to consider how to support preservice teachers CSLHM development so that they can help their students develop the same habits of mind.

Keywords: critical statistical literacy, habits of mind, preservice teachers

Preservice Secondary Mathematics Teachers' Enactment of Critical Statistical Literacy Habits of Mind: A Multiple Case Study

Data drives decision making in society. Students need to navigate the abundance of quantitative messages they will undoubtedly encounter in the world and be able to create and communicate their own messages supported by statistical investigation or the credible statistical investigation of others. These actions are often referred to as statistical literacy (SL; e.g., Gal, 2002). Traditionally, statistical content has been incorporated into the K-12 and undergraduate curriculum from a production orientation (i.e., calculation driven and/or employing the statistical investigative cycle) not a consumer orientation (i.e., sense making with statistics and/or employing the interrogative cycle). Following increased emphasis on statistics and data science in K-12 education, there has been a push for the inclusion of the consumer orientation (e.g., Bargagliotti, 2020; Boaler &

Levitt, 2019; National Governors Association Center for Best Practice & Council of Chief State School Officers, 2010; North Carolina Department of Public Instruction, 2020). For example, in North Carolina's newly adopted standards for Integrated Math 4, SP.1.4 reads: "Interpret non-standard data visualizations from the media or scientific papers to make sense of real-world phenomena" (North Carolina Department of Public Instruction, 2020, p. 20). From the consumer orientation, SL is conceptualized as the set of skills needed to effectively consume statistical messages presented in the real world, particularly the media. Given the multitude of data representations that people encounter daily, it is imperative to support students in becoming truly *critical* consumers of such representations so "they are not left vulnerable to people who are misrepresenting issues and data" (Boaler et al., 2021, p. 509). When these actions entail interrogating statistical content to inform action or change with a specific focus on power and equity, it is referred to as critical statistical literacy (CSL).

Teachers, including preservice mathematics teachers (PSTs), must be able to robustly enact CSL since they are responsible for supporting students' development of CSL from the consumer orientation. One way to think about how students and teachers make sense of statistical messages and data representations is to consider the habits of mind they employ when doing so. Habits of mind are the thinking behaviors that give rise to sense-making (Costa & Kallick, 2000a, 200b, 2008). Goldenberg (1996) posited that developing habits of mind is preferential to developing specific strategies to solve problems because habits of mind are transferable to new situations. This transferability is what makes habits of mind sustainable and powerful (Goldenberg, 1996). Critical

Statistical Literacy Habits of Mind (CSLHM) incorporate a critical lens[3] and are consumer oriented (Bailey, Chapter 2). The CSLHM are defined as the thinking behaviors called upon to make sense of statistical messages with a specific focus on how the statistics and/or statistical message are used to uphold or dismantle structures of inequity. Thus far, research has illustrated the differences between emergent and robust CSLHM enactment. Given the differences in enactment, it would be helpful to know what PSTs' robust enactment of the CSLHM looks like. The purpose of this study is to describe and contrast PSTs' CSLHM enactment.

Background Literature

The literature on SL reveals consensus on the importance of SL, but variety on what SL entails. Despite this variety, most SL models or frameworks include understanding statistical terminology, interpreting results, and valuing the need for data, generation of data, and possibility of data to produce conflicting conclusions (e.g., delMas, 2004; Gal, 2002; Garfield & Ben-Zvi, 2007). The overwhelming majority of SL models do not include a critical lens (i.e., emphasis on the relationship between literacy and power; see Lankshear & McLaren, 1993), thus such models and broader calls for SL have been criticized for ignoring or superficially incorporating a critical lens (e.g., Rubel et al., 2021). Gal's (2002) SL model is the one exception; he incorporated a dispositional element, critical stance. Gal recognized that "messages aimed at citizens in general may

[3] When I state "a critical lens" refers to any lens or collection of lenses that center issues of equity or power (acknowledging the plurality of critical lens)

be shaped by political, commercial, or other agendas which may be absent in statistics classrooms or in empirical enquiry contexts" (p. 15), and thus require the adoption of a questioning disposition to explore issues of power and equity.

There are several studies on PSTs' and in-service teachers' SL. Most revealed challenges and pointed to needed support for teachers (e.g., Tak et al., 2017). Nahdi et al. (2021) found that elementary preservice teachers exhibited difficulties drawing logical conclusions from data representations. Guven et al. (2021) studied elementary preservice teachers' SL before and after taking an introductory statistics course. Ultimately, they found that PSTs evidenced low SL, and SL was even lower on the post-test. In fact, PSTs performed better on data analysis questions devoid of context (purely procedural). Guven et al. raised concerns about PSTs with respect to thinking about the context within data investigations and analysis which is concerning given that context is a vital element of SL (e.g., Gal 2002, 2019). Similarly, Tak et al. (2017) examined secondary preservice mathematics teachers' understanding of sampling using Watson's (1997) statistical hierarchy. They found that teachers often disregarded the context with respect to sense making about sample variability. Other studies suggested that teachers prefer to rely on clean data instead of real, messy data (e.g., Muñiz-Rodriguez et al., 2020) which provides rich context, and others indicated that PSTs are not comfortable engaging in discussion or teaching of social justice issues (e.g., Simic-Muller et al., 2015). These studies suggest that PSTs and teachers are in need of support to develop SL.

There are fewer models that incorporate a true critical lens within SL. Scholars with data science literacy have included a critical lens with their models (e.g., D'Ignazio & Klein, 2020), however these models are production driven (and my focus is not on the

production orientation). With respect to SL, and from an explicitly consumer perspective, the critical lens is noticeably absent. Weiland (2017) synthesized literature on critical literacies and SL to build a CSL framework. He posited that CSL should incorporate both the production orientation and the consumer orientation with a Freirean emphasis on reading the world (i.e., making sense of statistical messages) and writing the world (i.e., the needed actions to combat issues of equity in the statistical messages). He suggested that future research should endeavor to determine how to support both students and teachers' development of CSL. My searches did not find any studies that explicitly report on PST or teachers' enactment of CSL.

Engaging in any field of study naturally elicits particular habits of mind. Habits of mind can be described as the thinking behaviors that experts employ to sense make (e.g., Costa & Kallick, 2000a, 2000b, 2008; Cuoco et al., 1996). Habits of mind scholars emphasize the methods over content, indicating that in our dynamic world content shifts and expires, but helping students achieve a repertoire of methods and thinking behaviors is far more useful (Cuoco et al., 1996). Within mathematics Cuoco et al. (1996) indicate that students should learn to become "pattern sniffers," "experimenters," "describers," "tinkerers," "inventors," "visualizers," "guessers," and "conjecturers" (p. 381). The emphasis is on engaging in behaviors that mathematicians engage in (not at the same level of content, but in the same behaviors). Within statistics, Lee and Tran (2015) have established statistical habits of mind focused on the behaviors statisticians employ when engaging in a statistical investigation which included thinking behaviors like "always consider the context of the data," "anticipate, look for, and describe variation," and "be a skeptic throughout an investigation". These habits are production oriented, and like SL

the critical lens is largely missing. CSLHM, on the other hand, is consumer oriented and incorporates a critical lens (Bailey, Chapter 2). Thus, this study draws on the CSLHM as the theoretical and analytical framework guiding the study.

Framework

The CSLHM (summarized in Table 4.1; Bailey, Chapter 2) resulted from the operationalization of Gal's (2002) critical stance within his SL model and Weiland's (2017) CSL framework. The CSLHM emphasizes that a questioning disposition is essential to effectively making sense of statistical messages from the real world, thus skepticism permeates each habit of mind. It is also important to note that it is possible to simultaneously enact the CSLHM.

Table 4.1

CSLHM Descriptions

CSLHM	Description
Questioning Sample Size and Methods	Individual demonstrates healthy skepticism regarding the sample, sample size, sampling technique, sampling bias, or lack of information regarding sampling that may lead to invalid inference on a target population. This includes considering who is missing, why, and how that influences the statistical message and the generalizability of the results, and the potential power of the message.
Recognizing Appropriate Statistics & Appropriate Representations	Individual questions whether the type of statistics and/or the way it is represented is the most appropriate for the data. This includes considering if the data representation employs techniques to mislead or deceive, thus questioning the motivation behind presenting the data in the way it was shared. Individual questions the role of outliers in the given representation. Individual questions whether the conclusions align with the selection of statistical test/procedure.
Desiring Additional Information	Individual demonstrates a need for additional information to draw a reasonable conclusion. Individual demonstrates healthy skepticism of the information, including the type of study, context of the study, the source (who collected the data and how), the author's motivation for sharing the statistical message (if not connected to appropriateness of the representation), and the credentials of the person/people sharing the statistical message.

Acknowledging Alternate Explanations	Individual acknowledges the potential for alternative interpretations for the meaning of findings or different explanations for what caused them, e.g., Was there an intervening moderator variable that affected the results? Are there additional or different implications that are not mentioned?
Recognition of One's *Own* Sociopolitical/ Critical Consciousness	Individual recognizes how one is integrating *their own* social, political, economic etc. understandings to make sense of injustice within the statistical message. Individual recognizes the degree to which one is engaged in critical reflection and critical action/active citizenry (see below). Individual recognizes the gaps in one's knowledge needed to interpret the statistical message.
Employing Active Citizenry	Individual is aware of inequities within the statistical message. Individual expresses desire to disrupt and dismantle inequities. Individual is motivated to act and describes next steps (action includes wanting to research the context, as education is an important part of being an active citizen).

I used the CSLHM framework as it operationalized the incorporation of a critical perspective in practice. The framework held the analytical power I desired, with guiding questions to support analysis. But more importantly, the use of a habits of mind lens elevated the importance of how robust enactment of CSL can result from the consistent and frequent use of developed thinking behaviors.

Methods

In a broader study of PSTs' CSLHM enactment (Bailey, Chapter 2), when making sense of data representations from the media it was found that most of the PSTs who participated enacted some CSLHM in an emergent manner. While the PSTs did enact some CSLHM, there was no evidence that this enactment was habitual. In fact, among the 17 PSTs only one enacted multiple CSLHM in a robust manner. Thus, this study aims to deeply examine the ways that PSTs enact CSLHM when making sense of data representations from the media. To do so, this study followed an instrumental multiple case study design (Bloomberg & Volpe, 2018). Two cases were selected from the broader study, one who exhibited typical CSLHM enactment (i.e., few CSLHM and emergently)

and the other unusual (i.e., many CSLHM robustly). Given the difference noted in the broader study, my purpose is to gain insight into the nature of the differences in enactment through the rich description that the case study methodology offers. The cases are bounded by time and context in that the data was limited to a single task-based interview in which PSTs were presented with 6 different tweets (i.e., a post on the social media platform Twitter) featuring a data representation.

The first case, Carrie (pseudonym), is a PST who exhibited CSLHM enactment typical of PSTs (i.e., the common case). The second case, Kate (pseudonym), was atypical in that she showed evidence of enacting the most of each CSLHM at least once and in a robust manner across multiple (not all) tasks (i.e., the unusual case). Specifically, this study aims to answer the following research questions:

1. How do the PSTs enact the CSLHM as they make sense of tweets featuring a data representation that has a social justice context?
    1. How does a typical PST enact the CSLHM when making sense of tweets featuring a data representation that has a social justice context?
    2. How does a PST who shows evidence of enacting many CSLHM robustly, enact CSLHM when making sense of tweets featuring a data representation that has a social justice context?
    3. What are the similarities and differences in the two PSTs CSLHM enactment when making sense of tweets featuring a data representation that has a social justice context?

Participant Selection

The case study participants, Carrie and Kate, were selected from a larger study of PSTs. The PSTs were recruited from 4-year universities in the southeast by asking mathematics education professors to send out a recruitment email including a link to the consent and a preliminary survey to their senior mathematics methods courses. Twenty PSTs completed the initial recruitment survey, and of those 17 (across three different universities) consented to and took part in the larger study. The participants completed a survey that included a set of demographic questions and the full Intermediate/Advanced Statistical Literacy form of the Levels of Conceptual Understanding in Statistics (LOCUS) assessment (Jacobbe et al., 2014) online (locus.statisticseducation.org). The LOCUS was designed for high school students in grades 10-12 which encompasses the content that PSTs will be expected to teach. It was included as a way to describe the PSTs' statistical knowledge at the time of the study.

Carrie and Kate were selected from this larger pool of participants. They attended different institutions but were both taking a senior level secondary mathematics course at the time of the study and would be student teaching the following semester. Carrie and Kate were similar in that they both self-identify as white females. They also have similar statistical backgrounds. Carrie and Kate took AP statistics in high school and introductory statistics at the university level. Carrie did not take any additional statistics content courses at the university level beyond the introductory course. On the LOCUS assessment she scored 90 out of 100 indicating she has a strong understanding of the statistical content taught in high school. Kate indicated that she had taken two additional statistics content courses at the university level. On the LOCUS assessment she scored 80

out of 100 indicating she has a strong understanding of the statistical content taught in high school. Neither had any previous knowledge of CSLHM before the study.

While they were similar in many respects, Carrie and Kate were selected for this study based on their differences. In the larger study Carrie exhibited CSLHM enactment typical of PSTs (i.e., the common case), which is to say she did not enact many of the CSLHM and what she did enact was not robust. In contrast, Kate was atypical in that she showed evidence of enacting each CSLHM at least once and in a robust manner across multiple (not all) tasks (i.e., the unusual case). Further details about how Carrie and Kate were selected for this case study are included in the sections that follow.

Data Collection

Data was collected using a semi-structured task-based interview (Goldin, 2000) via Zoom during the fall 2021 semester. The interviews included six tasks, each including a data representation that had been presented publicly via Twitter. Before starting the interview, the participants were asked to identify two people with whom they honestly discuss the media or news, one with whom they share similar beliefs and one who has dissimilar beliefs. Carrie and Kate did not identify specific people, but groups of people (e.g., Carrie's family or Kate's friends in the education program), thus moving forward I will refer to their selected people more generally as confidants with either similar or different beliefs. This information was used to tailor questions during the interview. At the start of the interview, I shared a task and asked them to think aloud: "As you are making sense of this tweet, please share what you notice." Once they finished talking aloud as she made sense of the initial task, they were asked what a conversation about the task would sound like with their confidant with similar beliefs. Then, they were asked the

same thing but with her confidant who has dissimilar beliefs. This process repeated for all six tasks. To ensure that Carrie and Kate naturally enacted the CSLHM, I limited my questions to clarification (e.g., I'm not sure I understand what you mean, can you explain that one more time or in a different way?) or elaboration (e.g., Can you tell me more about that?). After they cycled through all six tasks, they were asked if there were any which she would like to revisit.

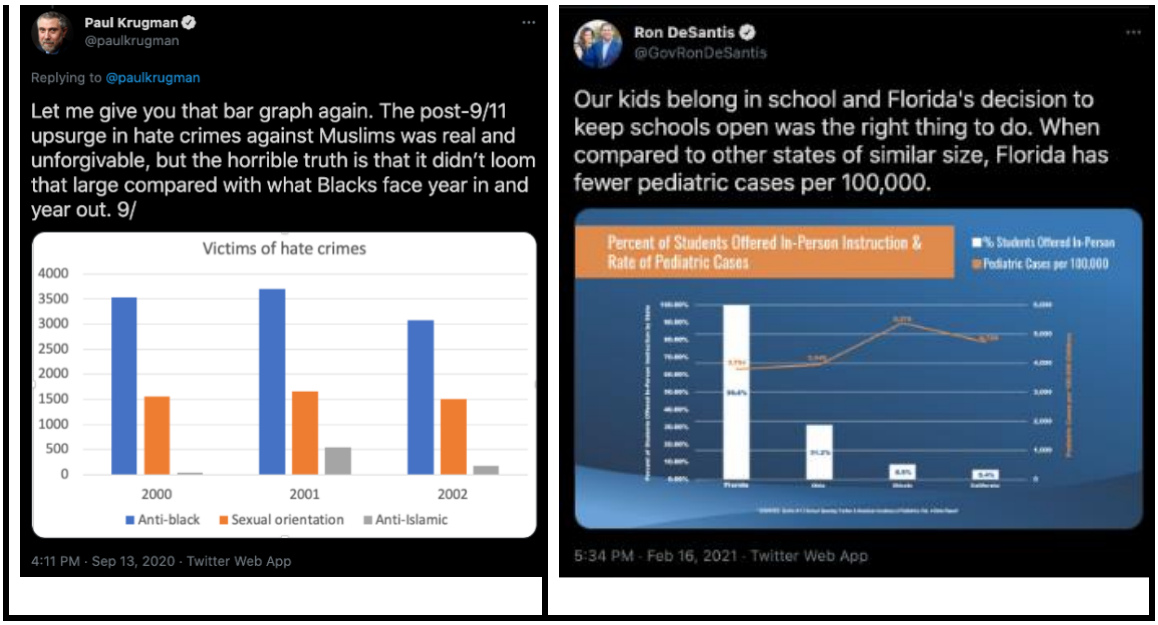
The Tasks

Each of the six tasks consisted of a single tweet related to a topic of social justice and featuring a data representation (five were static and one was dynamic; Butwell, 2020; DeSantis, 2021; JordanUhl, 2021; Krugman, 2020; Mobley, 2020; Purcell, 2017). Issues of social justice were selected as they naturally require critical analysis. Topics included: the gender wage gap, COVID-19 and education, flow of military gear to police, hate crimes, systemic racism among boys who grew up rich, and perceptions of deadly police encounters. I narrowed the focus of this study to tweets featuring a static or dynamic data representation since twitter is a common way to deliver statistical messages to the masses (Shearer & Mitchell, 2018). See Figure 4.1 for two examples.

Figure 4.1

Example Tasks Used in the Task-based Interviews

The Hate Crime Task is a tweet by Paul Krugman posted on September 13, 2020. He shares a bar graph on hate crimes between 2000 and 2002. His tweet was the ninth one in a thread in which he discussed his sentiments on hate crimes and specifically their connection to September 11, 2000.	The COVID and Education Task is a tweet posted by Governor Ron DeSantis of Florida on February 16, 2021. DeSantis shares a data representation that features two distinct graph types with two distinct y-axes on the same graph (same x-axis).



Analysis

In the first stage of coding, Carrie and Kate's interview data were deidentified and coded along with transcripts from the broader set of data. First, I used the CSLHM descriptions and guiding questions (see Table 4.1) as a priori theoretical coding which were developed according to the guidelines set forth by DeCuir-Gunby et al. (2011). The goal of this round of coding was to identify if and where PSTs enacted the CSLHM (existence of enactment). I asked a second researcher to independently code a randomly selected transcript for consistency. We compared our coding and discussed all discrepancies. This first comparison revealed that the coding and codebook were being used with strong consistency. I coded the remaining interviews independently and employed random spot checks with the second researcher to ensure consistent coding was upheld. Naturally, we were only able to code what the preservice teachers stated aloud; it is possible that the PSTs had thoughts that were not verbalized. While coding, I watched for the emergence of additional CSLHM; no new CSLHM emerged.

After the initial round of coding, I used the constant comparative method (Creswell & Creswell, 2018) to look for themes across the initial phase of coding. One main theme emerged: depth of enactment. By depth of enactment, I am referring to the level detail when applying a critical lens. I use the terms emergent and robust to describe the differences in depth of enactment. Emergent enactment of a particular CSLHM is characterized by vague wonderings (i.e., lack of depth and criticality). Robust enactment of a particular CSLHM is characterized by depth and criticality. For example, emergent enactment of the CSLHM *Questioning Sample Size and Methods* would include consideration of who was sampled but may fail to consider who is missing from the sample and why that matters. During this second phase of coding, each quotation was re-coded to determine if the coded CSLHM evidenced emergent or robust enactment (see the Appendix C for the full codebook). Again, I employed the use of the second researcher to ensure codes were applied consistently.

After the first two stages of coding, a summary table of all PSTs CSLHM enactment as either emergent or robust was created for each of the tasks (see Tables 4.2 and 4.3 for examples). These not only provided a summary, but also insight to how consistently the CSLHM were enacted for each PST across the 6 tasks. It was based on these tables that Kate and Carrie were selected as the focus of this case study. Kate was selected because the nature of her CSLHM enactment differed in depth and consistency from the other PSTs. Carrie was selected because she has a similar background to Kate (e.g., statistics course work), yet her enactment was very similar to the other 15 PSTs.

Table 4.2

Kate's CSLHM Enactment across the Six Tasks

	CSLHM					
Task	Questioning Sample Size/ Methods	Appropriate Stats & Representations	Additional Information	Alternate Explanations	Sociopolitical/ Critical Consciousness	Active Citizenry
Butwell (2020)						R
DeSantis (2021)	R	R	R	E	R	
JordanUhl (2021)			E		R	E
Krugman (2020)	R		R		R	R
Mobley (2020)	E		E	E	E	R
Purcell (2017)					R	R

Note: R indicates that Kate enacted the CSLHM robustly and E indicates that she did so emergently.

Table 4.3

Carrie's CSLHM Enactment across the Six Tasks

	CSLHM					
Task	Questioning Sample Size/ Methods	Appropriate Stats & Representations	Additional Information	Alternate Explanations	Sociopolitical/ Critical Consciousness	Active Citizenry
Butwell (2020)				R		
DeSantis (2021)	E			E		
JordanUhl (2021)						
Krugman (2020)			E	E		E
Mobley (2020)			E			
Purcell (2017)						

Note: R indicates that Carrie enacted the CSLHM robustly and E indicates that she did so emergently.

Looking across all six tasks, Kate often enacted the CSLHM robustly and across multiple (but not all) tasks (see Table 4.2). Kate most commonly enacted two CSLHM*:* *Recognition of One's Own Critical or Sociopolitical Consciousness* (5 of 6 tasks; 4 of which were robust) and *Employing Active Citizenry* (5 of 6 tasks; 3 of which were robust). It was less common for her to enact *Recognizing Appropriate Statistics and Appropriate Representations* (1 task robustly) and *Acknowledging Alternate Explanations*

(2 tasks emergently). While Kate did not robustly enact each CSLHM on each task, she did robustly enact five of six CSLHM at least once across the six tasks.

Kate's enactment was very different from the other PSTs including Carrie. Across the six tasks, Carrie most commonly enacted *Acknowledging Alternate Explanations* (2 tasks emergently; 1 robustly; see Table 4.3). Carrie never enacted *Recognizing Appropriate Statistics and Appropriate Representations* and *Recognition of One's Own Critical or Sociopolitical Consciousness*. She also rarely enacted *Questioning the Sample Size and Methods* (1 task emergently) and *Employing Active Citizenry* (1 task emergently).

Based on these initial findings I decided to focus on two tasks to more deeply understand the nature of Kate's and Carrie's CSLHM enactment: the Hate Crime Task (Krugman, 2020) and the COVID and Education Task (DeSantis, 2021) as these two illuminate the aforementioned difference most prominently. On these two tasks, Kate enacted the most CSLHM robustly (4 of 6 CSLHM robustly enacted on each task), and Carrie evidenced typical enactment (2-3 CSLHM emergently on each task). Thus, the third stage of analysis looked specifically at Kate and Carrie's enactment on these two tasks. I open coded each PSTs' enactment of a particular CSLHM on one task and repeated this process for each CSLHM and each task using a constant comparative method. This process revealed three major themes related to the attention to context, change in depth of enactment over time, and attention to the social issue. After these themes emerged, I open coded to examine these themes in more detail. Finally, based on this in-depth analysis I created detailed descriptions of each of the cases to create a complete picture of how the themes of attention to context, change in depth of enactment

during sensemaking, and attention to the social issue were related to enactment of the CSLHM. From these two cases, I completed a with-in cases analysis by answering the research questions for each case. Then I completed a cross case comparison (Yin, 2018) to better understand their similarities and differences.

Findings

The findings are organized by case. First, I will present the findings for the common case, Carrie. Since there are two tasks, I will begin by describing Carrie's enactment on the Hate Crime Task (see Figure 4.1; Krugman, 2020), and then I will describe Carrie's enactment on the COVID and Education Task (see Figure 4.1; DeSantis, 2021). After describing Carrie's enactment on each of the tasks, I will briefly discuss Carrie's enactment trends across the two tasks. I will repeat this process for Kate, the unusual case. Finally, I will present a cross case comparison in which I contrast Carrie's and Kate's enactment.

The Common Case: Carrie's CSLHM Enactment

Carrie was selected to represent the typical enactment evidenced by PSTs from the broader study (2-3 CSLHM emergently on each task; see Table 4.3). I will describe her enactment on the two tasks in subsequent sections.

Carrie's CSLHM Enactment on the Hate Crime Task (Krugman, 2020)

Carrie began unpacking the Hate Crime Task by expressing her surprise about the changes in hate crimes year to year:

> So, 2000 it's barely there [referring to the gray anti-Islamic bar], I guess it's not zero, but it's a very small number. And then here we have, it jumped to a little

> over 500 [looking at gray anti-Islamic bar for 2001]. And then I'm kind of surprised to see that it went back down pretty low. Um, only one year later. I'm just surprised it didn't last longer, I guess. Um, I wonder why from 2000 to 2001, the anti-black went up and then in 2002, it dropped down pretty, lower than it even was in 2000. So that seems to kind of, makes me wonder what that was about. Um, and again, even the sexual orientation jumped a little and then came back down so I'm wondering if 2001 was just because of the events that occurred that year, if everyone was just, um, I guess, hateful against anyone different than them.

In this unpacking Carrie briefly hints at how the hate crimes across all three categories could be related to the events of September 11—emergently enacted *Acknowledging Alternate Explanations*.

When considering what she would discuss with a confidant with similar beliefs, Carrie expressed a desire for more information: “I guess maybe, I'd like to see, um, maybe what these numbers look like now, especially the, um, anti-black hate crimes.” Carrie emergently enacted *Desiring Additional Information* by expressing curiosity about what this data would look like if the timeline was expanded. She did not explain why she wanted this information or why that would aid her sense-making of the data representation. She then emergently enacted *Acknowledging Alternate Explanations* by vaguely wondering what might have caused the decline in hate crimes in 2001: “Maybe look at what was going on in 2002 that made it go down because obviously that was a good thing. So, what was maybe what was going right in 2002, to decrease those hate crimes?” She then shifted back to emergent enactment of *Desiring Additional*

Information: “I guess I'm going to assume that this data is hate crimes in the US as a whole, so I guess it would maybe be interesting to look at it like state by state. Yeah, I think that's about it.” She expressed the desire to know if the data was national and to see the data disaggregated by state. Again, she did not explain why she wanted to see the data disaggregated.

Carrie’s CSLHM Enactment on the COVID and Education Task (DeSantis, 2021)

Carrie began this task by emergently enacting *Questioning the Sample Size* and *Methods*. She considered the states in the data representation and wondered if the sample was intentionally selected:

> We have Florida, Ohio, Illinois, and California. So, I guess first off, I mean, I understand why we chose Florida because that's what we're talking about, but I wonder, um, why we chose Ohio, Illinois, and California. Um, is that a strategic choice to make Florida look better? Or are those, is there another reason behind it? Like, um, it kind of looks like maybe Ohio had the next, maybe they were, um, had the next closest person of students offered in-person instruction or I guess I'm just wondering why they chose these three.

This enactment was emergent because Carrie did not explicitly consider if the sample was representative of the population. She then shifted to wanting to compare Florida’s educational testing data to the other states: “I think it would definitely be interesting to see, um, to compare some other things as well. Like see how maybe Florida's like, um, testing, like maybe their standardized testing compares to the other, um, states that didn't offer in-person instruction.” By wondering about the connection between the two variables in the data representation and adding in a third variable (i.e., standardized

testing) that seemed vital to consider, Carrie is enacting *Acknowledging Alternate Explanations*. When imagining a conversation with a confidant with similar beliefs, she revisited this idea: “I definitely think it would come up, like, wanting to look into, um, how Florida did, like how their students did compared to these other states.” She revisited this idea for a third time when imaging a conversation with a confidant with dissimilar beliefs: “I guess maybe one thing would be just like if, um, the, like I said, I would kinda like to see how Florida did compared to these other states with like their testing and student progress and things like that. And I guess maybe one thing would be like, is that even worth it?” She begins to allude to why she might be considering educational testing data in this last example, however she never explicitly states why this is important. Thus, her enactment in all three instances is emergent as it is missing an explanation of why or how this new variable influences how Carrie is interpreting the statistical message, nor does she explore multiple narratives. Carrie continued to emergently enact *Acknowledging Alternate Explanations* by wondering about the infection rates among the adults in the schools: “I guess also maybe want to look at like look at, um, we're looking at pediatric cases, but it would be interesting to look at adult cases as well because there are adults in schools.”

Carrie’s CSLHM Enactment across Both Tasks

Carrie enacted only two CSLHM on each task (three different CSLHM across these two tasks) and did so emergently. Her CSLHM enactment across both tasks was characterized by general wonderment and lack of justification or explanation. When she expressed a desire for more information, she did not discuss why such information was pertinent to her sense making. Similarly, when she shared a potential alternate

explanation, Carrie only considered one perspective and did not explicitly explain why this perspective is important to consider. When she suggested potential bias in the sample, she did not explicitly consider the representativeness of the sample. Each of these instances point to the vague wonderings characterized by emergent enactment of CSLHM. I will briefly discuss Carrie's CSLHM enactment with respect to the three themes.

Carrie's Attention to Context. Carrie's CSLHM enactment evidenced inconsistent and surface level integration of the context. For example, on the COVID and Education Task, Carrie did make contextual connections when considering how Florida's educational testing compares to other states. She did not explicitly discuss how such a comparison is important when weighing the risks and benefits of in-person schooling during a pandemic. In other instances, she seemed to ignore the context. When considering if the sample was intentionally selected, Carrie missed the opportunity to explore how politicians are worth scrutinizing and how deceit is possible. When engaging with the Hate Crime Task, Carrie did not discuss the broader implications of September 11, 2001, and terrorism in connection to hate crime rates. Across these two tasks, Carrie either ignored the context or made superficial connections.

Carrie's Attention to the Social Issue. Carrie danced around the social issues. She consistently used the language from the data representation, but often did not consider the broader implications of the issue or the connections to society. For example, on the Hate Crime Task, she used all the buzzwords from Krugman's graph such as "hate crime," "anti-black," and "sexual orientation," yet she did not explicitly discuss either the

context or issue within her enactment. For example, she did not explore hate crimes beyond a general wondering of why it happens and what caused the spikes or dips.

Carrie's Change in Depth during Sensemaking. Carrie's CSLHM enactment was consistently emergent. She did not change with respect to depth of enactment when making sense of the data representations in either task.

The Unusual Case: Kate's CSLHM Enactment

Recall that Kate was selected because her enactment deviated from the typical PSTs' enactment from the broader study. Kate enacted more CSLHM and often robustly (4 of 6 CSLHM robustly enacted on each task; see Table 4.2). I will describe her enactment on the two tasks in subsequent sections.

Kate's CSLHM Enactment on the Hate Crime Task (Krugman, 2020)

While unpacking the data representation that Krugman (2020) shared, Kate carefully read the entire tweet thread. In reading, Kate revealed how the lack of transparency about the definition of hate crime in this tweet thread and in the data representation is a vital consideration. She suggested that how this variable, hate crime, is defined is connected to both questions about the sample and methodology. Kate stated:

> Well, I think a hate crime is different from anti-Muslim sentiment. In fact, I would say most people with hate in their hearts aren't actively doing things that could get them arrested. Um, they're posting on Facebook or making a scene at Starbucks. And so even this, this graph doesn't do justice because this is also assuming that they're reported, um, reported documented crimes, which we know so many hate crimes are under reported. They're not reported whatsoever. And also, you know,

> harassment is not a crime in most instances, and you know, people love to hide behind free speech. And so, I think all of those fall into anti-Muslim sentiment.

Here Kate simultaneously enacted *Desiring Additional Information* and *Questioning the Sample Size and Methods*. There is implicit questioning of how hate crime is being defined and should be defined, thus Kate is emergently enacting the CSLHM *Desiring Additional Information.* Her enactment was considered emergent because she had yet to explore how this desire to know more about how hate crime is defined influences how she was making sense of the data representation. By comparing Krugman's use of the phrases "hate crime" and "anti-Muslim sentiment," Kate began to unpack that the operationalization of this variable matters and more specifically how these ideas relate to questions about the sample. She suggested that even if "hate crime" is clearly defined, there will be unreported hate crimes. This implication means that those unreported hate crimes cannot be captured in this sample, thus underestimating the severity and prevalence of hate crimes within this data representation. She robustly enacted *Questioning the Sample Size and Methods* by delving into thinking about who is not captured in this sample, why, and more importantly why that matters.

After reading the thread in its entirety, Kate shifted from emergently to robustly enacting *Desiring Additional Information.* Initially, she vaguely wondered about how hate crime was being defined. She shifted to not only justifying why she wants more information, but continuing to question the

operationalization of the variables more explicitly and in conjunction to how this influenced her sense making of the statistical message. Kate stated:

> Something I would be interested in looking at, number one, is that altered graph considering a larger, um, summary of, of what could fall into anti-Islamic. Especially because, um, there's, there is a lot of truth to, um, anti-Arab and anti-groups, quote, confused with Arabs, um, falling into that picture because it's the, the racism, the prejudice falls, um, it's not as nuanced as this guy is, is making it seem to be, especially when the people, um, acting out these hate crimes, probably don't or potentially don't have a lot of knowledge on, on the victims. And are largely spreading hate based on an appearance, [which is] why he's saying that this number is understated because other people could fall into a specific appearance.

Kate expressed the desire to see what this data representation would look like if the anti-Islamic category was extended not just to hate crimes against Islamic individuals, but if it were possible to capture any hate crime that was committed in an anti-Arab spirit. She aptly discussed that hate crimes are often committed based on superficial attributes like appearance without much knowledge of the victim. Her statements not only expressed a desire for more information, but more information to grasp a better understanding of the problem and prevalence of hate crimes as they are connected to racist ideology. She continued to desire additional information by wanting to consider more data:

> I would rather put focus on highlighting some lived experience and some more substantial data. I guess I have to, I feel pretty confident in the statement that hate crime is, um, not a broad enough category to explore, to determine that there was anti-Muslim sentiment. And also, I would want to, uh, consider highlighting the voices of Muslims who were really, really impacted. And the crossover between Muslims whose family members died in this horrific event and whose entire lives were turned upside down. I'd like to look at the individual impact as well as the macro impact. It seems like we need to zoom out a bit and also zoom in.

Kate expressed that one graph does not tell the whole story, but she also advocated for a variety of data types and levels to be able to understand this issue more fully. By suggesting we need both qualitative and quantitative data as well as the ability to zoom in and out, Kate demonstrated an understanding that data representations tell stories and to fully understand the story we must view it from several angles and slices. By wanting more data and explaining why, Kate robustly enacted *Desiring Additional Information*.

When I asked Kate what a conversation about this tweet and data representation would sound like if she was chatting with a confidant who holds different beliefs to herself, she continued to build upon her previous line of thought. Kate expressed:

> I would like to challenge this, this graph because I think it could potentially be misleading in how, I would like to see how this was gathered. Um, I would, I would like to be referencing the actual survey

> done. Like what's considered a hate crime? How are these quantified? Let's compare that to hate crimes that go un-, um, not unnoticed I unreported. Um, let's talk about, you know, newspaper headlines during that time or signs in restaurant windows expressing so much hate. Um, and simultaneously I would, I would want to zoom out and also zoom in and ask them to consider, you know, people's individual experiences and tell them that it wasn't that bad. Um, because this is not taking a very empathetic, it doesn't look like this is a really empathetic stance. And I think I would encourage, um, encourage someone to consider this with a little bit more human empathy. Um, because I don't like to look at everything as just a quantitative statistic. There's qualitative factors. There's human, human beings impacted by this. Um, and I also, I almost feel like the "it didn't loom that large compared with what blacks face year in and year out". I honestly feel like it's, that's a sentence weaponizing, two groups kind of against each other.

She began to vaguely consider how she would challenge her confidant's views, thus emergently enacting *Employing Active Citizenry* in addition to her continued robust enactment of both *Questioning the Sample Size* and *Methods* and *Desiring Additional Information*. She started to think about her next steps of crafting a discussion by expressing a desire to dive into both how this data was collected and also searching for additional data (e.g., what can we research about society's stance towards Islamic individuals based on newspaper headlines, can we find data that perhaps includes a survey asking about unreported hate crimes). Rather

than explicitly discussing how she would press on the viewpoint of her confidant, she has expressed a general plan of attack for that conversation. As she continued to describe this planned conversation, she also robustly enacted *Recognition of One's Own Sociopolitical or Critical Consciousness* and *Employing Active Citizenry*:

> And I, I would like to know, you know, well, okay, what do you want to do with this information? Are you, are you cool with like, let's say 2002 or 2000, are you cool with, uh, either of those graphs? Do those numbers not upset you? Are you, are you okay saying it's not that bad? And deciding to make no further effort or further stance or justice-based opinions or actions against that? Because that's when I start to enter the territory again, of like, I don't entertain racists or xenophobes, prejudiced people, I'm not going to put too much time into a conversation where someone is trying to convince me or essentially Gaslight me into not believing, um, the severity of hate that exists.

Her enactment of *Recognition of One's Own Sociopolitical or Critical Consciousness* and *Employing Active Citizenry* is intertwined here. While imagining what someone with different beliefs would in fact believe, Kate clearly identified her personal lens and feelings, but also explicitly acknowledged that there are groups of people with whom she would exert little energy when discussing this task. She implied that she was only willing to engage with individuals who recognize the broader social justice issue captured in this task. Her unwillingness to engage with people who do not recognize the social issue

that is clearly important to her, exemplifies robust enactment of *Employing Active Citizenry* by identifying boundaries of the circumstances surrounding when and how (or how not) she would press on the views of others. To craft this boundary, Kate drew on her feelings and beliefs (*Recognition of One's Own Sociopolitical or Critical Consciousness*).

Kate's CSLHM Enactment on the COVID and Education Task (DeSantis, 2021)

Before discussing what Kate shared while making sense of this task, it is important to note that Kate revealed that she was not sure who DeSantis is. She inferred that he was the governor of Florida by reading his Twitter handle (@GovRonDeSantis) and Florida focused tweet.

Almost immediately Kate questions the data representation and tweet posted by DeSantis as an issue of correlation confused with causation:

> I just think that it's a lot of, it's one of those instances of like, does, does this correlation mean that there's some causation and I think it's a tough, it almost feels like the argument is like keeping kids in school, maybe not necessarily that it's like preventing COVID cases, but that it's completely isolated from how many COVID cases children are getting. Um, I just think it's a big generalization…I don't think this, this chart doesn't speak to any other circumstances surrounding what may be causing pediatric cases of COVID-19. Um, which is what I'm, I'm assuming this is about, COVID-19 looking at the, the date of the tweet and the nature of the tweet. I'm guessing that they're referring to COVID-19. Um, nothing says that, but, well, that could be a point of contention. But, um, you know, for example, just, uh, thinking about California, for example, having such high cases

> and how, um, kind of how living conditions are a little bit tighter in California, potentially compared to Florida. I know there's a lot of rural Florida, potentially more than rural California. I wouldn't know enough to be conclusive about that, but I have a feeling that, um, in person instruction is, got nothing to do with having lower rates of COVID. If anything, we see that it has caused outbreaks.

Kate recognized that nowhere in the tweet or on the data representation does it indicate if this statistical message is about COVID-19. More importantly, she identified a potential issue of correlation confused with causation, thus robustly enacting *Recognizing Appropriate Statistics and Appropriate Representations*. In discussing this issue, she also emergently enacted *Acknowledging Alternate Explanations* by identifying an example of a variable (density of living conditions) that may influence the relationship between the percentage of students offered in person instruction and the pediatric COVID-19 rate. As she only considered one alternate explanation and not multiple narratives or viewpoints, she emergently enacted *Acknowledging Alternate Explanations*.

She continued her skeptical inquiry by robustly enacting *Questioning the Sample Size* and *Methods* and wondering if the selected states were curated to tell a particular story: “I'm curious to know why they picked these particular states. I think it's possible that they pick them because they have really high cases and really low school, uh, in person instruction. That's probably why, so the graph is incredibly polarizing.” Kate explicitly wondered if the sample was chosen to mislead viewers. She connected this back to the appropriateness of the representation: “And it's upsetting to me to see, uh, political figures taking something like pediatric cases of COVID and using it as bait or as fuel for their personal, um, beliefs or what they're personally supporting in terms of

legislation and state mandates." Kate discussed the connections between her questions regarding the sample selection and DeSantis' intentions displaying the data the way it was displayed, thus simultaneously and robustly enacting *Questioning the Sample Size* and *Methods* and *Recognizing Appropriate Statistics and Appropriate Representation*.

When I asked her to imagine a conversation with a confidant with similar beliefs to herself, she enacted *Desiring Additional Information*. First, she expressed the desire to learn more about the qualifications of DeSantis to share such information:

> We'd probably circle to some conversation on the qualifications of this guy to analyze these findings or lack thereof. I'm guessing he's not a statistician or a, what, doesn't study pandemics, epidemics, probably not, probably isn't as well versed in this. As you know, I would challenge the idea that his position can really support him having authority over this, versus if he were even, you know, trying to back a panel of scientists or doctors making this statement, it's him making this statement.

Her enactment was emergent as she did not connect this explicitly to why it would be important to know more about DeSantis' qualifications. Kate felt strongly that this issue needed to be addressed by drawing from experts in the field, and questioned whether the Governor would have the needed statistical, epidemiological, or virological knowledge to fairly analyze this data. Beyond wondering what DeSantis' qualifications are, she also expressed a desire to know more about the sources listed in the fine print:

> I don't know what the source is. I'd be interested in looking up what the Burbio K-12 school opening tracker and American academy of pediatrics, February 4th state report, where that's coming from and kind of what their stance is. Um, because it's

> very possible that this has been picked out of, you know, a web page or a study or a report, and is actually not in line with what the original report says. That happens a lot is that. Um, things are cited. But if you go to the original source, you're kind of surprised by the difference in conclusions that are being made. Um, so I'm, if I were speaking to somebody who I thought had really like opinions to me, I would, we'd probably be talking about, um, being critical of the original source, seeing the conclusions made there versus the conclusions of this tweet and also criticizing, um, Governor Ron DeSantis' ability to analyze what this information actually means in terms of a pandemic.

What makes Kate's enactment robust is the explicit justification of why this additional information is necessary. Kate drew on her experience and shared that some data representations are pulled from broader studies and how extracting information from an original report or source can be taken out of context.

When Kate transitioned to think about a conversation with a confidant with dissimilar beliefs, she wondered what the story would be with more data (i.e., more states) as well as questioned the magnitude of the difference in rates among states:

> But also, I would be interested to see how these, I would, I would, you know, open up the conversation about what does this look like with all the other states? How does this information relate to other states? Is it significantly worse than other states, for example, because what if, you know, 3,794 is starkly higher than, you know, two dozen other states are those two dozen other states like doing the wrong thing. I'd like to see, like what's their percent of students offering in-person instruction.

Kate expressed a desire for a broader sample to be able to better understand the effect size and in doing so robustly enacted *Desiring Additional Information.* Then she revisited the idea of being qualified to share such information by enacting *Recognition of One's Own Sociopolitical or Critical Consciousness*:

> Um, but this is a tough conversation. And it, I, I, um, I think I would openly say that I only feel so qualified to share. What's been reported by credible sources, by scientists and by, you know, credible organizations. And I, I don't think I would feel in the same way I'm criticizing the governor for making some type of statement about like, this means that it's good to have in person instruction. I would want to refer to professionals in the field before I try and make some conclusion, um, because it's just not super in my scope of expertise. And I think sometimes that makes people uncomfortable, um, to not feel like I'm the right person to be this authority of knowledge, especially when things turn into a debate, you'd like to be the authority of knowledge.

Kate recognized the gap in her own knowledge and identified that she would feel more comfortable gaining knowledge from professionals. She continues to explore this idea by recognizing that a gap in her knowledge is worthy of pausing the conversation. She also connected this to her experience as a student receiving notifications about potential COVID-19 exposure and how this experience may shape her unwillingness to engage with someone who believes that in-person instruction was the right call:

> I would probably take any, any pro in-person instruction arguments pretty lightly just in my personal opinion and just what I've seen with students in schools. And, you know, being someone at a university, we get COVID contact tracing emails

> when someone is like out sick with a case of COVID in one of our classes, that'll be an email sent to us as a potential COVID contact. And then the people who apparently sat really close to that person will also be asked to isolate. But, um, like just the amount of emails that I get based on that that's like someone in your class has COVID. We can't tell you who, but someone's got it. So good luck. Um, I know at the beginning of the semester, at least I was getting multiple of those emails a week and I'm only in three classes. And so, I don't think that it's, um, and that's obviously just a personal anecdote, but from my own experiences, I don't know that I'd have a lot of energy to try and listen to somebody tell me why in person instruction is so critical.

Kate was able to recognize how her beliefs about relying on professionals and how her experience of COVID-19 exposure influenced how she imagined the conversation with a confidant holding different beliefs and thus how it shaped her sense making of the data representation.

Kate finished her discussion of the DeSantis (2021) task by emergently enacting *Acknowledging Alternate Explanations*. She began to consider the broader impact by wondering if Florida's educational data supports Florida's decision to maintain in-person education:

> And I'd also like to see like where Florida ranks in education. Cause I know it's not that great… Um, but if you're looking at, um, Florida education ranking versus for example, California, um, I feel strongly confident that there there's a stark difference, um, because I I've, I've looked at it and I can't pull it off the top of my head, but, um, in person instruction is also not a measure of effective

> learning necessarily. So that's another, that's a whole, another issue is like, where does Florida rank and in education, how are they doing? Are they okay?

Her enactment of *Acknowledging Alternate Explanations* was classified as emergent as she only considered one perspective or narrative. She considered whether in-person instruction during a pandemic seemed to produce educational gains that may make the risk of infection worthwhile. She did not consider any other narrative other than from the stance of someone who believes in the risk of infection.

On this task, Kate again demonstrated elevation of enactment. Kate elevated from emergent enactment of *Desiring Additional Information* to robust enactment. She began by wondering about DeSantis' qualifications but did not explain why this would be important to know. As she wondered about his qualifications, she also examined the source. In doing so she explained that she would want to see the original studies to know if this piece of evidence that DeSantis was supplying was taken out of context. Her initial skepticism of DeSantis' qualifications led her to more carefully consider the sources listed in the fine print.

Kate's CSLHM Enactment across Both Tasks

Across these two tasks, Kate enacted all six CSLHM, and five of the six robustly. On each of these two tasks, Kate robustly enacted four of the six CSLHM. Kate's enactment across both tasks was characterized by detail, depth, and explanation/justification (with the exception of *Acknowledging Alternate Explanations* which she emergently enacted). I will briefly discuss her CSLHM enactment with respect to the three themes.

Kate's Attention to Context. Kate explicitly integrated the context. She did so in a variety of ways, but regardless of how, what Kate did consistently was connect her sense making to the context. Each quotation evidenced discussion of hate crimes or COVID and education beyond the key words on the data representations. When engaging with the COVID and Education Task, Kate talked about living conditions/population density, whether or not the data is an appropriate slice from a broader report (i.e., taken out of context), mask mandates, the implications on students and covid tracing emails, and much more that grounded her discussion within the context of the tweet and data representation.

Kate's Attention to the Social Issue. Kate also consistently attended to the broader social issues. For example, on the Hate Crime Task, Kate questioned how the variable hate crime was defined and connected this to the broader issue of what actually constitutes a hate crime and how that influences data collection and variable measurement. She clearly articulated the understanding of how hate crimes are reported results in underestimating their prevalence (e.g., not all hate crimes are reported to police or authorities). It is also important to note that when talking to an interviewer she did not know, she was not shy to share her beliefs, her political stance, and her feelings on these issues. More importantly, Kate shared why and how her beliefs and feelings influenced her sense making of the data representation.

Kate's Change in Depth during Sensemaking. Typically, Kate enacted a CSLHM and then immediately justified how the wondering influenced her sense making of the data representation (robust enactment). In the instances where she initially emergently enacted a particular CSLHM, she usually built upon her thinking and

eventually integrated the needed justification or explanation to enact the CSLHM robustly. This change from emergent to robust enactment as she continued to make sense of the data representations represents a change, or elevation, in depth during sensemaking. Across these two tasks, Kate elevated from emergent to robust enactment twice. While she did elevate her enactment of *Desiring Additional Information* and *Employing Active Citizenry*, she did not change her depth of enactment of *Acknowledging Alternate Explanations*.

Cross Case Comparison: Contrasting Carrie's and Kate's CSLHM Enactment

There are several important differences in how Carrie and Kate enacted the CSLHM. I will discuss these differences by theme in subsequent sections.

Attention to Context

The first difference is how context was integrated. Carrie often wondered vaguely without explicitly making connections to the context. For example, when she enacted *Desiring Additional Information* on the Hate Crime Task (Krugman, 2020) and wanted to see disaggregated data she did not mention the context or why this would be important to explore: "I guess I'm going to assume that this data is hate crimes in the US as a whole, so I guess it would maybe be interesting to look at it like state by state. Yeah, I think that's about it." With the exception of indicating that this is hate crime data, Carrie completely ignored the context. She did not make as many connections to the context beyond the elements within the data representation (i.e., used the exact language from the graph without digging into the context). In contrast, Kate continually drew from and integrated the context into her enactment; none of her comments were separated from the issue of hate crimes or COVID and education. Each thought Kate shared was directly tied

to the context. She discussed many topics (e.g., politics, her feelings, explicit discussions about who is qualified to share) in direct connection to the context. More prominent in the COVID and Education Task (DeSantis, 2021) was Kate's fearless approach to share her beliefs with someone she did not know (interviewer).

Attention to the Social Issue

Another stark difference was how the two PSTs talked about the social issue. Carrie danced around the issue. When thinking about the differences in the bars from year to year on the Hate Crime Task (Krugman, 2020), Carrie would state things like: "makes me wonder what that was about." She never labeled or discussed beyond the exact wording on the data representation (i.e., "hate crime," "anti-black," or "sexual orientation"). On the other hand, Kate was unafraid to discuss the issue. Kate intertwined political ideas (i.e., "free speech") and considered broader human impact. Kate explicitly talked about how the tweet by Krugman (2020) could "weaponize" two groups against each other.

Change in Depth during Sensemaking

The last difference lies in the depth of enactment during sensemaking. Carrie emergently enacted some CSLHM and never changed with respect to depth of enactment. However, Kate sometimes began with emergent enactment of a CSLHM and eventually enacted it robustly as she continued to make sense of the task. She evidenced this on the Hate Crime Task (Krugman, 2020) when enacting *Desiring Additional Information* and through continued sense-making elevated her enactment of *Desiring Additional Information*. Her initial desire for more information was general and disconnected from thinking about how wanting more information influenced her interpretations. As she

continued her sense making, Kate expressed why she wanted more information, thus elevating her enactment. Kate did not demonstrate the same elevation on the Hate Crime Task with respect to *Employing Active Citizenry* on this task. I will explore possible reasons for this in the discussion. On the COVID and Education Task (DeSantis, 2021), we again see evidence of Kate's elevation of enactment. Kate elevated from emergent enactment of *Desiring Additional Information* to robust enactment. She began by wondering about DeSantis' qualifications but did not explain why this would be important to know. As she wondered about his qualifications, she also examined the source. In doing so she explained that she would want to see the original studies to know if this piece of evidence that DeSantis was supplying was taken out of context. Her initial skepticism of DeSantis' qualifications led her to more carefully consider the sources listed in the fine print.

Discussion

The goal of this study was to provide an in depth description of what CSLHM enactment looks like for two PSTs who appear to enact it in very different ways. Carrie (who is representative of the typical PST) emergently enacted just a few CSLHM per task. Her enactment is characterized by general wonderment without justification or explanation. In contrast, Kate enacted more CSLHM robustly and with more consistency. Her enactment is characterized by reflection upon how her thoughts, wonderings, or feelings influence her sense making of the data representation. The most prominent differences between the common and unusual cases were with respect to the attention to context, attention to the social issue, and change in depth of enactment during

sensemaking. These differences, or three themes that emerged, are related to how the PSTs' enacted the CSLHM. The typical case (Carrie) revealed little to no integration of context and very little attention to the social issue as evidenced by heavy reliance on buzzwords. Kate's CSLHM enactment (the unusual case) was enhanced by consistent integration of both the context and social issue. She drew connections between current events, other factors (e.g., mask mandates or the effect of individual appearance on hate crimes), and her beliefs and feelings to make sense of the data representation. Furthermore, Kate elevated the depth of enactment of two different CSLHM from emergent to robust as she continued her sense making. While she did not always demonstrate such a change in depth, how she built upon her wonderings suggests a potential means to help other PSTs change their depth of CSLHM enactment. More specifically, scaffolding PSTs' sensemaking with probing questions could potentially help them build their own wonderings and elevate their CSLHM enactment; this warrants further research.

Before delving into the implications of this study, I want to address some specific limitations. First, I only focused on the two tasks in which Kate evidenced very different enactment to the typical PST. Looking at the other tasks in the broader study could provide more insight into why Kate, or other atypical PSTs, enacted CSLHM more consistently and more robustly on some tasks and not others. Future research should aim to unearth what prompts more consistent robust CSLHM enactment. Another limitation is having only 1 PST per case. In the broader study of 17 PSTs, only Kate was atypical, thus more research is needed to study other PSTs that deviate from typical enactment patterns. Another limitation is the nature of the questions I asked in the interview. As I wanted to

study how the PSTs enacted CSLHM, I intentionally limited myself to clarifying questions. It is entirely possible that some probing would change the depth of CSLHM enactment. Similarly, retrospective interviews might provide insight into why PSTs enacted CSLHM the way they did.

With rising calls to include SL from a critical and consumer perspective (e.g., Bargagliotti et al., 2020) and updated standards that include the consumer orientation (e.g., North Carolina Department of Public Instruction, 2020), teachers will be responsible for helping students digest statistical messages from the real world. With this responsibility comes the need for teachers (including PSTs) to be able to robustly enact the CSLHM. The findings of this study suggest several ways to help PSTs develop robust CSLHM enactment. I will discuss them based on the themes from the findings: attention to context, attention to the social issue, and depth of enactment over time.

Attention to Context

Context is vital to CSLHM enactment and SL in general (e.g., Gal, 2002, 2019). Prior research on SL enactment revealed that PSTs and teachers face difficulties when integrating context into sense making about sampling (Tak et al., 2017) and questions about data analysis (Guven et al., 2021). While such research focused on SL from a broader production orientation, the findings from this study corroborate the difficulty with context but from a consumer orientation.

The common case (Carrie) showed inconsistent, vague, and sometimes absent focus on context. Carrie rarely ventured beyond vague wonderings. She integrated the context on the COVID and Education Task with more consistency than on the Hate Crime Task. It is possible that as a future educator, Carrie felt more passionate about the

COVID and Education Task. It could also be related to the level of familiarity, experience, or knowledge of the particular context. In contrast, Kate (the unusual case) consistently connected her CSLHM enactment to the context. These connections often resulted in exploring other factors, questioning further, or considering qualifications, all of which deepened her sense making and likely influenced her depth of enactment. This difference between the common and unusual cases suggests that there are obstacles to consistently integrating the context when making sense of statistical messages from the media. Future research should aim to explore what these obstacles are, the relationship between considering the context and depth of CSLHM enactment, and the relationship between familiarity with, experience with, and knowledge of the context and CSLHM enactment.

Attention to the Social Issue

Research also suggested that PSTs are hesitant to discuss and teach social justice issues (e.g., Simic-Muller et al., 2015). The findings of this study support such hesitations. Carrie (the common case) did not talk about the issue beyond repeating the key words from the data representation, whereas Kate was unafraid to discuss such issues. Kate openly discussed racism and sexism in her interview. She did not know the views of interviewer, but openly identified herself as a liberal and freely expressed her feelings and beliefs. Her behaviors were atypical, thus there is an opportunity to explore why. It is possible that she has had practice engaging in social justice driven conversations; she often spoke about her education courses and the university's emphasis on anti-racist teaching practices during the interview. While we do not know the specific reasons that Carrie did not talk about the social issue, we do know that this was typical

among PSTs. Similar to attention to the context, it is possible that passion for the particular issue, personal experience with the issue, and knowledge of the issue influence PSTs willingness to discuss it. Kate's confidence speaking openly seems to suggest a connection between critical consciousness, comfort level in discussing social justice issues, and CSLHM enactment that warrants further study. Furthermore, her continual reference to a program that values anti-racist pedagogy suggests the need for explicit coursework that develops critical consciousness and exposes PSTs to critical pedagogies.

Change in Depth during Sensemaking

Kate demonstrated elevation of her initial emergent enactment to robust enactment on several CSLHM (as seen with *Employing Active Citizenry* on the Hate Crime Task and *Desiring Additional Information* on both tasks). With respect to *Desiring Additional Information*, she did so by explicitly connecting the initial emergent enactment back to how it influenced her sense making about the data representation. As elevating from emergent to robust on a particular task was not typical of the PSTs, we can learn from Kate. In these instances, Kate began with a vague question or wondering and eventually supplied justification or explanation for why those questions or wonderings were important to consider. This method of reflection on why the information is needed or how it influences sense making of the data representation can potentially serve as a starting point for supporting other PSTs in elevating their own CSLHM enactment. It is possible that asking PSTs to reflect upon how their question or wondering is connected to how they are making sense of the data representation may elicit robust enactment of some, if not all, CSLHM.

There were instances where Kate did not elevate her initial emergent enactment on other tasks. Since Kate was adept at elevating her enactment, it is worth exploring some potential reasons why she did not always do so. It is possible that robust enactment of some CSLHM, such as *Employing Active Citizenry*, is dependent upon a certain degree of passion for the social justice issue, and perhaps Kate did not hold the needed passion to robustly enact the CSLHM on a particular data representation. Future research should endeavor to better understand the obstacles to CSLHM enactment and potentially compare PST CSLHM enactment to that of other populations. Perhaps more importantly, future research should explore how to support PSTs' development of CSLHM. Given that I only asked clarifying questions, it is possible that intentional probes could deepen enactment. Lastly, research should investigate if explicit use of the CSLHM framework alleviates some of the aforementioned challenges cited in research on PSTs' SL enactment (Guven et al., 2021; Tak et al., 2017). In other words, if PSTs are taught to use the CSLHM framework, they may be more inclined to consider the context and discuss the social issue.

In conclusion, the aim of this study was to describe PSTs' CSLHM enactment when making sense of statistical messages from the media. More specifically, to examine the differences between a common and unusual case of CSLHM enactment. By providing a description of each, the field has some baseline research to move forward with trying to understand how to support the development of CSLHM enactment. It is my hope that this study inspires others to use the CSLHM framework in various ways to answer the continued calls for CSL from the consumer perspective.

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Appendix C

CSLHM Framework - Descriptions and Guiding Questions

Questioning Sample Size/Methods		
Description	**Emergent Guiding Questions**	**Robust Guiding Questions**
Individual demonstrates healthy skepticism regarding the sample, sample size, sampling technique, sampling bias, or lack of information regarding sampling that may lead to invalid inference on a target population. This includes considering who is missing, why, and how that influences the statistical message and the generalizability of the results, and the potential power of the message.	9. Were the sampling methods discussed? 10. Who was sampled and why? 11. How many were sampled? 12. The sample feels biased. 13. Were measures taken to reduce bias? 14. The sample was too small/ /large/convenient? 15. Discuss "cherry picking" without explicitly considering representation within the sample. 16. Where are the people in the sample from?	9. Were the sampling methods discussed? AND if not, why? 10. Who was sampled and why? AND Who is missing and why? Does that influence the results? 11. Could non-response or other sampling issues influence this data or the generalizability of the results? 12. How many were sampled AND why? 13. Were measures taken to reduce bias? 14. Was the sample too small? Too large? Convenient? AND why this matters? 15. Is the sample representative of the population? AND/OR was the sample intentionally selected to create a statistical message that misleads or deceives? 16. Where are the people in the sample from? Where is the data from? Who is the source, and do I trust them? (Note: questioning the data and source in these questions refers to the people/sample being studied)
Appropriate Stats and Appropriate Representations		
Description	**Emergent Guiding Questions**	**Robust Guiding Questions**
Individual questions whether the type of statistics and/or the way it is represented is the most appropriate for the data. This includes considering if the data representation employs techniques to mislead or deceive, thus questioning the motivation behind presenting the data in the way it was shared. Individual questions the role of outliers in the given representation. Individual questions whether the conclusions align with the selection of statistical test/procedure.	10. Are the reported statistics appropriate for this kind of data? AND does not justify why. o Why was the mean used instead of the median or vice versa? o Was a mean used to describe ordinal data? o What is the influence of outliers on the statistics used? 11. n/a 12. n/a 13. n/a 14. Do the variables measure what they are intended to measure? 15. Is the visualization appropriate for this kind of data and/or statistics? AND does not justify why. o Is the type of graph appropriate for the data? o Are the comparisons being shown appropriate? (e.g., comparing counts/frequencies vs. percentages) 16. Are the scales appropriate? Are the intervals/bins appropriate? 17. n/a 18. Were any techniques employed that are used to sway readers' opinions? For example, was the graph truncated in a way that exaggerates differences between	10. Are the reported statistics appropriate for this kind of data? AND justifies why. o Why was the mean used instead of the median or vice versa? o Was a mean used to describe ordinal data? o What is the influence of outliers on the statistics used? 11. Is correlation confused with causation? 12. Is the size of the difference described appropriately? 13. Is there evidence of Simpson's Paradox at play? 14. Do the variables measure what they are intended to measure? AND if so, who do they serve being measured in the way they are? 15. Is the visualization appropriate for this kind of data and/or statistics? AND justifies why. o Is the type of graph appropriate for the data? o Are the comparisons being shown appropriate? (e.g., comparing counts/frequencies vs. percentages) 16. Are the scales appropriate? Are the intervals/bins appropriate? 17. Why might the author of this message have chosen to display the information they did?

	groups? Or was the scale manipulated? AND does not justify why this seems like it is misleading or deceiving.	18. Were any techniques employed that are used to sway readers' opinions? For example, was the graph truncated in a way that exaggerates differences between groups? Or was the scale manipulated? AND justifies why this seems like it is misleading or deceiving.

Desiring Additional Information

Description	Emergent Guiding Questions	Robust Guiding Questions
Individual demonstrates a need for additional information to draw a reasonable conclusion. Individual demonstrates healthy skepticism of the information, including the type of study, context of the study, the source (who collected the data and how), the author's motivation for sharing the statistical message (if not connected to appropriateness of the representation), and the credentials of the person/people sharing the statistical message. **This does not include desiring additional information about the sample as that is captured in the *Questioning Sample Size/Methods* **This does not include desiring additional information about the construction of the data representation as that is captured in *Recognizing Appropriate Statistics* & Appropriate Representations. **This does not include desiring additional information related to the context as this is captured in *Employing Active Citizenry*	**Expressing a need for more information without justification as to why they want or need this information to make sense of the data rep** 10. Do I need to know more about the methods used? (without explaining why) 11. n/a 12. Do I need to know more about *how* the variables are defined/operationalized? 13. Was vital information about the context of the study omitted (e.g., when was the study performed)? Without explaining why that information is needed 14. How transparent was the author about the statistical message and/or methods? 15. How transparent was the author(s) about their personal lens (e.g., political affiliation or other beliefs) and/or motivation? 16. Vaguely questioning the source (e.g., I do/don't trust them) 17. Vaguely questioning the credentials of the individual sharing the statistical message. Who is this person? Are they qualified? 18. Vaguely wondering who the message serves	**Expressing a need for more information with justification as to why they want or need this information to make sense of the data rep** 10. Do I need to know more about the methods used? AND explaining why 11. Do I need to know more about how and if the assumptions were met? 12. Do I need to know more about *how* the variables are defined/operationalized AND why? AND Do I need to know more about if there are other ways to measure these variables that are more equitable? 13. Was vital information about the context of the study omitted (e.g., when was the study performed, does it only represent one slice in time)? AND explaining why that information is needed. 14. How transparent was the author(s) about the statistical message and/or methods? AND why does that matter? 15. How transparent was the author(s) about their personal lens (e.g., political affiliation or other beliefs) and/or motivation? AND why does that matter? 16. Who is the source of the message and/or data? Do I trust them? (Note: questioning the data and source in these questions refers to who created/collected the data) AND explaining why that matters. 17. What are the credentials of the individual sharing the statistical message? AND why does that matter? 18. Who does this message serve? AND why that is important

Acknowledging Alternate Explanations

Description	Emergent	Robust
Individual acknowledges the potential for alternative interpretations for the meaning of findings or different explanations for what caused them, e.g., Was there an intervening moderator variable that affected the results? Are there additional or different	**Only includes alternative explanations from one perspective/narrative** 4. Are there alternate interpretations that can be gleaned from the message? 5. Are there other variables that play a role that should have been considered and were excluded? 6. Did the author of the message fail to include vital information (e.g., did not address lurking or confounding variables) that I would need to make an informed decision?	**Includes alternative explanations from different perspectives/narratives AND/OR explains why those alternate explanation influence how the statistical message is interpreted** 4. Are there alternate interpretations that can be gleaned from the message? 5. Are there other variables that play a role that should have been considered and were excluded?

implications that are not mentioned?		6. Did the author of the message fail to include vital information (e.g., did not address lurking or confounding variables) that I would need to make an informed decision?

Recognitions of One's Own Sociopolitical/ Critical Consciousness

Description	Emergent	Robust
Individual recognizes how one is integrating *their own* social, political, economic etc. understandings to make sense of injustice within the statistical message. Individual recognizes the degree to which one is engaged in critical reflection and critical action/active citizenry (see below). Individual recognizes the gaps in one's knowledge needed to interpret the statistical message. **Simply stating one's beliefs is not enacting this CSLHM	6. n/a 7. n/a 8. n/a 9. Do I recognize if I am applying a particular lens (e.g., feminism)? 10. What is my understanding of this particular context? Are there gaps?	6. Am I considering the perspective of others? 7. How does my life experience shape how I think about this message? 8. How does my identity influence how I think about this message? 9. Do I recognize if I am applying a particular lens (e.g., feminism)? If so, how does that lens shape how I think about this message? 10. What is my understanding of this particular context? Are there gaps? Why is that important in how I make sense of the statistical message?

Employing Active Citizenry

Description	Emergent	Robust
Individual is aware of inequities within the statical message. Individual expresses the desire to disrupt and dismantle inequities. Individual is motivated to act and describes next steps (action includes wanting to research the context, as education is an important part of being an active citizen).	3. What do I need to read/research about to understand the context better? 4. Does this message unearth injustice or lack of equity (explicitly or implicitly)?	4. What do I need to read/research about to understand the context to be able to appropriately make sense of this message? Do I need to consult someone/an expert to make sense of the context, or the mathematics involved in understanding this statistical message? 5. Does this message unearth injustice or lack of equity (explicitly or implicitly)? How is this statistical message being used? Is it serving to further marginalize or privilege? Who benefits and/or profits from this statistical message? And more importantly, what are my actions in response to that? o How can this message be used to promote the dismantling or disruption of inequity? More specifically, what are my next steps? (e.g., do I need to read more on the topic to understand it? Do I need to write to a legislator? Do I need to spread the word about this issue? Can I share this with friends or family to push on their views?) o And conversely, could this message serve to uphold inequity in some manner? Or serve to maintain systems that marginalize? If so, what are my next steps?

		6. How will I press on others' views in conversations about this statistical message? Is it worth engaging? In what context would I decide not to engage and why?

CHAPTER 5: CONCLUSION

The purpose of this dissertation was to describe CSLHM and PST enactment of CSLHM. Chapter 2, the theoretical article, presented the CSLHM framework. I detailed drawing from literature to identify the initial habits of mind needed to enact CSL from the consumer orientation and the process of refinement based on qualitative interview data. Using quotations from interviews with statistics teachers, I illustrated what enactment of each CSLHM looks like in practice and illuminate the differences between emergent and robust enactment. This first article provided the grounding and framework for the subsequent two articles.

Based on the differences in enactment used to illustrate the CSLHM framework in article one, I developed research questions for the next article. Article two (Chapter 3) aimed to explore how preservice secondary (middle and high school) mathematics teachers enact critical statistical literacy habits of mind when engaging with a statistical message from the media. This study found that PSTs commonly enact the CSLHM emergently and sporadically (not habitually). This article then informed the development of research questions and case selection for the final article since there was one PST who enacted CSLHM very differently from the other 16 PSTs.

The final article (Chapter 4) presented a multiple case study with one common and one usual case from the broader study in Chapter 3. This study aimed to examine how PSTs enact CSLHM when making sense of data representations from the media by describing how each enact the CSLHM across two tasks and examining the differences between the two cases. Findings revealed CSLHM enactment differences with respect to

integration of context, attention to the social issue, and change in depth during sensemaking. In this chapter I will discuss the limitations of my study, implications for mathematics teacher educators, and plans for future research.

Limitations

The most obvious limitation of this study is that the sample of PSTs was not random. The PSTs who participated in the study may have been motivated by the call emailed to them by their mathematics methods professors that began with:

> I am conducting a research study for my dissertation, and I am interested in how future secondary mathematics teachers make sense of statistical messages in the real world. There is no statistical knowledge required to participate.

It is entirely possible that PSTs who are more interested in statistics or sharing their thinking agreed to participate. All PSTs were recruited from universities in the southeast United States. Thus, it is possible that there will be differences in PST CSLHM enactment with a broader sample. The sample of PSTs was not random, and all PSTs came from universities in the southeast.

This study focused on sense making of data representation within tweets. It is possible that CSLHM enactment is different when engaging with different types of statistical messages (e.g., podcasts, new reports). There are a variety of factors that likely influence how one may enact CSLHM such as the amount of time one has to digest the message (if you are watching the news in real time you have very little time to digest the data representation), the format of the message (auditory vs. visual or written), and the

purpose for consuming the message (scrolling twitter vs. actively seeking out a specific information). Thus, more research is needed to see if the CSLHM are enacted similarly across such conditions.

With respect to the PSTs in this study, it is also possible that if they recently took a statistics content course that emphasized statistics as a neutral field of study that may have influenced how they made sense of statistical messages. On the contrary, it is also possible that if PSTs took courses that emphasized critical issues that may have influenced how they made sense of statistical messages.

Implications for Teacher Educators

National organizations, researchers, standards, and guideline documents are all pointing to CSL from a consumer orientation (e.g., Bargagliotti, 2020; National Council of Teachers of Mathematics, 2018; North Carolina Department of Public Instruction, 2020). Given this emphasis and the findings from this set of studies, there are several important implications beyond the implications explicitly discussed in each of the three articles in this dissertation. As a reminder, the broad implications for the theoretical article (Chapter 2) pointed to both the analytical and conceptual promise of the CSLHM framework. The broad implications of the other two articles (Chapters 3 and 4) emphasize the need to support PSTs in developing CSLHM.

All three articles highlight the differences between emergent and robust enactment. More specifically, the last two articles point to PSTs evidencing emergent and inconsistent, thus not habitual, CSLHM enactment. This finding is particularly important knowing that prior research suggested that taking university level statistics content

courses did not appear to support PSTs' SL development (Tak et al., 2017). Specifically, similar to Tak et al.'s (2017) finding, many of the PSTs in this study took AP Statistics in high school and/or an introductory statistics content course at the university level (15 of 17 PSTs), and yet their CSLHM enactment was emergent and inconsistent. If taking statistics content courses at the university level does not help prepare PSTs to teach high school statistics content (Lovett, 2017, 2018), does not appear to help PSTs' development of SL (Tak et al., 2017), and similarly does not appear to help develop CSLHM enactment, mathematics educators need to carefully consider how to integrate CSLHM into education preparation programs. Using the CSLHM framework to plan and rehearse/execute instructional routines such as *data talks* (e.g., Boaler et al., 2021), *notice and wonder* (e.g., Rumack & Huinker, 2019), and *slow reveal graphs* (e.g., Laib, n.d.) are a potential way to integrate the CSLHM in meaningful ways to PSTs.

Previous research has shown that PSTs are often uncomfortable discussing or teaching social justice topics (e.g., Simic Muller et al., 2015) and struggle to integrate the context into sense making and sometimes even ignoring it (e.g., Guven et al., 2021; Tak et al., 2017). This study corroborated those findings. Most PSTs, including Carrie, inconsistently integrated the context when enacting CSLHM and were hesitant to discuss the broader social justice issues. Kate, on the hand, consistently integrated the context into her enactment. She fearlessly discussed social justice issues with an interviewer she did not know (and whose beliefs she did not know). Throughout the interview, Kate explicitly talked about her coursework and her program's strong emphasis on anti-racist pedagogy. This points to the need for a strong emphasis on such pedagogies and

opportunities for PSTs to discuss and grapple with critical issues and social justice within education programs.

Many scholars have advocated for the explicit attention to the development of critical consciousness among students with the intent of wielding mathematics as a tool that can be used to improve society (e.g., Frankenstein, 1983; Gutiérrez, 2002; Gutstein, 2003; Kokka, 2020; Skovsmose, 1994; Weiland, 2017). Kate's experience corroborates this call. Paulo Freire defined critical consciousness, or conscientização, as "learning to perceive social, political, and economic contradictions, and to take action against the oppressive elements of reality" (2000, p. 35). Kate's continual discussion of her teacher preparation program and the focus on anti-racist teaching pedagogy illuminated the explicit intent of her coursework to develop critical consciousness. Kate's experience points to the importance of developing critical consciousness among PSTs so that they can help their students develop an understanding of wielding mathematics and statistics as a tool that can be used to improve (or destruct) society. It is important to note that Carrie participated in a program that met 4 times a semester for a seminar focused on teaching math for social justice. Given the differences between Carrie's and Kate's CSLHM enactment, it is possible that outside support a few times a semester is not enough emphasis on social justice and critical consciousness to make a difference. It is possible that social justice, anti-racists pedagogies, and critical consciousness need to be intentionally integrated throughout a teacher preparation program to influence CSLHM enactment; this warrants further research.

Future Research

Now that the field has research on how PSTs enact CSLHM when making sense of data representations from tweets, there are several possible ways to move this work forward. I will briefly discuss how I plan to continue to study PSTs as well as expand my research to students and the general population.

My next step is to examine how to support PST development of CSLHM which is essential since PSTs will be responsible for helping their students make sense of statistical messages from the real world. I say *essential* given the continued calls (e.g., Bargagliotti, 2020) and inclusion of CSL from the consumer orientation within standards (e.g., North Carolina Department of Public Instruction, 2020). Mathematics teacher educators need research that examines supports for these curricular and standard changes. Thus, I plan to study how explicit introduction to the CSLHM framework can support PSTs planning for and rehearsing data driven instructional routines like data talks.

Given that teachers need to support students, both the teacher population (including PSTs) and mathematics teacher educators would benefit from knowing how K-12 students typically enact CSLHM as well as how to support students' CSLHM development. For example, such research could examine how the CSLHM framework used in conjunction with data driven instructional routines change the depth of CSLHM enactment for secondary students. The field would also benefit from adapting the CSLHM framework to align with elementary standards to provide a useable framework for K-5 students to use when making sense of statistical messages. Given that this would be of particular interest to teachers and PSTs, it would be important for such research to be disseminated in both research journals and outlets specific to teachers (e.g., practitioner journals).

As I mentioned in the limitations section, it is necessary to examine how PSTs make sense of data representation in different contexts. Thus, research should endeavor to examine CSLHM enactment using data representations within a variety of statistical messages such as news broadcasts, podcasts, and news articles.

I also plan to extend this research beyond the PST and student population to study how the general population enacts CSLHM. Specifically, I want to examine the differences between adults with varying statistical knowledge and varying critical consciousness (and I already have this data to move this research forward). This broader context can segue to research on how to support the general population in overcoming some of the pitfalls or barriers to making sense of statistical messages. The pandemic and polarized state of politics in the United States as revealed the need for people to explore how they are misled or deceived by data representations. Anecdotal evidence of using the CSLHM framework with my broader community and among local businesses trying to support CSL among employees has pointed to the CSLHM as a framework that can aid individuals in navigating conversations about data representations, politics, and social justice. Future research should endeavor to determine if the CSLHM framework is a means to help adults identify misinformation and navigate potentially polarizing conversations.

By describing CSLHM and PSTs' CSLHM enactment, I am advocating for emphasis on and adding to the call for SL from both a consumer orientation and a truly critical lens. By demonstrating the analytical potential of the CSLHM framework, I hope that the field continues to examine and explore CSLHM enactment of students, PSTs, teachers, and the general public. By demonstrating the conceptual potential of the

CSLHM framework, I hope that the field uses it to inform the development of supports and curriculum to cultivate CSLHM.

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