ABSTRACT

STARLING, TINA T. Comparing Discourse in Face-to-Face and Synchronous Online Mathematics Teacher Education: Effects on Prospective Teachers' Development of Knowledge for Teaching Statistics with Technology. (Under the direction of Dr. Hollylynne S. Lee).

This comparative study examined discourse and opportunities for interaction in two mathematics education methods classes, one face-to-face and one synchronous, online. Due to the content taught in the course, this study also sought to determine prospective mathematics teachers' understanding of variability and the role of discourse in each learning environment in developing statistical knowledge for teaching with technology in prospective mathematics teachers.

A qualitative research design was selected to help capture and analyze discourse and developing knowledge about teaching statistics with technology. Participants included forty-two students enrolled in one of two sections of a *Teaching Mathematics with Technology* course at a large public university. Each class was comprised of juniors, seniors, graduate students, and lateral entry students studying middle grades or secondary mathematics education. Three prospective teachers from each class were selected to be members of a focus group. Video recordings of each class session for both groups were collected and were used to analyze discourse opportunities throughout the five-week study. Then, based on questions related to statistical, technological, and pedagogical content, which were identified a priori, six episodes were selected for more detailed analysis of small group and whole group interactions. Each episode was coded line-by-line for direction, form, purpose, and topic of discourse. In addition, each transcript was coded for when and how prospective teachers discussed ideas related to variability, specifically describing distributions, deviation, and the law of large numbers. An external check of coding, along with several sources of

data, helped ensure credibility of the qualitative methods described. Supporting data included a pre-/post-assessment, written assignments, and interviews of focus group members.

A community of inquiry framework, which presents the social, technological, and pedagogical presences that make up an educational experience, was used as the theoretical lens for this study (Garrison, Anderson, & Archer, 2000). Findings indicate that the ways in which prospective teachers and the instructor interacted with one another and discussed notions of variability looked very different between the face-to-face and online environments. This was because in the synchronous, online environment, prospective teachers could interact with one another in a number of ways. They used non-traditional forms of communication such as the chat window and the interactive whiteboard to share ideas and to ask questions.

Despite these differences in how prospective teachers interacted with the instructor and with one another in each group, the substance of what they said about describing distributions, deviation, and the law of large numbers was strikingly similar across settings. Specifically, their usage of informal language was prevalent and comparable as they described center and spread throughout the study. In addition, the ways technology reportedly helped prospective teachers understand standard deviation and least squares regression was especially noticeable. While the ways they described the law of large numbers were also similar, other data pointed to differences in the groups' collective understanding. These differences caused the researcher to then look for ways that discourse affected prospective teachers' developing knowledge of variability and especially the law of large numbers. Factors that may have resulted in some difference in knowledge are technological issues in the online environment, physical distance that may have caused less productive discussions, less time for whole group and small group discussions, and more independent work due to time constraints.

Comparing Discourse in Face-to-Face and Synchronous Online Mathematics Teacher Education: Effects on Prospective Teachers' Development of Knowledge for Teaching Statistics with Technology

by
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DEDICATION

To my beautiful daughter, Victoria.

BIOGRAPHY

Tina Julian Tedder Starling was born in Statesville, NC on August 17, 1977. She is the daughter of Randy and Sherry Tedder. She graduated from North Iredell High School and attended NC State University as a recipient of the North Carolina Teaching Fellows Scholarship. She graduated from NC State University in May, 1999 with a Bachelor of Science degree in mathematics and in December, 1999 with a Bachelor of Science degree in mathematics education. In January, 2000, she was hired at Athens Drive High School in Raleigh, NC. The following summer, Tina married college sweetheart Ray Starling. Mrs. Starling continued teaching mathematics, earning the First Year Teacher of the Year Award (2001) and Teacher of the Year (2006) awards at her high school. During this time, Starling continued teaching full time while taking graduate courses. She completed her Master of Science degree in mathematics education at NC State in December, 2005. Her thesis study, advised by Dr. Lee Stiff, examined the effects of students' self-assessment on Algebra I achievement.

In August, 2006, she left public schools to teach in the Mathematics and Physics Department at Wake Technical Community College in Raleigh, NC. There, she taught a variety of courses including an online mathematical modeling course. During her time at Wake Tech, she was asked to help with the supervision of student teachers from NC State. Working as an adjunct supervisor allowed Tina to return to the public schools, which she missed, and opened her eyes to the joy in working with prospective mathematics teachers. In January, 2008, she began taking doctoral classes and decided to return to the doctoral program full-time in August, 2008. As a teaching/research assistant, Tina continued supervising student teachers. She also taught the *Teaching Mathematics with Technology* course and worked with the Preparing to Teach Mathematics with Technology (PTMT) research project under the direction of Dr. Hollylynne Lee and Dr. Karen Hollebrands.

Upon receiving her degree, Tina plans to continue learning about new technologies that can be used to teach mathematics and strategies for sharing that knowledge with prospective teachers through synchronous, online environments. In addition, she hopes to obtain a faculty position where she can work with prospective teachers in methods courses and during their professional, student teaching internships.

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To the prospective teachers who willingly participated in this study, I sincerely thank you for your patience and cooperative spirit. I learned so much from each of you and pray that your zeal for teaching students will remain with you for many years to come. Best of luck to all of you!

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CHAPTER 1: INTRODUCTION

Background of the Study

Over the past decade, there has been an increased focus to develop mathematics teachers' knowledge as it relates to teaching and learning with technology. At the same time, there has been a growing interest in constructivism and accepting the notion that knowledge is socially constructed and distributed (Putnam and Borko, 2000). The combination of these contemporary foci has resulted in some teacher education programs making concerted efforts to help prospective mathematics teachers collaborate and grow their knowledge about teaching with technology within settings that are designed to promote social interactions.

One possible approach to this challenge is offering online coursework to reach prospective teachers and practicing teachers where they are geographically located; but an internet search for online courses in mathematics education exposes the near absence of online course offerings. While other academic fields have been increasing their offerings in the online setting, mathematics education has moved more cautiously. Perhaps it should not be surprising then, that the research presented for online mathematics education is minimal as well. Therefore, the field of mathematics education knows little about how an online mathematics education course, especially a technology methods course, may compare to its face-to-face counterpart.

Specifically, how prospective teachers engage and interact with one another online and how that engagement and interaction affects knowledge is relatively uncharted territory in mathematics education research. If, as predicted, the market for online education continues to rise, colleges and universities will be doing more to meet this demand (Ginsburg, Gray, &

Levin, 2004) and mathematics education will need to learn how to adapt face-to-face methods courses so that prospective teachers will have the same online accessibility that students in other disciplines with online options are afforded.

Admittedly, preparation of today's prospective teachers is a complex task. In any learning environment, there is the challenge of developing prospective teachers' content knowledge as well as technological and pedagogical knowledge simultaneously. How do the ways in which they communicate their thoughts during mathematics education courses affect the development of this specialized knowledge? Knowing more about how prospective teachers may engage and interact with one another and the knowledge that is constructed in both face-to-face and online learning environments may help to answer questions surrounding whether there are important similarities and differences in these environments for mathematics teacher educators to consider.

Purpose of the Study

The purpose of this study was to analyze discourse and opportunities for interaction in two mathematics education methods classes, one face-to-face and one synchronous, online. In addition, due to the content of the unit of study, this study sought to determine the role discourse played in developing statistical knowledge, focused on the concept of variability, for teaching with technology in prospective mathematics teachers. Consequently, the goal was to answer the following research questions:

(1) What similarities and differences in discourse and opportunities for interaction exist between face-to-face and synchronous, online mathematics education courses?

(2a) What is the nature of prospective teachers’ understanding of variability and teaching concepts related to data analysis and probability with technology?

(2b) What is the role of discourse in face-to-face and synchronous, online environments in developing this understanding among prospective mathematics teachers?

Clarification of Terms

Due to the relative recent emergence of online education, particularly to the world of mathematics education, definitions of terms may be helpful. Some refer to online education as distance education; others use the term e-learning instead, and many use the terms interchangeably. Guri-Rosenblit (2005) admits there is some overlap, but the two are actually not synonymous. Distance education is typically associated with non-traditional students who live in remote locations, but it may be used by students of all ages, in any place (Guri-Rosenblit, 2005). Furthermore, online education could be set in an environment that includes both asynchronous and synchronous activities, which are terms to describe non-simultaneous and simultaneous activities respectively (Hodges, 2005). For this study, the **online setting** was a synchronous learning environment. Participants were able to engage and interact with others in real-time.

Participants in this study were **prospective teachers**. They were college students in mathematics education and had some degree of classroom experience from previous mathematics education courses. However, in this study, prospective teachers had not yet completed their student teaching semester.

One focus of this study was how prospective teachers engaged and interacted with others in the class (other prospective teachers and the instructor/researcher). **Discourse** encompassed all forms of communication to others. This included both verbal (i.e. talking, writing) and non-verbal (i.e. hand gestures, emoticons) forms of communication. In this study, prospective teachers were members of a technology mathematics education methods course. **Technology** refers to a collection of specific tools that may be used in teaching statistical and probabilistic ideas to students. This collection of technology tools included a *TI-83+/84* graphing calculator, Microsoft *Excel*, and three programs that provide linked representations of data (*TinkerPlots, Fathom,* and *Probability Explorer*). These programs allowed users to display data collection windows, tables, graphs and more. The dynamic nature of *Fathom* and *TinkerPlots* allowed these multiple representations to be linked; when a user changed one representation, other representations changed simultaneously.

Developing knowledge of statistical technology listed above was important for prospective teachers participating in this study. Developing knowledge of teaching statistics with technology was even more important. This knowledge included, at its foundation, **statistical content knowledge**. One key concept in statistics is variability (Ben-Zvi, 2004; Burgess, 2007). Prospective teachers' understanding of distributions, deviation, and the law of large numbers was addressed in this study.

Significance of the Study

There are two significant purposes of this study. First, with the increased importance of developing mathematics teachers' knowledge as it relates to teaching and learning with technology, there is still much to learn about how teachers may construct such knowledge. If one believes knowledge is socially constructed, how prospective teachers communicate and interact with one another certainly warrants additional research. Much of the current research literature on discourse is related to the mathematics classroom. For a number of years, there has been interest in discerning how it is that students of mathematics come to know what they know through social interactions (Yackel, Cobb, & Wood, 1991). What is missing, however, is how discourse may develop and manifest itself in mathematics teacher education and how it affects knowledge developed by prospective teachers.

Second, distance education coursework in higher education continues to grow, and it is no surprise that many academic disciplines have begun designing and implementing courses online. Although the jury is still out with regard to the quality of such courses currently being offered, no one can dismiss the efforts of such programs that have simply offered the online option. Because teaching online is a relatively new endeavor, much of the "research is in the form of case studies" (Engelbrecht & Harding, 2005, p. 268) and "most studies to date address learner perceptions and comparisons of online instruction to traditional course instruction" (Alexander, Lignugaris-Kraft, & Forbush, 2007, p. 201). Notwithstanding an undeveloped body of supporting research, it is likely that universities will continue to employ the use of online instruction until evidence surfaces that suggests such environments cannot be effective. Thus, it is also likely that the "research may reflect on practice rather than drive the practice" (p. 268).

As a result, the significance of this study is two-fold. One goal is to examine the nature of discourse in face-to-face and online mathematics education courses that impacts the development of prospective teachers' knowledge for teaching statistics with technology. A second goal is to identify, analyze, and describe prospective teachers' knowledge about teaching statistical and probabilistic concepts with technology as it develops in face-to-face and online settings. The results from this dissertation study will certainly fill a gap in the research literature for mathematics education.

Overview of Approach

To analyze discourse and prospective teachers' knowledge in face-to-face and online education learning environments, this study employed a qualitative methodology. The primary objective was to learn more about how discourse and interactions help develop prospective mathematics teachers' knowledge about teaching statistical and probabilistic ideas with technology. Since the form and strength of knowledge and processes cannot be completely measured or assessed quantitatively, utilizing qualitative methods through a multiple case study approach was necessary to allow for a more in-depth and detailed inquiry (Patton, 2002). A nested case study design was especially useful for this specific project because it permitted an in-depth investigation of two groups simultaneously and took advantage of individual cases.

Chapter Summary and Organization of the Study

Since there has been little interest in moving mathematics education courses (particularly technology methods courses) online, there was a need to develop and study the effectiveness of one such course. The purpose of this work is to analyze the nature of discourse related to constructing knowledge for teaching statistics with technology in face-to-face and online learning environments. In the following chapter, an abbreviated snapshot of important, relevant literature is presented which provides insight into how this study was designed in order to answer the research questions. Chapter three will explain, in greater detail, the methodology used in this study to address those questions. Specifically, information about the research design, site selection and sample, data collection and analysis, issues surrounding validity and reliability, bias, and ethical processes are reported. Chapters four and five present the cases of the online and face-to-face environments respectively. Cases include a detailed description of the contextual setting of discussions and analyses of multiple sources of data collected to study discourse and understanding of variability. Chapter six provides a cross-case analysis of the online and face-to-face classes. Similarities and differences regarding curriculum implementation, discourse, and prospective teachers' understanding of variability are reviewed. Finally, chapter seven presents a deeper discussion of the findings. Among other things, the answers to the research questions, limitations, implications of the study and recommendations for future research are shared. Following a list of references, the appendices include assessments, grading rubrics, interview guides, and the Institutional Review Board proposal.

CHAPTER 2: LITERATURE REVIEW

Introduction

The current study aimed to examine the nature of discourse, in both face-to-face and online environments, that impacts the development of prospective teachers' knowledge for teaching statistics with technology and to identify that knowledge in prospective teachers. For this literature review, numerous research and theoretical articles related to distance education, mathematics education, and statistics education were used. However, by no means is this review an exhaustive account of all literature pertaining to any of those disciplines. Instead, this literature review takes a broad look at three key presences in any mathematics education course – social, cognitive, and teaching. A community of inquiry framework (Garrison, Anderson, & Archer, 2000), with its social, cognitive, and teaching presences, will be formally introduced in the next section and will serve as a lens for reviewing literature related to this dissertation study.

The social presence includes opportunities for discourse and interaction. Thus, literature related to these opportunities with face-to-face and online learning environments was reviewed. The cognitive presence includes opportunities for developing knowledge associated with teaching statistical and probabilistic concepts with technology. Understanding the research on how teachers think about variability was particularly important to this study. Specifically, three big areas of variability - describing distributions, understanding deviation/error/residuals, and understanding the law of large numbers – was the focus in this study. Additionally, the nature of discourse and interactions that seems to contribute to that knowledge was important. The teaching presence includes carefully crafted lessons so that opportunities exist for prospective teachers to engage and interact with others on important statistical ideas, using dynamic statistical technology tools. Literature associated with facilitating this type of community of inquiry was also reviewed. This literature review concludes with a summary discussion on the implications of the literature cited.

Community of Inquiry Framework

The community of inquiry framework (Figure 1) attempts to understand the social, technological, and pedagogical processes that lead to collaborative knowledge construction (Garrison, Anderson, & Archer, 2000). It reveals the behaviors and processes necessary to cultivate knowledge construction through various forms of “presence.” The social, cognitive and teaching presence components that encompass this framework do not exist in isolation. Therefore, while the presences may be discussed independently, it is how they work together in order to foster a unique type of learning-community that is most important.

Figure 1. The community of inquiry framework (Garrison, Anderson, & Archer, 2010, pg. 6).

Because this study looked at effects of discourse on prospective teachers' developing knowledge about teaching statistics with technology, it was important to consider all presences of the learning environment. Therefore, the community of inquiry framework was used to provide organizational structure and continuity in reviewing literature related to the social, cognitive, and teaching presences in both face-to-face and online mathematics education courses.

Social Presence

The idea that knowledge is constructed by an individual through interactions is not a new one (Bruner, 1966, 1986, 1990; von Glasersfeld, 1984, 1989; Wertsh, 1985). In the mid-1900s, Piaget's theory about social interaction incorporated the view that the social world has an important role to play in the developmental process (Tudge & Rogoff, 1989). Bruner (1966), who once described his teacher as a "human event not a transmission device," asserted that reality is constructed by individuals through the use of other people and assistive tools (pg. 126). Vygotsky (1978) believed that it was the social interaction and dialectic process which furthered existing knowledge and promoted new insights. Interpretations of his notion of the zone of proximal development have emerged that apply, extend, and reconstruct his original ideas (Daniels, 2001). The constructivist philosophy supports the belief that learners construct unique knowledge based on their own experiences and understanding.

More recent research in mathematics education supports that discourse and interaction are important components of any learning experience (Clement, 1997; Groves & Doig, 2004; Picollo, Harbaugh, Carter, Capraro & Capraro, 2008; Yackel, Cobb, & Wood, 1991). Socio-cognitive learning theory states that “learning is a social activity and that individuals learn more from their interactions with others than from reading materials alone” (Richardson & Swan, 2003, p. 43). Therefore, cognition is “social and distributed” (Putnam & Borko, 2000, pg. 5). This notion that knowledge is socially constructed is not limited to the mathematics classroom; it applies to mathematics teacher education classes as well. “The view that knowledge is socially constructed makes it clear that an important part of learning to teach is becoming enculturated into the teaching community – learning to think, talk, and act as a teacher” (Putnam & Borko, 2000, pg. 5, 9). This type of knowledge construction potentially results from purposeful discourse and interaction.

Discourse and Interaction

A major contributor to interactivity is discussion during class. “There must be time to talk about the mathematics, to develop models, to analyze and synthesize ideas, and to develop an atmosphere in which they can think for themselves…there must be adequate resources in an environment in which exploration and discovery are supported and ideas are valued and can be freely stated” (Sliva, 2002, pg. 80). Among any group of novices, reflective discussion about their practice can be helpful. Teachers are no different. McCrory, Putnam, and Jansen (2008) assert that in professional communities “teachers learn through sustained discourse with other teachers, sharing their expertise and learning from the expertise of others” (pg. 157). Prospective teachers, however, have little expertise to draw upon, which can be an obstacle for productive discourse. Feiman-Nemser (2001) reminds us that teacher discourse is not naturally productive or, as she describes, professional:

> “The kind of conversation that promotes teacher learning differs from usual modes of teacher talk, which feature personal anecdotes and opinions and are governed by norms of politeness and consensus. Professional discourse involves rich descriptions of practice, attention to evidence, examination of alternative interpretations and possibilities” (pg. 1043).

Productive or professional discourse requires much more than prospective teachers simply grouped together. Therefore, it is critical that strategies are in place during class to assist prospective teachers in developing the practice of giving rich descriptions, attending to evidence, and considering alternate approaches as Feiman-Nemser (2001) suggests.

For mathematics teacher educators, much can be gleaned from existing research regarding strategies that have been shown effective in promoting constructive discourse with mathematics students. Research on effective whole-class discussions (Nathan & Knuth, 2003), questioning (Piccolo et al., 2008), and implementing effective small discussion groups (Elbers, 2003; Webb, 1991; Webb, Nemer, & Ing, 2006; Kazemi & Franke, 2004) may be transferred to teacher education. For example, we know that often ideas and/or actions are discussed and then become objects of discussion in their own right (Yackel & Cobb, 1996). This type of discourse is natural and healthy in developing knowledge for teaching.

Much has also already been learned about interaction from face-to-face experiences with prospective and practicing mathematics teachers (Leikin & Zaslavski, 1997; Rosales, Orrantia, & Vicente, 2008; Piccolo et al., 2008). Leiken and Zaslavski (1997) remind instructors to increase task-related interactions to promote learning. Rosales et al. (2008), accounting for cognitive processes involved during interactions with both prospective and practicing mathematics teachers, confirmed the idea that a less teacher-centered lesson results in higher levels of interaction among students. Piccolo et al. (2008) agree. “Students need the opportunity not only to hear what the teacher is teaching, but actually converse and articulate their own understanding of the content being presented" (pg. 404). In their work, they also stressed the effect questioning from students has on discourse and interactions. When students ask questions, "they are thinking about their thinking" (pg. 381). Those types of reflective activities are important, especially for prospective teachers.

It is important to note that simply organizing prospective teachers into small groups does not imply social and distributed learning is taking place. Even online, networked interaction or social interaction, by itself, is insufficient to the development of a community of inquiry (Larreamendy-Joerns & Leinhardt, 2006; Shea & Bidjerano, 2009). As can be expected, in any learning environment, there are variations in the level of participation from prospective teachers and group functioning is not identical (Koehler & Mishra, 2005).

Several researchers have provided recommendations for teacher educators to help maximize the small-group learning experience. First, small group discussions in teacher education should be focused on content and students' thinking (Cady & Rearden, 2009; Groth, 2007; Stephens & Hartmann, 2004). Within the context of this study, discussions focused on tasks that were designed to develop content, technological, and pedagogical knowledge simultaneously (Lee, Hollebrands, & Wilson, 2010). Second, Stipek, Givvin, Salmon, and MacGyvers (2001) further recommend teachers "engage in practical inquiry, try new things, and reflect in a collaborative setting" (pg. 225). Experiencing a task together, working through a solution, and discussing the process allows prospective teachers to critically reflect on a common experience while sharing alternate viewpoints which permits them to anticipate more about how their future students may approach a task differently and what they may do in response.

Discourse and Interaction Promoted Synchronously. The interactions in an online environment will undoubtedly be different from those in a face-to-face environment. But, results from research show that the computer-mediated environment can be powerful in initiating and maintaining a learning forum for mathematics teachers (Chinnappan, 2006). Web-conferencing programs such as *Elluminate*[1] allow users to chat, view live demonstrations, interact with presentations, and more. Advantages of using a synchronous learning environment include real time sharing of knowledge and immediate access to the instructor, and others, to ask questions and receive answers. In particular, with regard to environments such as *Elluminate*, students tend to favor features such as emoticons, hand raising, a shared whiteboard, polling, and application sharing as points of personal engagement. McBrien, Jones, and Cheng (2009) found that with these features, students talked about an enhanced learning experience with improved communication, high levels of satisfaction with the course, and strong group cohesion when compared to earlier asynchronous designs of online courses.

Stephens and Mottet (2008) also showed that increased interactivity with tools such as the ones described in *Elluminate* enhances participants' satisfaction with the learning environment. Therefore, the interactivity of the students in a synchronous, online environment is critical. Online instructors must employ strategies that encourage interaction to allow students to feel socially present in the lesson. One feature in web-conferencing

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[1] *http://www.elluminate.com/Services/Training/Elluminate_Live!/?id=418*

programs like *Elluminate* that shows promising gains is “Application Sharing” (Cady & Rearden, 2009). With this feature, participants have the opportunity to view live demonstrations and even take control of the instructor’s mouse if permission is requested and granted.

Existing Frameworks For Analyzing Discourse and Interaction

Some researchers have used Vygotsky’s zone of proximal development explicitly as a framework for analyzing discourse. For example, Goos and Bennison (2004) used data from classroom observations and interviews to study scaffolding involved in teacher-student interactions, collaboration in student-student interactions, and interweaving of intuition and formalized concepts. Other researchers have broadened their perspective to include “constructs derived from symbolic interactionism and ethnomethodology… to account for and explicate the development of general classroom social norms” (Yackel & Cobb, 1996, pg. 459). They describe the basis of explanations offered by mathematics students. According to their work, students use explanations as descriptions of actions and as objects of reflection to construct sociomathematical norms (Yackel & Cobb, 1996).

Using the works cited above as a guide, more recent research efforts have found frameworks that have emerged from data to be helpful during analysis. Hufferd-Ackles, Fuson, and Sherin (2004) report three themes and the relationships among them to be central to their work in analyzing discourse in a mathematics classroom: “evidence of mathematics community, teacher actions, and student actions” (pg. 87). To further study the emerging themes, they created developmental trajectories which followed students’ questioning, explaining of mathematical thinking, source of mathematical ideas, and responsibility for learning. The levels of their “Math-Talk Learning Community framework went through cyclical revisions” (pg. 87).

Krussel, Edwards, and Springer (2004) also draw upon previous works. Their framework, designed to assist in understanding discourse moves, “synthesizes elements of several other discourse frameworks” (pg. 307). One such framework comes from Sfard (2000), who studied the foci of mathematics students’ talk. Sfard noted students’ pronounced and attended (either explicit or implicit) foci and the perceived intended focus from their words and actions. “A careful examination of the pronounced and attended foci, together with the researcher’s interpretation of the participants’ intended foci at various points in the discourse, provides insight into the process of developing mathematical understanding” (Krussel et al., 2004, pg. 308). Knuth and Peressini (2001) also introduced the distinction between univocal discourse and dialogic discourse, which is evaluation authority given to the teacher and to the group respectively. Using existing frameworks, including the ones described above, Krussel et al. (2004) described discourse by purpose, form, and consequences. Their framework was used, among other things, to study discourse in a geometry course for teachers. Especially interesting in light of the current study, in their concluding comments, the authors claim that “although much of the discourse literature clearly concerns face-to-face settings, the framework can also be applied to discourse in distance courses” (Krussel et al., 2004, pg. 311).

It is true that much of the discourse literature comes from face-to-face research and that literature related to online discourse overwhelmingly involves asynchronous discussion boards. However, techniques for studying asynchronous, online discourse and interaction may also be helpful for studying synchronous, online and face-to-face discourse and interaction as well. Topco and Ubuz (2008) found that coding messages and scoring them based on rubrics is critical for analysis of comments and interactions by prospective teachers. Their categorization of interactions included the following labels: “acknowledgment, question, compare, contrast, evaluation, idea to example, example to idea, clarification, and cause-and-effect” (pg. 7-8). Similar labels for synchronous and face-to-face interactions are possible.

Nandi, Chang, and Balbo (2009) adapted frameworks from other distance education research to create their own way of selecting criteria for quality in prospective teachers’ online discourse about content and interaction. Specifically, they looked for evidence of: “clarification and critical assessment, justification or judgment, inferencing or interpretation, application of knowledge, prioritization of key knowledge, breadth of knowledge, critical discussions of contributions, new ideas/solutions, sharing outside knowledge, use of social cues or emotions, and participation rate” (Nandi et al., 2009, pg. 669). This type of framework, they claim, can be helpful to online facilitators and researchers to assess discourse and interaction. Using these criteria to also evaluate quality of discourse and interactions in a face-to-face setting may be fitting, particular in discussions focused on content.

If knowledge is, in fact, social and distributed, then discourse and interaction may play an even more important role in the development of prospective teachers than research currently suggests. Implementing whole-group and small-group discussions in ways that elicit understanding beyond simply drawing upon personal experiences is a challenge.

However, by allowing prospective teachers to focus on content and students' thinking, they have a common ground on which to stand and base their conversations. Researchers have tried to develop frameworks for analyzing the social presence and its effects, each attending to different nuances that exist. They do seem to agree on one thing, however. The way students communicate and interact with one another, either face-to-face or online, may affect the knowledge they develop. In the current study, the knowledge being developed in prospective teachers was knowledge about teaching statistics with technology.

Cognitive Presence

In the previous section, the importance of socio-cognitive learning theory or social presence was stated. The notion that students construct knowledge from interactions with other students, the instructor, and the content is not new; and the practice of allowing students to learn from one another is widely accepted in both face-to-face and online settings (Ling, 2007; Richardson & Swan, 2003).

The community of inquiry framework, in its own right, encourages inquiry on the part of students. "Inquiry means to ask questions, investigate, acquire information or search for knowledge" (Fuglestad, 2007, p. 250). The design of a course certainly influences how inquiry manifests itself with students. Nevertheless, students' prior experiences cannot be ignored. Their metacognitive knowledge, knowledge about their own cognitive processes, can influence how they explore, identify, and examine their thinking and learning (Topcu & Ubuz, 2008). This skill of self-awareness can ultimately affect whether or not accurate knowledge is constructed. In a study of prospective teachers, Topcu & Ubuz (2008) found metacognitive knowledge was an important determiner of the variance of engagement in meaningful discussions. Teachers with high metacognitive knowledge were more “aware” of the mathematical and pedagogical content in the messages and were better able to make connections between ideas presented in the discussions and their own. These teachers also tended to initiate interaction more frequently, perhaps ultimately leading to an even more satisfactory performance in the course. Prospective teachers with lower metacognitive knowledge benefited from tools or activities that could scaffold their thinking. They especially performed better with discussions that caused them to judge knowledge of self.

With the prior knowledge and attitudes students bring with them, cognitive presence may be more difficult to assess than the social and teaching presences. Regardless of the setting in which learning takes place, today’s prospective mathematics teachers need to develop a specialized type of knowledge. A literature search for fundamental knowledge of mathematics teachers results in parallel, yet independent results (Ball, Thames, & Phelps, 2008). Over time researchers have made claims about that which teachers should know in order to be effective in the classroom, but no consensus has been reached (Ball & Forzani, 2009; Fennema & Franke, 1992; Reynolds, 1992). Yet, there is enough evidence to suggest that mathematics educators do, in fact, require a specialized knowledge (Ball & Forzani, 2009; Ball et al., 2008). This knowledge may be a careful combination of knowledge related to content, technology, and pedagogy. The following sub-sections will look more closely at a possible combination of these types of knowledge.

Existing Frameworks For Analyzing TPSK

Just as it is not effective for teachers to have content knowledge without pedagogical knowledge, it is also not enough for them to have only technology knowledge without a proficiency in content and pedagogy. Researchers believe that learning to teach mathematics is a developmental process the teacher will need to create – a process where different types of knowledge build on each other (Hiebert, Morris, & Glass, 2003; Simon, 1994). With the onset of easy-to-access interactive tools, the development of a unique knowledge comprised of technology, pedagogy, and content (TPACK) in prospective teachers is one goal of 21st century teacher education. TPACK is a way to represent what teachers need to know about technology and requires the design of a curricular emphasis that is all-encompassing, not a focus on technology, pedagogy, and content at individual moments. Mishra and Koehler (2006) described it this way:

> "TPACK is the basis of good teaching with technology and requires an understanding of the representation of concepts using technologies; pedagogical techniques that use technologies in constructive ways to teach content; knowledge of what makes concepts difficult or easy to learn and how technology can help redress some of the problems that students face; knowledge of students' prior knowledge and theories of epistemology; and knowledge of how technologies can be used to build on existing knowledge and to develop new epistemologies or strengthen old ones" (p. 1029).

Teachers who have a developed TPACK understand that "technology is not merely applied to the pedagogy of the past. It has implications for how we teach and what we teach" (Koehler & Mishra, 2005, pg. 144).

Mathematics educators are beginning to understand that technological tools do not just enhance knowledge, they help transform it (Putnam & Borko, 2000). Important to note, however, is that prospective and practicing teachers often have difficulty transferring their knowledge of the technology into their instructional practices (Harper, Schirack, Stohl, & Garofalo, 2001; Olive & Leatham, 2000; Zbiek, Heid, Blume, & Dick, 2007). Therefore, careful attention should be paid to how teachers learn and consequently, how teacher education programs facilitate that learning.

Mishra and Koehler's (2006) popular TPACK framework highlights the intersection of three distinct types of knowledge in a Venn diagram. Lee and Hollebrands (2011) specialized the notion of TPACK for the knowledge needed to teach data analysis and probability, the content focus of this study. Rather than viewing technological, pedagogical, statistical knowledge (TPSK) as the intersection of three distinct circles in a Venn diagram like the TPACK framework, they considered TPSK as layered circles. The TPSK framework allows researchers to consider statistical knowledge as the foundation of specific TPSK (Figure 2).

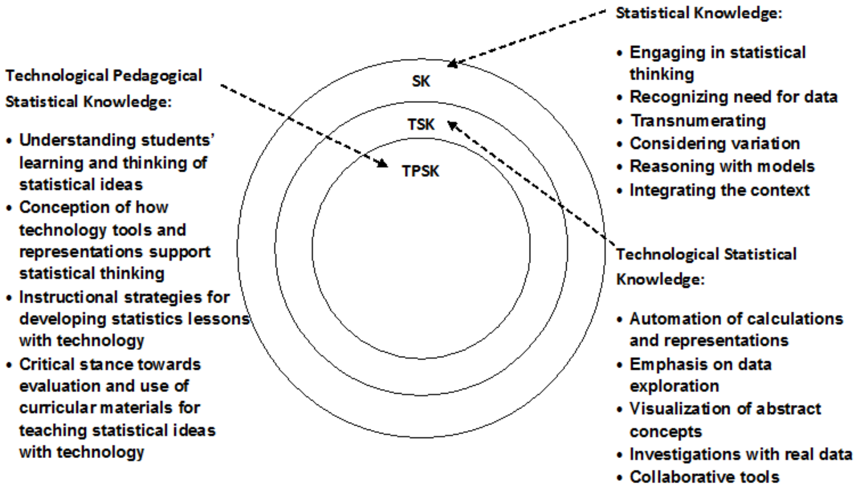


Figure 2. TPSK framework (Lee & Hollebrands, 2011, pg. 362).

The textbook and materials used for this study (Lee et al., 2010) are designed to engage prospective teachers through opportunities for exploring interesting data. Throughout the materials, teachers' knowledge related to technology, pedagogy, and statistics is developed simultaneously. “After teachers have engaged in examining a statistical question with technology, they are asked pedagogical questions concerning how technology and various representations can support or hinder students’ statistical thinking” (pg. 4). Specifically related to statistical ideas about variability, Lee et al. claim the following:

> “The materials examine distributions graphically and characterize the data before computing statistical measures. Questions promote the comparison of distributions as means to transition to thinking about data as aggregate and help teachers conceptually coordinate center and spread. Studying data in a univariate context helps students consider measures of variation (e.g., residuals, sum of squares) in a bivariate context when modeling with a least squares line” (pg. 4).

Using the TPSK framework above, prospective teachers’ statistical content knowledge is developed in purposeful ways as they engage in tasks using technological tools. Pedagogical issues are woven into the activities once participants have some similar ideas about concepts being discussed.

Statistical Knowledge

Among teacher educators, there is little dispute over the belief that teachers need to be mathematically proficient (Hiebert et al., 2003; National Research Council, 2001; Stipek et al., 2001; Usiskin, 2001). But, only a few studies have found and tested appropriate levels of content knowledge required for teaching (Ball et al., 2008). In statistics education, there are even fewer related research efforts.

For this study, prospective teachers were engaged with curriculum materials related to data analysis and probability. Specifically, their knowledge of variability was assessed in light of three big ideas: analyzing/comparing distributions, understanding of deviation/error/residuals, and articulating the Law of Large Numbers. There is now a growing body of literature that focuses on postsecondary students' understanding of statistics and probability (e.g. Hammerman & Rubin, 2004; Heaton & Mickelson, 2002; Madden, 2008; Makar & Confrey, 2004; Chance, delMas, & Garfield, 2004; Reading & Shaughnessy, 2004; Garfield, 1995). From these studies, we can begin to learn more about how prospective teachers may think about the ideas central to this study.

First, consider the terms variability and variation. Understanding variation has been identified as a key piece of statistical knowledge teachers must acquire (Ben-Zvi, 2004; Burgess, 2007). Reading and Shaughnessy (2004) maintain there is a distinct difference between variation and variability, but many researchers use the terms interchangeably (Slauson, 2008). Just as Makar and Confrey (2004) made no distinction between reasoning about variability and reasoning about variation, this study also made no distinction. Variation and variability will be used interchangeably due to the fact that variation/variability encompasses so much. "Variation encompasses more than a measure, although measuring variation is an important component in data analysis. In considering variation, one must consider not just what it is (its definition or formula), or how to use it as a tool (related procedures), but also why it is useful within a context (purpose)" (Makar & Confrey, 2004, pg. 28).

One well-known problem with research related to statistics education is that topics are undeniably intertwined (Chance, delMas, & Garfield, 1999). This makes studying a single statistical idea difficult. So, while the following parts of this section include distinct headings, it is believed that most research findings can be applied to each of the three big ideas of variability being assessed in this study.

Describing Distributions. Recall that this study assessed prospective teachers' statistical knowledge about variability related to three big ideas: analyzing/comparing distributions, understanding of deviation/error/residuals, and articulating the Law of Large Numbers. Often with the materials used in this study, teachers were asked to analyze graphical representations. Leavy (2006) found that, initially, prospective mathematics teachers in a methods course preferred quantitative methods over graphical representations. By the end of the study, however, prospective teachers were much more likely to attend to effects of sample size and to benefits and drawbacks for certain statistical measures and graphical representations. According to Slauson (2008), Leavy's study highlighted the "connection between statistical inquiry, concepts of distribution, and recognizing the importance of variation" (pg. 33).

While the Leavy (2006) study shows promising results, using graphical displays to describe variation is generally not an easy task for novice students of statistics or their teachers (Jacobbe & Horton, 2010; Makar & Confrey, 2004). Perceiving data as a series of individual cases rather than as an aggregate can be problematic (Bakker, 2004; Baker, Corbett, & Koedinger, 2002). However, allowing informal descriptions of distributional behavior is a great start to addressing the problem (National Council of Teachers of Mathematics, 2000; Konold & Higgins, 2003; Bakker & Gravemeijer, 2004; Makar & Confrey, 2004). Instead of exclusively using sophisticated statistical terminology, Makar and Confrey (2004) found that secondary mathematics teachers articulate the notion of variability in multiple ways. During their study, some teachers were able to use correct, standard statistical language to describe a distribution. Others referred to variability in nonstandard ways that inherently provided beneficial means of analyzing data. More importantly, however, were the reported long-lasting benefits in conceptual understanding with such informal statistical thinking.

Of particular interest to the current study were the ways in which students and prospective teachers described center and spread. In Makar's dissertation study, she found prospective elementary teachers describing spread by using relative grouping with phrases and words such as "bulk of this data," "scattered," and "bunched" (Makar & Canada, 2005, pg. 276). In Canada's dissertation study, he also found the same non-standard statistical language with prospective secondary teachers. They used words and phrases like "clustered," "evenly distributed," and "wider spread" to describe the spread or distribution of data (pg. 276). He also found prospective teachers were less likely to discuss center, despite having the means marked on the graphs that were shown to them. The lack of attention to center has caused others to make suggestions for developing an understanding of center in an alternative way. Konold et al. (2002) suggested developing the idea of a modal clump as a way to characterize the center of a distribution.

With potential problems in mind, purposeful development of prospective teachers' ability to analyze distributions is necessary. In her dissertation study, Madden (2008) used a collection of tasks and activities designed to promote statistical reasoning with distributions. Prospective teachers reportedly had more difficulty interpreting box plots than practicing teachers initially. In a pre-interview, one of her participants was able to compare center and spread of two distributions. He was also able to identify potential outliers, which he called "glips" (pg. 244). But, Madden reported this about the prospective teacher's preference of graphical representations:

> "He preferred to reason from dot plots because the actual data were preserved for his view. His second graphical preference for comparing distributions was histograms and he least preferred box plots. He acknowledged that he had only learned about box plots during the last 10 years and did not have great experience interpreting distributions with the box plot representation" (pg. 244).

By the end of her professional development, however, she found that "teachers' responses indicated a generally greater disposition toward a distributional view of the data and awareness of variability" (pg. 204).

Another effort to develop prospective teachers' understanding of center and spread was the curriculum materials used for this study (Lee et al., 2010). A recent study reported how prospective teachers used interval reasoning to coordinate center and spread through specific activities from the text (Lee & Lee, 2011). The way intervals were chosen varied. For example, one prospective teacher chose intervals that would include all possible values instead of trying to find an interval that would include most values. Another prospective teacher's interval was symmetric about an expected value. While that particular study was centered around a probabilistic context, the notion of using intervals to help describe a distribution was an important part of the curriculum design in each chapter and is, therefore an important form of developing understanding of center and spread for the current study.

Understanding Deviation. Often, the term deviation is used when describing distributions and variability, in general. But, some research has been specifically designed to capture understanding of standard deviation. Liu and delMas (2005) used a computer interface to try to develop students' conceptual understanding of the relationship between standard deviation and histograms. They argued that a foundational knowledge of distribution and arithmetic mean is critical if students are to develop proper understanding of standard deviation. In their analysis, they found eleven broad categories of justification: “balance, bell-shaped, big mean, contiguous, equally spread out, far away, guess and check, location, mean in the middle, mirror image, and more bars in the middle” (pg. 63). By the end of their study, students were moving from “simple, one-dimensional understandings of the standard deviation that did not consider variation about the mean to more mean-centered conceptualizations that coordinated the effects of frequency (density) and deviation from the mean” (pg. 55).

Slauson (2008) also tried to conceptualize how college students understood standard deviation. In her dissertation study, she used reform-type labs with her treatment group and discovered several different interesting responses. In answering questions, some students did not expect variability at all. Others knew that variability was expected but did not understand standard deviation was an appropriate measure to use.

In preparing the conceptual framework for their study, which looked at understanding students’ covariational reasoning, Zieffler and Garfield (2009) reported that research shows college students have difficulty understanding several statistical ideas. One such difficulty lies in understanding the correlation coefficient. “Students realized that the absolute value of the correlation coefficient was related to the magnitude of the relationship, but did not relate that idea to the spread of scatter around the regression line” (pg. 9). In the mathematics classroom, teachers also often have difficulty explaining ideas related to standard deviation, correlation, residuals, and more. After observing three practicing teachers, Casey (2010) offers suggestions for the statistical content knowledge needed for teaching association. Specifically, about the correlation coefficient, she claims that teachers need to understand how the formula creates a statistic that describes the strength and direction of a linear association and why r is zero when points follow a horizontal line or some non-linear pattern.

Understanding the Law of Large Numbers. Research has shown that prospective mathematics teachers have limited understanding of probability (Lee & Lee, 2011; Liu, 2005; Ives, 2009). In particular, prospective teachers often hold misconceptions about the Law of Large Numbers (Carter & Capraro, 2005; Ives, 2009; Konold, 1995). In her multiple-case study, Ives a prospective teacher was asked to clarify what she meant by a “good” experiment. She responded:

> “The main point that I'm trying to make in my paper is to get kids to realize that the more experiments you do the closer the experimental probability will be to the theoretical probability. Because like if something has a probability of like 11%, then let‘s say you only do 20 samples and something happens twice out of the 20 times, that will only give you 10%. But if you were to do maybe 100 samples or 100 experiments then it will happen maybe around 11 times so it will be closer to the 11% than your experimental probability was when you only did a few experiments” (pg. 92).

The prospective teacher seems to understand that a greater number of experiments should give closer results to the assumed known probability. What is missing, however, is evidence that she expects more variability with n=20 and that her sample of n=100 may not be large enough to result in the expected probability. This single instance of misunderstanding described above is one of the many examples Ives reports.

Lee and Lee (2011) also present findings about prospective teachers’ understanding of sampling variability as it relates to sample size. In their study, they used the curriculum materials used in this study (Lee et al., 2010) with three groups of prospective teachers. A different curriculum was used with the control group. Post-test results showed tremendous improvement across all implementation and comparison groups with regard to a particular question related to law of large of numbers. They concluded that “merely engaging in learning about data analysis and probability may be helpful in one’s ability to correctly respond to that question, regardless of curriculum material” (pg. 43).

Assessing Understanding of Variability. Efforts in assessing the understanding of variability in elementary and secondary students have increased in the past decade (Reading & Shaughnessy, 2004). For example, Watson and Moritz (2003) developed a measure to assess understanding of variability in younger students. Their instrument includes a hierarchy of understanding ranging from prerequisite knowledge to critical reasoning. Unfortunately, less attention has been given in trying to understand what prospective teachers know and understand. For that reason, some researchers are applying assessments designed for younger students to different circumstances. Madden (2008) is one such example. She used the Watson and Moritz (2003) assessment mentioned above with practicing high-school teachers. In her study, she used four levels of understanding to “help clarify teachers’ level of understanding of variability” (pg. 38). With careful construction of her professional development sessions, through task selection, group facilitation and the use of dynamic technology, Madden found teachers had an improved understanding of variability when comparing distributions.

Technological Statistical Knowledge

Some “lists” on required knowledge for teachers presented in the literature do not include technology explicitly (Ball & Forzani, 2009; Ball et al., 2008, Hiebert et al., 2003; Usiskin, 2001). Mishra and Koehler (2006) said this in response to Shulman’s (1986) popular pedagogical content knowledge (PCK) framework:

> “Although Shulman did not discuss technology and its relationship to pedagogy and content, we do not believe that these issues were considered unimportant…technologies weren’t foregrounded to the extent that they are today….What has changed since the 1980s is that technologies have come to the forefront of educational discourse primarily because of the availability of a range of new, primarily digital, technologies and requirements for learning how to apply them to teaching” (p. 1023).

Without doubt, some technology knowledge is necessary for the “work of teaching” (Ball & Forzani, 2009). Common technological use for productivity, including the use of word processors, equation editors, computer projectors, and more, are likely the norm rather than the exception. Today’s teachers are more comfortable and competent at using technology as a general pedagogical and performance tool (Putnam & Borko, 2000). More importantly, mathematics teachers are gaining an awareness of specific tools such as internet applets and dynamic mathematical software programs. This type of awareness brings to light new ways of thinking about how and when technology can be used during instruction. Furthermore, technological tools for learning mathematics provide a means for interactive and collaborative experiences and are a product of the social constructivist approach to teaching and learning (Maor, 2003).

The participants for this study were enrolled in a *Teaching Mathematics with Technology* course. The technologies used in this course, particularly in the focus unit of the current study, are related to statistics and probability. Therefore, the technological knowledge of interest is that which is aligned to those technological tools used in the curriculum. Specifically for teaching and learning statistical concepts, tools that allow for interactive and collaborative experiences include, but are not limited to, *TinkerPlots* (Key Curriculum Technologies, 2005), *Fathom* (Key Curriculum Technologies, 2007), and *Probability Explorer* (Stohl, 2002).

There is now a growing base of research on how these technologies may be used with students during instruction. Nearly a quarter of a century ago, Pea (1987) described two constructs of technology tools, which he called cognitive tools. First, he suggested that when cognitive tools are used to amplify, they accept the goals of curriculum and work to enhance understanding and achievement. Second, when cognitive tools are used to reorganize, they alter the curriculum. The way a cognitive tool is used is then determined by the teacher and/or school (Zbiek et al., 2007). However, in light of the current study, it is important to note here that technological “tools by themselves don’t reduce the complexity of data analysis, nor do they solve the problem of variability” (Hammerman & Rubin, 2004, pg. 18). Instead, teacher development efforts will need to include opportunities for prospective teachers to develop an applied knowledge of the technologies they are learning. By using the technology to engage in tasks, teachers in this study experienced dynamically linked representations, visualizations of abstract ideas (e.g. standard deviation), and real data that would be cumbersome to work with otherwise. In doing so, they will know first-hand the power in using technology as pedagogical tools (Lee & Hollebrands, 2011). Technological statistical knowledge encompasses more than just a prospective teacher’s understanding about how to perform statistical functions with technology. TSK includes using technology to visualize abstract ideas and help make connections between representations.

Technological Pedagogical Statistical Knowledge

"Knowledge of general pedagogical principles is a necessary component of teachers' knowledge" (Fennema & Franke, 1992, pg. 147). But, the focus of pedagogical knowledge in this study will be on prospective teachers' ability to demonstrate the use of technology in order to effectively teach statistical and probabilistic concepts. Defining pedagogical knowledge is tricky as multiple descriptions surface in the literature. Some describe it as the knowledge of students and how to best transfer new mathematical ideas to them practically; others describe it as approaches to, procedures for, and presentation of mathematics; still, others describe it as the knowledge of assessing students and making the most of curricular materials (Van der Sandt, 2007). Without a single definition, it is difficult to prescribe to teachers exactly what it is they need to know with regard to pedagogy. Suggesting the seamless integration of technology during instruction adds another layer to the definitions above. Yet, the potential descriptions above serve as a springboard for looking at what technological, pedagogical, content knowledge may encompass.

Scholars suggest that knowledge of how students think and learn is imperative knowledge for teachers (Fennema & Franke, 1992; Van der Sandt, 2007). Teachers need to be able to "see content from others' perspectives" (Ball & Forzani, 2009, pg. 500) and "represent mathematical ideas in a manner that is understandable by the students" (Fennema & Franke, 1992, pg. 153). Exactly how to do this and what strategies should be used vary according to research. The literature is full of recommendations for questioning, using multiple representations, cooperative learning vs. individualized learning, motivation, and more. But, as Ball et al. (2008) contend, “high-quality instruction requires a sophisticated, professional knowledge that goes beyond simple rules such as how long to wait for students to respond” (pg. 391).

Curriculum knowledge has also been thought of as an important part of pedagogical content knowledge for some time (Shulman, 1986; Van der Sandt, 2007), but standards-based curricula and the recent introduction of reform curricula with high expectations of technology usage add another level to consider. They encourage a new level of discourse, inquiry-based tasks, and student-centered lessons for the mathematics classroom (NCTM, 2000; White-Fredette, 2010). It is likely some combination of many techniques is most effective. Despite efforts from teacher education programs, teachers often resort to traditional approaches because they are comfortable and safe. It is well-known that past traditional beliefs about instruction lead to traditional, classroom practices (Hiebert et al., 2003; Stipek et al., 2001). As such, much of teachers’ pedagogical knowledge is acquired outside of teacher education programs or professional development – making it difficult for mathematics education researchers to analyze fully.

In this study, the pedagogical knowledge being developed by prospective teachers includes using technology in the teaching and learning of statistics. Recall the nested circles of the TPSK framework (see Figure 2). Lee and Hollebrands (2011) said this about TPSK:

> “The inner-most layer represents elements of TPSK and is a subset of the sets in the outer two circles, meaning TPSK is founded on and developed with teachers’ knowledge in the outer two sets of technological statistical knowledge (TSK) and statistical knowledge (SK). In addition, developing TSK and SK is essential to, but not sufficient for, teachers having the specialized TPSK” (pg. 361).

Once statistical knowledge and technical knowledge are developed, prospective teachers can think about how students learn and understand statistical ideas and how affordances of technology may help students' conceptual understanding.

Teaching Presence

Recall the community of inquiry framework (Garrison et al., 2000) described at the beginning of this chapter. It attempts to characterize the processes that lead to collaborative knowledge construction by revealing the behaviors necessary to cultivate knowledge construction through various forms of "presence." The social, cognitive and teaching presence components that encompass this framework do not exist in isolation. So far, the social and cognitive presences have been discussed independently. However, there are obvious overlaps between how prospective teachers are communicating and interacting with one another and the content of their discussions and the subsequent knowledge they are developing and sharing. Similarly, the teaching presence does not exist in isolation. How it works together with the social and cognitive presences is important in fostering this type of learning community.

Indeed, the teacher educator must be mindful in making decisions that support productive discourse and learning among prospective mathematics teachers. Some decisions are thought out well in advance, others are spontaneous; both have potentially great effects. Aligned with Vygotsky's notion of zone of proximal development and the ideas of constructivism, many mathematics educators agree that careful support during instruction is beneficial. Nathan and Knuth (2003) describe two types of scaffolding that should occur.

One type is analytic scaffolding. It includes the scaffolding of mathematical concepts and ideas in order to support learning during class interactions. “Examples of analytic scaffolding include a teacher re-describing student contributions to a discussion in more precise mathematical terms or a teacher highlighting particular aspects of student contributions in light of their potential utility for introducing more advanced mathematical ideas” (pg. 178). This requires that teacher educators carefully consider contributions from students. Yackel and Cobb (1996) remind instructors to pay special attention to the quality of those contributions. Often prospective teachers are driven by wanting students to reach a certain point of understanding. According to Tzur (2001), they quickly dismiss the differences in process by which students come to an agreement. Teacher educators, then, need to remind prospective teachers to pay close attention to interesting and important differences among their students’ ideas.

A second type of scaffolding described by Nathan and Knuth (2003) is social scaffolding. It includes establishing norms for social behavior and expectations regarding whole-class and small-group discussions in order to better facilitate class interactions. “Examples of social scaffolding include asking students to provide explanations for solutions to problems or eliciting contributions to whole-class conversations from all students” (pg. 178). While Nathan and Knuth’s ideas of analytic and social scaffolding were built around mathematics classes, they may certainly be applied to mathematics teacher education as well. There needs to be a balance between facilitating a welcoming environment where prospective teachers can share freely without fear of judgment and one where acceptable mathematical explanations are expected. As a result, teacher educators have the responsibility of modeling the use of three elements involved in creating opportunities for students to learn mathematics: “the management of learning, sensitivity to students, and the mathematical challenge” (Zaslavsky & Leikin, 2004, pg. 7).

Aligned with constructivist learning theory, discourse plays an important role in the construction of knowledge. For teacher educators, providing analytic and social scaffolds described above is an important part of creating the desired learning environment. They must be intentional about their own participation in conversations and interactions. Often the most effective strategy for teacher educators to implement is to encourage prospective teachers to reflect and share with one another, stepping in when appropriate. Sliva (2002) claims this practice of “scaling back,” providing less and less guidance, is an appropriate technique for modeling good responses and promoting strong, thoughtful discussion. Sing and Khine (2006) give further advice for facilitating discussions:

> “For in-depth knowledge building discourse to happen within the context of teacher professional development, the teachers need to challenge the cultural/professional norm of niceness; be able to detect gaps in understanding; have adequate knowledge about the context of another teacher’s classroom; have the necessary social skills in putting across the critical comments; and assumes a new identity of knowledge producer…. Educators have to carefully engineer the social, cultural, cognitive dimensions of the learning environment before they can reap the benefits afforded by technologies – it depends on how skillful the facilitator is. There also seems to be no prescription available on how to form the desired learning environment. It seems that teacher educators or the online facilitators need to constantly model the skills” (pg. 259).

Several research studies have articulated what the goals of mathematics teacher educators should be when it comes to specifically helping prospective teachers learn about integrating technology into mathematics instruction (Cannings & Stager, 1998; Quesada, Wheland, & Zachariah, 2001). According to research, prospective teachers should be exposed to rich, standards-based mathematical content, various educational technologies, and innovative methods to mathematics delivery and the assimilation of that technology into the mathematics classroom. These future teachers should also receive assistance in building a community of support. It is the instructor's duty to facilitate the discussions and provide adequate feedback so that the discourse within the course is positive and productive. It is paramount then, that instructors create a comfortable and professional learning environment.

Hammerman & Rubin (2004) remind their readers that there are "new demands on teachers to assess the validity of the arguments that students are making with these [technological] representations, and to facilitate conversations in productive ways" (pg. 18). Therefore, the goal is not to teach new technology skills exclusively, although it is important for teachers to see technology being modeled by their instructors (Goodell & Yusko, 2005). More importantly, the goal is "to encourage teachers to reflect on and discuss the affordances of technology in helping students learn mathematics" (Stephens & Hartmann, 2004, pg. 61). Reflecting on action has been recommended by others as well (Garcia, Sanchez, & Escudero, 2006). Helping prospective teachers connect theory and practice is critical if effective implementation of technology with students is to be achieved.

Implications of Synchronous Learning for the Instructor

A variety of activities in the synchronous environment help supply the platform for teachers to be able to personally construct mathematical knowledge and discuss pedagogical issues related to using technology with students (Cady & Rearden, 2009). Web-conferencing programs such as *Elluminate*, allow the instructor to create breakout rooms for small group discussions. For the instructor, knowing when to join a breakout is difficult (Goos &

Bennison, 2008). But, most research reports that some level of instructor intervention is beneficial (Bender, 2003). Vlachopoulos and Cowan (2010) remind their readers that online facilitators are “required to adopt a number of roles in online discussions, which include a social, a pedagogical, and an intellectual role” (pg. 33). Instructors need to provide timely guidance and feedback, stepping in with responses and new questions when necessary (Hou, Chang, & Sung, 2008). Just as in face-to-face settings, online “learners may experience bottlenecks, such as insufficient information or inadequate deduction. When this occurs, teachers’ assistance will be needed” (pg. 23).

Similar to their face-to-face counterparts, online instructors have many things to consider when designing and participating in a lesson. In developing prospective teachers’ TPSK, teacher educators should include time for individual work/reflection, time to learn and practice using a new technology, and time to collaborate with other prospective teachers in thinking about using the strategies presented to teach students.

Chapter Summary

A community of inquiry framework (Garrison, Anderson, & Archer, 2000) was formally introduced and served as a lens to review literature pertaining to discourse and the development of prospective teachers’ knowledge about teaching statistics with technology. This framework includes social, cognitive, and teaching presences. These presences, denoted by circles in a Venn diagram, are not isolated. Rather, they overlap and seem to intersect seamlessly in some instances. So, while each presence was reviewed independently, the researcher does not claim that they are altogether separate entities.

Social presence, assembled by interactions, is how individuals communicate with one another. Based on constructivism, discourse and interaction are essential to the construction of knowledge. However, discussions, in and of themselves, do not facilitate the kind of inquiry necessary to complement the social presence, particularly in the online learning environment (Larreamendy-Joerns & Leinhardt, 2006; Shea & Bidjerano, 2009). Instead, prospective teachers must be given thoughtful prompts by teacher educators that scaffold their responses and guide them to a deeper understanding. Once this happens, teachers in training are then equipped to reflect upon, question, and revise ideas related to content and pedagogy.

Cognitive presence includes all parts of the community of inquiry that correlate with assessing knowledge. Over time researchers have made claims about that which teachers should know in order to be effective in the classroom, but no consensus has been reached (Ball & Forzani, 2009; Fennema & Franke, 1992; Reynolds, 1992). Yet, there is enough evidence to suggest that mathematics educators do, in fact, require a specialized knowledge (Ball & Forzani, 2009; Ball et al., 2008). This knowledge may be a careful combination of knowledge related to pedagogy, mathematical content, and technology. The cognitive presence of a course includes opportunities for prospective teachers to develop this specialized knowledge. The current study looks at prospective teachers' constructed knowledge related to teaching data analysis and probability with technology. Therefore, the TPSK framework (Lee & Hollebrands, 2011) may be particularly helpful in understanding teachers' developing knowledge for teaching statistical and probabilistic concepts with technology.

The teaching presence requires that instructors facilitate discussions such that the social and cognitive objectives are met. One important role for the teacher educator is to provide scaffolds. Both analytic and social scaffolds are acceptable means to facilitating a productive classroom discourse (Nathan & Knuth, 2003). Prospective teachers should be encouraged to reflect on practices and their implications for students. Paying close attention to the differences in processes students may use, they need modeled examples as to how such differences may be used to deepen the collective understanding of students and how theory connects with practice.

CHAPTER 3: METHODOLOGY

Introduction

Because discourse and constructed knowledge cannot be completely identified and analyzed quantitatively, a qualitative research methodology was desirable (Patton, 2002). This chapter presents a detailed description of the research methodology through sections regarding the following: two research questions; a research framework; description of a pilot study; explanation of the research design; site selection and sample; description of methodologies related to data collection and data analysis. Moreover, a discussion about research validity and reliability as well as safeguards against researcher bias, ethical issues, and limitations of the study follow.

Research Questions

The purpose of this study was to analyze discourse patterns and opportunities for interaction in two mathematics education methods classes, one face-to-face and one synchronous, online. In addition, due to the content in this course, this study sought to determine the role discourse played in developing statistical knowledge for teaching with technology in prospective mathematics teachers. Consequently, the goal was to answer the following research questions:

(1) What similarities and differences in discourse and opportunities for interaction exist between face-to-face and synchronous, online mathematics education courses?

(2a) What is the nature of prospective teachers' understanding of variability and teaching concepts related to data analysis and probability with technology?

(2b) What is the role of discourse in face-to-face and synchronous, online environments in developing this understanding among prospective mathematics teachers?

Frameworks

The development of mathematics teachers' knowledge about teaching and learning statistics with technology is complex. University education related to teaching data analysis and probability with technology should include opportunities for prospective teachers to develop the specialized knowledge discussed in Chapter 2 which includes a combination of statistic content knowledge (SK), technological statistical knowledge (TSK), and technological pedagogical statistical knowledge (TPSK). For the current study, this knowledge development was analyzed from interviews, assessments, and discourse in the face-to-face and online learning environments. The emphasis on discourse as a key part of knowledge development is important for this study, as it used ideas from constructivism as conceptual underpinnings.

Constructivist learning theory, sometimes referred to as simply constructivism, is derived from the sociocultural constructivist perspective (Vygotsky, 1962), which assumes that participation in discussions and activities that make up classroom discourse supports knowledge construction. Specifically, the constructivist learning theory promotes the idea that a student's potential capacity for gaining knowledge is enhanced by scaffolding through interaction (Ling, 2007). It also recognizes that prospective teachers may internalize knowledge differently. This is a familiar idea in mathematics education.

The community of inquiry framework, described in Chapter 2, is an attempt to understand the social, technological, and pedagogical processes that lead to collaborative knowledge construction (Garrison, Anderson, & Archer, 2000). It reveals the behaviors and processes necessary to cultivate knowledge construction through various forms of "presence." With the Venn diagram representation of this framework (Figure 1), it is important to note the social, cognitive, and teaching presence components do not exist in isolation. Because this study focused on effects of discourse on prospective mathematics teachers' developing knowledge about teaching variability with technology, the educational experience denoted by the intersection of social, cognitive, and teaching presences is especially important.

For the current study, an effort was made to anticipate important pieces of the educational experience denoted in the community of inquiry framework. The result was a conceptual framework for studying discourse and how it affects knowledge related to teaching variability with technology. The instructor selected activities and facilitated discourse and interactions in order to encourage teachers to grow three types of knowledge simultaneously. This selection of activities and facilitation of the class was a critical component of the resulting discourse and interactions among prospective teachers and the knowledge they developed. As the teachers' knowledge (SK, TSK, and TPSK) grew, they developed a repertoire of content and technology skills for solving problems and acquired a growing awareness of how such tools may be used with students effectively. This, in turn, potentially affected how they communicated and interacted with their instructor and with one another.

To study the interactions and the constructed TPSK that occurred among prospective mathematics teachers during this study, the researcher drew heavily on the work of previous research. The first research question required the researcher to analyze discourse by which prospective teachers come to know what they know. Using a modification of Krussel et al.'s (2004) framework, discourse was analyzed to identify the direction, form, purpose, and topic of a participant's contribution to discourse. By examining whole group and small group discussions line-by-line with these four categories, the researcher was able to capture the following: 1) who was talking and to whom he/she was talking (direction), 2) how he/she chose to communicate with others (form, i.e. chat, talk, use of whiteboard), 3) whether he/she wanted to share an idea, ask a question, etc., and 4) what he/she focused on in their statements (topic, i.e. statistical content, technology issues, pedagogical considerations). Special attention was given to opportunities for interaction, the ways in which prospective teachers and the instructor communicated with one another, discussion patterns and evidence that prospective teachers were considering SK, TSK, or TPSK in the ways in which they talked about variability.

The second research question entailed the use of important components of variability as related to describing distributions, deviation, and the law of large numbers. In general, how prospective teachers attended to shape, center, and spread of a distribution was noted. Further analysis of their focus and use of statistical language as it pertained to center (mean, median, mode, and midrange), deviation (outliers, connection of center and spread, use of standard deviation, interpretation of sum of squares, interpretation of residual plot), and law of large numbers (effects of sample size) was completed.

While this study did not attempt to understand the ways in which prospective teachers used technology tools in statistical tasks, general comments about affordances and constraints of technology were also noted (technology is faster, color, multiple representations, dynamic nature versus static representations, thought about statistical idea in a new way, provides a good visual). In addition, due to the setting of this study (a *Teaching Mathematics with Technology* course), prospective teachers had opportunities to discuss ideas related to content and technology and how what they were learning would affect students' conceptual understanding. The researcher attended to times when prospective teachers seemed to be focusing on technological, pedagogical, statistical knowledge.

Pilot Study

In the Fall 2010 Semester, a pilot study was conducted with a section of a technology methods course, *Teaching Mathematics with Technology*, at a large public university. The class was taught by the researcher, and was divided into two groups for the data analysis and probability unit (a five-week study) – one face-to-face group and one online group that met synchronously at a time different from the normal class time so that the instructor could continue teaching the face-to-face group.

Twenty-two students (juniors, seniors, graduate students, and lateral entry students studying mathematics education) were enrolled in the course. There were 17 female students and 5 male students in the class. This class was selected due to the curriculum focus of teaching data analysis and probability with the integration of technology that makes up one-third of the course. No other course at this university was using this curriculum at the time of the pilot study.

The face-to-face class met on Mondays from 4:10-6:55 p.m. in a computer lab on campus. Each student had her/his own computer and the instructor had a computer and projector to use during class demonstrations. Data analysis and probability makes up approximately one-third of the course. Prior to the study, the twenty-two students learned about teaching algebra and geometry with technology in a face-to-face setting during the first eight weeks of the semester.

Data analysis and probability was studied for 5 weeks, starting the week of October 18 and ending the week of November 15. For this unit, the class was divided into two groups. One group continued coming to class once a week for face-to-face instruction. The second group followed the same curriculum through both asynchronous and synchronous environments. The online group was selected by the instructor based on students' availability outside of the regular class time. Thirteen of the twenty-two students volunteered to study the data analysis and probability curriculum online. However, the largest number of students that could meet for three consecutive hours outside of the usual class time was eight. These students met synchronously for a 3-hour block per week for the five weeks of the pilot study using the *Elluminate* web-conferencing software program.

During the pilot study, there were many lessons learned for the researcher. Separate lesson plans were necessary for the online class to highlight key differences in implementing the curriculum with an online group. First, sharing content of the course was sometimes more challenging online. This was mostly because in the online environment, prospective teachers were using their own computers, web-browsers, programs, and calculators. In the face-to-face setting, all students were working in a lab in which computers had equivalent settings and programs, and graphing calculators were provided by the instructor when needed. Facilitating discussions and demonstrations regarding graphing calculators, when a few members of the class were using a different model calculator, was tough particularly since the prospective teachers' calculators could not be seen by the researcher or other members of the face-to-face group. Another challenge was nuances between using Macintosh computers versus other personal computers. While most problems that surfaced were addressed, either by the researcher or by a classmate, the time it took to troubleshoot and solve those technological issues was significant. Knowing to expect some of the above problems helped the researcher, in the present study, address some issues before they ever surfaced during class time, though she was unable to entirely eliminate all technological difficulties.

Creating a productive environment for discussing pedagogical implications of using technology with students was also different in the online environment. Prospective teachers, in the synchronous setting, could communicate by using the chat window, the microphone, or writing on the interactive white board. While a couple of students were more than willing to speak during a whole-group discussion, more students seemed to prefer the chat window when a question was posed by the researcher. Sometimes, this delayed response seemed awkward. Other times, it was clear that their hesitancy to respond was proper wait time in action as prospective teachers' correctly to questions. Several times throughout the five-week unit, prospective teachers were asked to join a small group, called a break out room. They seemed much more comfortable talking with the microphone in that setting, similar to students talking in small groups of a face-to-face class.

In addition to lessons learned regarding the presentation of content and facilitating discussions, there was also much learned with regard to moderating an Elluminate session. Failed internet connections and audio mishaps occurred during the first three sessions. However, when the researcher or a prospective teacher was temporarily removed from the session, the problems were always resolved quickly. While the technology glitches were undoubtedly distracting, the lessons were able to continue. And, there were no problems during the final session at all.

Four of the five synchronous lessons were recorded. These recordings show everything that happened in the “Main Room” of Elluminate, but break out room discussions could not be recorded. At the end of the five weeks, prospective teachers informally shared some of their likes and dislikes. They liked “everything,” “breakout groups,” “giving control of mouse to student,” and “working on my own computer – i feel like i see it better when looking directly at my computer.” They disliked “[having to] have extra time for kinks” and “technology problems.”

Research Design

The current study employed a qualitative methodology. The primary objective was to learn more about how discourse compares in face-to-face and online mathematics education courses, and how it may affect the knowledge constructed by prospective teachers. As a result, utilizing qualitative methods through a multiple case study approach was necessary to allow for a more in-depth and detailed inquiry (Patton, 2002).

A nested case study design was especially useful for this specific project because it permitted an in-depth investigation in two distinct learning environments simultaneously.

Data about individuals (e.g. things they said during discussions, interviews, written assessments) were then aggregated to analyze each learning environment as a single case. Thus, this study included two cases, the case of a face-to-face environment for learning to teach data analysis and probability with technology, and an online environment for learning the same content.

Because the class time was the determining factor in which of the two classes would meet online, the two groups were selected by convenient sampling. However, the second level in the nested case study included six cases selected by purposeful sampling (see Figure 3). Three prospective teachers were carefully selected from each of the face-to-face and online groups to form two focus groups. Each focus group was comprised of prospective teachers with similar attributes (gender, age, and major) in order to insure comparability. These students were observed carefully during the five-week unit. More about why and how they were observed follows in the data collection section.

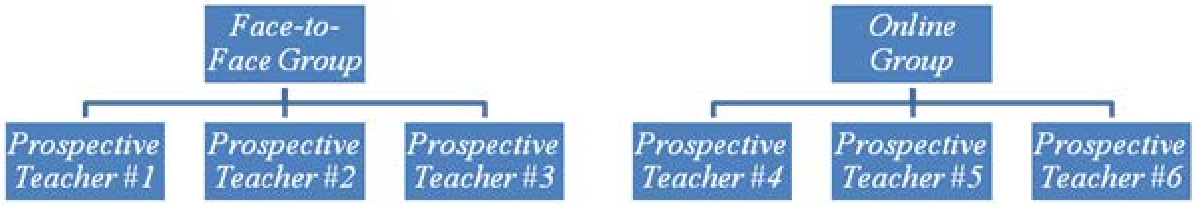


Figure 3. Nested case study design.

Site Selection and Sample

This comparative study examined discourse and constructed knowledge in one face-to-face section and one partially-online section of a mathematics education methods courses. Forty-two students enrolled in one of two sections of a *Teaching Mathematics with*

Technology course at a large state university - juniors, seniors, graduate students, and lateral entry students studying mathematics education or middle grades mathematics education. These classes were selected due to the curriculum focus of teaching data analysis and probability with the integration of technology that makes up one-third of the course. The two classes were studied intensely for the data analysis and probability unit of the course (a five-week study). Curriculum used (Lee et al., 2010) and learning outcome expectations were identical for both sections of the course.

At the beginning of the semester, both classes met face-to-face. During the first 3-4 weeks of the semester, students in both sections learned technological and pedagogical skills for teaching Algebraic concepts with technology in a face-to-face setting. The researcher was the instructor for a section of the course that met twice weekly for 1.25 hours each day. The second section of the course, taught by another instructor, met once a week for 2.75 hours. The researcher attended the first two weeks of the semester in order to get to know students in that class. Given the time of day, the researcher felt it would be easier for the late afternoon/early evening group of prospective teachers to meet synchronously online. Prospective teachers in the twice-a-week morning class had other classes meeting directly before or after the class time allotted. Thus, the second section of the course that met once a week for 2.75 hours in the evening was selected to be the online group. During the 5-week study, the researcher continued teaching the twice-weekly section in the face-to-face setting and began teaching the once-a-week section online. The start time of the online class was changed slightly so that prospective teachers had time to return home from campus (some of them had a long commute).

Data analysis and probability, the unit of focus for this study, was studied for five weeks in each learning environment. Participants in both classes were required to purchase the textbook and CD (Lee et al., 2010) for this unit and thus had access to the same written and electronic materials. The curriculum also required the use of multiple technologies (*TinkerPlots, Fathom, Probability Explorer,* Microsoft *Excel*, and the *TI-83+/84* graphing calculator). These were all listed on the course syllabus for both sections of the course. Prospective teachers in the face-to-face class who used the desktop computers in the classroom had access to all programs while on campus. Prospective teachers in the same class who chose to bring their laptops to class each day had to either purchase the software or make a reservation in the university's remote Virtual Computing Lab (VCL) in order to have access to the technologies during class. Participants in the online class had that choice as well. They needed to either purchase the software or use a VCL reservation. Access to graphing calculators was the biggest challenge. For the face-to-face class, the instructor provided enough *TI-83+/84* calculators so that each prospective teacher could have their own. In the online class, participants did not have such access. Instead, many of them simply used the calculator they did have access to, which was often not the one listed on the course syllabus (e.g. *TI-86*, *TI-89*).

Online Group

Seventeen students (prospective teachers) were enrolled in the section of the course that served as the online group, meeting once a week for 2.75 hours using *Elluminate*, a synchronous, web-conferencing program. The online class was taught by the researcher during the unit of study. To maintain continuity for the prospective teachers in the course, the primary instructor for this section participated in each of the online class sessions.

To use *Elluminate*, each participant needed access to speakers and a microphone to participate during the online class meetings. As mentioned above, prospective teachers also needed a *TI-83+/84* graphing calculator, Microsoft *Excel*, all dynamic software programs used in the materials (*TinkerPlots, Fathom,* and *Probability Explorer*), and the textbook (Lee et al., 2010) for the duration of the curriculum unit. In *Elluminate*, prospective teachers could interact with one another and with the instructor. Live technology demonstrations with the programs above were a regular part of the online class.

Three prospective teachers of the online group were selected to be members of a focus group. These participants, Abby, Les, and Sally, were all juniors in secondary mathematics education. During the first two weeks of the semester, the researcher observed their participation during whole group discussions in the face-to-face setting. Abby and Sally were regular contributors to discussions, whereas Les rarely shared ideas with the large group. Their background in statistics varied, based on their reported experiences on the pre-assessment. Abby reported having no experience with statistics, and Sally and Les had taken an introductory course at the university.

Face-to-Face Group

Twenty-five students (prospective teachers) were enrolled in the section of the course that met face-to-face, twice weekly for 1.25 hours each, in a computer lab on campus. Each student had her/his own computer and the instructor had a computer and projector to use during class demonstrations. As mentioned above, prospective teachers needed a *TI-83+/84* graphing calculator, Microsoft *Excel*, all dynamic software programs used in the materials (*TinkerPlots*, *Fathom*, and *Probability Explorer*), as well as the textbook (Lee et al., 2010) for the duration of the curriculum unit. Prospective teachers in the face-to-face class who used the desktop computers in the classroom had access to all programs while on campus. Prospective teachers in the same class who chose to bring their laptops to class each day had to either purchase the software or make a reservation in the university's VCL in order to have access to the technologies during class.

Three prospective teachers, who incidentally were ones who brought their laptops to class each day, were selected to be members of a focus group. These participants, Ava, Carrie, and Sam, were all juniors. Two of them, Carrie and Sam, were secondary mathematics education majors. Ava was a middle grades mathematics education major. During the first four weeks of the semester, the researcher observed their participation during whole group discussions in the face-to-face setting. Carrie and Sam were regular contributors to discussions, whereas Ava rarely shared ideas with the large group. Their background in statistics varied, based on their reported experiences on the pre-assessment. Sam reported having no prior college experience with statistics, though he was concurrently taking a probability course. Ava and Carrie reported having taken an introductory and calculus-based introductory statistics course at the university respectively.

Curriculum

Approximately one-third of the *Teaching Mathematics with Technology* course, which was the setting of this study, is devoted to data analysis and probability. The curriculum for this part of the course uses an integrated approach (Lee et al., 2010). This means that materials were designed to develop prospective teachers' technological, pedagogical, and content knowledge simultaneously. Throughout the text, "teachers are engaged as learners and doers of statistics" (pg. viii). The following table of contents is taken from the text and gives an overall sense of big ideas in the materials:

Chapter 1 Center, Spread, and Comparing Data Sets
Section 1: Asking Questions and Visually Exploring Data
Section 2: Describing the Aggregate
Section 3: Examining Center, Spread, and Shape with Box Plots
Section 4: Comparing Two Distributions
Section 5: Importing Data to Explore New Questions
Section 6: Using *TinkerPlots* to Build Conceptual Understanding of Measures of Center

Chapter 2 Analyzing Students' Comparison of Two Distributions Using *TinkerPlots*
Section 1: Exploring the Data
Section 2: Reflecting on Your Analysis of Data and Use of Technology
Section 3: Anticipating Students' Thinking
Section 4: The Videocase: Jordan and Kathy
Section 5: Analyzing Students' Work

Chapter 3 Analyzing Data with *Fathom*
Section 1: Asking Questions from Data
Section 2: Examining Univariate Distributions
Section 3: Comparing Distributions Using Center and Spread
Section 4: Understanding Spread of a Distribution

Chapter 4 Analyzing Bivariate Data with *Fathom*
Section 1: Examining Relationships Between Two Quantitative Variables
Section 2: Conceptualizing Correlation
Section 3: Using a Line to Describe a Relationship Between Two Quantitative Variables
Section 4: Visualizing the Residuals
Section 5: The Least Squares Regression Line
Section 6: Exploring Additional Attributes on a Scatterplot
Section 7: Exploring the Effects of Outliers on Correlation and the Least Squares Line
Section 8: Data Collection for New Questions

Chapter 5 Designing and Using Probability Simulations

Section 1: Using Data to Estimate Probabilities and Design Simulations
Section 2: Simulating Randomness in Technology Tools
Section 3: Simulating Events with a Graphing Calculator
Section 4: Simulating Events with a Spreadsheet

Chapter 6 Using Data Analysis and Probability Simulations to Investigate a Problem

Section 1: Examining Birth Data in North Carolina
Section 2: Using a Simulation to Test Likelihood of an Event
Section 3: Conducting a Simulation and Constructing a Distribution of Sample Proportions
Section 4: Using the Binomial Formula to Compute a Theoretical Probability

Each section in a chapter contains tasks and activities which allow prospective teacher to explore some statistical idea with a specific technological tool. There are two types of questions presented in the text, one called *Engaging with Content* and another called *Considerations for Teaching*. These questions can help develop prospective teachers' statistical knowledge (SK), technological statistical knowledge (TSK), and technological pedagogical statistical knowledge (TPSK). Specific activities from the textbook were identified, a priori, by the researcher as potential sources of evidence of prospective teachers' developing knowledge of variability in describing distributions, deviation, and the law of large numbers. (Table 1; Appendix A).

Table 1. Opportunities for revealed understanding of variability identified a priori (exact questions can be found in Appendix A).

Timeline	Textbook Materials	SK and/or TSK	TPSK
Week 1	**Chapter 1** • Analyzing Distributions	Q20 (pg. 10) Q40 (pg. 17) Q45 (pg. 18)	Q30 (pg. 13)* Q42, Q43 (pg. 17)**
Week 1	**Chapter 2** • Analyzing Distributions	Q1, Q3 (pg. 34) Q7 (pg. 36)	Q11, Q12 (pg. 37)*** Q24 (pg. 42)***

Table 1 Continued

Week 2	**Chapter 3** - Analyzing Distributions - Understanding Deviation/Error/Residuals	Q8 (pg. 54) Q18, Q20 (pg. 59) Q27, Q30, Q31 (pg. 64)	Q10 (pg. 55)* Q21, Q22 (pg. 59)** Q24 (pg. 61)** Q32 (pg. 65)* Q33 (pg. 65)*
Week 3	**Chapter 4** - Understanding Deviation/Error/Residuals	Q25 (pg. 84) Q27 (pg. 85)	Q26 (pg. 84)** Q29 (pg. 86)**
Week 4	**Chapter 5** - Law of Large Numbers	Q4 (pg. 107) Q16, Q17 (pg. 113)	Q7 (pg. 107)* Q19 (pg. 114)* Q34 (pg. 123)**
Week 5	**Chapter 6** - Analyzing Distributions - Law of Large Numbers	Q3 (pg. 133) Q9 (pg. 136) Q1 (pg. 130) Q28 (pg. 148)	Q15 (pg. 137)* Q15 (pg. 137)* Q23 (pg. 141)** Q38 (pg. 151)**

* whole-class discussion, ** small-group discussion, *** homework

In planning the study, the researcher used her prior experiences in teaching this course and using the curriculum text to decide the setting in which the questions listed above might be answered. A blend of whole-group and small-group discussions was anticipated. And, the only questions planned to be assigned for homework were ones in Chapter 2, the videocase chapter.

Data Collection

Multiple sources of data were collected over the course of the study in order to address each of the two research questions. The remainder of this section will describe each data source.

Audio and Video Recordings of Class Sessions

In the face-to-face environment, there were two stationary video cameras. One of them was positioned so that the instructor's computer with the projection screen was in clear view during an entire class meeting. The other video recorder was positioned on members of the face-to-face focus group so that their small group work could be analyzed. After poor audio was obtained of the focus group discussions the first week of the study, an audio recorder was also utilized during times of small group discussion. An undergraduate student, who was not a member of the class, assisted with the setup of video equipment and controlled the audio recorder during class sessions in order to capture small group discussions in their entirety.

That type of equipment was not necessary in capturing video and audio recordings of the online group. *Elluminate* sessions were digitally recorded and stored on the university's *Elluminate*-resource website. With that recording, one could replay events that happened in the "main room" of the environment. For this reason, during times of small group work, focus group members were asked to remain in that "main room" so that their discussion could be recorded. While the digital recordings of the class sessions were saved in the university's *Elluminate* archives, it was not possible to download the recorded sessions in a way that preserved all recorded interactions (e.g., chat window, shared applications, whiteboard, participant dashboard that indicated various actions done by users). Thus, once selected episodes were chosen for careful analysis, as described in a later section, a screencapture program was used to record selected episodes and create a stand-alone digital video file.

Assessments

In addition to video data, this study also included multiple sources of written data. The table below provides a brief summary of such data and how that information was used in terms of describing prospective teachers' developing knowledge of variability. Details about the questions and scoring rubrics for each data source follows.

Table 2. Brief summary of written data sources.

Written Data Sources	**Describing Distributions**	**Understanding Deviation**	**Understanding the Law of Large Numbers**
Pre-/Post-Assessment	☑	☑	☑
Chapter 2 Homework	☑		
Final Exam Performance Task	☑	☑	☑

Pre-/Post-Assessments. Prior to the beginning of the study, both groups were meeting face-to-face. In order to measure pre-existing content knowledge about data analysis and probability, an eight-question pre-assessment (Appendix B) was given the week prior to the data analysis and probability unit of study. This assessment consisted of two background questions to obtain information about prospective teachers' statistical history and five questions designed to capture some of their understanding of analyzing distributions, deviation/error/residuals, and the law of large numbers. Questions on the pre-/post-assessment, taken from Madden (2008), came from a variety of research efforts that increased the validity of the assessment in her study. The table below provides a summary of the five content questions, how each was scored, and whether or not each one was included in analysis for describing distributions, deviation, and the law of large numbers. The numbers correspond to the pre-assessment (recall questions 1 and 2 were background questions). The post-assessment included only the five content questions.

Table 3. Summary of pre-/post-assessment questions, scoring rubrics and corresponding constructs of variability.

Question *(Numbers correspond to Pre-Test)*	**Content Goal**	**Scoring Rubric**	**Included in Item-Analysis**
#3	Describe similarities and differences between two distributions	4-point scale See Appendix C	Yes Included in analysis for describing distributions
#4	Reasoning on the basis of sample size; smaller samples are more variable.	Multiple choice 4-points if correct, 0-points if incorrect.	Yes Included in analysis for understanding the law of large numbers
#5	Recognizing the need to attend to variation	4-point scale See Appendix D	No
#6	Thinking about variability as deviation from a central anchor point	4-point scale See Appendix E	Yes Included in analysis for understanding deviation
#7	Reasoning with box plots and numerical summaries of two unequal-sized groups	5-point scale was used on each statement (up to 15 points) See Appendix F	No

The purpose of the pre-assessment was simply to further establish that prospective teachers participating in this study had similar existing content knowledge initially about the specific data analysis and probability concepts listed above. The week following the data analysis and probability unit a post-assessment instrument (pre-assessment sans background questions) was administered in a face-to-face setting with both groups (the online group returned to a face-to-face setting with their assigned primary instructor for the remainder of the semester). The purpose of the post-assessment was to measure changes in statistical content knowledge at the individual teacher's level.

Chapter 2 Homework. Prospective teachers in both classes completed Chapter 2 of the curriculum text outside of class and submitted documents through their class's *Moodle* website. In Chapter 2, participants worked through a task using *TinkerPlots* and reflected on their own use of technology. They also anticipated how middle-school students might approach the task. Finally, they were asked to watch a short video of two middle-school students working on the same task and to consider how they perceived the technology to be a help or hindrance to them. Two *Engaging with Content* questions from this assignment were selected for deeper analysis (Lee et al., 2010, pg. 34).

> "Q1. Describe the distribution of graduation rates for NC schools.
> Q3. Based on the graduation rates for public and private schools, which type of school do you think is better at helping their students graduate within six years? Use the various plot tools to investigate the spread and centers of the graduation rates. Provide a detailed description of your comparison of the two data sets and decisions that you made."

Responses to these questions were graded holistically using the same 4-point scoring rubric (see Appendix C) for pre-assessment item #3, which was also part of the analysis for describing distributions.

Performance Tasks. On the final exam for the course, prospective teachers were allowed to choose three of five questions. Out of the five questions, three of them were related to statistical and probabilistic concepts discussed during the study (Appendix G). Each question contained three major parts – technology, pedagogy, and advanced technology. Because prospective teachers were primarily demonstrating technological skills in their responses to the first and third sections, the pedagogical section, which required more discussion about statistical ideas, was used for analysis. The table below provides a summary of these three questions, how each was scored, and how each one fit into the analysis for describing distributions, deviation, and the law of large numbers.

Table 4. Summary of final exam performance tasks, scoring rubrics, and corresponding constructs of variability.

Question	**Content Goal**	**Scoring Rubric**	**Construct of Variability**
#1	Given two box plots, create a list of questions to help students compare the two distributions	3-point scale was used on each statement (modified from pre-/post-assessment) See Appendix H	Describing distributions
#2	Create examples to address students' understanding of correlation	6-point scale See Appendix I	Understanding deviation
#3	Describe an introductory activity for the law of large numbers	5-point scale See Appendix J	Understanding the law of large numbers

Prospective teachers, in answering the exam question(s), were asked to submit a document with their responses as well as any supporting technology files (e.g. *TinkerPlots, Fathom, Probability Explorer,* and/or *Excel*).

Interviews

The six prospective teachers who made up the two focus groups, three from each class, were interviewed during the study. Two types of data were collected during interviews. First, an audio recorder was used to capture the exact words said during each interview. Second, *CamStudio* was used to collect a screen capture of the prospective teachers' work with *TinkerPlots* and *Fathom* during the interviews. The interviews were semi-structured as interview guides were utilized to ensure consistency across interviews. The instructor served as the interviewer since questions contained content related to teaching data analysis and probability with technology. Five of six prospective teachers in the focus groups were interviewed after week two of the study and again after week five. The sixth focus group participant had a scheduling conflict and was interviewed only once. This interview was longer as two interview guides were used to address questions other participants were asked to answer across two interviews. The instructor prepared the first interview guide (Appendix K) upon reflection of the first two class meetings when prospective teachers had learned data analysis with *TinkerPlots*. The second interview guide (Appendix L) was prepared following the first round of interviews and the second interviews were conducted after week four when participants had studied statistical ideas with Fathom. The sixth participant who was interviewed only once was interviewed at that time.

While the interviews certainly had a statistical focus, the researcher also asked prospective teachers about their impressions regarding learning and working in either a face-to-face or online setting. The following table provides a brief summary of the types of questions asked and how responses were used in analysis. The first three questions (white) were asked during both interviews. The questions with lighter and darker shades of gray are examples of questions from the first and second interviews respectively.

Table 5. Examples of interview questions related to discourse and understanding variability.

Question	Discourse	Understanding of Variability		
		Describing Distributions	Understanding Deviation	Understanding the Law of Large Numbers
Did you have any technology problems?	☑			
What are your general feelings right now?	☑			
What would be helpful for you as a learner?	☑			
How would you describe that distribution?		☑		
How would you use this data to address students' misconceptions about average?		☑		
How would you describe a "typical" cat length?		☑		
How might you describe that box plot to students?		☑		
What do you think the correlation would be?			☑	
How would you describe that residual plot to students?			☑	
How is the structure of the class?	☑			

Noticeably, there were no questions asked during interviews that explicitly asked prospective teachers about the law of large numbers. This was due to the placement of the interviews (once after Chapters 1 and 2, and once after Chapters 3 and 4).

Data Analysis

Much data was collected over the course of the study in order to address each of the two main research questions. A description of the analytic methods used for data related to discourse and understanding variability follows.

Analyzing Discourse

Video data from all class sessions in both groups were a primary source of data for the discourse analysis. Several phases of analysis of the class session data occurred: 1) creating holistic representations of activities in each lesson, 2) carefully choosing episodes from each chapter for in-depth analysis, 3) examining interaction patterns among instructor and prospective teacher, and 4) line-by-line coding in each episode to examine direction, form, purpose, and topic of each contribution made by a participant. Interviews of focus group members were also used in the analysis of discourse.

Creating Lesson Graphs. Recall that the online group met once a week for approximately 3 hours and the face-to-face group met twice a week for 1.25 hours. Therefore, the decision was made to identify a “lesson” as the class time dedicated to activities focused on material for each chapter, resulting in six lessons for each group. Upon reviewing each class session multiple times, the researcher created detailed lesson graphs of each class session. Lesson graphs are used to display the “content and structure of the lesson” (Jacobs, Hollingsworth, and Givvin, 2007). In the lesson graph, each row represented approximately one minute of class time. Cells were appropriately merged and color-coded (e.g. blue for small group) to capture the essence of each section of chapters used for class meetings. These lesson graphs provided a detailed timeline of events and described key conversations that occurred. However, the researcher found it helpful to create additional, more-condensed representations of the lesson graphs. First, a pie chart was created for each chapter of the curriculum text, for each of the online and face-to-face groups, to determine the percent of time spent for direct instruction, small group discussion, whole class discussion, independent work, and addressing technical issues (e.g. Figure 5). Second, a color-coded timeline was created where the sizes of the rectangles were proportional to the time spent during class in that type of activity (e.g. Figure 6). These were essentially streamlined representations of the lesson graphs.

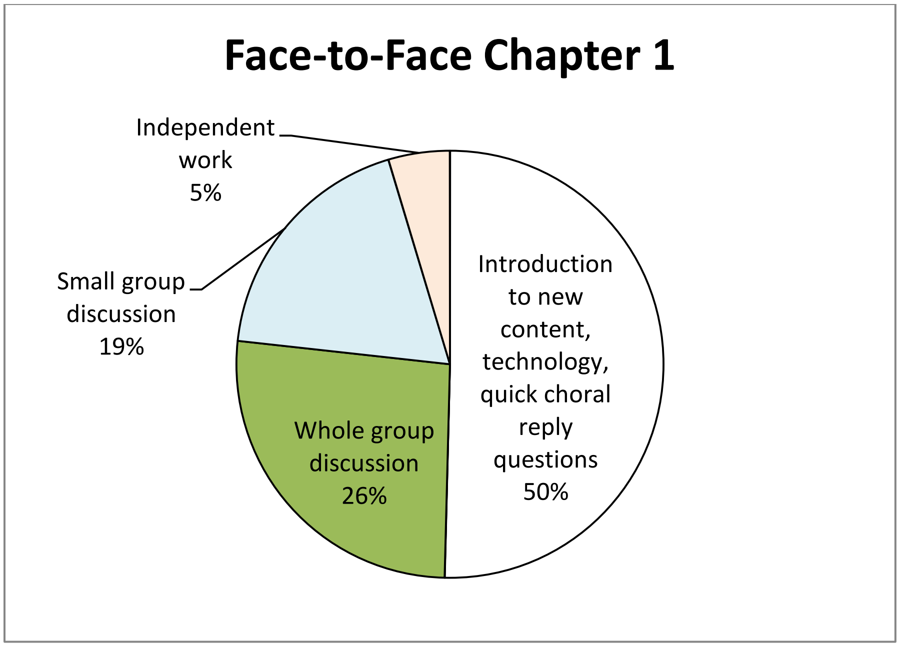


Figure 4. Opportunities for interaction in Chapter 1 of the face-to-face class.

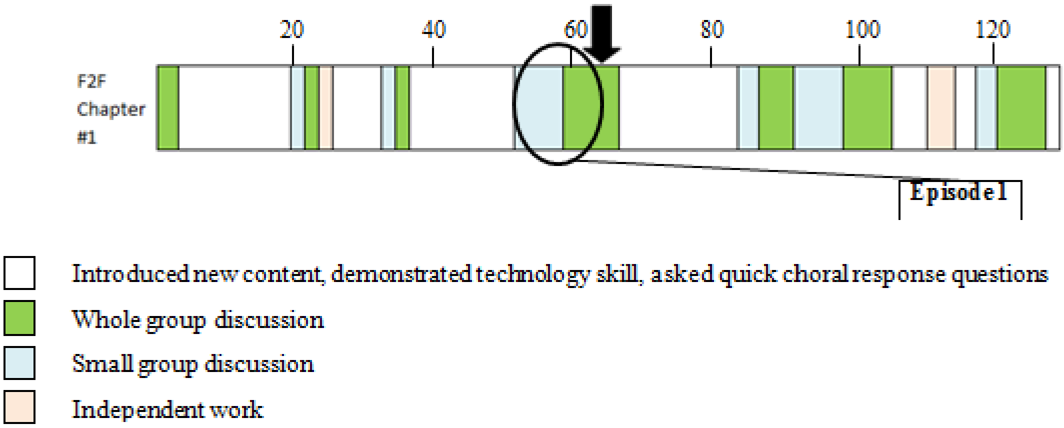


Figure 5. Timeline for Chapter 1 of the face-to-face class.

These representations were helpful in seeing an overall picture of the types of opportunities for interaction that occurred across chapters of the curriculum within a learning environment and across learning environments. Special attention was given to the placement of opportunities for interaction and similarities and differences across chapters and across settings were noted.

Choosing Episodes. Hours of audio and video recordings were collected. After carefully viewing all recordings paying attention to the activities and tasks of the curriculum text that were identified a priori (see Table 1), the instructor identified six episodes, one for each chapter in the text, for deeper analysis. When possible, special attention was given to episodes that contained both small group and whole group activities so that discourse related to TPSK could be studied on each level of the nested case study. However, not all episodes contained both types of discussions. It is important to emphasize that these episodes followed the same structure in both the online and face-to-face class environments. In other words, the content focus of small group and whole group discussions in the face-to-face class was similar to that of discussions in the online class. Each episode, listed below in chronological order, targets one of the three concepts of variability mentioned above:

Episode 1: Describing distributions (Chapter 1)
Episode 2: Describing distributions (Chapter 2)
Episode 3: Describing deviation (Chapter 3)
Episode 4: Describing deviation/residuals (Chapter 4)
Episode 5: Anticipating variation of empirical probability (Chapter 5)
Episode 6: Describing the effects of sample size (Chapter 6)

These recordings were also used to analyze prospective teachers' understanding of variability by looking closely at the ways in which they discussed content in whole group and small group settings. To further analyze understanding of variability, written assessments were collected. Lastly, interviews with focus group members from each group provided additional information regarding discourse trends prospective teachers' understanding of variability.

Examining Discussion Patterns. Once episodes from each chapter were selected, additional analysis occurred. For each small group and whole group discussion in episodes for all six chapters of the text, transcripts were created. Discussion patterns in whole group discussions were an important part of describing overall class structure between chapters within a case and across cases. Using the transcripts, discussion pattern representations were created for all whole group discussions. These visuals portrayed the number of exchanges during a discussion, as well as the sequence of communication and times when multiple prospective teachers were talking or chatting at once (denoted by the X and ↔ for face-to-face and online classes respectively in Figure 6 below).

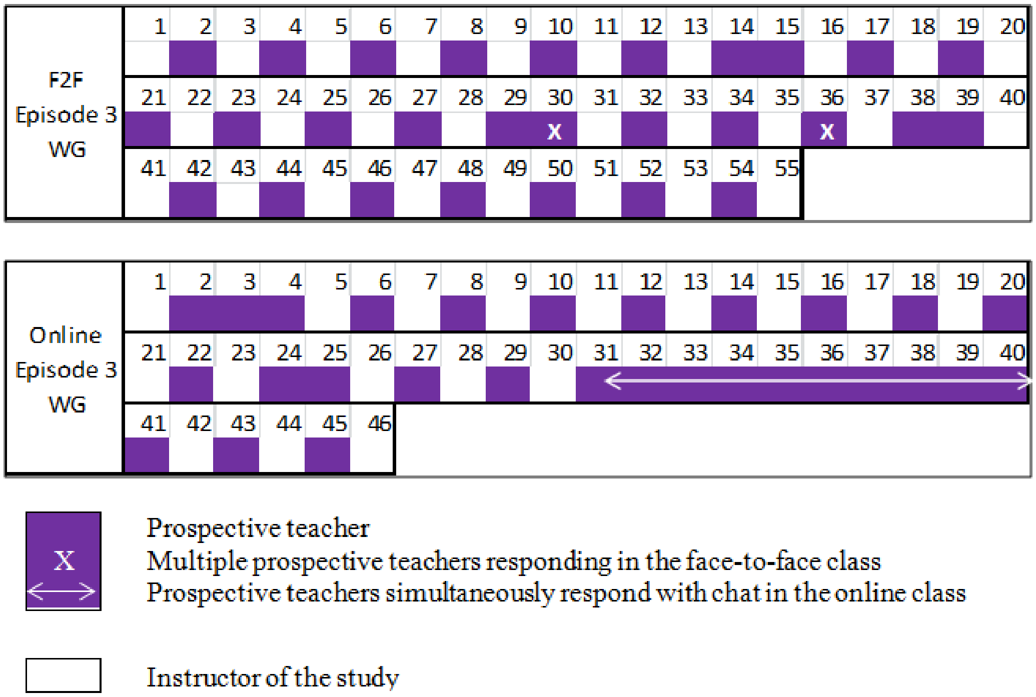
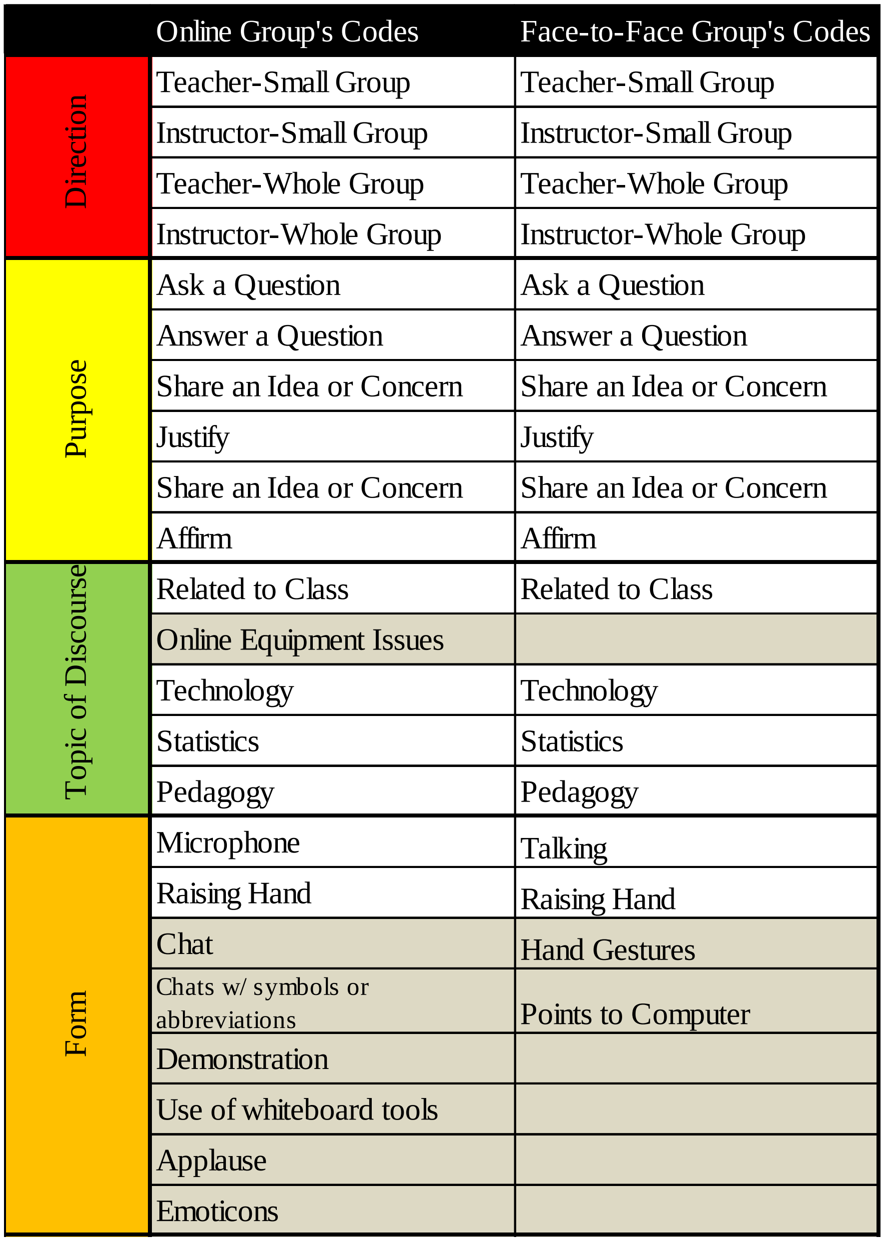


Figure 6. Example of discussion pattern representations.

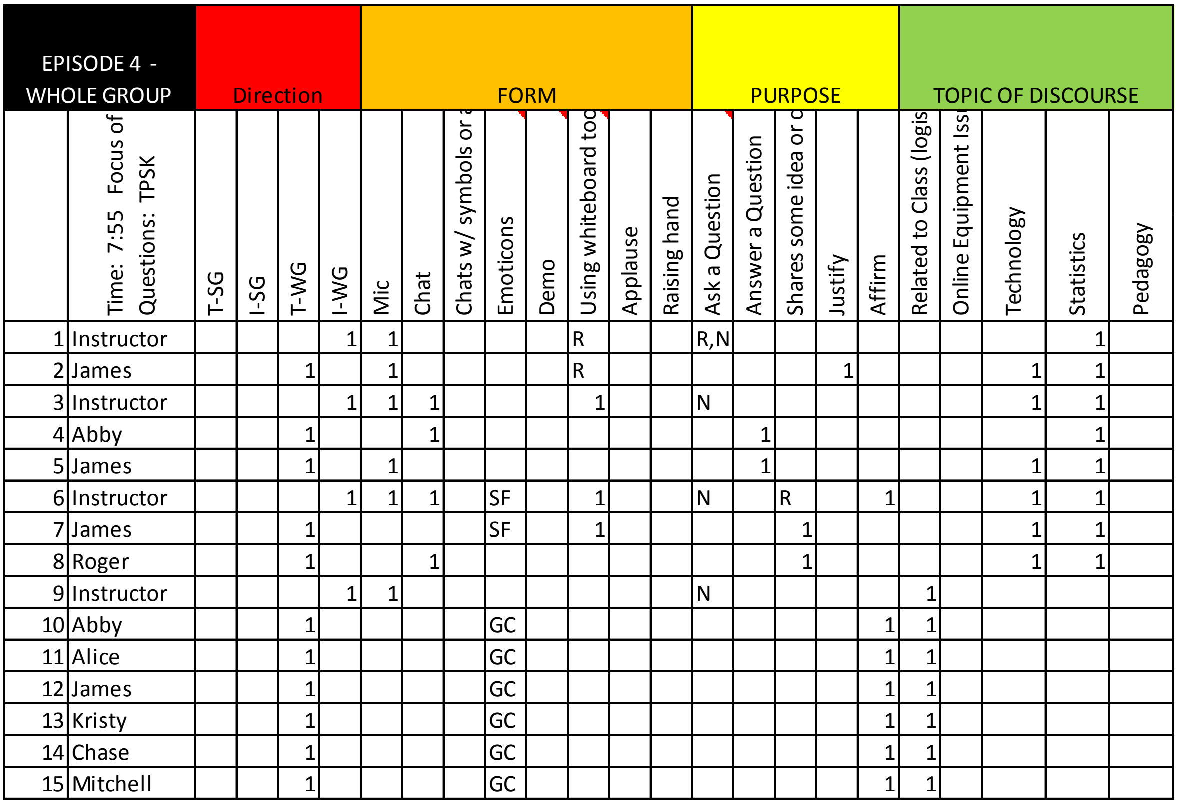
Line-by-Line Coding. With transcripts and recordings together, codes were created to use in analysis. As suggested by Miles and Huberman (1994), for qualitative research the researcher started with codes, applied them, and modified them during the analysis phase. In order to analyze discourse, a modification of Krussel et al.'s (2004) framework was used to code the instructor's and prospective teachers' direction, form, purpose, and topic. Special attention was given to opportunities for interaction, the ways in which prospective teachers and the instructor communicated with one another, discussion patterns and evidence that prospective teachers were considering SK, TSK, or TPSK in the ways in which they talked about variability. Specifically, with regard to discourse, transcripts were coded line by line using the codes seen in Table 6 and Table 7 for both whole group and small group discussions. There were obvious differences in the ways in which prospective teachers communicated in each learning environment (form) and sometimes in the focus of their communication as well (topic of discourse). These differences resulted in different codes, which are highlighted (in gray) in Table 6.

Table 6. Codes for discourse used in line-by-line analysis of episode transcripts.



	Online Group's Codes	Face-to-Face Group's Codes
Direction	Teacher-Small Group	Teacher-Small Group
	Instructor-Small Group	Instructor-Small Group
	Teacher-Whole Group	Teacher-Whole Group
	Instructor-Whole Group	Instructor-Whole Group
Purpose	Ask a Question	Ask a Question
	Answer a Question	Answer a Question
	Share an Idea or Concern	Share an Idea or Concern
	Justify	Justify
	Share an Idea or Concern	Share an Idea or Concern
	Affirm	Affirm
Topic of Discourse	Related to Class	Related to Class
	Online Equipment Issues	
	Technology	Technology
	Statistics	Statistics
	Pedagogy	Pedagogy
Form	Microphone	Talking
	Raising Hand	Raising Hand
	Chat	Hand Gestures
	Chats w/ symbols or abbreviations	Points to Computer
	Demonstration	
	Use of whiteboard tools	
	Applause	
	Emoticons	

A partial coding of a whole group discussion in the online class (Chapter 4) is shown below and serves as an example of how discourse codes were applied. The full coding and accompanying transcript for that particular episode can be found in Appendix M.



EPISODE 4 - WHOLE GROUP		Direction				FORM								PURPOSE					TOPIC OF DISCOURSE				
	Time: 7:55 Focus of Questions: TPSK	T-SG	I-SG	T-WG	I-WG	Mic	Chat	Chats w/ symbols or	Emoticons	Demo	Using whiteboard to	Applause	Raising hand	Ask a Question	Answer a Question	Shares some idea or	Justify	Affirm	Related to Class (logis	Online Equipment Iss	Technology	Statistics	Pedagogy
1	Instructor				1	1					R			R,N								1	
2	James			1		1					R						1				1	1	
3	Instructor				1	1	1				1			N							1	1	
4	Abby			1			1								1							1	
5	James			1		1									1						1	1	
6	Instructor				1	1	1		SF		1			N		R		1			1	1	
7	James			1					SF		1					1					1	1	
8	Roger			1			1									1					1	1	
9	Instructor				1	1								N					1				
10	Abby			1					GC									1	1				
11	Alice			1					GC									1	1				
12	James			1					GC									1	1				
13	Kristy			1					GC									1	1				
14	Chase			1					GC									1	1				
15	Mitchell			1					GC									1	1				

Figure 7. Partial coding of discourse for online whole group discussion related to Chapter 4.

A number "1" was used to indicate the direction, form, purpose, and topic of contributions to discourse so that frequencies could be easily calculated. Sometimes, other codes were used to better depict what was happening at that particular time. For example, in the emoticon column, "SF" was used when prospective teachers used a smiley face and "GC" was used when they shared a green check. Often the green checks were solicited by the instructor. Smiley faces, on the other hand, were generally shared without any prompting. In this example, other codes were also used for use of the whiteboard tools (R = referencing a drawing on the interactive whiteboard), asking a question (N = asking a new question, R = restating a question from the textbook), and sharing an idea (R = restating a previously shared answer or idea). When non-numeric codes were applied, the researcher manually counted the frequencies for those categories of form and purpose.

Interviews of Focus Group Members. Six focus group participants, three from each class, were interviewed during the study. As previously mentioned, the audio of these interviews was recorded. These recordings were then transcribed and notes were taken about how prospective teachers viewed working/learning in the face-to-face and online environments (e.g. liked small group discussion, liked the structure of the class)

Analyzing Understanding of Variability

Multiple phases of analysis also occurred for understanding of variability. First, video data from all class sessions in both groups were a primary source of data for the understanding of variability analysis. Codes were applied within the episode analysis to indicate the nature of the statistical ideas related to variability being discussed. Second, analysis of written documents included looking for ways prospective teachers described their understanding, particularly as it related to describing distributions, deviation, and the law of large numbers. The following documents were analyzed: 1) pre-/post-assessment, 2) a homework assignment, and 3) final exam performance tasks. Finally, interviews of focus group members' work on statistical tasks with technology were also analyzed.

Line-by-Line Coding. In order to analyze prospective teachers' understanding of variability, additional codes were applied. The second research question entailed the use of the Technological, Pedagogical, Statistical Knowledge (TPSK) framework to study prospective teachers' knowledge for teaching variability with technology (Lee & Hollebrands, 2011). In the layered circle design of the TPSK framework, one must begin by thinking about statistical knowledge. The focus of this study was not on the intricacies of all statistical knowledge developed by prospective teachers, but on one big idea in statistics, variability. Therefore, codes were created related to prospective teachers' focus on describing distributions, deviation, and the law of large numbers (Table 7).

Table 7. Codes for variability used in line-by-line analysis of episode transcripts.

Focus/Use of Statistical Language	Describing a Distribution	Attention to shape	
		Attention to center	
		Attention to spread	
	Measures of Center	Use of mean	Specific value
			General location
			Symbol
		Use of median	Specific value
			General location
			Symbol
		Use of mode	Specific value
			General location
			Symbol
			Reference to a modal clump
		Use of midrange	Specific value
			Symbol
	Deviation	Use of outliers	Reference only
			Extreme values
			Statistical outliers
		Connection of center and spread	
		Interpretation of box plot	
	Law of Large Numbers	Effects of sample sizes in probability	

Table 7 Continued

Focus on Technology	Faster Than Doing Things "By Hand"	
	Color	
	Multiple Representations	Making multiple representations
		Linking multiple representations
		Dynamic
		Static
	Dynamic Nature is Better Than Static	
	Helped Me Think About It in a New Way	
	Gives a Good "Visual"	

The codes above were first used to simply describe the frequency in which prospective teachers were attending to statistical ideas such as center and spread, outliers, and sample size. Like the coding for discourse, the researcher used a “1” to indicate the focus and use of statistical language as well as the focus on specific affordances of technology. Sometimes, other codes were used to better depict what was happening at that particular time. For example, prospective teachers in both groups discussed center in a variety of ways and codes were used to discern those differences during analysis (G = described a general location, M = just mentioned center in their comment, SV = used a specific value, S = referenced a symbol displayed within the technology tool, MC = referenced a modal clump). When non-numeric codes were applied, the researcher manually counted the frequencies for those foci and uses of statistical language. Then, having codes linked to particular lines in transcripts of episodes allowed the researcher to look for similarities and differences in the way prospective teachers described those ideas. This allowed the researcher to give rich descriptions of how they “talked” about the constructs of variability highlighted in this study.

Document Analysis. As stated above, written documents included pre-/post-assessments, a homework assignment, and performance tasks on the final exam. The pre-assessment consisted of two background questions. Prospective teachers were first asked to describe their statistics coursework/experience. Data from that question were sorted by level of experience (no experience, introductory elsewhere (0-25 years ago), introductory course, calculus-based introductory course). The second background question asked prospective teachers to rate their comfort level with seven statistical topics (1=low, 5=high). Descriptive statistics were used to compare comfort levels across topics within a class and also across classes, but due to the categorical nature of the question, distributions of comfort levels for each of the seven topics, and for both groups, were compared as well.

The pre-/post-assessment also contained statistical content questions intended to capture some of prospective teachers’ understanding about variability. Questions were scored using rubrics, and overall scores for the pre- and post-assessments were recorded. A Wilcoxon rank-sum test was used with each group’s scores to test for significant improvement. Three questions were identified to specifically aid in better understanding prospective teachers’ knowledge of variability, in particular, knowledge related to describing distributions, deviation, and the law of large numbers. Participants’ responses to those questions were studied even more closely, and similarities and differences in the ways they wrote about these ideas were noted. A comparison of distributions and descriptive statistics was part of the analysis, and when applicable, the Wilcoxon rank-sum test was also used to test for significant improvement from the pre-assessment to the post-assessment.

The Chapter 2 written homework assignment was also graded holistically using the same rubric as a similar question on the pre-/post-assessment (see Appendix C). The distribution of scores along with descriptive statistics gave an initial impression of prospective teachers' understanding of describing a single distribution and describing similarities and differences between two distributions. The ways in which prospective teachers used informal language to describe ideas of center and spread were noted.

On the final exam for the course, there were three performance tasks – one for each of the constructs of variability highlighted in this study. Separate scoring rubrics were used for each question (see Appendices H-J). Distributions of scores for each question were created. Descriptive statistics were used in analysis of the third performance question (i.e. Law of Large Numbers). In addition to assigning a rubric score, the researcher also paid close attention to the ways in which prospective teachers were writing about variability.

Interviews. Six focus group participants, three from each class, were interviewed during the study. As previously mentioned, these interviews were recorded with an audio recorder. These recordings were then transcribed and notes were taken about how prospective teachers spoke about variability. Specifically, how they described distributions, explained correlation and discussed a residual plot were of particular interest. Evidence from interviews was used to corroborate themes and ideas about discourse and understanding variability from other data sources. Screen captures of their technology work with

TinkerPlots and *Fathom* were collected as well. These were only used to confirm technology moves that seemed to be described during the interviews.

Research Validity and Reliability

Validity and reliability "address issues about the quality of the data and the appropriateness of the methods used."[2] They insure the arguments presented are truthful and that the design of the study is replicable. While the mathematics education academic community is growing more and more accepting, some are still skeptical of qualitative research methods. The critics argue that the trustworthiness and rigor of qualitative studies are questionable. Thus, validity and reliability are important parts of this study. Due to the qualitative nature of this study, triangulation of data and analysis was important. Two different types of triangulation were used in this study (Denzin, 1978 as cited in Patton, 2002). First, data triangulation, using a "variety of data sources in a study," was used. Multiple sources of data for each of the online and face-to-face group included pre-/post-assessments, transcripts of whole group and focus group discussions for each episode, transcripts of interviews, and written assessments (homework, performance tasks). Also adding to the validity of the study was the use of different types of data (e.g. self-assessment, written assessment, discussion, and interview). Themes and trends that emerged in different sources provided opportunities for very rich descriptions of the ways in which prospective teachers' described variability.

The second type of triangulation was investigator triangulation, which is the use of "several different researchers or evaluators" (Patton, 2002, pg. 247). The researcher had a

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[2] *Retrieved from http://www.qmu.ac.uk/psych/RTrek/study_notes/web/sn5.htm, June 28, 2010.*

great deal of experience grading for this course and using grading rubrics. Therefore, assessments from both sections of the course were graded only by the researcher. However, evaluations from a faculty member who was familiar with the study and another graduate student (Cathy) who was not familiar with the study ensured the validity and reliability of the coding techniques and interpretation of themes and trends from the line-by-line analysis of episodes.

The researcher asked Cathy to select a number between 1 and 6. When Cathy selected the number 4, the researcher emailed her transcripts of the small group and whole group discussions in Chapter 4 for the online group as well as a spreadsheet for coding. That evening, the researcher and Cathy met synchronously in *Elluminate* for a training session. The coding scheme was discussed and a sample transcript was coded. Once the training session ended, Cathy was asked to code the online episode (small group and whole group discussions) for Chapter 4. Results showed 100% agreement between Cathy and the researcher in codes related to direction, 98% agreement on form, 85% agreement on purpose, and 84% agreement on topic of discourse. Having little to no knowledge about the curriculum used in the study, or the study itself, these percentages revealed a straightforward coding scheme that was both valid and reliable. The case study design, coupled with data and investigator triangulation, increase the credibility of the study (Patton, 2002). Furthermore, the use of multiple, existing frameworks insure that results from the current study may be compared to those of existing research.

Safeguards Against Researcher Bias

With nine years of classroom teaching experience, I have certainly developed beliefs and attitudes that have influenced my Ph.D. studies. As a high school teacher, I was always interested in using technology in the mathematics classroom. I believed that students could learn best through discovery learning and that technological tools helped facilitate that type of learning environment. I carried this belief with me as a community college instructor and continued increasing my knowledge and use of technological tools for teaching mathematics. I even began teaching mathematics online – a completely new experience for me.

In the Ph.D. program, I accepted an assistantship to teach the *Teaching Mathematics with Technology* course for seven semesters. My assistantship also included working as a research assistant with the PTMT (Preparing to Teach Mathematics with Technology) project team. This team developed the curriculum materials used in this study and I have been involved with their work since July, 2009. Combining previous beliefs and experiences with new ones as a graduate student, I developed a strong interest in designing and testing an online unit for mathematics education regarding teaching with technology. My hope is that this effort will lead to a future online course about Teaching Mathematics with Technology.

My past teaching experiences and beliefs about implementing technology in the mathematics classroom created the knowledge and enthusiasm I brought to this study. While I view this as a strength, there may be some that view it as a limitation. They may be skeptical of my abilities to have a critical eye toward the data I have collected since I longed to show the online environment can be used effectively in mathematics education. Triangulation of data and analysis will help minimize this skepticism. My little experience in teaching mathematics education online was my biggest weakness. I have taught mathematics online, received training from my employer and through attending distance education conferences, taken a course at the university called Teaching in the Online Environment, and conducted a pilot study during the Fall 2010 semester. While the pilot study certainly did not replace the experience of actually teaching a full mathematics education course online, I do believe it provided sufficient knowledge to carry out a sound study.

Ethical Issues

Participants’ confidentiality and anonymity is important to the researcher. Therefore pseudonyms were used for all participants in the study. All digital recordings were saved on the researcher’s personal computer and saved periodically to an external hard drive. All original recordings were deleted from the recording devices and no one outside the researcher and the two additional evaluators had access to the data. There were no physical, emotional, or psychological risks associated with this study. The curriculum used in this study was part of the Preparing to Teach Mathematics Project (PTMT) sponsored by the National Science Foundation (NSF DUE 04-42319). Therefore, the current study fell under Institutional Review Board (IRB) approval for the PTMT project (#996-09-6). That particular IRB was modified to allow for video recordings of class sessions and was approved in November, 2010 (see Appendix N). All participants in this study signed an IRB informed consent form (see Appendix O).

Chapter Summary

Discourse that helps develop prospective teachers’ understanding of variability and TPSK, in general as it relates to data analysis and probability, cannot be captured solely from a quantitative measure. Therefore, this chapter began with an argument for qualitative research methods followed by a detailed explanation and justification of procedures. A nested case study design was selected. The researcher used two *Teaching Mathematics with Technology* classes – a face-to-face group and an online group. Therefore, there were two cases, an online case and a face-to-face group. At the whole group level, discussions were analyzed. Individual assessments were also used to describe the aggregate. At the small group level, discussions of focus groups were also analyzed. In addition, members of the focus groups were interviewed in order for the researcher to probe further some trends that were noticed during class sessions.

Multiple sources and types of data were collected for this study. After looking at all content data and looking for patterns and themes associated with discourse and understanding variability, the researcher developed a coding scheme for line-by-line analysis of transcripts from discussions of selected episodes. Codes and interpretations were assessed by two additional evaluators. Triangulation of data and reviewers increase the validity and reliability of the study. All participants involved with the study were treated respectfully. They are protected through the use of pseudonyms and compliance with the Institutional Review Board. The following two chapters present the online and face-to-face cases, where data and analyses related to discourse and prospective teachers' understanding of variability will be shared. Chapter 6 provides a cross-case analysis, reporting important similarities and differences between the online and face-to-face cases with regard to discourse and variability. Finally, the discussion in Chapter 7 includes, among other things, explicit responses to the research questions, limitations, and implications of this study.

CHAPTER 4: THE CASE OF TEACHING AND LEARNING IN AN ONLINE ENVIRONMENT

Introduction

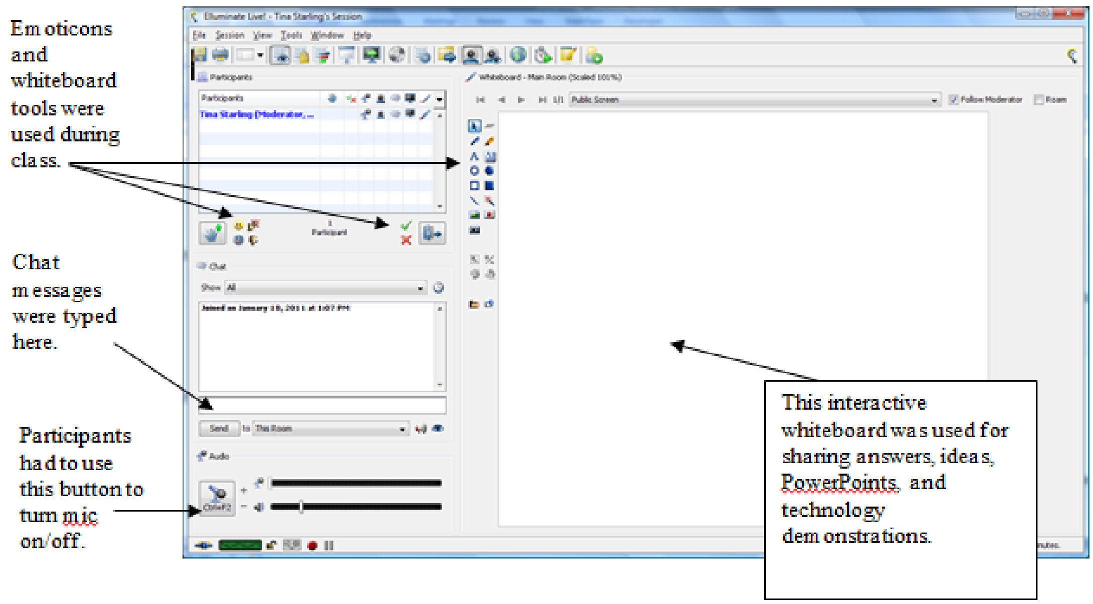
Using a synchronous online environment is still a relatively new idea for both instructors and learners, particularly in mathematics education. Therefore, this case begins by describing the online environment, *Elluminate,* which was used in this study. An overview of the setting, class structure, and curriculum is also provided in order to situate the contexts of episodes chosen for deeper analysis. The remainder of this case is divided into two large sections, one to address each research question.

First, this study considered how prospective mathematics teachers interacted with one another and with the instructor with curriculum focused on teaching data analysis and probability with technology. Questions related to activities throughout the curriculum text were identified a priori as potential sources of evidence in identifying prospective teachers' TPSK related to variability. Specifically, in this unit of study, prospective teachers were asked to describe distributions, deviation, and the law of large numbers. After carefully reviewing responses to the aforementioned questions, the instructor identified six episodes, one for each chapter in the text, for further analysis. Each episode contained opportunities (whole group and small group discussions) for analyzing discourse. Follow-up interviews with three focus group participants provided further information on discourse and general attitudes about learning and working in the synchronous, online setting. A summary of discourse trends related to form, purpose, topic, and TPSK are discussed.

Second, this study also aimed to understand how prospective teachers thought about variability, specifically related to describing distributions, deviation, and the law of large numbers. Episodes for each chapter also contained opportunities for analyzing understanding. Follow-up interviews, pre-/post-assessments, homework, and final exam tasks provided further insight into how prospective teachers were thinking about variability. Analysis for this case is divided by topic of variability (e.g. describing distributions, deviation, and law of large numbers) and sub-divided by data source (e.g. assessments, discussions, interviews). A short summary of trends related to prospective teachers' understanding for each topic of variability is included. The chapter concludes with an overall summary of the online case.

Context and Overview

Seventeen prospective teachers enrolled in the section of the course that served as the online group. The class met approximately three hours, once a week, for the duration of the five-week study. *Elluminate*, a synchronous web-conferencing program, was used as the online classroom so that live interactions and technology demonstrations could take place. During each class session, prospective teachers could participate in class through the use of emoticons, interactive whiteboard tools, a chat window, or their microphone. Figure 8 below shows a screenshot from a sample *Elluminate* session.



Joined on January 18, 2011 at 1:07 PM

Emoticons and whiteboard tools were used during class.

Chat messages were typed here.

Participants had to use this button to turn mic on/off.

This interactive whiteboard was used for sharing answers, ideas, PowerPoints, and technology demonstrations.

Figure 8. Sample *Elluminate* session.

Each prospective teacher was expected to have access to speakers and microphone equipment to participate during the online class meetings. In addition, access to a TI-83+/84 graphing calculator, Microsoft Excel, and all dynamic software programs used in the curriculum materials (*TinkerPlots*, *Fathom*, and *Probability Explorer*) was expected. All technologies, except the TI-83+/84 graphing calculator, were available through the university's virtual computing laboratory (VCL). The textbook was also listed as a required text on the syllabus. Based on assignments that were submitted, it seemed that all prospective teachers did have access to the curriculum materials. An overview of content and implementation for each chapter of the curriculum text is provided below.

Chapter One

Working through activities from Chapter 1, participants revisited statistical ideas and concepts related to introductory data analysis and measures of center and spread through exploring a relevant data set while learning a new dynamic technology, *TinkerPlots*. Prospective teachers also used the idea of “typical” teacher salary from the data set as a springboard for a discussion regarding the potential benefits and drawbacks of approaching certain statistical topics informally. The instructor introduced the notion of Exploratory Data Analysis (EDA) that would be prevalent throughout the curriculum unit and provided live demonstrations with Application Share in *Elluminate* of some of the skills in *TinkerPlots* related to reading, organizing, graphing, and analyzing data.

Prospective teachers were instructed to try skills on their own and engage with the class through various means of communication in *Elluminate*. Participants used the microphone, chat window, emoticons and the interactive whiteboard to share ideas. In addition, small-group discussions occurred in private breakout rooms while the instructor moved from room to room. Most of sections 1, 2, and 3 of Chapter 1 in the textbook were studied during class. Section 4 of Chapter 1 was briefly mentioned and Chapter 2 was assigned for homework.

Figure 9 below summarizes how instructional time (136 minutes) was spent during instruction for Chapter 1. The first online class meeting was not without technological challenges. Fifteen percent of the time spent was used to troubleshoot issues surrounding the use of *Elluminate* or accessing *TinkerPlots* through the VCL of the university. Only 8 percent of class time was used for small group discussion. About half of the time (49%) was used by the instructor to introduce new skills and concepts from the curriculum text or demonstrate a skill in *TinkerPlots*, particularly since this was the first time prospective teachers had ever used the data analysis program. Participants often responded to choral response questions from the instructor throughout the technology demonstrations. Whole group discussions occurred when no new content was being presented (28%).

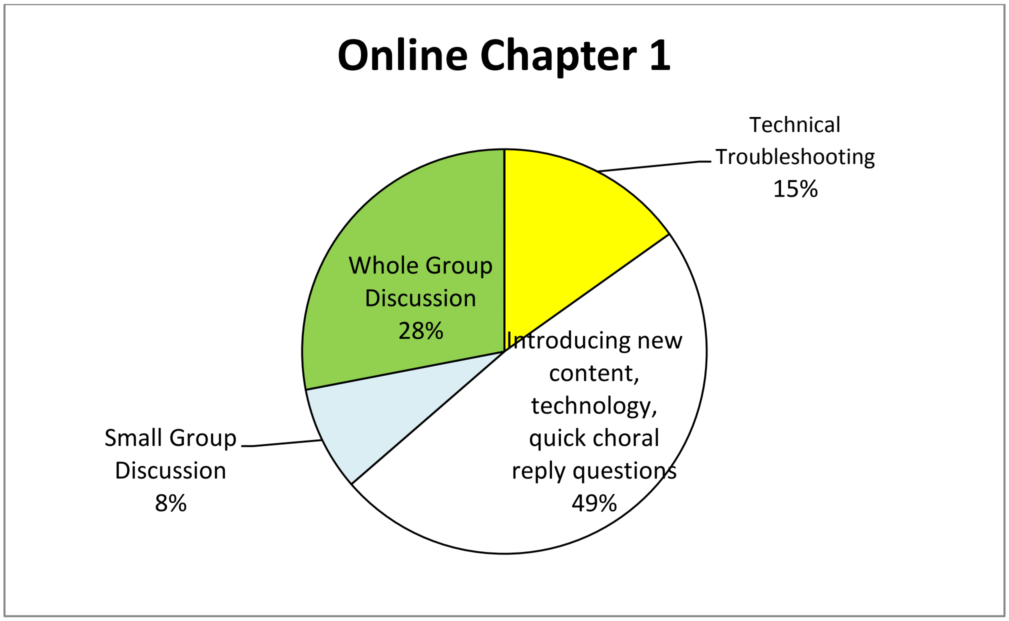


Figure 9. Opportunities for interaction in Chapter 1 of the online class.

The timeline below (Figure 10) shows the sequence of opportunities for discourse in the class meeting of Chapter 1. The entire bar represents the total number of minutes, 132, spent studying Chapter 1 and the colored cells depict the proportion of time for each type of activity as it occurred during class time. Small markings above the timeline provide indicators of 20-minute increments of time. As the timeline shows, most technical difficulties occurred at the beginning of the first class. Then, there were periods of instruction followed by whole group discussions as prospective teachers began learning *TinkerPlots* for the first time. The small group and whole group discussions circled in the timeline indicate where, during the online class, Episode 1 occurred.

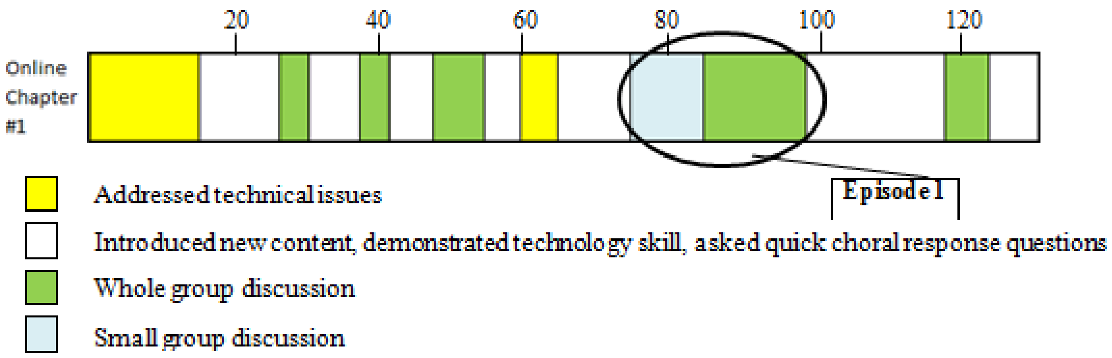
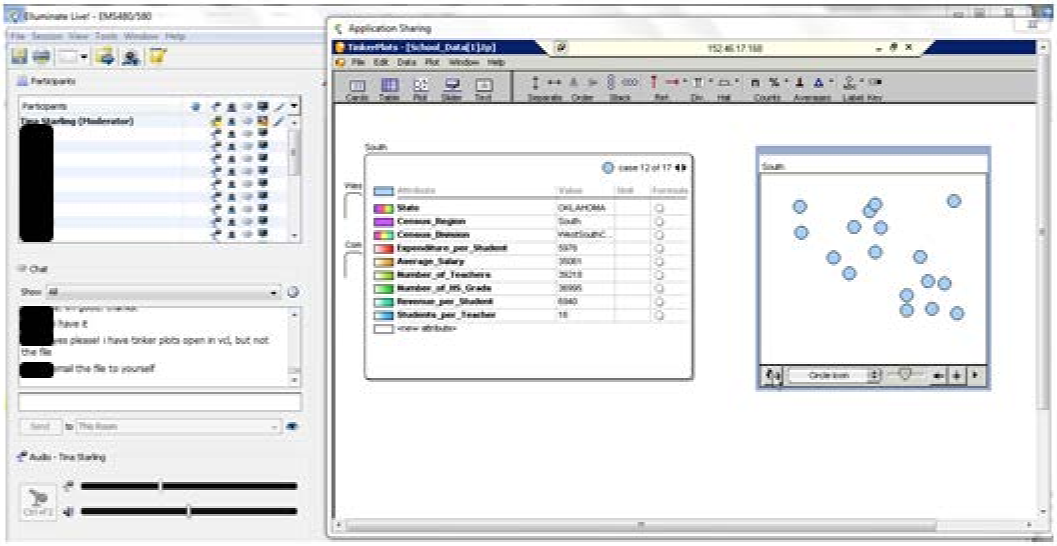


Figure 10. Timeline for Chapter 1 of the online class.

Setting of Episode One of the Online Class. This episode contained one activity in Chapter 1 of the curriculum text in which prospective teachers used measures of center to describe a distribution. Prospective teachers had been introduced to *TinkerPlots* with a pre-created data set that accompanied the text. During the introduction, the instructor provided a demonstration of exploring data cards, and discussed quantitative versus qualitative attributes, the general use of color and the dynamic nature of *TinkerPlots*. Prospective teachers created graphical representations, including a “random” plot and participated in a whole class discussion about exploratory data analysis. Figure 11 below shows these representations and how they were displayed through application sharing within *Elluminate*. Prospective teachers also used *TinkerPlots* to create a bar graph for a qualitative attribute (Figure 12).



[redacted]

[redacted] i have it
yes please! i have tinker plots open in vd, but not the file

[redacted] email the file to yourself

Figure 11. Data cards and default plot in *TinkerPlots*; sharing an application within *Elluminate*.

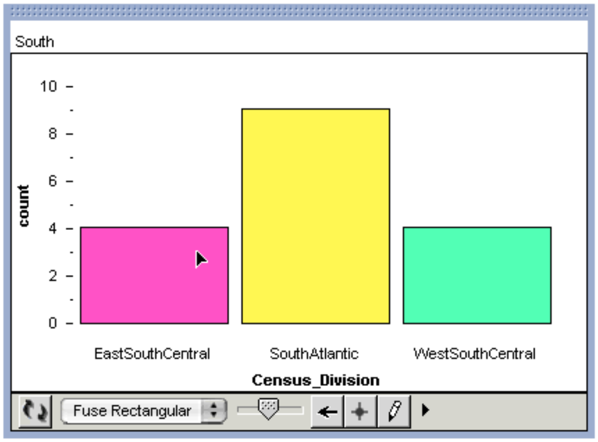


Figure 12. Bar graph of a qualitative attribute in *TinkerPlots*.

Prospective teachers were asked to create a dot plot of average teacher salary, a quantitative attribute. The instructor demonstrated creating a fully-separated dot plot for average teacher salary (Figure 13), and allowed time for prospective teachers to create their own dot plot.

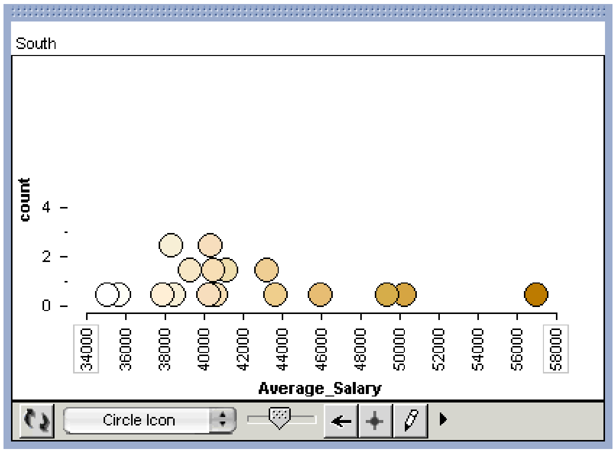


Figure 13. Dot plot of a quantitative attribute in *TinkerPlots*.

After hearing from several prospective teachers about how they might describe the distribution for average teacher salary, the instructor used one of the interactive tools in Elluminate. She shared control of her mouse with one prospective teacher who demonstrated the use of the divider tool to shade the top 50% of the data (Figure 14 left) and the middle 50% of the data (Figure 14 right).

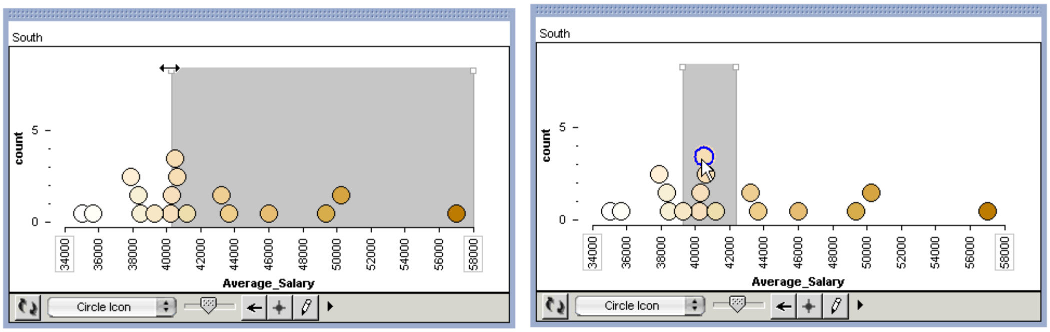


Figure 14. Divider tool used by a prospective teacher for a quantitative attribute in *TinkerPlots*.

After the group of prospective teachers had time to try this skill out on their own, the instructor asked them to include a vertical reference line and to move it to a “typical” teacher salary for the data represented. The instructor again shared control of her mouse to another prospective teacher, Les (a focus group participant), who assisted in demonstrating the vertical reference line and how to display measures of center in *TinkerPlots*, specifically the mean, median, mode, and midrange.

Episode 1 began with the instructor inviting prospective teachers to move into small breakout rooms (previously assigned) to discuss measures of center of the teacher salary data. Specifically, they were using the dot plot (see Figure 13) along with statistical measures to describe a “typical” teacher salary. They also discussed how displaying measures on a graphical display in *TinkerPlots* may help students understand these measures in relationship to each other and to the distribution of data. Members of small groups then shared ideas for a “typical” teacher salary in a follow-up whole group discussion. Following these discussions, the instructor posted a survey question for the class. She asked, “If you had to choose a measure to describe a ‘typical’ average teacher salary what would you choose?” The choices were A) Midrange, B) Mean, or C) Median. The results were as follows:

Figure 15. Survey of measures of center in the online class.

Chapter Two

Given some of the issues from the first online session, the instructor began this session by setting some ground rules about “raising hands” (a tool in *Elluminate*) to reduce audio feedback. While the technical issues were not entirely absent, the time spent by the instructor addressing the issues was considerably less than in the first online class meeting. She reminded prospective teachers that interaction during class should utilize chat, microphones, interactive whiteboards, and emoticons. At the beginning of the class meeting, the instructor also revisited *TinkerPlots* skills previously learned. Specifically, she reviewed how qualitative and quantitative data are displayed in *TinkerPlots* with static images of bar graphs, dot plots, and box plots as well as a live demonstration of constructing histograms.

Activities and questions surrounding the videocase in Chapter 2 of the curriculum text were completed by prospective teachers outside of class time for homework. Prior to coming to the class session, participants reflected on their own use of technology and how they perceived the technology to be a help or hindrance to two middle school students who were viewed in a video clip, part of the Chapter 2 materials. At the beginning of the second week's online class meeting, approximately twenty-one minutes were spent discussing the task and videocase, with about half of the time in small groups, and the other half in a whole group discussion. Figure 16 below summarizes the opportunities for interaction during instruction for Chapter 2.

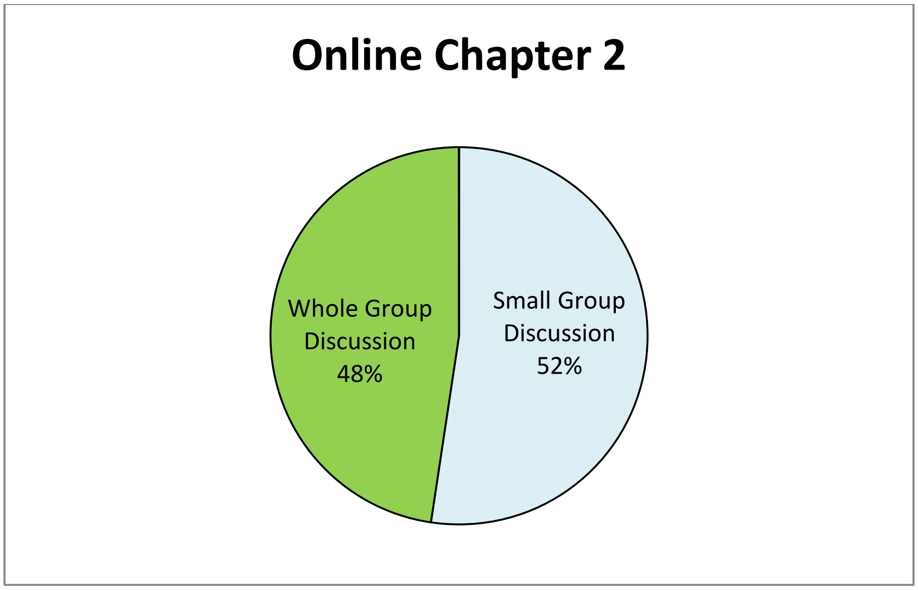


Figure 16. Opportunities for interaction in Chapter 2 of the online class.

The creation of a timeline for Chapter 2 was omitted since determining the amount of time prospective teachers spent outside of class was not possible. But, with regard to Episode 2, it is important to note the sequencing of discussions that occurred during class; the small group discussion preceeded the whole group discussion.

Setting of Episode Two of the Online Class. Chapter 2 was assigned for homework and allowed prospective teachers to practice skills for *TinkerPlots* from Chapter 1. They were given a pre-created file which accompanied the curriculum text. This *TinkerPlots* file contained 2004-2005 data for thirty North Carolina colleges; there were seventeen attributes given for each college. The task began with this quotation: “Public schools are usually bigger and less expensive (especially if you live in-state) than private schools. Private schools tend to be more selective and offer more individualized attention” (Lee, Hollebrands, & Wilson, 2010, pg. 33). Prospective teachers were asked to specifically explore the graduation rates for public and private schools in the given set (see Figure 17, taken from Les's assignment).

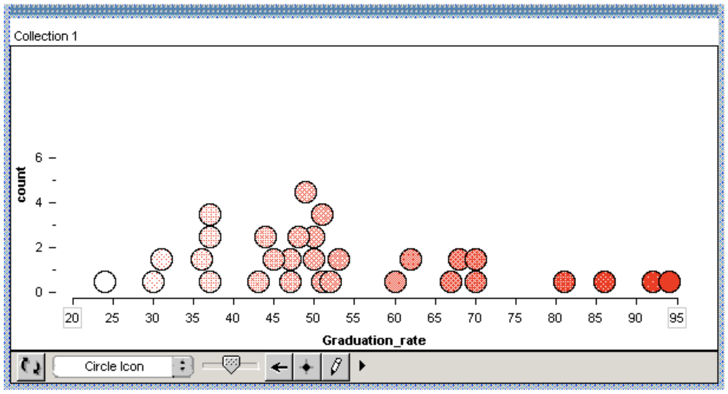


Figure 17. Dot plot of graduation rates for public and private schools.

The first question in the task asked them to describe the distribution of graduation rates for NC schools. Prospective teachers were then directed to click on the Public_Private attribute name so that each data point changed colors according to whether that school was private or public (see Figure 18, taken from Les's assignment).

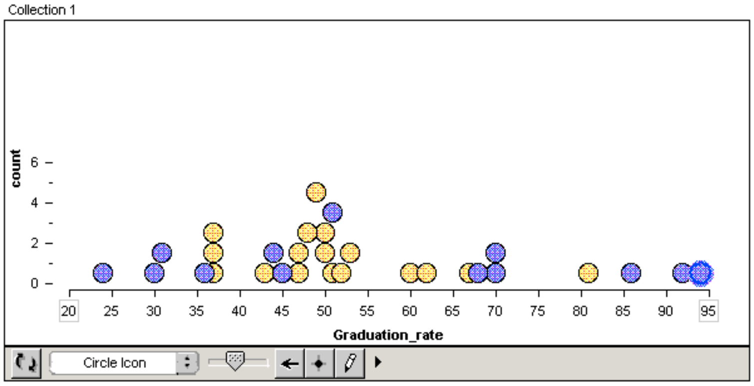
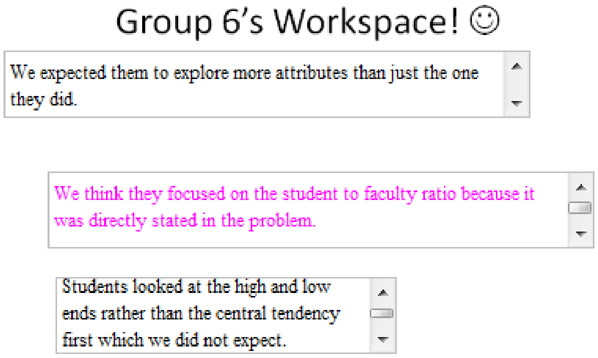


Figure 18. Dot plot of graduation rate with the setting attribute selected (purple represents private and yellow represents public).

With this representation created, prospective teachers were asked, based on the graduation rates for public and private schools, which type of school was better at helping their students graduate within six years. They were encouraged to use various plot tools to investigate the spread and centers of the graduation rates. In addition, after reflecting on their own uses of *TinkerPlots* to answer questions in the task, prospective teachers were asked to try and anticipate how students might approach the task. Specifically, they were to consider what difficulties they thought students might encounter with this task and how the use of color in *TinkerPlots* might help or hinder student thinking. Finally, the curriculum text included a video case and copies of student work of two middle school students working on the same task for prospective teachers to view and use in analyzing students' work.

At the beginning of Episode 2, the instructor gave prospective teachers ten minutes in their small groups to discuss their work on the Chapter 2 assignment. The focus group used the interactive whiteboard space to record some of their ideas (Figure 19).



We expected them to explore more attributes than just the one they did.

We think they focused on the student to faculty ratio because it was directly stated in the problem.

Students looked at the high and low ends rather than the central tendency first which we did not expect.

Figure 19. Online focus group's whiteboard for Chapter 2 discussion.

After ten minutes, a timer sounded and indicated to prospective teachers that it was time to move back into the main room of *Elluminate*. The instructor had technology problems with her connection at this time and the primary instructor for the course started and facilitated the whole group discussion. Group 6, the focus group, tried to restate and elaborate on ideas posted on the interactive whiteboard, which was viewed by the whole group. Members from other groups also shared ideas and affirmed one another during the discussion.

Chapter Three

Work with activities and tasks in Chapter 3 began immediately after the Chapter 2 discussion above. With information about some vehicles manufactured in 2006, prospective teachers were asked to consider how Exploratory Data Analysis might be used with students. Many of the same statistical concepts related to measures of center and spread were discussed once again. However, with the 2006 vehicle data, prospective teachers explored data with the dynamic program *Fathom*. Because a new technology was introduced in Chapter 3, approximately thirty-four percent of class time (123 minutes) in Chapter 3 was used by the instructor to introduce new content or demonstrate a skill in *Fathom*. Thirty-six percent and sixteen percent of class time for whole group and small group discussions respectively, were spent discussing ideas surrounding the new technology, statistical content, and/or pedagogical issues.

The absence of addressing technological issues in Figure 20 below does not imply that technological issues were absent, but that they were addressed in a different manner. The instructor introduced more purposeful times for prospective teachers to work independently on practicing new technology skills following live demonstrations. Prospective teachers were asked several times, throughout the class, to step away from *Elluminate* to perform different tasks, from answering questions in their textbook to practicing new skills in *Fathom*. This provided time for the instructor to troubleshoot, with individual prospective teachers, any problems they may have been having with *Fathom* or *Elluminate*. However, prospective teachers were predominantly troubleshooting and correcting issues on their own. Most of sections 1, 2, and 3 of Chapter 3 in the textbook were studied during class. Section 4 of Chapter 3, a section related to building the understanding of standard deviation, was assigned for homework.

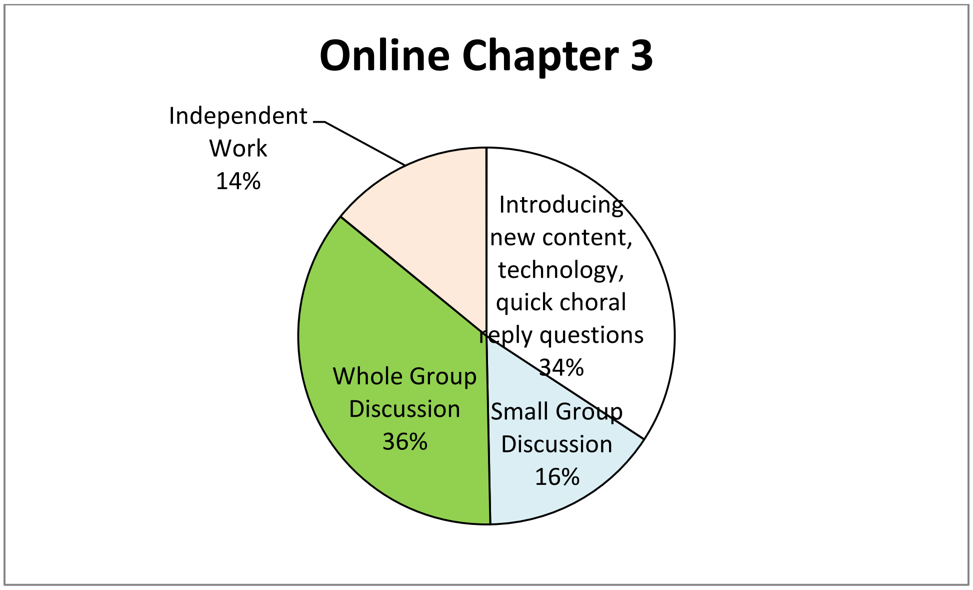


Figure 20. Opportunities for interaction in Chapter 3 of the online class.

The timeline below (Figure 21) depicts the sequence of activities during the 123 minutes spent studying Chapter 3. The arrow indicates where, over the course of the chapter, one class meeting ended and the next one began. Therefore, for Chapter 3 of the online class, the arrow represents prospective teachers working independently on Chapter 3, Section 4 and the seven days that passed before episode three began.

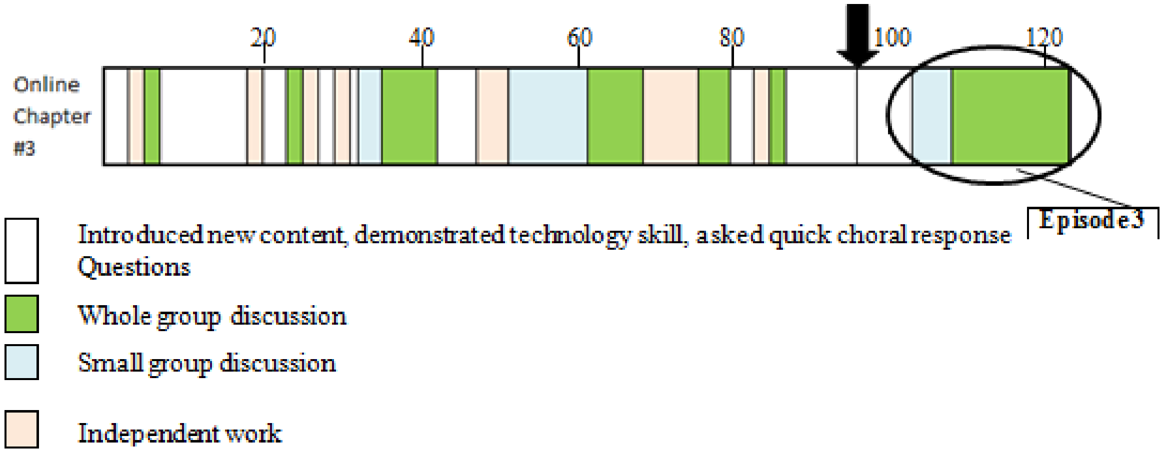


Figure 21. Timeline for Chapter 3 of the online class.

Setting of Episode Three for the Online Class. Chapter 3 material spanned two class sessions. As indicated in Figure 21, there were about 7 minutes spent with the instructor reviewing univariate data analysis skills studied during the previous class session. The question related to a “typical” value with three static images of graphical representations from *Fathom* (dot plot, box plot, and histogram) shown on the interactive whiteboard. The instructor reminded the group to describe distributions with information related to center and spread.

This episode contained discussion surrounding an activity from the homework assignment (Chapter 3, Section 4) in which prospective teachers were asked to explore standard deviation with data in *Fathom*. Specifically, prospective teachers created a dot plot and added a movable line. They were then prompted to add squares (Figure 22).

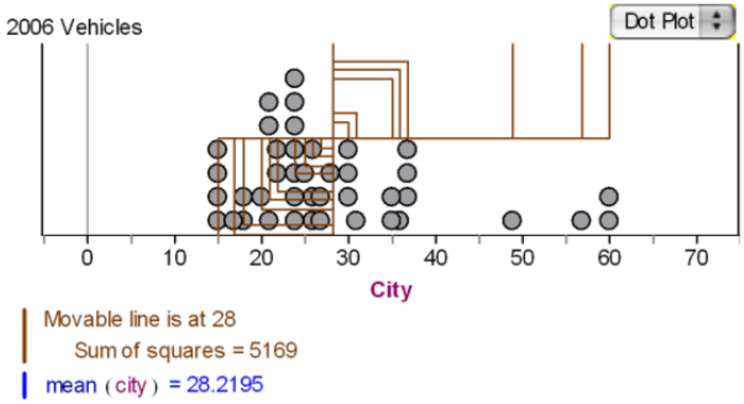


Figure 22. Movable line and squares with one attribute in *Fathom*.

The *Fathom* features pictured above, from Chapter 3, Section 4, were also displayed through *Elluminate*. The instructor noted the formula for standard deviation in the text. The participants then had a brief small group discussion to discuss their successes and failures at interpreting the formula and the *Fathom* file simultaneously. A whole group discussion followed, in which the instructor tried to make explicit connections between the standard deviation formula and the dynamic representation in *Fathom* by asking prospective teachers to think about parts of the formula and how they might be explained by the representation seen in Figure 22.

Chapter Four

Immediately following the discussions about standard deviation from episode three, the online class began discussing content from Chapter 4. While Chapter 4 of the curriculum text built on previously learned skills in *Fathom*, the statistical content of the text shifted as prospective teachers connected their understandings of univariate data analysis with ideas surrounding bivariate data analysis. They considered a relationship between two quantitative attributes by first looking at the attributes individually through a univariate lens. Prospective teachers created a representation to illustrate how two quantitative attributes co-vary, similar to one pictured below (Figure 23), and considered the affordances and potential drawbacks of having the dynamic linkage of representations.

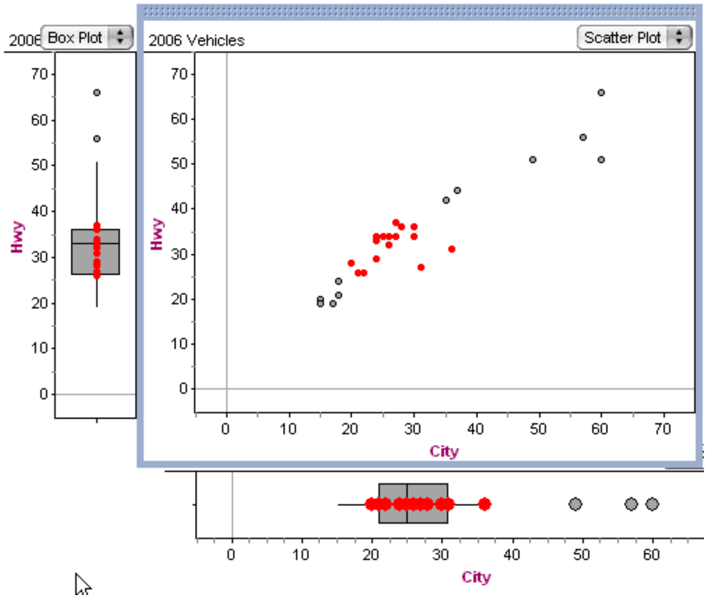


Figure 23. Building bivariate understanding in *Fathom*.

Prospective teachers also worked independently with a pre-created Fathom file (Figure 24) from the text which prompted them to adjust a slider so that the resulting scatter plot was similar to that from the City/Hwy data seen above. A summary table was also introduced so that prospective teachers could determine the exact value of the correlation coefficient, *r,* for the City/Hwy data (Figure 25).

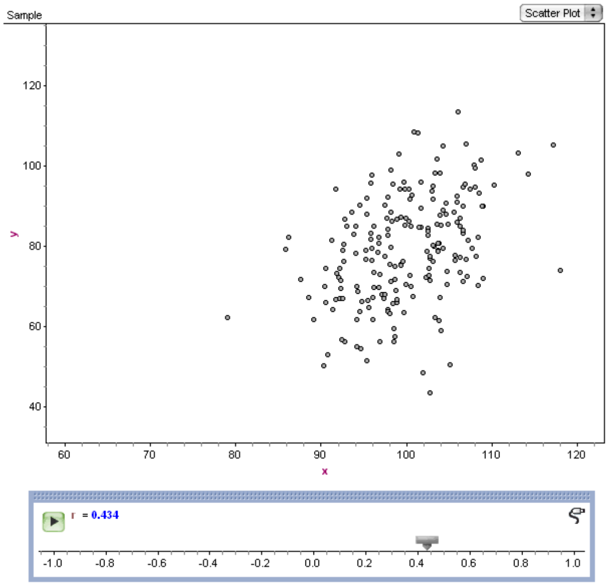
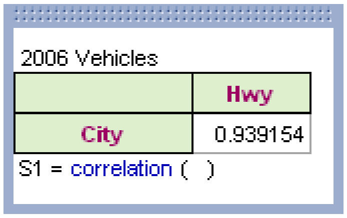


Figure 24. Pre-created *Fathom* file for the curriculum text.



2006 Vehicles

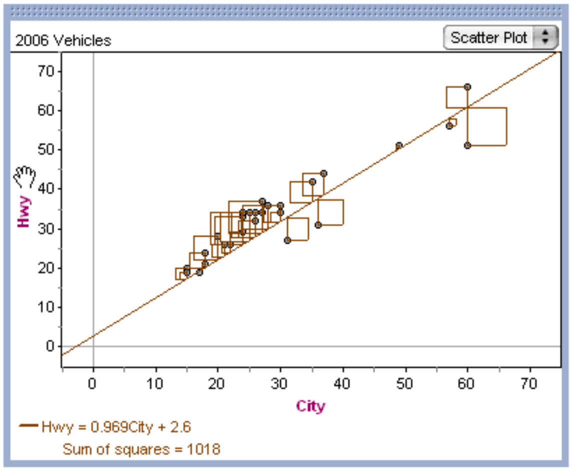
	Hwy
City	0.939154

S1 = correlation ( )

Figure 25. Summary table in *Fathom* for the correlation coefficient.

Prospective teachers then inserted a movable line in Fathom, and placed it in a location that “best represented the data.” The instructor facilitated a lengthy group discussion about the equation for that movable line and how the slope and y-intercept should be interpreted given the context of the data. A misconception that the slope was always equal to the correlation coefficient was addressed with counterexamples. Finally, the instructor engaged prospective teachers in a review of residuals and how to calculate those values.

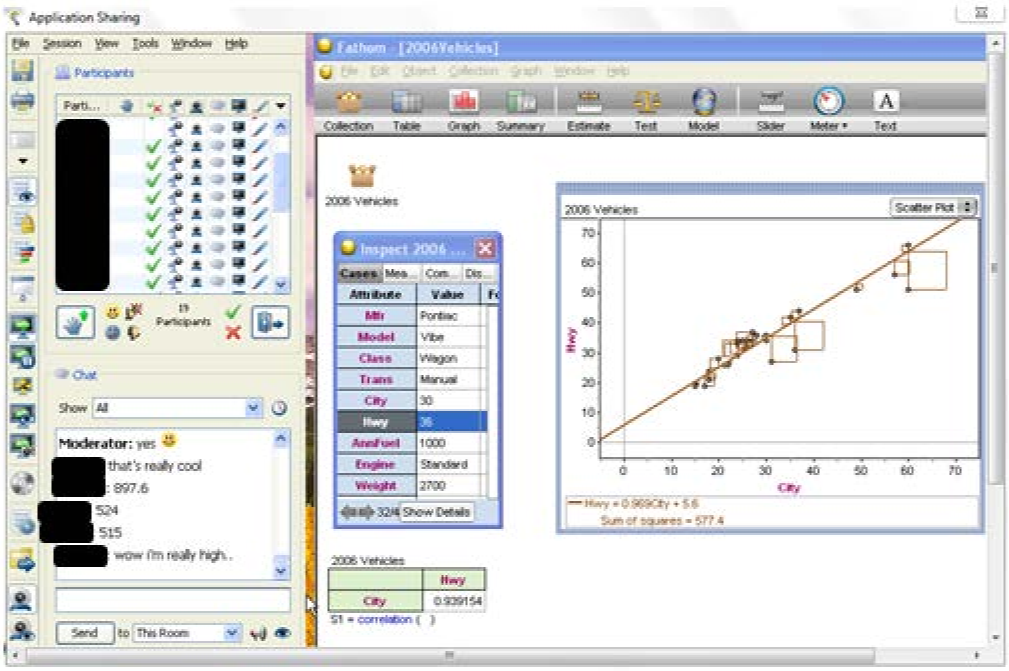
Prospective teachers were asked to insert squares into their graphical representation (Figure 26). The instructor said the squares were “very much connected to the ideas of squares with standard deviation” that the group had previously worked with.



Hwy = 0.969City + 2.6
Sum of squares = 1018

Figure 26. Movable line with squares in *Fathom*.

Prospective teachers were given time to try this new skill and were asked to give a “green check” in *Elluminate* if they were able to successfully add squares to their movable line in *Fathom*. They were instructed to move their movable lines in order to minimize the sum of squares. Figure 27 below shows a screen capture of the *Elluminate* session, including some of the green checks by prospective teachers and an unsolicited sharing, in the chat window, of sums of squares that individual prospective teachers were obtaining by moving the lines on their plots.



[redacted] [redacted] [redacted] [redacted]

Moderator: yes
[redacted] that's really cool
[redacted]: 897.6
[redacted] 524
[redacted] 515
[redacted] wow i'm really high..

Attribute	Value
Mfr	Pontiac
Model	Vibe
Class	Wagon
Trans	Manual
City	30
Hwy	35
AnnFuel	1000
Engine	Standard
Weight	2700

	Hwy
City	0.939154

S1 = correlation ( )

Figure 27. Chapter 4 *Elluminate* session.

A brief whole group discussion regarding pedagogical issues surrounding this type of technology tool followed. All in all, most of sections 1 through 6 of Chapter 4 were studied during class. Sections 7 and 8 of Chapter 4 were omitted.

Partly because of the shift in content focus, thirty-nine percent of class time (179 minutes) spent in Chapter 4 was used by the instructor to introduce new content or demonstrate a skill in *Fathom.* Thirty-five percent and fourteen percent of class time for whole group and small group discussions respectively, were spent discussing ideas surrounding the new technology, statistical content, and/or pedagogical issues. Once again, the absence of addressing technological issues in Figure 28 below does not imply that technological issues were absent from the online class, but that they were addressed during times of independent work (12%). As in Chapter 3, prospective teachers were predominantly troubleshooting and correcting issues on their own.

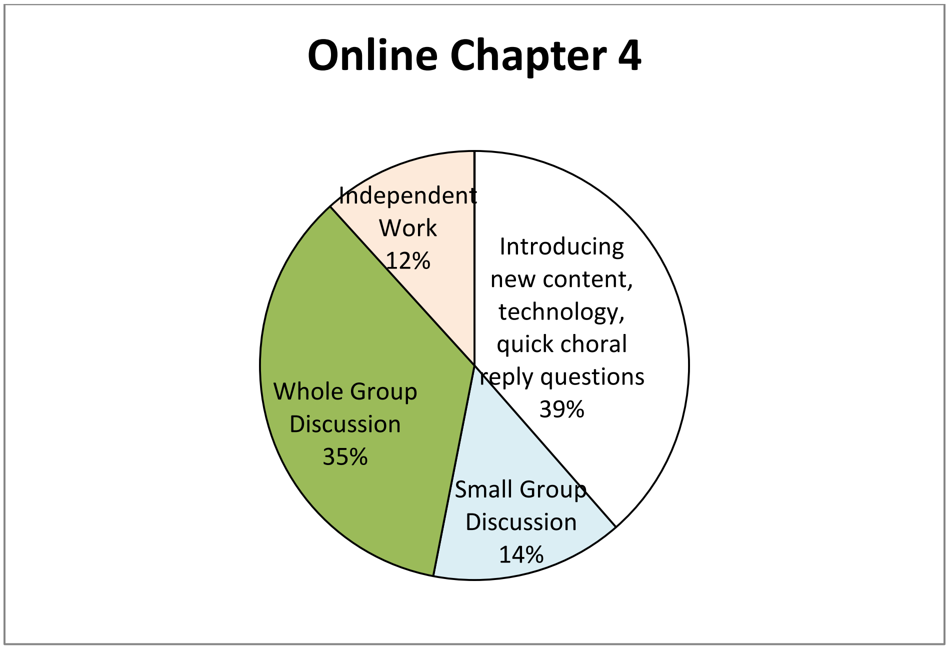


Figure 28. Opportunities for interaction during Chapter 4 of the online class.

The timeline below (Figure 29) indicates the sequential use of independent work, whole group discussions, and teacher-led instruction of new content. There were four brief opportunities for small group work in this chapter. The whole group discussion in the episode that was analyzed was separated by a brief period of time that was more instructor-centered, denoted in Figure 29 with a darkened arrow. This was also where one class session ended and another one began. In other words, Episode 4 spanned across two online class meetings. Thus, the arrow indicates when, over the course of the chapter, one class meeting ended and the next one began. Therefore, for Chapter 4 of the online class, the arrow represents the seven days that passed in the middle of episode four.

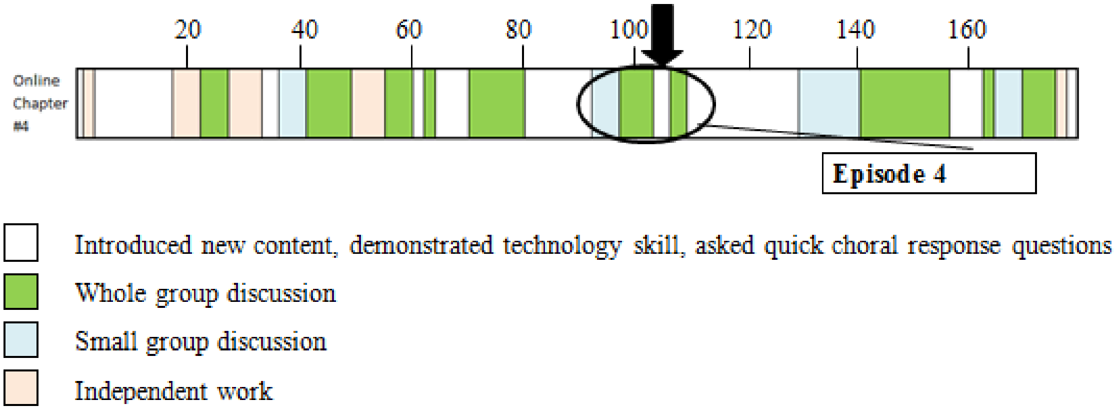
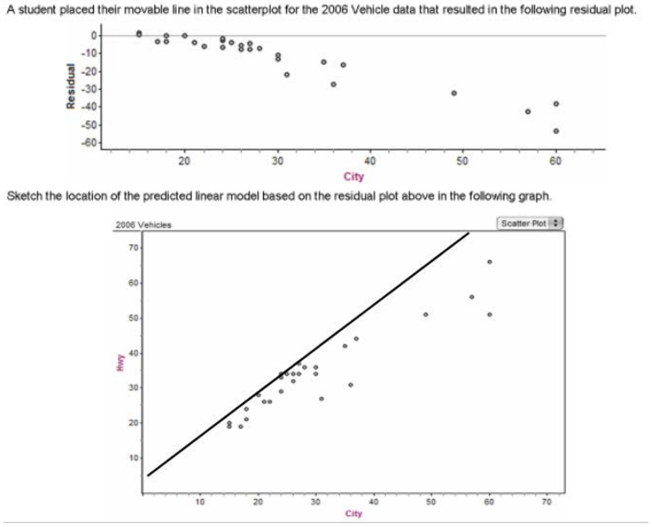


Figure 29. Timeline for Chapter 4 of the online class.

Setting of Episode 4 for the Online Class. This episode contained two activities in which prospective teachers were exploring sums of squares and residual plots with a movable line in *Fathom*. Prospective teachers were then asked to move into small groups to answer the following questions:

> “(1) Consider the residual plot for your linear model. What does the residual plot reveal about the usefulness of your linear model for predicting Hwy mpg for various vehicles? (2) Estimate the location of the predicted linear model based on the residual plot [in the text]. (3) Describe some of the conceptual difficulties students may have in interpreting and using the residual plot. How will you help them understand the residual plot and its usefulness in analyzing a linear model?” (Lee et al., 2010, pg. 85-86).

Time ran out before the focus group was able to finish their discussion. A whole group discussion followed with one prospective teacher sharing his interpretation of the residual plot on the interactive whiteboard. He placed the darkened line shown in Figure 30 to represent the location of the linear model that would produce the preceding residual plot.



A student placed their movable line in the scatterplot for the 2006 Vehicle data that resulted in the following residual plot.

Sketch the location of the predicted linear model based on the residual plot above in the following graph.

Figure 30. Prospective teacher’s interpretation of a residual plot in the online class.

Due to time constraints, this whole group discussion had to be continued at the beginning of the next class meeting. In the next class meeting, a whole group wrap-up discussion of residuals occurred. Prospective teachers shared what they had learned in terms of minimizing the sum of residuals and looking for a residual plot that did not contain a pattern when finding a best-fit model.

Chapter Five

Beginning with activities in Chapter 5, online class discussions were focused on probability. The *TI*-84 graphing calculator and *Excel* were utilized in running simulations to begin to show effects of sample size on variability between samples and variability between empirical and theoretical probabilities. First, the instructor shared the *TI-84+ Smartview* application in *Elluminate* to provide live demonstrations of steps for the graphing calculator simulations. Several participants were using other models of the graphing calculator and had difficulty following along. Still others, despite having a calculator in the *TI-83+* family, were unable to troubleshoot on their own. Therefore, much time was spent by the instructor helping those individuals. As Figure 31 shows, twenty-two percent of the online class time for Chapter 5 (78 minutes) was spent addressing technology issues.

Still, due to the changes in content and technologies from previous chapters, nearly half of class time (49%) was more instructor-directed and only twenty-seven percent of class time was used for whole group discussion. Independent work continued in Chapter 5, but was minimal because of the number of technical issues that had to be adddressed. Noticeably, however, small group work was completely missing during class time for Chapter 5. Activities in sections 1, 2, and 3 of Chapter 5, which rely heavily on the use of the graphing calculator, were completed during class. Section 4 of Chapter 5, which exclusively promotes the use of *Excel* for simulations, was assigned for homework.

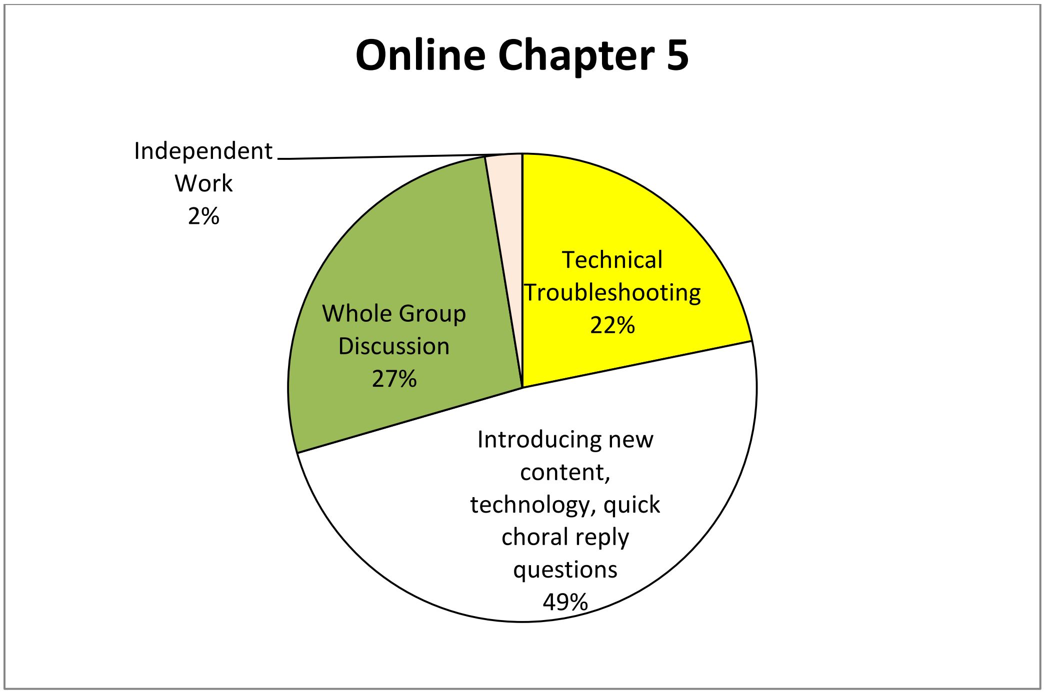


Figure 31. Opportunities for interaction in Chapter 5 of the online class.

The timeline below (Figure 32) illustrates the sequence of whole group discussions and introduction of new content, with the large continuous chunk of time dedicated to technical difficulties with participants using their own graphing calculators. Whole group discussions circled in the timeline indicate where, during the online class, Episode 5 occurred. The arrow indicates when, over the course of the chapter, one class meeting ended and the next one began. Therefore, for Chapter 5 of the online class, the arrow represents prospective teachers working independently on Chapter 5, Section 4 (the *Excel* work) and the seven days that passed before Chapter 5 concluded.

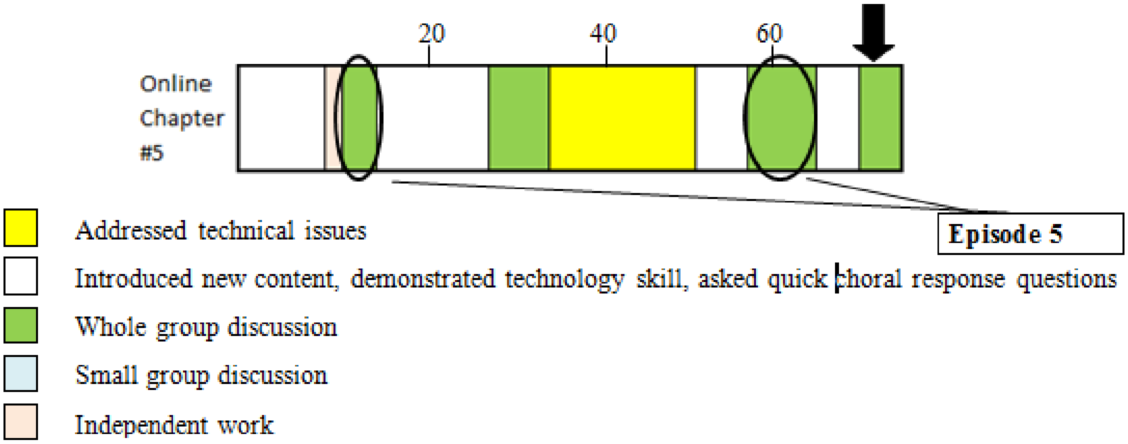
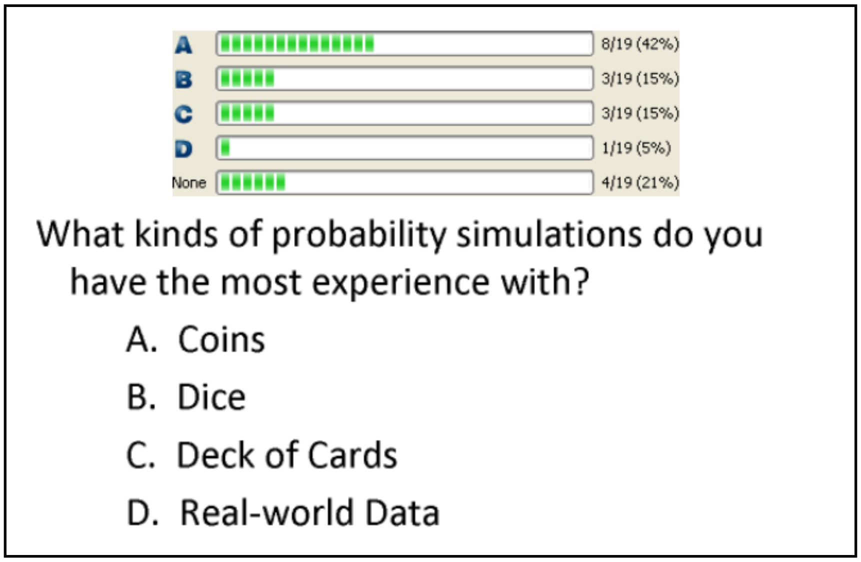


Figure 32. Timeline for Chapter 5 of the Online class.

Setting of Episode 5 for the Online Class. This episode contained two activities that asked prospective teachers to begin thinking about difficulties students often have with probability. The episode begins shortly after a quick review of deterministic and stochastic functions and a survey of prospective teachers' prior experiences with probability simulations (Figure 33).



A	8/19 (42%)
B	3/19 (15%)
C	3/19 (15%)
D	1/19 (5%)
None	4/19 (21%)

What kinds of probability simulations do you have the most experience with?

A. Coins
B. Dice
C. Deck of Cards
D. Real-world Data

Figure 33. Online survey of prior experiences with probability simulations.

The instructor posed a problem, from the text, in which prospective teachers considered a familiar data set (one they had used in *TinkerPlots*) and how they might simulate, with a coin, the freshmen retention rate for Chowan College (Figure 34).

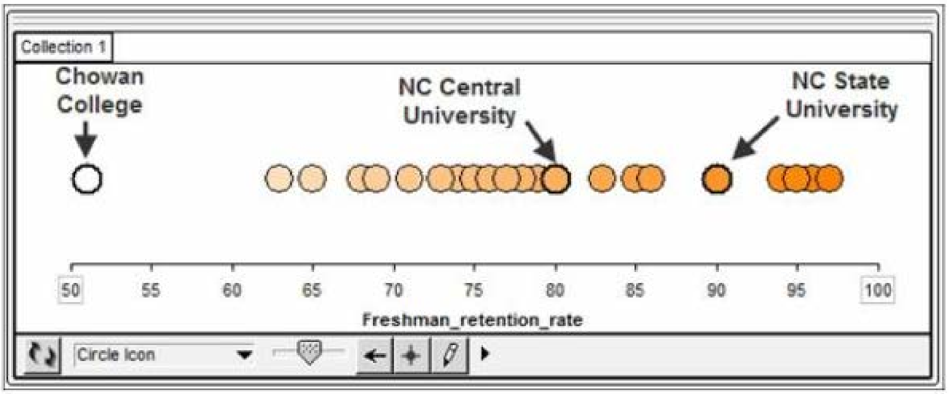


Figure 34. Freshmen retention rate shown in *TinkerPlots*.

When asked the probability of a randomly selected freshman returning at Chowan College, prospective teachers responded with values near 50%. With that in mind, it was decided that in a coin-toss simulation, "heads" would indicate the freshman returned and "tails" would indicate the freshman did not return. The instructor had initially planned to use an online coin-toss simulation, but due to comments from some prospective teachers about there being "a lot going on," she decided to keep things more simple. Prospective teachers, therefore, were encouraged to literally flip a coin thirty times, find the proportion of freshmen returning, and indicate their value on the interactive whiteboard (Figure 35).

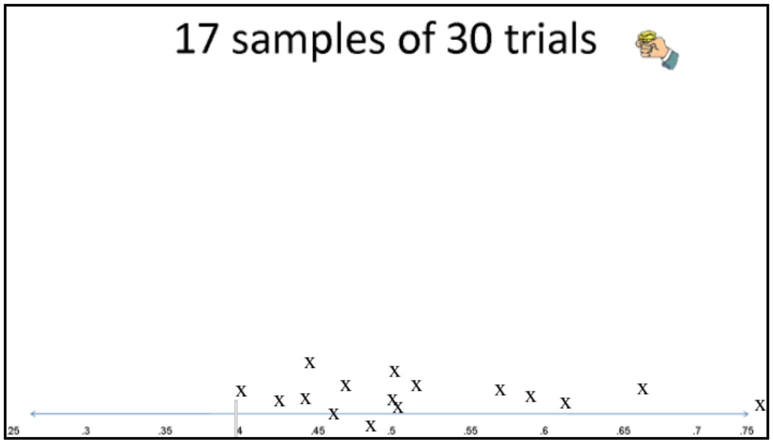


Figure 35. Results from coin simulation in the online class.

A whole group discussion followed. Using a reference to the gray divider tool in *TinkerPlots* as a way to think about possible values, the instructor asked prospective teachers to share where that gray box might be placed if the simulation was performed again. Then, the class continued with demonstrations of simulating this problem with the *TI*-84 graphing calculator. Finally, prospective teachers continued the problem of freshmen retention rate for Chowan College (50%) with varying sample sizes. They recorded their results on the interactive whiteboard of Elluminate (Figure 36) and the whole group discussion, which was the focus of the Chapter 5 episode, continued. During this discussion, prospective teachers described similarities and differences they were noticing between distributions.

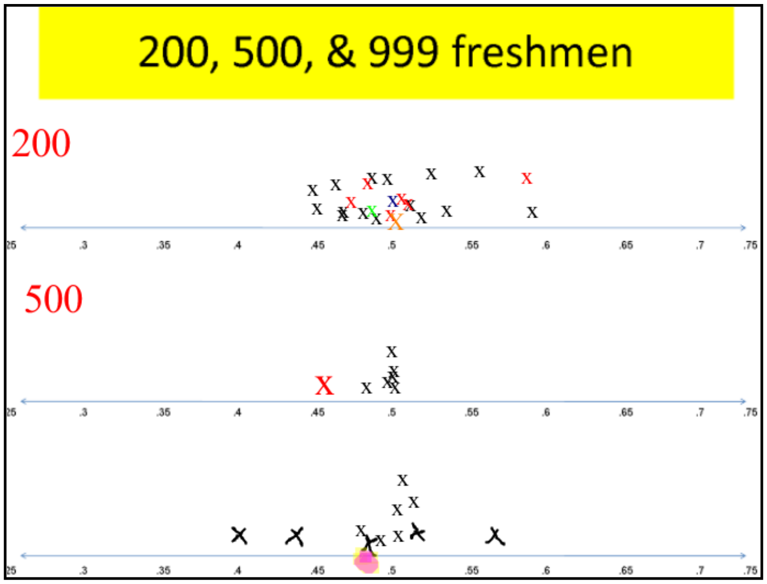


Figure 36. Probability simulation with different sample sizes in the online class.

Chapter Six

Prospective teachers' work with probability simulations continued with the programs *Probability Explorer* and *Fathom*. Participants used real-world data to influence the assumptions underlying each simulation. Specifically, birth data – the number of boy births versus the number of girl births in North Carolina during the year 2004 – were used in this lesson. Prospective teachers imported birth data from a website into *Fathom* and learned skills to take repeated samples in order to think about their distributions and whether or not certain outcomes were unusual. With *Probability Explorer*, prospective teachers used special tools to simulate births in two counties in North Carolina. Real-time changes assisted them with answering questions related to probability. Most of sections 1, 2, and 3 of Chapter 6 in the textbook were studied during class. Time expired before Chapter 6, Section 4, a section utilizing the graphing calculator, could be introduced.

While the class had studied many features of *Fathom* in Chapters 3 and 4, many new skills for probability simulations were presented in Chapter 6. In addition, *Probability Explorer* was a new software for prospective teachers. Hence, thirty-nine percent of the Chapter 6 class time (137 minutes) was spent introducing new content and technology skills surrounding probability simulations. Other opportunities for discourse included twenty-five percent for whole group discussions, fifteen percent for small group discussions, and fourteen percent for independent work. Technical troubleshooting unfortunately resurfaced during Chapter 6 due to many difficulties with Macintosh users having compatibility issues with *Probability Explorer*.

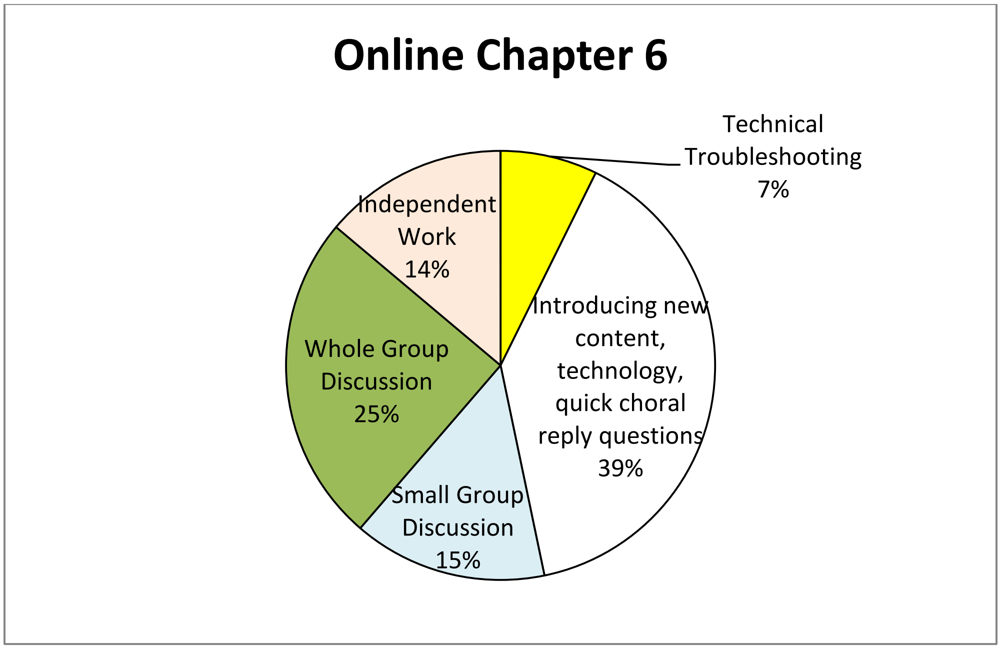


Figure 37. Opportunities for interaction in Chapter 6 of the online class.

The timeline below (Figure 38) indicates how whole group and some small group discussions were interspersed in the instruction on new content. The whole group and small group discussions circled below occur after prospective teachers have independently created a dot plot in *Fathom* based on real-world data imported from a website.

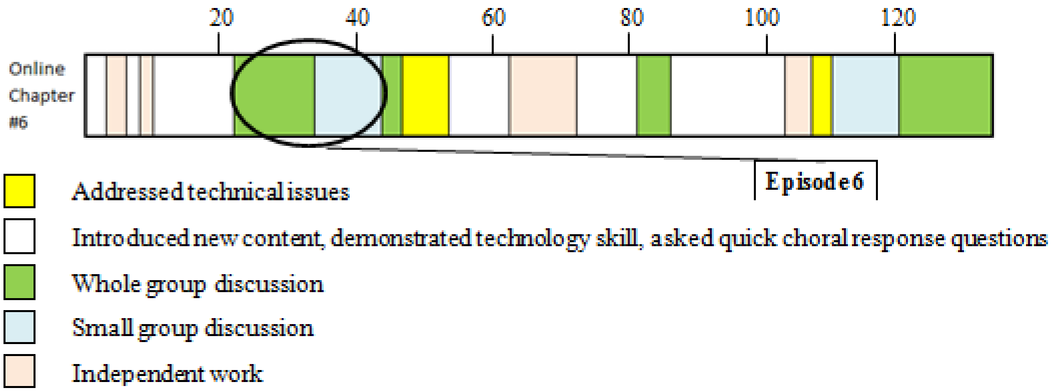
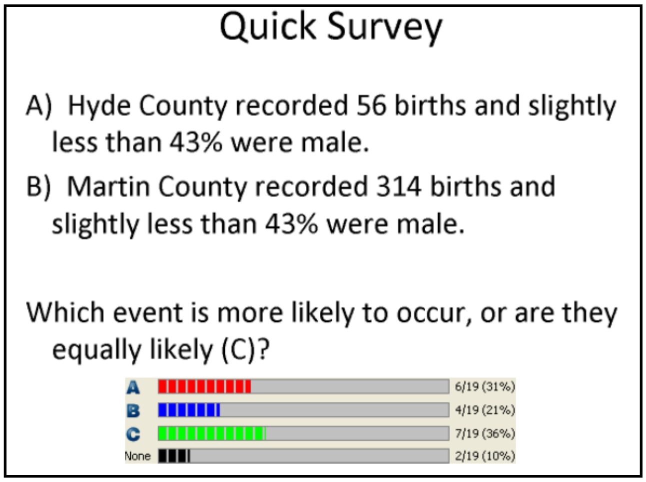


Figure 38. Timeline of Chapter 6 for the online class.

Setting of Episode 6 for the Online Class. This episode contained one activity regarding the proportion of male births in North Carolina. Based on real-world data, the probability of a live male birth is 0.51. Through different questions, prospective teachers were asked to consider the effects of sample size on distributions of proportions of male births. The technology used was *Fathom*.

The instructor began by conducting a quick survey of the class. Only 6 out of 19 prospective teachers in the online class answered correctly. The question and results are shown below:



Quick Survey

A) Hyde County recorded 56 births and slightly less than 43% were male.

B) Martin County recorded 314 births and slightly less than 43% were male.

Which event is more likely to occur, or are they equally likely (C)?

A	6/19 (31%)
B	4/19 (21%)
C	7/19 (36%)
None	2/19 (10%)

Figure 39. First Chapter 6 survey for the online class.

2004 birth data for each of the 100 counties of North Carolina were gathered from the internet and used to make a new collection in Fathom. Prospective teachers followed directions to create a new attribute called "PerctMale" to consider the proportion of males born. Then, they were asked to create a dot plot of this new attribute (Figure 40).

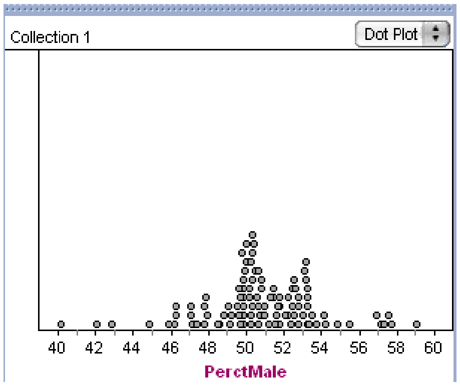


Figure 40. Dot plot of the PerctMale attribute in *Fathom*.

The instructor asked prospective teachers to examine the distribution of percent of male births. Specifically she asked, "what do you notice about the spread and where the data seems to cluster?" A whole group discussion followed as prospective teachers described how the cluster appeared to be slightly greater than 50.

Next, prospective teachers were instructed to add a vertical reference line to place at 50 and then at 51. Also, the highest (and lowest) counties were highlighted to show, once again, the linked representations within *Fathom* and to look for similarities among those counties. The instructor asked the group to overlay the "Total" attribute on the dot plot, which added more information to the plot through the use of color (Figure 41). Purple represents the counties with the smallest populations while red represents counties with the largest populations.

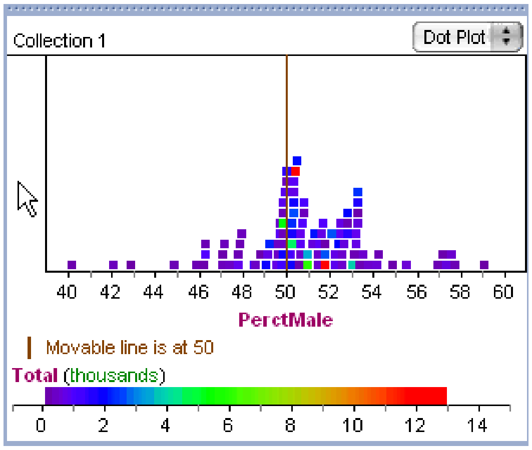


Figure 41. PerctMale dot plot with vertical reference line and Total attribute overlay in *Fathom*.

Whole group discussion continued around this representation. Prospective teachers considered the general trend in the spread of the data as the total number of births increased. Adding the Total attribute to the y-axis (Figure 42) made the trend even easier to describe.

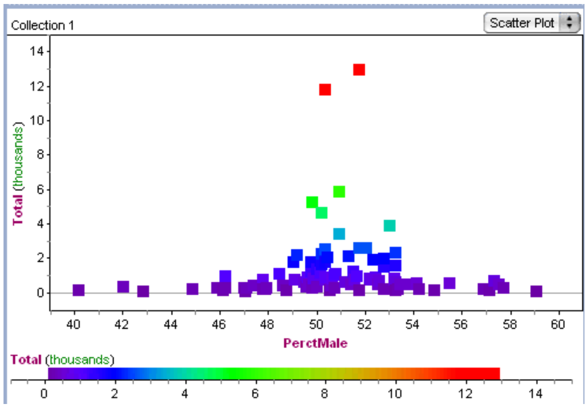


Figure 42. PerctMale dot plot with Total overlayed and played on the y-axis in *Fathom*.

At this point in the lesson, prospective teachers moved into their small groups to discuss several questions:

> “(1) Considering the mean percent of males and the spread of the data with counties with a large number of births (over 2000), what do you think is a reasonable estimate for the probability that a male is born? Justify your estimate. (2) Does assuming a 51% chance for males change your response to whether event A (43% with 56 births) or B (43% with 314 births) is more likely or if they are equally likely? Why or why not? (3) Why is it important in a probability task to have students state and understand the implications of assumptions about the likelihood of an outcome?” (Lee et al., 2010, pg. 136-137).

Discourse

While some details of the discourse that occurred during the five-week study were included in the overview and episode descriptions above, this section includes a much more descriptive and analytic account of the opportunities for interaction in the online class meetings. Based on analyses of lesson graphs and timelines of entire lessons, and transcripts of episodes from each chapter, this section contains findings about opportunities for discourse at the whole-group and small-group levels. A summary, for each episode, of how prospective teachers chose to participate, is followed by additional information that was useful in explaining some of the number of occurrences stated. Data collected from individuals are also presented for triangulation to corroborate trends found from discussions that occurred during online class meetings.

Chapter One

Episode One – Small Group. Using a data set which included information about teacher salaries in the southern region of the United States, prospective teachers were asked to consider pros and cons of each of the measures of center being used to describe a “typical” teacher salary. During the small group discussion of Episode 1, thirty-two exchanges of communication occurred during approximately nine minutes. The table below provides an overview of the direction, form, purpose, and topic of discourse during this small group discussion. Note that while each exchange has one direction and one form, it often contained more than one purpose and/or topic.

Table 8. Online Episode 1, Small group discourse.

Episode 1, Small Group Discourse (32 exchanges, 9 min)		# of occurrences
Direction	Teacher-Small Group (T-SG)	24
	Instructor-Small Group (I-SG)	6
	Teacher-Whole Group (T-WG)	1
	Instructor-Whole Group (I-WG)	1
Form	Microphone	26
	Chat	
	• Chatted with words	4
	• Chatted with symbols or abbreviations	2
Purpose	Asked a Question	
	• Asked a new question	5
	• Restated a question from the text	4
	Answered a Question	6
	Shared an Idea or Concern	11
	Justified an Idea or Response	6
	Affirmed an Idea or Response	9

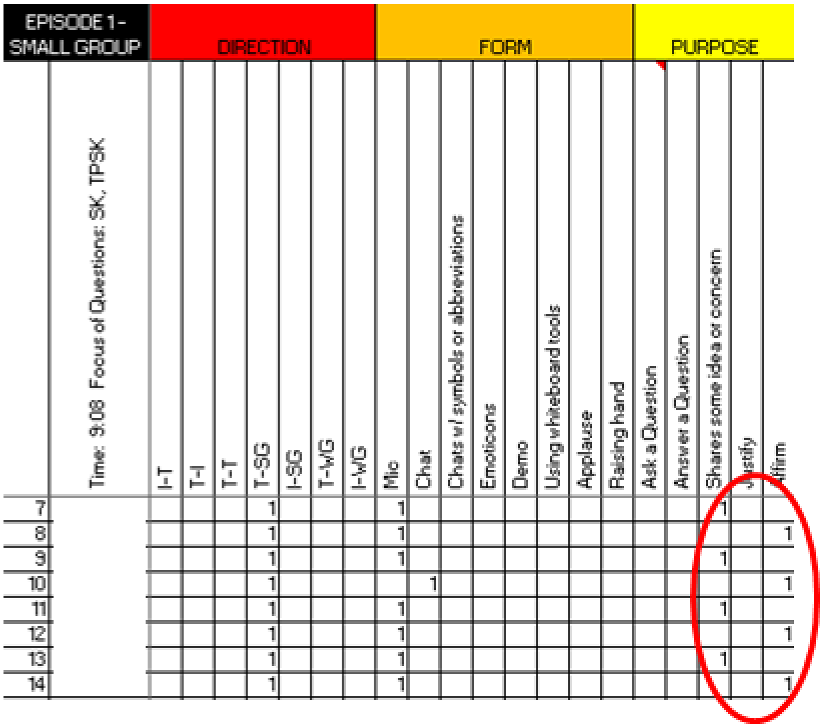
Table 8 Continued

Topic	Class (logistical, curriculum text, etc.)	6
	Online Equipment Issues	6
	Statistics • Statistical Knowledge	15 (6)
	Technology • Technological Statistical Knowledge	11 (6)
	Pedagogy • Technological Pedagogical Statistical Knowledge	4 (3)

During these nine minutes, the majority of discussion was with focus group participants using the microphone. The distribution of purpose was fairly evenly spread, but sharing ideas and affirmations was slightly higher. In this first small group episode, a noteworthy cycle emerged between the two focus group participants present for that class (see excerpt from episode transcript below).

> *7 Sally (mic): I've been having trouble with this all night so if I just randomly cut off, I'll refresh it so just bear with me.*
> *8 Abby (mic): no problem.*
> *9 Abby (mic): I'm kind of pondering the questions at this point.*
> *10 Sally (chat): samw, same**
> *11 Abby (mic): Well, right now, looking at this, I'm kind of thinking the mode is probably closer to the actual, the more typical teacher salary to me. What do you think?*
> *12 Sally (mic): I agree.*
> *13 Sally (mic): I believe the midrange is kind of skewed from the data because of the two outliers, the one at the very beginning and the one at the end.*
> *14 Abby (mic): I agree.*

Shortly after the small group discussion began, Sally shared an idea or concern and Abby followed with an affirmation. This form and purpose cycle continued three additional times. These cycles began with conversation surrounding technical difficulties Sally was having with her microphone, but carried on through discussion surrounding a pedagogical question raised in the curriculum text. The circled portion of Figure 43 below shows how this portion of the transcript was coded and highlights the cycle described above.



EPISODE 1 - SMALL GROUP		DIRECTION							FORM								PURPOSE				
	Time: 9:08 Focus of Questions: SK, TPSK	I-T	T-I	T-T	T-SG	I-SG	T-WG	I-WG	Mic	Chat	Chats w/ symbols or abbreviations	Emoticons	Demo	Using whiteboard tools	Applause	Raising hand	Ask a Question	Answer a Question	Shares some idea or concern	Justify	ffirm
7					1				1										1		
8					1				1												1
9					1				1										1		
10					1					1											1
11					1				1										1		
12					1				1												1
13					1				1										1		
14					1				1												1

Figure 43. Partial coding for online Episode 1, small group.

Focus group participants discussed statistical content most, followed by technology and pedagogical issues. In Table 8, and subsequent tables in this section, the bulleted topic items of SK, TSK, and TPSK help identify the number of occurrences of these types of knowledge out of the total occurrences of statistics, technology, and pedagogy.

Episode One – Whole Group. Immediately following the small group discussion above, prospective teachers were invited to share ideas that were discussed regarding measures of center and a “typical” teacher salary. During the whole group discussion of

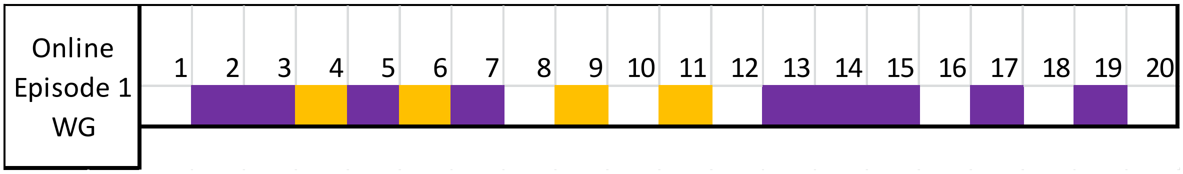
Episode 1, twenty exchanges of communication occurred during approximately seven and a half minutes. The table below provides an overview of the direction, form, purpose, and topic of discourse during this whole group discussion. Note that while each exchange has one direction and one form, it often contained more than one purpose and/or topic.

Table 9. Online Episode 1, whole group discourse.

Episode 1, Whole Group Discourse (20 exchanges, 7.5 min)		# of occurrences
Direction	Teacher-Whole Group (T-WG)	13
	Instructor-Whole Group (I-WG)	7
Form	Microphone	17
	Chat • Chatted with words	 3
Purpose	Asked a Question • Asked a new question • Restated a question from the text	 8 1
	Answered a Question	6
	Shared an Idea or Concern	5
	Justified an Idea or Response	3
	Affirmed an Idea or Response	3
Topic	Class (logistical, curriculum text, etc.)	9
	Online Equipment Issues	3
	Statistics • Statistical Knowledge	6 (3)
	Technology • Technological Statistical Knowledge	7 (1)
	Pedagogy • Technological Pedagogical Statistical Knowledge	3 (2)

The number of new questions asked is high due to the manner in which the instructor facilitated this discussion. While transitioning between groups' responses, she would ask the next group to share. These questions were coded as a new question but were also coded as ones related to class in terms of the topic of discourse. Only on two occasions did prospective teachers from a different small group from the one presenting participate and, on both of those occasions, the discourse occurred in the form of a chat. Thus, much of the whole class discussion was a reporting out of ideas that had been discussed in the small groups, with little group-to-group interaction about the ideas being shared.

Seven different prospective teachers (41% of the whole group), along with the instructor and primary instructor of the course, participated in this discussion either by using the microphone or the chat window. As Figure 44 shows, the times when the instructor spoke (white) were often followed by a single reponse either from a prospective teacher (purple) or the primary instructor.



Online Episode 1 WG	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	⬜	🟪	🟪	🟨	🟪	🟨	🟪	⬜	🟨	⬜	🟨	⬜	🟪	🟪	🟪	⬜	🟪	⬜	🟪	⬜

🟪 Prospective teacher

🟨 Primary instructor of the course

⬜ Instructor of the study

Figure 44. Online Episode 1 whole group discussion pattern.

Chapter Two

Episode Two – Small Group. In small groups, prospective teachers reflected on their use of technology and how that was similar to and different from the work of Jordan and Kathy from the videocase. During the small group discussion of Episode 2, eighteen exchanges of communication occurred during approximately nine minutes. The table below provides an overview of the direction, form, purpose, and topic of discourse during this small group discussion. Note that while each exchange has one direction, it sometimes contained more than one form, purpose, and/or topic.

Table 10. Online Episode 2, small group discourse.

Episode 2, Small Group Discourse (18 exchanges, 9 min)		# of occurrences
Direction	Teacher-Small Group (T-SG)	18
Form	Microphone	5
	Chat • Chatted with words • Chatted with symbols or abbreviations	 11 1
	Whiteboard Tools • Typed comment on the interactive whiteboard	2
Purpose	Answered a Question	3
	Shared an Idea or Concern	8
	Justified an Idea or Response	2
	Affirmed an Idea or Response	6
Topic	Class (logistical, curriculum text, etc.)	5
	Online Equipment Issues	2
	Statistics • Statistical Knowledge	5 (0)
	Technology • Technological Statistical Knowledge	5 (0)
	Pedagogy • Technological Pedagogical Statistical Knowledge	5 (5)

While the time allowed for small group discussion in Chapter 2 was equivalent to that allowed in Chapter 1, there were considerably fewer exchanges in this episode. In this small group episode, the three focus group participants communicated primarily through the chat window. This may be due, in part, to the fact that the instructor remained in the “main room” of *Elluminate* trying to minimize the chances of technical issues related to the online environment. There were also long periods of time where no new idea was being presented or discussed. Rather, two prospective teachers in the small group were having difficulty typing ideas through the interactive whiteboard. Their unfamiliarity with the whiteboard tools in *Elluminate* seemed to be the cause of the discussion not moving more quickly. At the end of the small group discussion time, after spending a great deal of time trying to type and re-type ideas on the whiteboard, Abby and Sally shared their frustrations in the chat window:

> *Abby (chat): "shoot, im having trouble using the board and correcting what i write"*
> *Sally (chat): "i had trouble too"*

One similarity between this small group discussion and that in Chapter 1 is the number of exchanges which included prospective teachers sharing an idea or concern, often followed by another person affirming the idea (e.g. Yes, I agree).

Episode Two – Whole Group. Immediately following the small group discussion above, prospective teachers shared ideas about Jordan and Kathy's statistical understanding and how *TinkerPlots* helped or hindered their work in the videocase. During the whole group discussion of Episode 2, fifty exchanges of communication occurred during approximately nine minutes. Due to technical difficulties the instructor was having with her microphone and with *Elluminate*, the primary instructor for the course (who participated in each online class meeting) acted as facilitator for the majority of this discussion. Therefore, times when he was clearly facilitating the group were coded as I-WG. The table below provides an overview of the direction, form, purpose, and topic of discourse during this whole group discussion. Note that while each exchange has one direction and one form, it often contained more than one purpose and/or topic.

Table 11. Online Episode 2, whole group discourse.

Episode 2, Whole Group Discourse (50 exchanges, 9 min)		# of occurrences
Direction	Teacher-Whole Group (T-WG)	37
	Instructor-Whole Group (I-WG)	13
Form	Microphone	19
	Chat • Chatted with words	28
	Whiteboard Tools • Referenced the ideas shared on the interactive whiteboard • Doodled on the interactive whiteboard	28 2
	Raised Hand in *Elluminate*	1
Purpose	Asked a Question • Asked a new question	10
	Answered a Question	24
	Shared an Idea or Concern	16
	Justified an Idea or Response	3
	Affirmed an Idea or Response	8
Topic	Class (logistical, curriculum text, etc.)	7
	Online Equipment Issues	1
	Statistics • Statistical Knowledge	12 (1)
	Technology • Technological Statistical Knowledge	12 (3)
	Pedagogy • Technological Pedagogical Statistical Knowledge	10 (8)

A new form, Raised Hand in *Elluminate*, appeared in Chapter 2 following instructions given at the beginning of class, asking prospective teachers to raise their hand (i.e. click on the hand-raising icon) to be recognized before using the microphone to speak to the large group. This procedure was put in place to reduce the amount of audio feedback experienced when multiple participants tried to speak simultaneously, something that was experienced during the first online class. The fact that this form only surfaced once may possibly be attributed to the technical difficulties the instructor had with microphone equipment during this discussion; she was not present to remind prospective teachers to use this tool. The primary teacher for the course, who participated throughout the five-week study, fortunately stepped in to facilitate the majority of this whole group discussion. Multiple references to the focus group's ideas displayed on the interactive whiteboard in *Elluminate* were made during the whole class discussion. Of the ten questions asked, four of them were from three prospective teachers. Fourteen different prospective teachers (82% of the whole group), along with the instructor and primary instructor of the course, participated in this discussion either by using the microphone or the chat window. As Figure 45 shows, the times when the primary instructor spoke were often followed by multiple responses from prospective teachers, many times simultaneously (#15-26 and #28-36). These were instances when prospective teachers responded to a question by typing in the chat window. Responses appeared in the chat window within seconds of each other; this is denoted by the double arrowed lines below.

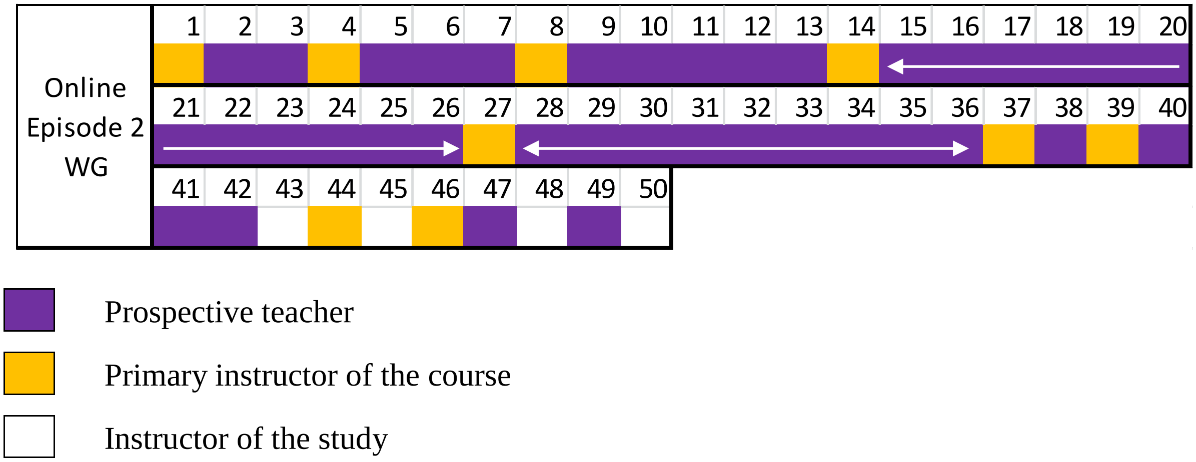


Figure 45. Online Episode 2 whole group discussion pattern.

Chapter Three

Episode Three – Small Group. Having completed Chapter 3 Section 4 for homework, prospective teachers were given time to share with one another their trials and successes in exploring deviation with *Fathom*. During the small group discussion of Episode 3, only eight exchanges of communication occurred during approximately four and a half minutes. The table below provides an overview of the direction, form, purpose, and topic of discourse during this small group discussion. Note that while each exchange has one direction and one form, it often contained more than one purpose and/or topic.

Table 12. Online Episode 3, small group discourse.

Episode 3, Small Group Discourse (8 exchanges, 4.5 min)		# of occurrences
Direction	Teacher-Small Group (T-SG)	8
Form	Microphone	7
	Chat • Chatted with words	1
Purpose	Asked a Question • Asked a new question	2
	Answered a Question	2
	Shared an Idea or Concern	3
	Affirmed an Idea or Response	5
Topic	Class (logistical, curriculum text, etc.)	3
	Online Equipment Issues	2
	Statistics • Statistical Knowledge	2 (2)
	Technology • Technological Statistical Knowledge	2 (2)
	Pedagogy • Technological Pedagogical Statistical Knowledge	0 (0)

Although short in length of time, the small group discussion in Chapter 3 provided the instructor with valuable information related to discourse between two members of this online focus group (the third group member, Sally, was absent during this discussion). First, when coding for purpose, all exchanges coded as sharing an idea or concern in this group consisted of statements that revealed prospective teachers' lack of confidence in discussing the topic assigned to them. The following excerpt comes from two focus group participants discussing a movable line and squares in *Fathom* while thinking about standard deviation:

> *Les (mic): …That's the way I think about it, but I'm not sure if that's how it is supposed to be like looked at or anything like that though.*
>
> *Abby (mic): …it's been a long time since I've dealt with probability, statistics, that sort of thing. So, standard deviation, you know, is still kind of a sketchy topic for me…it's kind of confusing to me just a little bit…*
>
> *Les (mic): …standard deviation is really complicated and everything and like just like being able to actually see it was like totally different from the way I was taught to like do it and stuff.*

The second piece of valuable information followed their discussion of the movable line in *Fathom*. A long pause was followed by the following:

> *Les (mic): I don't know, I was thinking, do you find it weird to this talking to a computer screen and all? I'm not used to this whole thing of like talking and all like that. Like, do you agree?*
>
> *Abby (mic): Yeah, it's definitely taking some getting used to but at the same time I kind of like it. I've always liked various technologies. You know I've used Skype and other forms of video chat so I'm fairly familiar with it.*

This brief exchange revealed differences in comfort levels with using *Elluminate* to communicate between two focus group participants and could provide some explanation for long, awkward pauses in the previous small group episode.

Episode Three – Whole Group. Immediately following the small group discussion above, the instructor facilitated a whole group discussion to give prospective teachers a chance to share ideas and perhaps further understand the standard deviation formula. During the whole group discussion of Episode 3, forty-six exchanges of communication occurred during approximately fourteen minutes. The table below provides an overview of the direction, form, purpose, and topic of discourse during this whole group discussion. Note that while each exchange has one direction, it often contained more than one form, purpose, and/or topic.

Table 13. Online Episode 3, whole group discourse.

Episode 3, Whole Group Discourse (46 exchanges, 14 min)		# of occurrences
Direction	Teacher-Whole Group (T-WG)	29
	Instructor-Whole Group (I-WG)	17
Form	Microphone	24
	Chat • Chatted with words	18
	Emoticon • Red “x”	1
	Whiteboard Tools • Typed comment on the interactive whiteboard • Referenced the ideas shared on the interactive whiteboard	4 1
	Raised Hand in *Elluminate*	3
Purpose	Asked a Question • Asked a new question	12
	Answered a Question	23
	Shared an Idea or Concern	5
	Justified an Idea or Response	2
	Affirmed an Idea or Response	18

Table 13 Continued

Topic	Class (logistical, curriculum text, etc.)	11
	Online Equipment Issues	1
	Statistics • Statistical Knowledge	28 (19)
	Technology • Technological Statistical Knowledge	9 (6)
	Pedagogy • Technological Pedagogical Statistical Knowledge	4 (3)

The high number of chats during this whole group discussion can be partly attributed to a question asked by the instructor which prompted the class to answer a question by typing their responses in the chat window. Other choral-response type questions were answered both by chat and microphone use. The emoticon present in the summary above, a red “x,” was shared by Abby (one of the focus group participants) when the instructor asked the Online class if they could see a pointer tool she was trying to use on the interactive whiteboard in *Elluminate*.

Fourteen different prospective teachers (82% of the whole group), along with the instructor, participated in this discussion either by using the microphone, chat window, or an emoticon. As Figure 46 shows, the times when the primary instructor spoke were often followed by a single response from a prospective teacher. However, there were three instances where the instructor’s comment or question was followed by multiple responses from prospective teachers, many times simultaneously. In exchanges 31 through 40, prospective teachers responded to a question by typing in the chat window. Responses appeared in the chat window within seconds of each other; this is denoted by the double arrowed lines below.

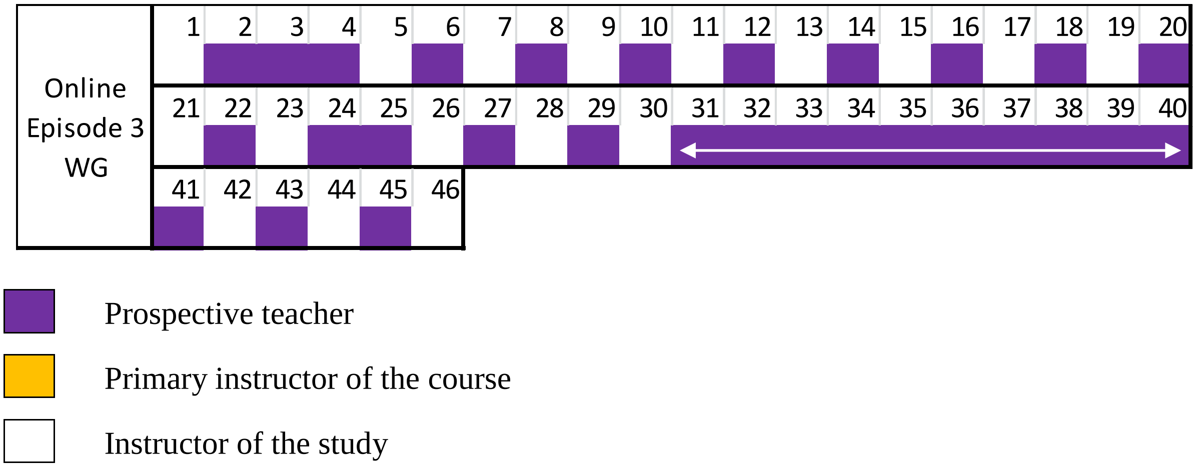


Figure 46. Online Episode 3 whole group discussion pattern.

Chapter Four

Episode Four – Small Group. Prospective teachers were asked what the residual plot revealed about the usefulness of their linear model in making a prediction and what conceptual difficulties students may have with interpreting that plot. During the small group discussion of Episode 4, twelve exchanges of communication occurred during approximately four minutes. The table below provides an overview of the direction, form, purpose, and topic of discourse during this small group discussion. Note that while each exchange has one direction and one form, it often contained more than one purpose and/or topic.

Table 14. Online Episode 4, small group discourse.

Episode 4, Small Group Discourse (12 exchanges, 4 min)		# of occurrences
Direction	Teacher-Small Group (T-SG)	12
Form	Microphone	5
	Chat • Chatted with words	 7

Table 14 Continued

Purpose	Asked a Question	
	• Asked a new question	1
	• Restated a question from the text	1
	Answered a Question	1
	Shared an Idea or Concern	2
	Justified an Idea or Response	1
	Affirmed an Idea or Response	6
Topic	Class (logistical, curriculum text, etc.)	2
	Statistics	4
	• Statistical Knowledge	(0)
	Technology	4
	• Technological Statistical Knowledge	(3)
	Pedagogy	1
	• Technological Pedagogical Statistical Knowledge	(1)

This small group discussion began almost identically to that of Episode 3. Abby restated the first question. Then, Les gave his response which included an answer to the question followed with "But, I'm not quite sure." Abby shared an idea and both Les and Sally quickly affirmed her answer in the chat window.

> *Sally (chat): I agree*
> *Les (chat): yea I agree*

This type of discourse, sharing an idea followed by short affirmations, immediately followed after Sally shared an idea.

> *Abby (mic): I agree completely*
> *Les (chat): same here*

It should be pointed out, however, that Abby did continue during this episode by justifying why she agreed with Sally, which was an uncommon practice among these prospective teachers.

Episode Four – Whole Group. Using questions from the text, prospective teachers were asked to determine where the predicted model would be based on the residual plot of the graph and what qualities of the residual plot imply that the model may be useful. During the whole group discussion of Episode 4, forty-four exchanges of communication occurred during approximately eight minutes. The table below provides an overview of the direction, form, purpose, and topic of discourse during this whole group discussion. Note that while each exchange has one direction, it often contained more than one form, purpose, and/or topic.

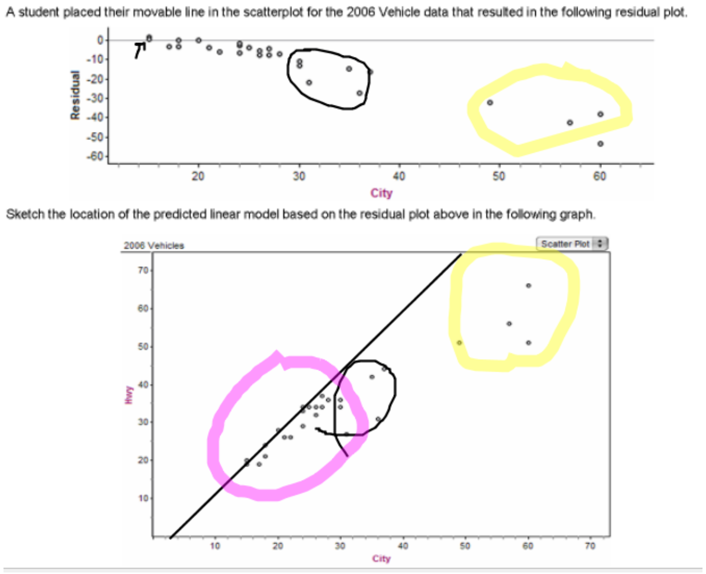
Table 15. Online Episode 4, whole group discourse.

Episode 4, Whole Group Discourse (44 exchanges, 8 min)		# of occurrences
Direction	Teacher-Whole Group (T-WG)	35
	Instructor-Whole Group (I-WG)	9
Form	Microphone	11
	Chat • Chatted with words	 15
	Emoticon • Smiley Face • Green check mark	 5 20
	Whiteboard Tools • Typed comment on the interactive whiteboard • Referenced the ideas shared on the interactive whiteboard	 3 2
Purpose	Asked a Question • Asked a new question • Restated a question from the text	 5 1
	Answered a Question	12
	Shared an Idea or Concern	9
	Justified an Idea or Response	1
	Affirmed an Idea or Response	21

Table 15 Continued

Topic	Class (logistical, curriculum text, etc.)	21
	Statistics • Statistical Knowledge	21 (13)
	Technology • Technological Statistical Knowledge	10 (9)
	Pedagogy • Technological Pedagogical Statistical Knowledge	0 (0)

The interactive whiteboard was utilized during this discussion. In response to a question that asked prospective teachers to estimate the placement of the least squares line based on a residual plot, one prospective teacher volunteered a solution by drawing a line on the interactive whiteboard (Figure 47). Using the prospective teacher's line as a reference, the instructor drew an arrow on the residual plot and circled a group of points in both graphical representations to help prospective teachers see the relationship between the two. The prospective teacher, who had voluntarily shared a line, seemed to assist in the explanation by circling a second group of points, using the yellow marker tool, in the scatterplot and the residual plot. He also circled a third group of points, using the pink marker tool, as the instructor discussed how the line fit that data.


A student placed their movable line in the scatterplot for the 2006 Vehicle data that resulted in the following residual plot.

Sketch the location of the predicted linear model based on the residual plot above in the following graph.

Figure 47. Use of the interactive whiteboard during an online discussion of Chapter 4.

There is an association between the high number of teacher-whole group exchanges, and the high number of emoticons and affirmations in this episode. At the conclusion of the discussion regarding interpretation of residuals and a residual plot, the instructor asked prospective teachers to give her some indication of their confidence with that content; she asked for a “green check” or some type of emoticon response. Also interesting is the topic of discourse in this episode. Although the questions identified a priori were TPSK questions, the summary above reveals that discussion remained centered around statistical ideas.

Fifteen different prospective teachers (88% of the whole group), along with the instructor and primary instructor of the course, participated in this discussion either by using the microphone, chat window, or an emoticon. In most instances, as Figure 48 shows, the times when the instructor spoke were often followed by multiple responses from prospective teachers, many times simultaneously (#10-27, 32-35, and 37-39). These were instances when prospective teachers responded to a question by typing a chat. Responses appeared in the chat window within seconds of each other; this is denoted by the double arrowed lines below.

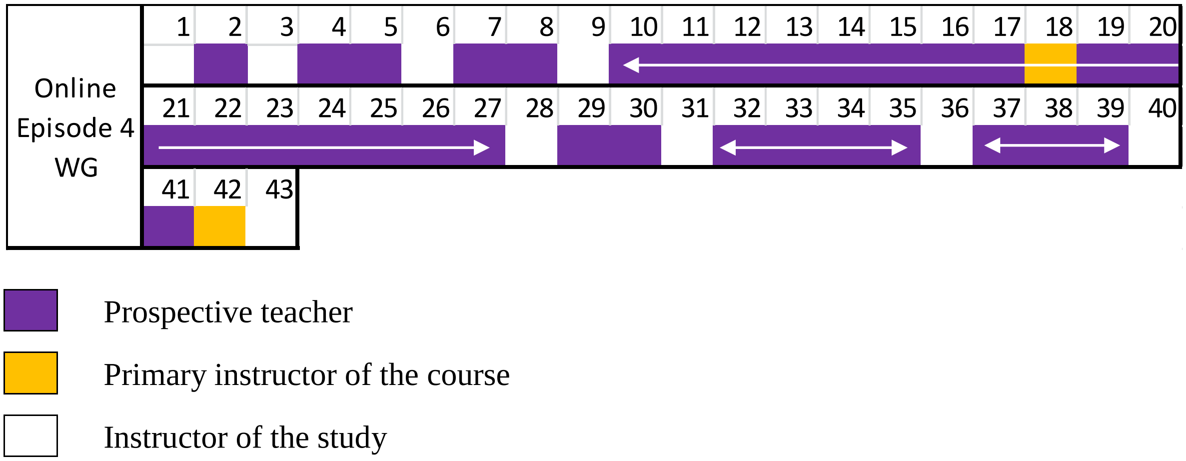


Figure 48. Online Episode 4 whole group discussion pattern.

Chapter Five

Episode Five – Whole Group. Prospective teachers, having just completed the coin simulation, were asked to anticipate an “acceptable” range if the simulations were run again. Discussion also focused on describing distributions of increasing size. During the whole group discussions of Episode 5, thirty-one exchanges of communication occurred during approximately seven and a half minutes. The table below provides an overview of the direction, form, purpose, and topic of discourse during this small group discussion. Note that while each exchange has one direction, it often contained more than one form, purpose, and/or topic.

Table 16. Online Episode 5, whole group discourse.

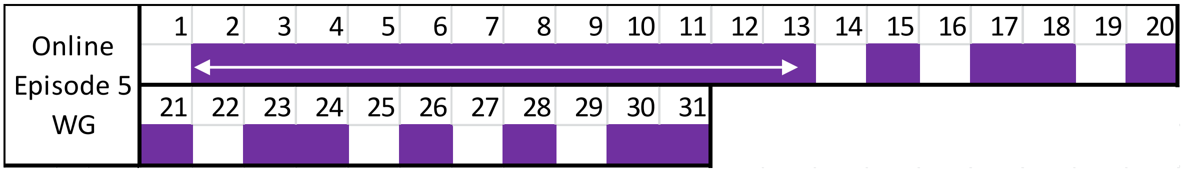
Episode 5, Whole Group Discourse (31 exchanges, 7.5 min)		# of occurrences
Direction	Teacher-Whole Group (T-WG)	24
	Instructor-Whole Group (I-WG)	7
Form	Microphone	9
	Chat • Chatted with words	22
	Whiteboard Tools • Referenced the ideas shared on the interactive whiteboard	2
Purpose	Asked a Question • Asked a new question	2
	Answered a Question	18
	Shared an Idea or Concern	6
	Affirmed an Idea or Response	6
Topic	Class (logistical, curriculum text, etc.)	4
	Statistics • Statistical Knowledge	28 (22)
	Technology • Technological Statistical Knowledge	7 (5)
	Pedagogy • Technological Pedagogical Statistical Knowledge	4 (1)

The higher number of chats can be attributed to prospective teachers' tendency and preference to use the chat window. Although the instructor did not explicitly ask prospective teachers to respond to a statistical question through chat in Episode 5, twelve of them did. Thus, twelve of each of the occurrence totals for "Chat," "Answered a Question," and "Statistics" were affected by that instance. Another close look behind the numbers shows that the instructor was responsible for sharing four of the six affirmations with the whole group.

Notice, once again, the lack of focus on TPSK. In Chapter 5, prospective teachers were beginning to think about the law of large numbers through technology simulations.

Despite the new technology skills, content remained the focus of discussion. When pedagogical issues were addressed, only once did anyone include some level of consideration for using technology with students. And, that particular instance was initiated by the instructor. Prospective teachers did not seem ready to discuss TPSK.

Fourteen different prospective teachers (82% of the whole group), along with the instructor, participated in this discussion either by using the microphone or chat window. The episode began, as Figure 49 shows, with multiple responses from prospective teachers following the instructor's question (#2-13). These were instances when prospective teachers responded to a question by typing a chat. Responses appeared in the chat window within seconds of each other; this is denoted by the double-arrowed line below. Then, a pattern of one or two responses from participants can be seen for the remainder of the episode.



🟪 Prospective teacher

🟨 Primary instructor of the course

☐ Instructor of the study

Figure 49. Online Episode 5 whole group discussion pattern.

Chapter Six

Episode Six – Whole Group. Having completed the birth data survey task (for Hyde and Martin Counties), prospective teachers described the distribution of birth data for all 100 counties of North Carolina emphasizing noticeable differences between smaller and larger counties. During the whole group discussion of Episode 6, thirty-five exchanges of communication occurred during approximately seven minutes. The table below provides an overview of the direction, form, purpose, and topic of discourse during this small group discussion. Note that while each exchange has one direction and one form, it often contained more than one purpose and/or topic.

Table 17. Online Episode 6, whole group discourse.

Episode 6, Whole Group Discourse (35 exchanges, 7 min)		# of occurrences
	Teacher-Whole Group (T-WG)	23
	Instructor-Whole Group (I-WG)	12
Form	Microphone	14
	Chat • Chatted with words	20
	Raised Hand	1
Purpose	Asked a Question • Asked a new question	5
	Answered a Question	10
	Shared an Idea or Concern	8
	Justified an Idea or Response	1
	Affirmed an Idea or Response	15
Topic	Class (logistical, curriculum text, etc.)	8
	Statistics • Statistical Knowledge	26 (15)
	Technology • Technological Statistical Knowledge	11 (11)
	Pedagogy • Technological Pedagogical Statistical Knowledge	0 (0)

All questions in this discussion were asked by the instructor. Once again, the instructor did not explicitly ask prospective teachers to respond to a statistical question through chat in Episode 6, but many of them chose to do so. There were also notably more instances of affirmation than other purposes coded. Seven of these affirmations came from the instructor, but that means that eight (nearly ¼ of the 35 exchanges for this episode) affirmations were initiated by prospective teachers.

There were no instances of TPSK in this whole group discussion. This was not surprising, particularly in light of whole group discourse results from Episode 5 and knowing that the content focus in Episode 6 remained that of the Law of Large Numbers. There was, however, a greater focus on TSK than that which occurred for Episode 5. This is likely due to the use of a new program, *Probability Explorer*, and more advanced skills in *Fathom*. Ten different prospective teachers (59% of the whole group), along with the instructor and primary instructor of the course participated in this discussion either by using the microphone or chat window. As Figure 50 shows, there were many times when the instructor's question or comment was followed by a response from one prospective teachers. There were, however, four instances where there were mutiple, often simultaneous, responses from prospective teachers (#6-11, 19-23, 27-28, and 30-32). These were instances when prospective teachers responded to a question by typing a chat and are denoted by the double arrowed lines below. Responses appeared in the chat window within seconds of each other; this is denoted by the double arrowed lines below.

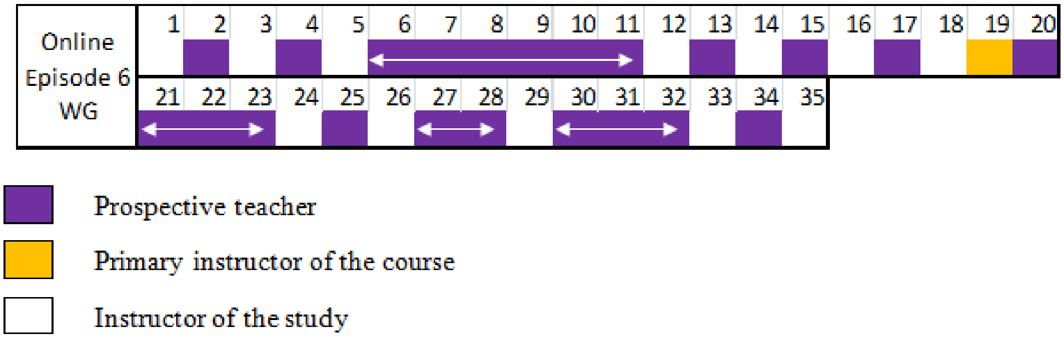


Figure 50. Online Episode 6 whole group discussion pattern.

Episode 6 – Small Group. Immediately following the whole group discussion above, prospective teachers were asked to discuss a reasonable estimate for the probability of a male birth and the importance of assumptions in probability problems. During the small group discussion of Episode 6, twenty-three exchanges of communication occurred during approximately eight minutes. The table below provides an overview of the direction, form, purpose, and topic of discourse during this small group discussion. Note that while each exchange has one direction and one form, it often contained more than one purpose and/or topic.

Table 18. Online Episode 6, small group discourse.

Episode 6, Small Group Discourse (23 exchanges, 8 min)		# of occurrences
Direction	Teacher-Small Group (T-SG)	22
	Instructor-Small Group (I-SG)	1
Form	Microphone	10
	Chat • Chatted with words • Chatted with symbols or abbreviations	 12 1
Purpose	Asked a Question • Asked a new question • Restated a question from the text	 3 3
	Answered a Question	6
	Shared an Idea or Concern	5
	Affirmed an Idea or Response	8
Topic	Class (logistical, curriculum text, etc.)	8
	Statistics • Statistical Knowledge	13 (12)
	Technology • Technological Statistical Knowledge	1 (1)
	Pedagogy • Technological Pedagogical Statistical Knowledge	0 (1)

Focus group participants seemed to prefer the use of chat over using the microphone during this discussion. But more interestingly, the continued focus on statistical content and the absence of focus on TPSK and TSK is telling. These prospective teachers lacked understanding of the Law of Large Numbers. This was clearly evident in the small group discussion for Episode 6, when all three members of the online focus group disagreed about the correct response to the survey question at the beginning of the online class meeting (see Figure 39). In a multiple choice question with three choices, prospective teachers in this group each selected a different response. Interestingly, they never attempted to justify their ideas. Instead, each member of the group simply stated his or her idea.

Trends and Patterns in Discourse

For each chapter of the curriculum used in this study, function of the online class in *Elluminate*, implementation of the curriculum, and opportunities for discourse have been analyzed. This was done by analyzing episodes of whole group and small group (when possible) discussions within each chapter. It was difficult to compare themes and patterns across chapters since only a portion of the online class surrounding that chapter was studied closely. However, there are trends that are worth noting that certainly affected discourse in the synchronous, online environment. Some of those trends resulted from decisions made by the instructor in designing and managing the class. Others resulted from preferences and personalities of prospective teachers participating in the study and are presented as trends in form, purpose, and topic.

Trends in Class Design and Management

Viewing timelines for all chapters (sans Chapter 2) simultaneously confirms that the placement of discussion activities seemed to be consistent. Whole group discussion time was interspersed throughout each lesson (Figure 51), and was kept to less than fifteen minutes in most instances. In addition, throughout the study, each time small group discussion occurred it was immediately followed by whole group discussion.

The figure below also highlights the fact that technical troubleshooting was unavoidable, despite efforts from the instructor to address individual's issues during times of independent work. The cells shaded yellow in the timelines for Chapters 5 and 6 reveal multiple issues with prospective teachers using *TI* graphing calculators and with software compatibility problems for Mac-computer users. Using class time to address these problems was necessary in order for the class to move forward with later activities.

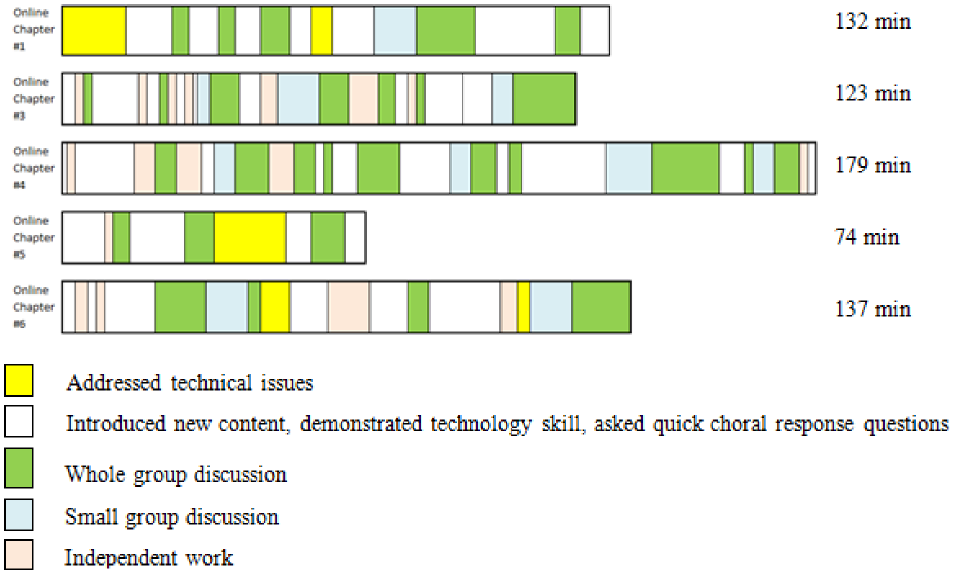


Figure 51. Comparison of timelines for online chapters.

The role of the instructor, as facilitator, also surfaced during the whole group discussions for each chapter. In episodes corresponding to work in the first five chapters, when small group discussions preceded whole group discussions, the instructor worked to solicit responses from each group. Sometimes, however, multiple prospective teachers would respond simultaneously in the chat window to questions that were not directed toward one participant or one group. The instructor would rarely recognize all responses orally, but many times she would acknowledge prospective teacher's responses and ask them to elaborate on their shared idea.

The instructor interacted with the online class predominantly through the use of a microphone. The times when she used chat to correspond with prospective teachers, she was usually helping someone with a logistical matter in the class. For example, during the whole group discussion for Episode 3, she asked the group member with the shortest name to be the spokesperson for their group. While one prospective teacher began to share his ideas, the instructor addressed the following concern:

> *Alice (chat): Mitchell's mic isn't working so what should group 5 do?*
> *Instructor (chat): oh yeah, sue can go for group 5 this week.*

At times during whole group discussions, the instructor would not comment on the correct or incorrect nature of a group spokesperson's response but would rather leave it open for further discussion from the next group or someone from the class at large.

Asking different members of each small group to report back to the whole group was just one way the instructor tried to elicit responses from different prospective teachers. She also used the "green check" emoticon frequently during each class session to receive feedback from prospective teachers about their ability to perform technological skills and understand statistical and probabilistic concepts on their own. In an interview with Abby, one of the focus group participants, the instructor asked how she liked the structure of the class. Abby responded, “Overall, I really give it some good thoughts. I wouldn’t know how to improve it. You give instruction, we try it ourselves, then we look at things and think about things ourselves, and then our groupwork…The order of things is working well. I really like it. I’ll be honest, I’m enjoying it.” This is an indication that at least one participant clearly was cognizant of the structure and felt it worked for her.

Trends in Form

During whole group discussions, prospective teachers interacted in a variety of ways. The instructor shared ground rules during the second week of the study asking participants to “raise their hand” in *Elluminate* before using the microphone to minimize audio feedback. The number of times prospective teachers wanted to ask a new question or share a new idea during more instructor-centered times of the lessons, however, was minimal. During times of whole group discussion, someone speaking for a particular group would always use the microphone. Interestingly, other prospective teachers rarely used the chat window during this time. Yet, when the instructor asked a choral-response type question, most participants seemed comfortable sharing an idea with a quick chat window response.

Emoticons and whiteboard tools within *Elluminate* were rarely accessed by prospective teachers unless directed by the instructor. Across episodes for the six chapters, no participant asked to demonstrate a technology skill for the class, despite having the knowledge that the instructor’s computer mouse could be shared with any of them.

During small group discussions, prospective teachers continued to interact in a variety of ways. Aside from the occasional faulty microphone equipment, members of the focus group seemed to strike a balance between microphone and chat window use. And, as described in Episode 3 above, they had different levels of comfort in communicating online with one another. This difference in comfort levels, coupled with a lack of confidence in statistics and probability (which will be described in a later section), seemed to cause lengthy awkward, pauses in their small group discussions at times. The small number of exchanges in each episode often contained brief responses and also confirms the difficulties prospective teachers may have been having in discussing statistical and probabilistic ideas through *Elluminate*. The interactive whiteboard was used occasionally to record ideas during small group discussions, but members of the focus group, admittedly, found typing and editing comments on that whiteboard problematic.

The trends in form described here were confirmed during interviews with focus group participants. For some prospective teachers (e.g. Les), this was the first online class setting they had ever experienced. In Les's first interview, which occurred after the first online class meeting, the instructor asked if there was anything differently she might do to help him learn better in this environment. He responded with, "it's hard when to know to type a response in or when to talk. I guess I don't know when to speak up and when not to. I guess there's a way to raise a hand, right?" Thus, the plan of "raising hands" was an adjustment made to ensure prospective teachers knew when to speak with the microphone.

Other comments from focus group members during interviews gave insight into their attitudes about learning technology in a synchronous, online environment. The ability to share a live technology demonstration proved to be an effective tool. Les said, “I like how the instructor can share her screen.” Sally said, “It took me a while to get used to all of the commands. I like that you could see everything going on, with the mouse, and then trying it out ourselves. So, we don’t always do that in class.” She described working online as being “hands-on.” Abby also commented, “I like how you share your screen. I could literally fix my screen so that it’s side by side and do it while you’re doing it… I like it. I really do.” Technologically, she was able to resize her program windows on a single monitor so that she could see what the instructor was doing while trying a new skill herself. This is somewhat similar to what could happen in a face-to-face environment, although the screen and monitor would not be side-by-side.

Having multiple ways of sharing ideas was also a feature of *Elluminate* that prospective teachers seemed to appreciate. All three members of the focus group explicitly mentioned interacting with others as something they enjoyed and perhaps did not expect.

> *Abby (Interview #1): I like the whiteboard where people can post stuff. I like how you can have voice discussion and chat discussion too… I like the small group work – I never would’ve guessed you could do small group work in an online class…I learn about things from others.*
>
> *Les (Interview #2): It’s been neat how you can interact with people. You can still talk and communicate with people.*
>
> *Sally (Interview #2): I really like our groups. Everybody has a little bit of a say.*

Despite the alluded enjoyment of group work, it is concerning that during group work often ideas were accepted without question, even when there were obvious discrepancies in responses. The small group discussion in Episode 6 showcases three different ideas from each of the three focus group members, yet no consensus or shared understanding was reached at that time. Thus, though they seem to value being able to engage in small groups, their actions in the group settings illustrate a lack of ability to persevere on a question and to construct arguments or justifications for a point of view within group discussions. One hypothesis may be that the online environment provided enough social distance that did not prompt them to engage in such activities. However, these three students may have had very similar conversations if they were in a face-to-face setting. Since they were not observed in such a setting it is difficult to assess the impact of the online environment.

Trends in Purpose

Table 19 below shows the percentage of exchanges in which specific purposes were coded. Note that a single exchange was often coded as having multiple purposes. Thus, the percentages for each row (a small group or whole group discussion) may not necessarily add up to 100%. Looking across episodes and discussions for each chapter, however, there are several interesting results. First, four episodes included times when prospective teachers shared an idea or concern more frequently than they answered a question. These four episodes were all small group discussions (from Chapters 1, 2, 3, and 4). Second, justification occurred the least. Instead, prospective teachers would often share ideas without returning to them in order to offer their reasoning.

Table 19. Online exchange percentages of types of purpose.

	Ask a Question	Answer a Question	Share an Idea or Concern	Justify	Affirm
Episode 1 – SG	28%	19%	34%	19%	28%
Episode 1 – WG	45%	30%	25%	15%	15%
Episode 2 – SG	0%	17%	44%	11%	33%
Episode 2 – WG	20%	48%	32%	6%	16%

Table 19 Continued

Episode 3 – SG	25%	25%	38%	0%	63%
Episode 3 – WG	26%	50%	11%	4%	39%
Episode 4 – SG	17%	8%	17%	8%	50%
Episode 4 – WG	13%	26%	19%	2%	49%
Episode 5 – WG	6%	58%	19%	0%	19%
Episode 6 – WG	14%	29%	23%	3%	43%
Episode 6 – SG	26%	26%	22%	0%	35%

Third, one surprising result in purpose was the number of affirmations. Most of these affirmations came directly from microphone use, particularly from the instructor during whole group discussion and from prospective teachers in small group discussion. However, 35% of all affirmations in the online episodes showcased here were in the form of a chat and 18.5% were from the use of emoticons when prompted by the instructor.

Trends in TPSK

Another trend that emerged from analysis of six episodes was the attention given to statistics and probability in prospective teachers' discussions. The following table lists the focus of questions in activities for each episode studied and the corresponding percent of interactions for statistical knowledge (SK), technological statistical knowledge (TSK), and technological pedagogical statistical knowledge (TPSK) associated with those questions. Again, row totals below may not add up to 100% if there were exchanges that had a non-TPSK focus (e.g. affirmations, comment or question about a technological issue).

Table 20. Online percent of exchanges, summary of purpose (SK, TSK, TPSK).

	Focus of Questions	SK	TSK	TPSK
Episode 1 – SG	SK, TPSK	19%	19%	9%
Episode 1 – WG	SK, TPSK	15%	5%	10%
Episode 2 – SG	TPSK	0%	0%	17%
Episode 2 – WG	TPSK	2%	6%	16%
Episode 3 – SG	TSK	25%	25%	0%
Episode 3 – WG	TSK	41%	13%	7%
Episode 4 – SG	TPSK	0%	30%	10%
Episode 4 – WG	TPSK	28%	15%	0%
Episode 5 – WG	SK, TPSK	71%	16%	3%
Episode 6 – WG	SK, TSK	43%	31%	0%
Episode 6 – SG	SK, TPSK	52%	4%	0%

In most episodes, the topic of discourse was predominantly statistical in nature. The only time pedagogy was the main focus was in episodes surrounding Chapter 2, which included the videocase analysis. Four discussions contained no evidence of TPSK at all. While two of those can be expected (Episode 3 – SG and Episode 6 – WG) as the focus of discussion questions was not on TPSK, the other two (Episode 4 – WG and Episode 6 – SG) absences of TPSK were surprising. These particular discussions were centered on understanding residuals and the law of large numbers, respectively. The lack of focus on TPSK during these discussions may indicate a lack of confidence in articulating those statistical ideas, thereby making it difficult to consider related pedagogical issues.

Understanding of Variability

Before the online study began, a pre-assessment was given to obtain information on the seventeen prospective teachers. Two questions, one about their statistics background and one about their comfort level with statistics, revealed varied experiences with statistics and probability. Regarding the statistics background question, one prospective teacher, who was a member of the online focus group, had “no background in statistics at all,” having not yet taken a statistics course in college, nor one in high school. Nine prospective teachers listed having taken ST101, an introductory course at the university, though one of them added that she “could not tell you 5 things about statistics.” Five prospective teachers had taken an introductory statistics course at a community college or other university, some as long ago as twenty-five years. And, based on the assessment, only two prospective teachers (one of whom was a member of the online focus group) in the class had taken a calculus-based statistics course in college.

The second background question on the pre-assessment asked prospective teachers to rate their comfort level on seven statistical topics (with 1 being very low or none, and 5 being high comfort). Because the background survey question was categorical in nature, viewing distributions of prospective teachers’ self-assessments of comfort with each topic was helpful (Figure 52). The following figure shows a distribution of responses for each of the seven background questions. Each graphical display, created in *Fathom*, also includes a plot of mean (vertical blue line) and median (vertical green line) values.

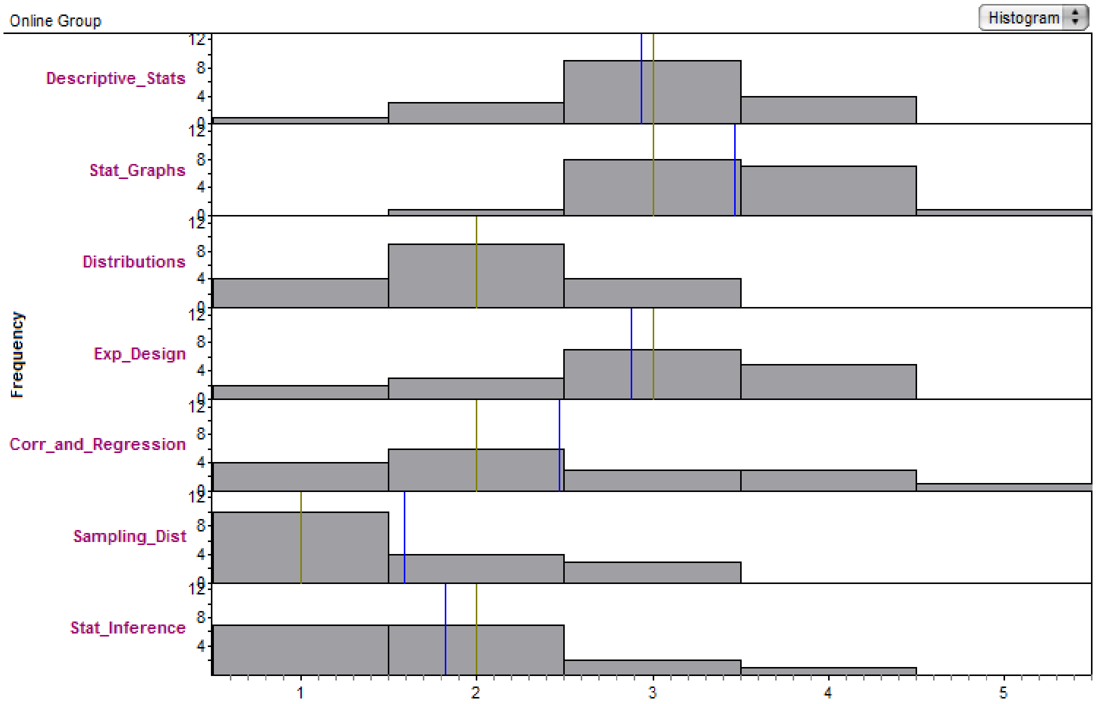


Figure 52. Online statistical background responses (1 low comfort, 5 high comfort).

As one might expect, many prospective teachers rated themselves most confident with descriptive statistics and statistical graphs. Across all seven categories, however, a self-assessment score of four or five was rare. Only one prospective teacher, a member of the focus group, even used a score of 5. Participants rated themselves least confident with sampling distributions and statistical inference. The following table provides a summary of statistical topics along with the mean, median, and standard deviation of scores from seventeen prospective teachers.

Table 21. Online pre-assessment summary of statistics comfort levels.

Online Pre-Assessment (n=17) Statistics Comfort Level (1=low, 5=high)	*Mean*	*Median*	*Standard Deviation*
Descriptive Statistics (mean, standard deviation, z-score)	2.94	3	0.83
Statistical Graphs (histogram, boxplot, bar graph)	3.47	3	0.72
Distributions (normal, chi-square, probability density functions)	2	2	0.71
Experimental Design (surveys, blocking, bias, sampling methods)	2.88	3	0.99
Correlation and Regression (least squares, R^2, residuals, outliers)	2.47	2	1.23
Sampling Distributions (Central Limit Theorem)	1.59	1	0.80
Statistical Inference (t-tests, confidence intervals, chi-square tests, power, Type II error, ANOVA)	1.82	2	0.88

As with any self-rated data, there are surely some discrepancies between what was reported in the comfort level question and the statistics background. For example, one prospective teacher reported her only experience with statistics was the ST101 course mentioned earlier. Yet, she rated her comfort level of experimental design as being a 4. Perhaps the terms “surveys” and “bias,” which are popular words in today’s culture, in the description of experimental design misled her. The only other topic she rated with a 4 comfort level was descriptive statistics. Statistical graphs received a rating of 3 on her pre-assessment and all other topics received a rating of 1.

At any rate, it is important to point out that the mean comfort level for all topics except for statistical graphs was less than 3. In other words, this group of prospective teachers rated themselves with low to medium comfort levels with these statistical topics.

Recall, from the discourse analysis (see Table 15), that much online class time was spent discussing statistical content. Perhaps prospective teachers' lower comfort levels contributed to the need for statistical discussions.

This study did not attempt to address content related to each of the seven topics listed in the pre-assessment background questions. Instead, the focus of this study was limited to prospective teachers' understanding of variability. Along with background questions, the pre/post-assessment also contained statistical content questions intended to capture some of their understanding about variability. Using scoring rubrics (Madden, 2008), each test was graded. The maximum number of points prospective teachers could score on a single test was 31. Figure 53 below shows distributions for the pre/post-assessments in this online group. As a group, their performance on the post-test showed signs of improvement. A closer look at the gains, with a mean of 1.76, shows that most prospective teachers made some improvement in understanding of variability over the 5-week study (Figure 54). However, results from a Wilcoxon rank-sum test revealed that the improvement was not significant (*P-value* = 0.157).

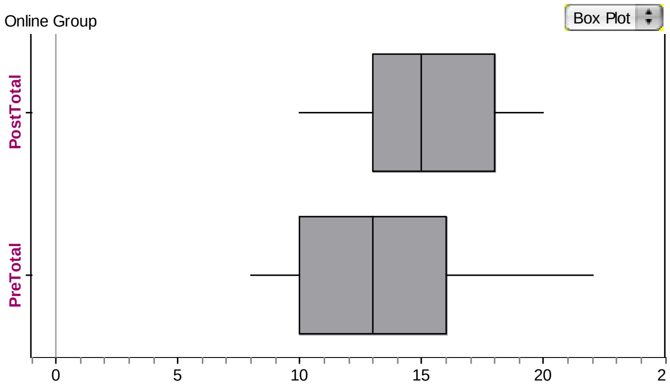
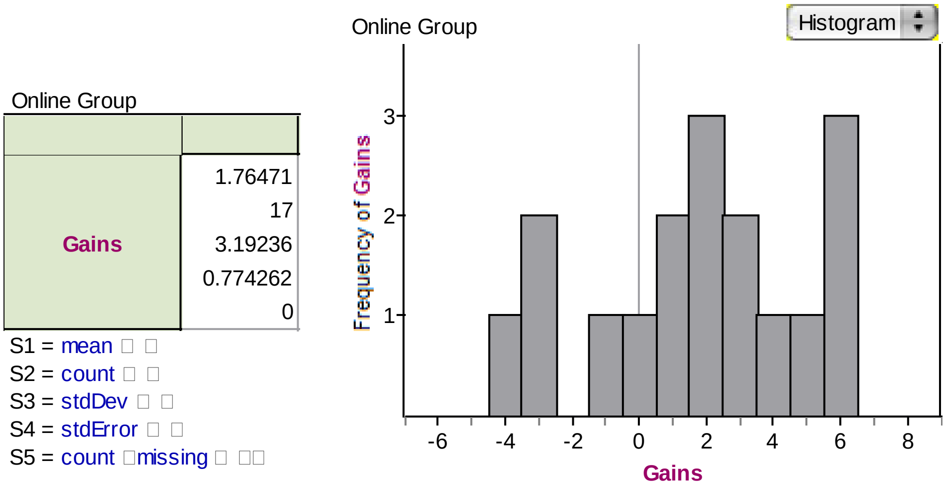


Figure 53. Box plots of the online groups' performance on the pre/post-assessments (out of possible 31 points).



Online Group

Gains	1.76471 17 3.19236 0.774262 0

S1 = mean □ □
S2 = count □ □
S3 = stdDev □ □
S4 = stdError □ □
S5 = count □missing □ □□

Figure 54. Gains from assessment scores in the online group.

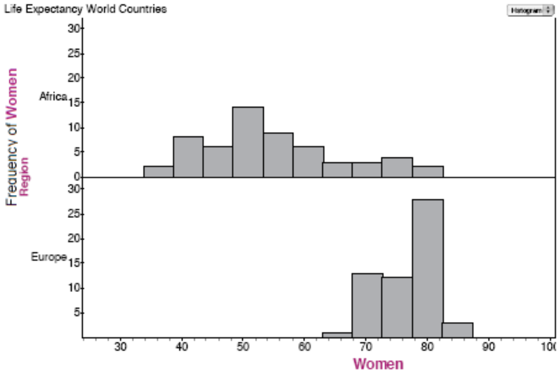
From Figure 54 above, one can see that four prospective teachers had negative gains from their pre-assessments to post-assessments. Two of those teachers' gain scores (e.g. the -4 and one of the -3s) can be attributed to their post-assessment being incomplete. These particular teachers failed to explain their reasoning on two questions. It is not clear what caused the other two prospective teachers with negative gains to have lower post-assessment scores. Also, all of the positive gains were true gains since these prospective teachers answered all questions for both the pre- and post-assessments fully.

The pre/post-assessment was only one piece of data collected from each prospective teacher. The remainder of this section is organized by the three areas of variability: describing distributions, understanding deviation, and understanding the law of large numbers. For each section, analysis will include findings from assessments, discussions, and interviews.

Describing Distributions

The ability to correctly describe distributions plays a key role in understanding variability. There were multiple times, over the 5-week study, when prospective teachers were asked to describe distributions as a part of a task they were completing. Therefore, there were multiple data sources associated with this statistical idea. Assessment data collected and analyzed included a pre/post-assessment question (n=17), a performance task (n=6), and a Chapter 2 homework assignment (n=16). Discussion data from episodes described earlier include whole group and small group discussions from Episodes 1, 2, 5, and 6. Finally, data from follow-up interviews with focus group participants provided further evidence of prospective teachers' understanding of describing distributions. The following analyses are organized by data source.

Pre-/Post-Assessment. A short-answer question on the pre/post-assessment asked prospective teachers to describe how two distributions displayed as histograms (Figure 55) were similar and different. A 4-point scoring rubric (see Appendix C) was used to grade the question holistically.



(Source: *Fathom2*; Taken from www.overpopulation.com which cites McDevitt, Thomas M., World population profile: 1998. Washington, DC: U.S. Government Printing Office, 1999)

Figure 55. Pre/post-assessment item for describing distributions (Madden, 2008, pg. 397).

While the post-assessment score (mean=2.18, stdev=0.53) showed a small improvement from the pre-assessment mean score (mean=1.82, stdev=0.53), the shift in scores was minimal (Figure 56). A Wilcoxon rank sum test (*P-value* = 0.065) confirmed the shift was not significant.

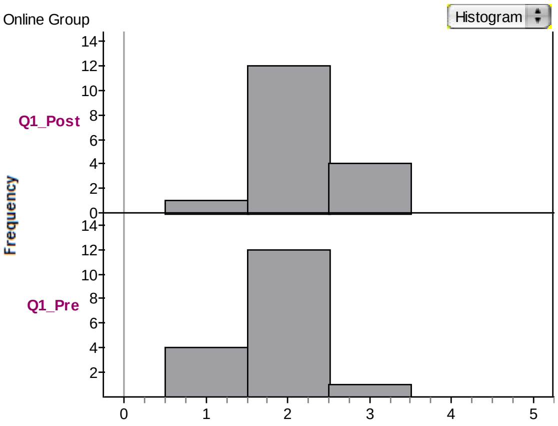
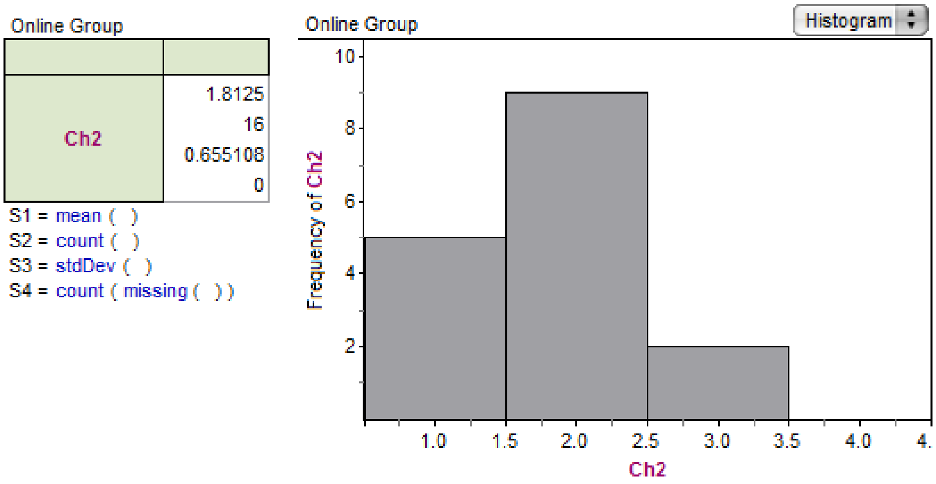


Figure 56. Online pre/post describing distribution assessment (4-point scale).

Most prospective teachers did not use statistical language to describe the similarities and differences between the two histograms. Rather, they kept references to statistical ideas on the descriptive level and used informal phrases such as “more spread out” and “tighter” to explain differences. While some prospective teachers explicitly addressed center, spread, and shape in their post-assessment responses, most remained at the descriptive level and seemed comfortable describing distributions more informally.

Chapter 2 Homework. Sixteen prospective teachers in the online class completed questions from Chapter 2 outside of class time. Two questions from the homework assignment were identified to provide additional information on prospective teachers’ understanding of distributions and their ability to describe them. The questions were part of the initial task in Chapter 2 of describing the distribution of graduation rates (see Figure 17) and comparing graduation rates between public and private schools (see Figure 18). Prospective teachers' responses were graded holistically using the same scoring rubric used on the pre-/post-assessment above (a 4-point scale). The following figure shows the distributions of scores. Mean, median, and standard deviation values are displayed in the summary table.



Online Group

Ch2	1.8125 16 0.655108 0

S1 = mean ( )
S2 = count ( )
S3 = stdDev ( )
S4 = count (missing ( ))

Figure 57. Online Chapter 2 assessment (4-point scale).

Like the pre/post-assessment, this homework assignment showed that most prospective teachers did not use statistical language to describe a single distribution or the similarities and differences between the two distributions. Four prospective teachers' work included no references to formal, statistical ideas at all. Most teachers kept references to statistical ideas on the descriptive level and used informal phrases such as "scattered" and "cluttered" to describe the spread. Only two prospective teachers included statistical language and seemed to intentionally address center, spread, and shape in their responses.

Performance Task. Six prospective teachers chose to complete a task related to describing distributions as part of their final exam. After showing competence in performing various skills in *TinkerPlots*, which included creating box plot representations (Figure 58), they were asked the following:

> "Consider the display that contains the boxplots for body weight by gender. Create a list of questions you would ask students to help them compare these distributions. Be specific."

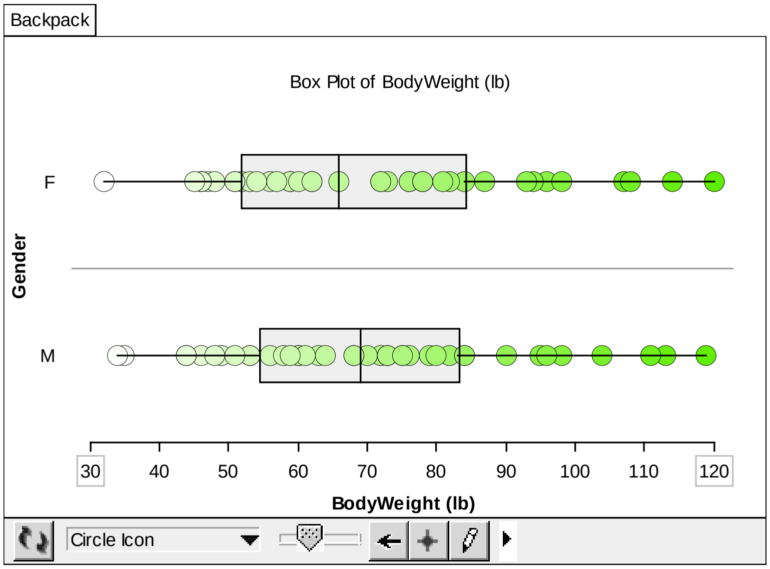


Figure 58. Sample performance task figure for comparing distributions (*Backpacks.tp* data from *TinkerPlots* resources).

A scoring rubric was used to grade their lists of questions (see Appendix H). Data showed a variety of competency levels in prospective teachers' ability to design questions for students related to describing distributions. Each row of Figure 59 below represents one prospective teacher who chose to complete this problem on the exam (n=6). Percentages for each row add up to 100% and provide information about each how each teacher's overall score was determined. Each column represents the number of questions receiving a rubric score of 1, 2, or 3. A quick look across the columns below shows that many questions considered only one box plot or distribution. Also, there were more questions focused on a comparison of the mean than a comparison of spread. But, a quick look at the rows below shows that only three of the six teachers who chose to complete this question made it clear they were thinking about both center and spread with the questions they wrote for students. Regardless, in all cases, prospective teachers posed questions in the order presented in Figure 59. In other words, all questions coded as “1” with the rubric were followed by questions coded “2” and/or “3.” This may mean that prospective teachers were trying to scaffold questions by getting students to consider one distribution at a time before focusing them on comparing centers and/or spreads.

There was also a notable difference in the number of questions prospective teachers chose to write. One thing the rubric failed to capture were the questions written that did not relate to distributional ideas at all. For example, prospective teacher #3 actually wrote four questions in his response. However, only one of them was captured in this analysis. The other three questions asked students to explore the data more in order to offer an explanation for the trends he was noticing in *TinkerPlots*. These questions were coded “n/a” and can be viewed in Figure 59.

Figure 59. Describing distributions performance task results for online class.

Discussion Analysis. Describing univariate distributions requires attending to center, spread, and shape. Four of the six episodes (Episode 1, Episode 2, Episode 5, and Episode 6) for the online case contained small group and whole group discussions that provide insight into prospective teachers' ability to do one or all of those things. Specifically, in thinking about center, discussions revealed a tendency to consider the effects of higher and lower values on measures of central tendency. If center was used to describe a distribution, usually no attention was given to spread and shape unless directions to do so were given. In fact, prospective teachers frequently described distributions in only one way. And, regardless of how prospective teachers thought about them, distributions were often described in informal, more descriptive language.

First, when thinking about measures of center, some prospective teachers considered the effects of higher and lower values on those measures. In the whole group discussion for Episode 1, which asked prospective teachers to share arguments for each of the measures of center (mean, median, mode, and midrange) to describe a "typical" value, one prospective teacher stated, "One of the things we discussed was that median was really a better way to talk about the data because that data point really skewed the mean." Another student later said, "Well, we were talking about how we thought the mean might be a good way to determine the typical salary because if you look at the median, if the data is skewed to the left or to the right, the median could potentially be very far from what the average could be." Each statement indicated they understood how high and low values affect the mean and median, but does not provide evidence that they were considering the role of context of the data as to whether one measure of center may be more representative than another.

This type of focus on the "high" and "low" values happened eleven times in the episodes related to describing a distribution, including during small group discussions. In one example, from Episode 1, Sally states in her small group, "I believe the midrange is kind of skewed from the data because of the two outliers, the one at the very beginning and the one at the end." Aside from the misuse of skewed, this statement revealed her attention to very high or very low values as the class had not discussed statistical outliers or how they might be displayed in *TinkerPlots* when this discussion took place. This is confirmed later in the discussion when she makes the following comment:

> "I think it would be interesting to see how the, um, average would change if you took out that one big outlier, that very high number, and see how it would move. Because I think having that, that really small outlier and the very large outlier, I think that can

> really make a difference on where that average sits. And right now, it’s having it sit on the mode since most of the clumps of the colors are [inaudible]. I just think it would be interesting to see how the average would change if you took out the outliers.”

Interestingly, Sally invoked a dynamic motion of imagining how the average marker would move if an outlier was removed. She seemed to be considering how the average was related to the aggregate and that each case was contributing to its value.

Episode 2, which contained discussions involving the videocase task, provided evidence that some prospective teachers expected middle school students to approach the task the way they did, including looking at measures of center. During the small group discussion, Abby shared, “Personally, I expected them to explore more attributes than just the student_to_faculty ratio. You know, because the first thing I did was checked out all these things.” She thought that since she had looked at multiple attributes, students would do the same. In the whole group discussion she also based her ideas of what students might do on how she was thinking about the distribution. She said,

> “I think personally I expected them to look at the central tendency first because that’s where you’ve got the biggest clump of data. To me, looking at the high and low ends would you know give me more information about how accurate the data is, but not necessarily help me make a total decision. At least not the first impression for my decision.”

This episode also highlighted the ways in which prospective teachers chose to describe parts of a distribution informally. Since the curriculum material also used this type of language, this was expected. Therefore, Abby’s reference to a “clump” of data was not uncommon. A line-by-line analysis of transcripts of episodes, in which discussion focused on measures of center, revealed that informal language was used to describe the center of a distribution 87.5% of the time (or 28 out of 32). To describe center, they said things like “it really looks like its right smack in the middle of all of the values” and “heavier on the right.” Prospective teachers also described the spread of the data in informal, more descriptive ways (31 out of 33 times, or 93.9% of the time). Words such as “shrinking” and “scattered” were frequently used. In the whole group discussion in Episode 5, for example, the instructor asked the class to share some of the things they were noticing about the distributions (see Figure 35) from three simulations of different sample sizes. Their responses follow:

> *James (chat): 500 is less spread*
> *Thomas (chat): shrinking*
> *Alice (chat): it looks less scattered*
> *Les (chat): less spread*

Prospective teachers also seemed to think about the shape of the distribution, often comparing it to a normal distribution. In describing the birth distribution in Episode 6, one member of the focus group said,

> “There seems to be like a rise and then a decline, kind of like a bell-shaped curve. As it gets closer, as the population gets, as the birth numbers gets higher the closer it gets to the 50%. And as it drops down again in the other direction, it gets a little further away. So, almost a bell-shaped curve.”

Throughout discussions and assessments, there were multiple mentions of a normal distribution although this terminology was not introduced through the curriculum materials, nor did the instructor spend class time focusing on this familiar topic.

The snapshot from Episode 5 above also corroborated the notion that prospective teachers rarely described a distribution in more than one way. From the coding analysis of related episodes, it was determined that describing a distribution only by center, shape, or spread occurred nearly 63% of the time. However, by Chapter 6, questions from the text explicitly ask them to address center and spread. In the whole group discussion for Episode 6, where prospective teachers were examining the distribution of percent of male births (see Figure 40), the instructor asked, “What do you notice about the spread and where the data seems to cluster?” One student replied,

> “Well the first thing that I notice is that, as we would expect, the center is right around 50 and it looks to be, it’s pretty evenly spread – the percentage never drops below 40 and never above 60. But, at the same time it’s kind of spread on the uh, it’s kind of skewed, I think to the right or maybe to the left – I get mixed up on those. But, most of the data seems to fall a little higher than 50% actually.”

In this single response, he addressed center by generally locating it to be “around 50” and addressed spread by saying it “never drops below 40 and never above 60” approximating the minimum and maximum values in the dot plot. This prospective teacher also begins to address the shape by introducing that the distribution is “skewed.”

Finally, analysis of episodes related to describing distributions showed that the number of times prospective teachers were thinking about how they would use technology with students was minimal. Only twice (once for both the small group and whole group discussions of Episode 1) did prospective teachers explicitly address how a program like *TinkerPlots* might be helpful in forming conceptual understanding of measures of center. In both cases, they described what was helpful for them and implied that similar affordances of the technology would help students as well. In answering a question about how displaying graphical measures can help students, Abby said,

> “Well I mean visually it allows them to, you know, to make certain adjustments while moving the data itself to just kind of see how they separate out. Anything visual is an aid to help understand sometimes. So, as far as, I don’t know, with data like this, it’s pretty easy to see where they clump up without having to look at raw calculations. It makes assessing the data a little easier.”

In her response, Abby draws upon the dynamic nature of *TinkerPlots* and describes the tool as one that makes analysis faster while providing a good visual. Martha also pointed out the visual aspect of the tool in describing the symbols that *TinkerPlots* displays for measures of center. Specifically, in her argument for midrange in the "typical value" discussion, she said, "the midrange, was just a nice way when you saw that green little thing, and you saw that was the midrange, it was just a nice way for students to quickly see where all that data fell, which in this case was more to the left." Again, based on the fact that it was helpful for her, Martha seemed to be describing how seeing statistical measures on the graphical representation was going to be helpful for students as well. Specifically, she explained how students will be able to think of the data as an aggregate while considering how values affect the position of the green symbol *TinkerPlots* uses for midrange.

Interview Analysis. Interviews with focus group participants provided further evidence to support claims about prospective teachers' knowledge and understanding of distributions. It also gave more insight into their developing TPSK. First, interview data show that distributions are often described one way and that descriptions include more informal language. In a follow-up interview after Chapter 2, the instructor asked focus group participants to describe a distribution of a quantitative attribute (body length of cats) displayed as a dot plot in *TinkerPlots* (Figure 60). Abby focused on the center of the distribution as she answered,

> "Wow, there's really no definite center because we've got so much separated out. I'm not very good at statistics so I'm not sure how to describe it. The greatest length is about 21 inches, next to that is about 17. The center point appears to be about 19 and

> about an equal number of representations on either side of the center point but there's no central cluster, I don't know."

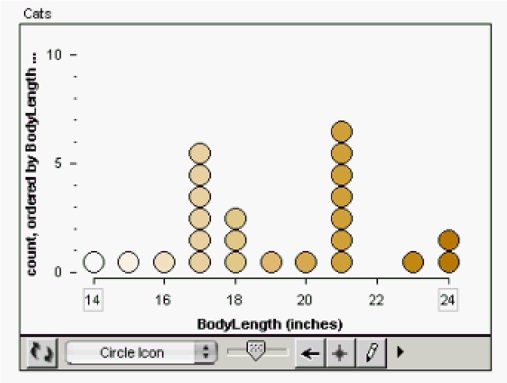


Figure 60. Cats *TinkerPlots* file used in interview #1.

While she clearly identified the two highest frequency lengths, the bimodal distribution appeared to cause some uncertainty for Abby. Her usual preference of finding the modal clump was less effective in this example, perhaps because she tried to describe the distribution by its center. She had difficulty, therefore, in describing this distribution. Yet, when asked how she might use this data set to address misconceptions students may have about average, she responded,

> "The fact that there's no central cluster will allow students to see that averages is more than just where everything seems together. They would see about the same number of points on either side of the center dot. Can I use the tools? … [Shows the average] Oh, ok so the average is 19."

In talking through how she would use this example with students, she addressed her own misconception. The ability with technology to quickly display measures on the graph was helpful to her understanding of average and its relationship (or in this case, its lack of relationship) to a modal clump.

Like Abby, Les also described the distribution one way. He answered,

> “Um, I guess kind of like, uh what’s the word? It’s got two peaks I guess and all so like I guess, like bi… bimodal? It kind of goes up and down and down and stuff. So I guess bimodal is the way I would describe it.”

While Les correctly identifies the bimodal shape of the distribution, he ignores the center and spread of the distribution. He also demonstrated some confusion with measures of center when he was asked how he might use this data set to address misconceptions students may have about average. He responded,

> “You could always add up the number for each one and then divide it by the total number and find where it falls. So then, I guess that would give you the, I don’t know, the median? And all, so then you would be able to find where the middle of the data was. And finding the middle value would help them find typical I guess.”

When asked the same question, Sally responded with, “The majority lies between 17 and 21, but they are mainly 17 or 21. We have a lower outlier of 14 and an upper outlier 24.” Knowing Sally’s tendency to use that terminology, the instructor followed up by asking to describe what she thought about outliers. Sally responded by describing the type of cat and age of cat associated with the values she was considering as outliers. So the instructor asked another question, “How do you decide that 14 and 24 [the minimum and maximum values] are outliers?” Sally replied, “Because they don’t lie with the mode, the average, the mean of the body length.” This further confirmed that Sally was using the term outlier to represent the highest and lowest values and was not thinking about whether or not a value was truly a statistical outlier. With this in mind, one could argue that Sally was beginning to think about center and spread of the distribution. Sally’s response is also further support to the idea that prospective teachers tended to consider the effects of higher and lower values on measures of central tendency.

Summary of Understanding of Describing Distributions. Much data was collected and analyzed to determine how prospective teachers' thought about and described distributions. Each data source provided evidence which, when viewed collectively, give a picture of prospective teachers' understanding during this study. Data show that prospective teachers often described distributions by addressing one of the following: center, spread, and shape. For example, if center was used to describe a distribution, usually no attention was given to spread and shape unless directions to do so were given. Some responses on the performance task, however, were exceptions. Data showed that some prospective teachers were considering center and spread when designing questions for their future students. In all responses there was a tendency to informally describe measures (e.g. clump, separated out, goes up and down).

Also regarding measures of center, it was evident that many prospective teachers considered the effects of higher and lower values on those measures. They would often explicitly state the highest and lowest values of data, sometimes inaccurately calling them outliers without determining whether or not those values were actually statistical outliers. They seemed astute at recognizing the effects "extreme" values had on the mean, median, and midrange.

Understanding Deviation

Curriculum materials used for this study allowed prospective teachers to think about deviation in a variety of contexts. Assessment data collected and analyzed from the pre/post-assessment question (n=17) provided insight into how prospective teachers were thinking about univariate deviation (e.g. deviation from a mean). A performance task on the final exam (n=6) provided information about their understanding of bivariate deviation (e.g. deviation from a line of best-fit). Discussion data from Episodes 3 and 4 revealed how prospective teachers were thinking about both univariate and bivariate deviation respectively. Finally, data from follow-up interviews with focus group participants provided further evidence of prospective teachers' understanding of bivariate deviation. The following analyses are organized by data source.

Pre/Post-Assessment. A short-answer question on the pre/post-assessment asked prospective teachers to decide which of two distributions had the largest variability (Figure 61). A scoring rubric was used to grade the question (see Appendix E).

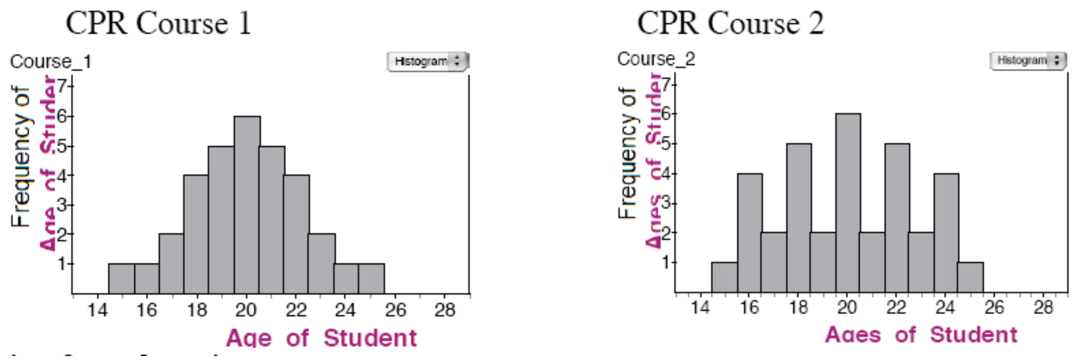


Figure 61. Pre/post-assessment item for understanding deviation (Madden, 2008, pg. 403).

Figure 62 below shows the distributions of scores for this question on the pre- and post-assessments. There was some shift in the distribution of scores but the post-assessment (mean = 1.823, stdev = 0.636) showed no improvement from the pre-assessment mean score (mean = 1.823, stdev = 0.529). The lack of gain from the pre-assessment to post-assessment can be partly explained by the fact that three prospective teachers chose the correct distribution but did not explain their answer and therefore had a lower score for this item on the post-assessment.

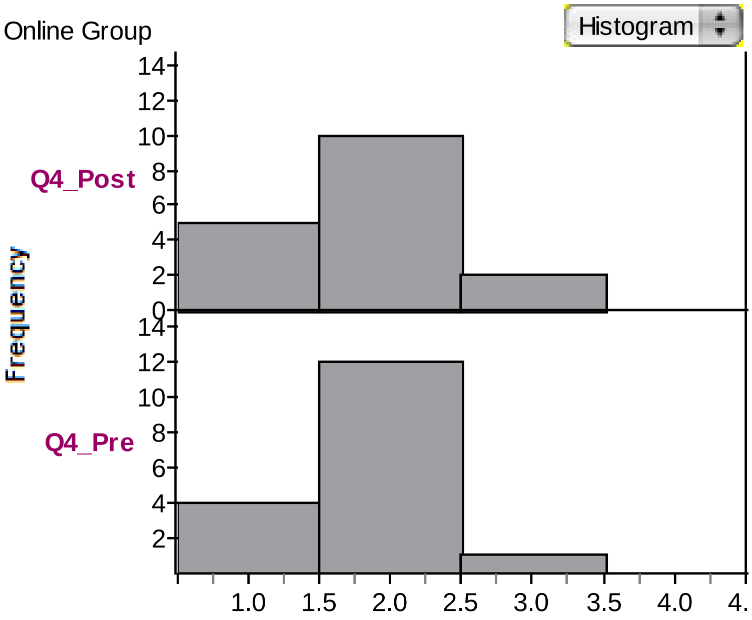


Figure 62. Online pre/post understanding deviation assessment (out of 4 points).

Prospective teachers in this online group were mostly thinking about variability as “bumpiness” and described the “up and down” shape of the second distribution in their answers. Some also included the lack of normal curve or bell curve shape as part of an explanation for greater variability in the second distribution. Only a few students seemed to focus on some measure of center and think about how values deviated from the center. When they did, they wrote things informally such as “Avg age will be approx. the same but in CPR2 the concentration in age will be all over.” Overall, results from this assessment showed prospective teachers seemed to not make a connection between variability and deviation in comparing two univariate distributions.

Performance Task. Six prospective teachers chose to complete a task related to bivariate deviation as part of their final exam. After showing competence in performing various skills in *Fathom*, they were presented with the following question:

> "Two students are arguing about correlation and slope. Jack says that the number for the correlation coefficient tells you how good a straight line fits the data and that a 0 correlation with no pattern would be a horizontal line with slope 0, and that a correlation of 1 would be a perfect relationship so the slope must be 1. Jill disagrees and thinks that slope of the least squares line is only related to the correlation in that a positive correlation means a positive slope and a negative correlation means a negative slope.
>
> What examples might you use in class to capitalize on these two students' points of view and further develop their and the other students' understanding of correlation and the slope of a least squares line? Be specific."

A 6-point scoring rubric was used to grade their responses (see Appendix I). Data showed a variety of competency levels in prospective teachers' ability to create examples specifically addressing students' understanding of correlation (see Figure 63).

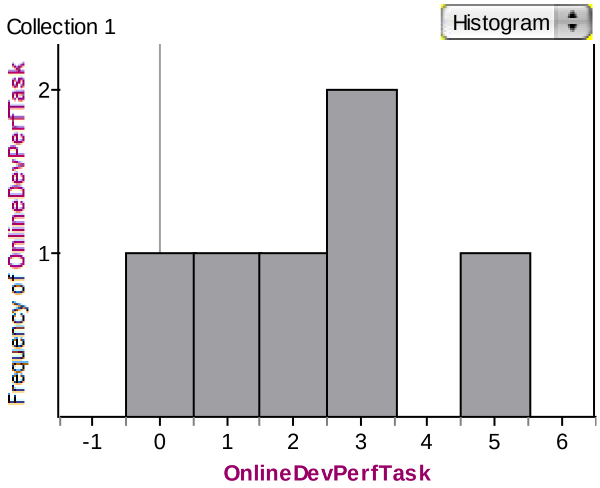


Figure 63. Online performance task assessment of deviation (out of 6 points).

One prospective teacher earned a score of 0 because he failed to address Jack's misconception at all. Instead, his examples were geared to show that Jill was correct. The prospective teacher who earned a score of 1 also failed to give good examples that might be used with these students. In her response (see below), however, she did provide additional information about her understanding that was not captured by the rubric scoring.

> "I would start the discussion by asking the students to think more about what correlation means. What measures or calculations determine the correlation coefficient. What does correlation show? Does it show a number or does it show a relationship? If they answer it's a relationship between attributes, then ask them more about the mean of those attributes. The correlation coefficient is based off of numbers and the mean; think about what happens if you have very high or low extremes. What happens to the mean? If you remove those extremes, does your mean increase or decrease? So if the correlation coefficient is based off of the mean, is it still possible for you to have a semi strong coefficient but data values that don't fit quite a straight line? The line is the best fit through all data points but what role would outliers play? On the other hand, if there is a – correlation and you look at the model of the data, what can you say about whether it's – or +? Negative typically means decrease, so if the means are spread out and the sum of the squares is very large with the data points being way below the mean, then it's correct to think that there is a negative correlation. Slope of a line is y=mx+b. And if you set x to 0, and solve for y you get y=y(bar on top) so the least squares line always passes the mean but that doesn't say that it's going to be a good fit for all data points."

This response shows an understanding that students' misconceptions stem from misunderstanding correlation overall. Instead of jumping to examples, she described an attempt to re-explain the underpinning ideas of correlation. She seemed to understand that correlation was measuring deviation from a best-fit line. So, while her rubric score for this task was low, she provided more evidence that pointed to a deeper understanding than those prospective teachers who gave clear examples and scored higher. Therefore, the scores above only indicate a prospective teacher's ability to give concrete examples that address Jack and

Jill’s understanding. They are not, unfortunately, an indicator of deep understanding of deviation from a best-fit line.

Discussion Analysis. There were two components of understanding deviation for this study. Like the pre/post-assessment, one component of understanding is at the univariate level. And, like the performance task, the other component is understanding of deviation at the bivariate level. Two of the six episodes (e.g. Episode 3 and Episode 4) for the online case contained small group and whole group discussions which provided some insight into prospective teachers’ understanding of each of those components. Specifically, prospective teachers seemed comfortable generally describing technology moves (e.g. the squares are bigger or smaller, some squares are above and some are below, the line moves right and left) and were quick to admit seeing statistical concepts visually in *Fathom* was unlike their previous experiences in statistics. Perhaps for this reason, they seemed less confident in connecting those moves with more abstract statistical formulas initially. The following paragraphs take a closer look at the episodes individually and the progression of deviation from univariate to bivariate contexts.

Recall that Episode 3 involved prospective teachers using *Fathom* and a movable line with squares (see Figure 22) to help conceptualize the standard deviation formula. Prospective teachers appreciated the visual aspect technology provided. On seven separate occasions, they explained how *Fathom* provided a good “visual.” In the discussions, two teachers also explicitly highlighted the dynamic nature of the movable line. This episode was grounded by a TSK question so prospective teachers naturally shared their experiences with the technology in their responses. For example, in the small group discussion, Les said, "Well, I kind of thought like the movable line thing and the squares and stuff was neat. And it showed like the larger the square the greater it was, like the greater the deviation was. And like the smaller the square and all it was closer to zero I guess." Les's response correctly identified the association between the sizes of the squares and the corresponding deviations.

Other prospective teachers made general comments about the "visual" nature of *Fathom* and how it can be helpful in understanding standard deviation. In the whole group discussion, prospective teachers said the following:

> *Thomas (mic): Um, yes, well we both really liked it, and me especially, because I'm a visual learner. So it definitely helps you when you want to see something like that. I personally, if you just give me a formula, then I'm not necessarily going to know what it's supposed to be doing and what it all means. I might can plug and chug the answers in and get you the correct answer out but it doesn't mean I have any idea what is going on. I thought those squares and all were a great example…*
>
> *James (mic): Going off of what Thomas said, we also liked the visual aspect of it. We thought about it in a way like, communication is about more than just words. We can sit and talk about abstract ideas all day and some students will just click and they'll get it and they'll be able to visualize it in their heads. But sometimes we can take these, with the visuals we have here, we can take that abstract idea and we can make it concrete. We can say, you know that abstract idea that we've been theorizing about – it's real, like right here you can see like a real representation of it. And lots of students will like that, I feel like, or our group felt like.*
>
> *Roger (mic): Well, we thought that it was definitely a good way to approach standard deviation in a new way. We have never really seen a visual way to approach this. It's always been approached with a formula that's never really been explained. We also thought that would be something good for the students also, that with that complex formula that you're looking at to relate what's going on there to what's going on inside that graph itself. I mean, you see a bunch of squares and you're moving this line back and forth, and you see the numbers at the bottom, but you've got to relate that back to the* X_i*s and the Xs, there's just a lot going on in that formula and explaining each part of that and how it affects the standard deviation is also important.*

Roger's comment summarizes perhaps the feelings of other prospective teachers. Understanding the formula for standard deviation is no easy feat for beginning students of statistics. Yet, he explicitly stated how the formula should be connected to what was being visualized in *Fathom*. In the small group discussion after describing the movable line and squares, Les and Abby both revealed a lack of confidence in working with standard deviation (see transcript from Episode 3 below).

> *Les (mic): That's the way I think about it, but I'm not sure if that's how it is supposed to be like looked at or anything like that though.*
>
> *Abby (mic): Yeah, same here. I thought it was pretty cool, you know, the way you can kind of visualize it you know Especially for me, considering I haven't, it's been a long time since I've dealt with probability, statistics, that sort of thing. So, standard deviation you know is still kind of a sketchy topic for me anyway simply because I don't know, I just haven't dealt with it much and I haven't had statistics yet… So, you know, it's kind of confusing to me just a little bit, but I think it will be helpful for sure.*
>
> *Les (mic): Yeah, I definitely agree. Like I took statistics here like two years ago I think, or maybe it was last year I don't remember when, but standard deviation is really complicated and everything and like just like being able to actually see it was like totally different from the way I was taught to like do it and stuff. Like they give you the formula, but like he never really used a formula from what I remember (inaudible). It was just more or less plug in some numbers into the calculator and find it that way. That's the way I've always done it and stuff. So this just gave it a whole new meaning to it for me almost.*
>
> *Abby (mic): Yeah, it definitely adds a new dimension to understanding what it's about you know?*
>
> *Les (chat): agreed*

Perhaps a lack of confidence explains why some prospective teachers eluded to making a connection between the "visual" in *Fathom* and the formula for standard deviation, but no one offered to do so before being probed by the instructor. Even then, there was an obvious reluctance in volunteering to do so. After Roger brought up connecting the formula with the visualization in *Fathom*, the instructor responded “Thanks, Roger. And so you’ve brought up a good point. Can somebody talk us through that formula and how that relates to the picture you see here?” In response, one prospective teacher typed “nope” in the chat window. After a period of silence, the instructor jokingly said “I love it when I ask a question and it’s like ‘crickets.’ So, let me ask a smaller question then. Can somebody tell me what the $(X_i - \overline{X})$ is in this picture?” By scaffolding the discussion, the instructor was able to obtain correct responses for each of the “smaller” questions she asked about standard deviation.

The lack of confidence displayed in discussions focused on univariate deviation was also evident during the class centered on bivariate deviation. Before Episode 4 began, the instructor asked prospective teachers to rate their understanding of least squares regression along a continuum using the interactive whiteboard of *Elluminate*. Figure 64 below shows the self-assessment of some teachers in the online group.

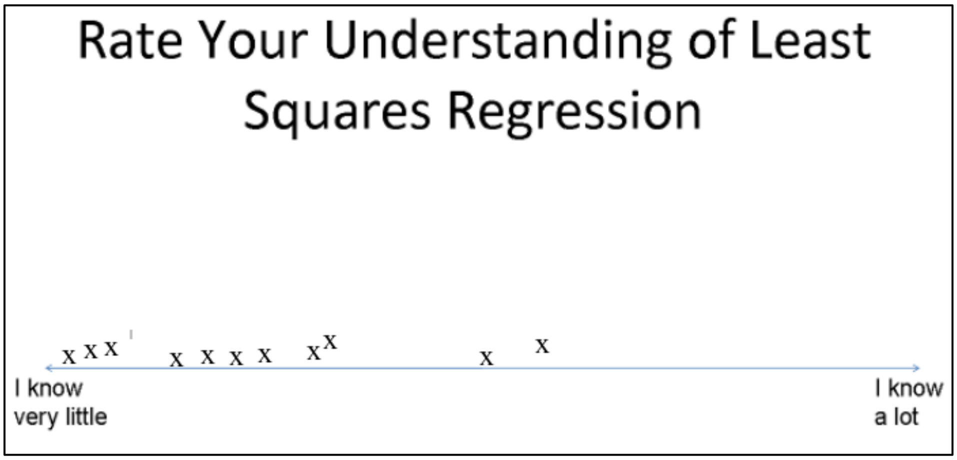


Figure 64. Online pre-survey of least squares regression understanding.

When deciding if a linear model is good, one looks at a number of things, including a residual plot. Analysis of discussions in Episode 4 show that prospective teachers were correctly identifying residuals and were considering different aspects of the residual plot in terms of the appropriateness of their linear model. They shared ideas such as “when it’s just a flat horizontal line, like the dots you can see how far away it is from it” and “none of the points are directly on the line but they are pretty equally scattered about the line.” These comments show they were looking at residual distances and whether or not there was a balance of positive and negative residual values. Further evidence of their knowledge in what to look for in a residual plot came at the beginning of the following class. Members of the online class shared these ideas:

> *Alice (chat): how linear it is*
> *James (chat): Even spread of points above and below the residual line*
> *Sally (chat): how far the point[s] are from the line*
> *Martha (chat): for residual want some points above and below the line*
> *Alice (chat): compared to spread out points*
> *Mitchell (chat): the points weren't way above or below the line*
> *Thomas (chat): small residuals*
> *Primary Instructor (chat): no pattern in the residuals*

Unlike the focus of Episode 3 (which was TSK), the focus of Episode 4 was intended to by TPSK. However, only one comment was shared that acknowledged how students might understand residual plots using *Fathom*. In the small group discussion, Sally thought “students would have trouble understanding how to relate the two graphs.” Her focus group members agreed with her but offered no suggestion on how to help students conceptualize the plot. The lack of focus on TPSK (1 out of 59 exchanges or 1.6%) confirms the lack of confidence displayed in the earlier self-assessment.

In a line-by-line analysis of Episodes 3 and 4, the general trend of focusing on statistical knowledge held precedent. It was apparent that prospective teachers were less comfortable with the content in the corresponding chapters. Looking at comments and responses for the two episodes collectively, teachers focused on TPSK four times (3.5%), TSK twenty times (17.7%), and SK thirty-four times (30%). Perhaps the attention to statistical content and being allowed to "visualize" some ideas in a new way attributed to the collective shift in prospective teachers' self-assessment of understanding seen in Figure 65.

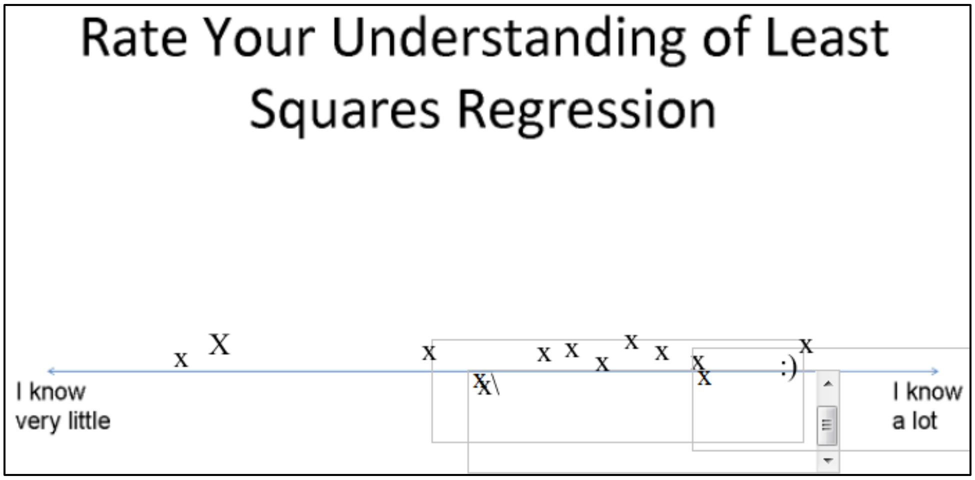


Figure 65. Online post-survey of least squares regression.

Interview Analysis. Interviews with focus group participants provided further evidence to support claims about prospective teachers' knowledge and understanding of deviation. In follow-up interviews, two participants were able to correctly connect the residual plot with a linear model. Sally, on the other hand, described her plot this way: "Seems that most of the data ranges between 0-60, I really don't understand. Why is it negative 60?" After further instruction on residuals, the instructor asked her to interpret a cluster of data points with small residual values. Sally replied, "My line had a really close residual to the data." While the wording is somewhat awkward, her notion of "close" revealed more understanding in describing bivariate deviation in that example.

Interviews also gave more insight into focus group participants' developing TPSK. They were asked to generate a question students could explore using a pre-created data set (bears.ftm from *Fathom*'s sample files) and describe the mathematics or statistics involved. Les created a question that used one quantitative attribute and one qualitative attribute. He stated it would address content ideas such as mean, median, making box plot, and describing and comparing distributions.

Abby and Sally created questions that would require their students to think about the correlation of two quantitative attributes, but Abby went further to explain her plan of asking students to "do a horizontal box plot and a vertical box plot so you have three representations of the same data." This technique was one explored during the online class meeting for Chapter 4 and was one that resonated with her. Later in the interview, she replied,

> "One of my favorite tricks was doing the box plots on the different axes - being able to create multiple representations of the same set of data allows you to cover diverse styles of learning when it comes to students. Some students may be able to look at this, these individual points plotted, and understand it better than a box plot or histogram (and vice versa). You're able to correlate it's basically equal to another. It allows them to see how much they are the same. And it allows them to see uses of individual graphs (center and spread vs. prediction). Also, you can cover everything in less time than with paper and pencil. It's easier manipulation."

In describing the box plot representation, Abby highlighted the benefit of using multiple representations with students in building bivariate understanding. She agreed that by showing students two univariate distributions initially, either box plots or histograms, they would be better equipped at seeing similarities for her particular example, "How is the length of a bears head correlated to the length of its body?" This, in turn, would help them better understand how the two quantities covaried.

While all members of the focus group adequately provided a question to use with students, two interesting things stand out. First, despite having just completed the online class focused on scatterplots and least squares regression Les reverted back to perhaps a more comfortable analysis of comparing distributions. Second, while all members of the focus group were able to demonstrate proficiency with using *Fathom*, only Abby made explicit connections as to how and when she might utilize features of the dynamic environment.

Summary of Understanding of Deviation. Each data source provided evidence which, when viewed collectively, give a picture of prospective teachers' understanding of univariate and bivariate deviation during this study. Data show that prospective teachers often thought about variability as shape and not deviation from a mean. With regard to univariate deviation, they used informal language such as "bumpiness" and "cluttered" and rarely provided evidence that they were considering the distance from a mean.

The same was true for bivariate deviation. They seemed to have a strong understanding of the directions of a correlation (positive, negative, none) but lacked understanding about how correlation was a measure using each individual point and its difference from means. One exception was with a prospective teacher's response on the performance task of the final exam. While her rubric-based score was low, she was clearly thinking about the line of best-fit and tried to articulate how she might explain that idea to her students.

Prospective teachers, in general, came in to this study with little content knowledge of least squares regression. Much time was spent building the ideas of standard deviation and residuals with movable lines and squares in *Fathom*. Prospective teachers could accurately describe the actions made with technology, but struggled initially to make connections to formal statistical equations (e.g. standard deviation, regression line). Despite this, most of them seemed to appreciate the "visual" aspect of *Fathom* and discussed openly the differences between the dynamic movements they were learning and the formula-based, "plug-and-chug" methods they learned in their introductory statistics courses. However, most prospective teachers did not offer explicit connections to the underlying concept of deviation that was being visualized with tools like the movable lines and squared deviations (from a mean or a linear model).

Understanding the Law of Large Numbers

Understanding the law of large numbers requires one to understand the variability from sample to sample, and variability between theoretical and empirical probabilities, and the effect of sample size on the magnitude of this variability. The number of opportunities for assessing prospective teachers' understanding of the law of large numbers was more limited than the other ideas of variability previously presented. Assessment data was collected and analyzed from the pre/post-assessment question (n=17) and a performance task (n=11). Data from whole group and small group discussions in Episodes 5 and 6 were also analyzed to better understand how prospective teachers were thinking about the law of large numbers. The following analyses are organized by data source.

Pre/Post-Assessment. A multiple-choice question on the pre/post-assessment asked prospective teachers the following question:

> "A certain town has two hospitals. In the larger hospital about 45 babies are born each day, and in the smaller hospital about 15 babies are born each day. As you know, about 50% of all babies are boys. However, the exact percentage varies from day to

> day. Sometimes it may be higher than 50%, sometimes lower. For a period of 1 year, each hospital recorded the days on which more than 60% of the babies born were boys. Which hospital do you think recorded more such days?
>
> A) The larger hospital
> B) The smaller hospital
> C) About the same number of days (within 5% of each other)
> D) Can't tell."

The matrix below shows frequencies of responses for each answer choice. Six prospective teachers correctly identified B, the smaller hospital, as the answer to the question on the pre-assessment. One of those six changed his/her response to A, the larger hospital, in the post-assessment. Three prospective teachers changed their incorrect pre-assessment responses to B on the post-assessment. But, along a diagonal, one can see that seven prospective teachers did not change their incorrect responses at all, insinuating that perhaps the curriculum materials and/or instruction had little to no effect on their understanding of the law of large numbers.

Post – Assessment Responses

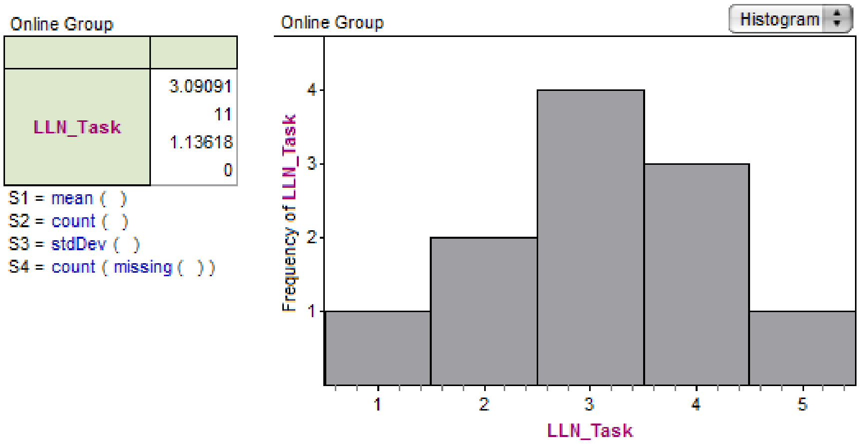
Pre – Assessment Responses

	A	**B**	**C**	**D**	**Total**
A	2				2
B	1	5			6
C	1	2	4		7
D		1		1	2
Total	4	8	4	1	17

Figure 66. Online pre/post-assessment for law of large numbers (correct response B).

The minimal gain in number of correct responses for this question was particularly surprising as the wording of this question was very similar to activities prospective teachers had completed in Episode 6 (see Figure 39).

Performance Task. Eleven prospective teachers chose to complete a task related to probability and the law of large numbers as part of their final exam. After showing competence in performing various skills with *Probability Explorer*, they were presented with the following question: “Briefly describe how you would introduce the concept of the Law of Large Numbers to students.” Tasks were graded with a 5-point scoring rubric (see Appendix J). The overall distribution of scores for this task (Figure 67) is nearly symmetric. Only one prospective teacher provided an introductory activity that addressed each point in the rubric. Instead, most teachers’ plans included some simulation (usually with coins) of varying sample sizes. Interestingly, before class discussions about probability occurred in this study, prospective teachers listed using coins most in their prior experiences with simulations (recall Figure 33). Coins were also used to model a scenario in Chapter 5 of the textbook.



Online Group

LLN_Task	3.09091 11 1.13618 0

S1 = mean ( )
S2 = count ( )
S3 = stdDev ( )
S4 = count (missing ( ))

Figure 67. Online performance task for law of large numbers (out of 5 possible points).

Another commonality in prospective teachers' responses for this performance task was the exclusion of discussion about variability one might expect from sample to sample. Instead, most of them (91%) described how they wanted students to see in multiple simulations that, as the sample size grew, the values became closer to "what was expected." Some of them (45%) explicitly referred to a theoretical probability. Strangely, however, several prospective teachers explicitly used the term "odds" in their responses although statistical odds were not discussed in class or in the curriculum materials. It is quite possible they were using the term incorrectly, in place of probability.

Discussion Analysis. Two of the six episodes (Episode 5 and Episode 6) for the online case contained whole group and small group discussions which provided some insight into prospective teachers' understanding of each variability as it related to the law of large numbers. First, from analysis of the episodes, it was apparent that prospective teachers had different ideas about the variability one might expect with a small sample. Recall that Episode 5 began with a coin-toss activity. Prospective teachers were asked to describe what they believed an acceptable range might be for seventeen samples of thirty tosses (see Figure 35). Answers varied (see transcript below).

Sue (chat): ".4 - .6"
Kelly (chat): ".45-.6"
James (chat): ".25-.75"
Peggy (chat): ".4 - .6 but of course you will have the exception of outliers"
Kristy (chat): ".4-.55"
Martha (chat): ".45-.55"
Mitchell (chat): ".4-.6"
Alice (chat): ".4-.6"
Sally (chat): ".45-.6"
James (chat): "(in a set this small, the deviation will be higher)"
Ruby (chat): ".45-.55"

Abby (chat): ".4-.52"

Only two prospective teachers acknowledged that there may be values that fall outside a 0.4-0.6 range. When asked to explain his idea further, James said,

> "I was just thinking, like for example, if we only flip the coin 5 times, we could theoretically have 100% heads. So, when you're thinking about probability like this, while 50% would be what you would predict at, at thirty trials, you could feasibly get, I mean it would be really rare, but you could get all heads or the vast majority heads or tails. I don't know, that was just my thinking about it."

Here, James tries to explicitly connect the theoretical probability with variability he expected in the empirical probabilities by pointing out the effect of a small number of trials. Following his statement, two prospective teachers offered their own coin tossing results in the chat window (e.g. "my first five tosses were all heads" and "I got 7 tails in a row! And 23 of 30"), perhaps to give examples for what James described.

Second, Episode 5 also provided more details about how prospective teachers thought about how sample size affects the differences one might notice between theoretical and empirical probabilities. After using technology to simulate larger samples, the class seemed to accurately describe some of the differences in sampling distributions among distributions for n=200, n=500, and n=999. When asked to display results on the interactive whiteboard for a simulation of n=999, most markings were added very near the 0.5 point of the line (Figure 36). However, one particular teacher added five equally-spaced markings between 0.4 and 0.6. It is likely he did not participate by doing his own simulation. If he believed those markings would represent "real" results, it would show a misunderstanding of how unlikely those particular results were.

Episode 6 further confirmed that some prospective teachers did not completely pick up on the idea of sample size affecting the variability between theoretical and empirical probabilities. An opening survey and results for the online group were shown in Figure 39 and only 31% of the class correctly answered the question, similar to the 35% who answered the related pre-assessment question correctly. Simulations in *Fathom* provided prospective teachers with color to help them relate sample size with the center and spread of the sampling distribution (see Figure 42). When describing the general trend in the spread of the data as the total number (of births) increased, prospective teachers shared ideas like "it's closer to 50," "gets closer to 50," "actually 51," and "the deviance decreases" in the chat window during a whole group discussion. The small group discussion which followed also showed prospective teachers accurately describing the trend. They related the shape of the graphical display to that of a "bell-shaped curve."

Despite many positive aspects of the whole group and small group discussions, members of the focus group disagreed on the earlier survey. At the end of their discussion, they came back to the survey. The following transcript reveals some confusion they had in thinking about law of large numbers.

> *Sally (mic): "…isn't B more likely to occur than A since it has a greater number of births?"*
>
> *Abby (mic): "yea I would say that B is actually the less likely to happen compared to A because of the fact, like you said, the greater number of births; the more births the closer it should be to the actual 51%."*
>
> *Abby (mic): "so yea, I would say A is more likely."*
>
> *Les (mic): "I kind of see it that they are both equally likely to happen or whatever because like on the graph if they have less than 1000 births, they are all pretty much*

> *purple and so like, I don't know the purple are pretty spread out so you get that 50%. So, I kind of feel like they're both equally likely to happen, you know?"*

Abby was correct in thinking that an increase in sample size will result in a decrease of variability between the empirical and theoretical probabilities, but the issue was not resolved as a group at that time. However, shortly after this exchange, Abby explains her thinking this way:

> "So, the larger the set of numbers, you're dealing with, the more likely you're going to have a more even spread. Like in the case of births – the larger the population the closer it was to say that 50%. When you have a small, small population or small number of births, you know you're a little bit more likely to be a little bit off, kind of like when we did the coin toss last week. You know you can flip a coin 5 times and get 5 in a row, but what's the chances of you getting 30 in a row? That's a lot less likely. The set size is what makes the biggest difference as far as assumptions."

Les and Sally then agreed with her in the chat window. It is unclear from this transcript what, exactly, Les and Sally concluded. Time ran out before Les and Sally were able to articulate how they were thinking about the problem after Abby's remarks. However, on the post-assessment item related to the law of large numbers, all three members of the focus group answered correctly. Perhaps this means that their "agreed" was a true affirmation and not just a quick acknowledgement of Abby's idea without understanding.

Questions that were the focus of discussions in Episode 5 (WG) and the small group for Episode 6 were identified as TPSK questions. Therefore, the fact that no prospective teacher addressed using technology to teach these ideas with students was surprising. The only TPSK focus was from the instructor. This, coupled with the fact that there was a 55% focus on SK (49 out of 89), a 19% focus on TSK, and only a 1% focus on TPSK reveals that prospective teachers were likely just as uncomfortable with this content as they were with deviation.

Summary of Understanding the Law of Large Numbers. Data collected revealed an unsettling difference in prospective teachers' understanding of the law of large numbers. From performance task responses and discussions surrounding activities during class, it was clear that some prospective teachers had a strong understanding that as a sample size grows, the variability between empirical and theoretical probabilities decreases. Sometimes probabilities were named as such; at other times, prospective teachers informally described how the results became "closer" to a value that was "expected." Other prospective teachers did not provide evidence that this concept was understood. In addition, few of them provided evidence of their understanding of the variability one might see from sample to sample, depending on the size of sample.

One surprising result was the lack of improvement from the pre- to post-assessment question related to law of large numbers. Nearly 55% of class time was allotted to addressing this statistical topic and the assessment question was situated in a context (birth data) similar to what prospective teachers had experienced in Chapter 6 of the curriculum text. Their low scores show that prospective teachers still lacked understanding in this area.

Likely due to the lack of statistical knowledge (SK) regarding the law of large numbers, prospective teachers rarely considered pedagogical issues unless forced to do so. When they were (e.g. final exam performance task), many of them reverted back to coin-toss simulations to introduce this topic with students. This may reveal a lack of technological knowledge (TSK) as well.

Trends in Understanding Variability

In this study, prospective teachers were asked to work through activities related to describing distributions, univariate and bivariate deviation, and the law of large numbers. At the same time, they were learning new technology skills with dynamic programs such as *TinkerPlots, Fathom,* and *Probability Explorer* and trying to think about pedagogical issues related to the content and to using technology with students. Much data was collected and prospective teachers' SK, TSK, and TPSK were analyzed. Common themes which emerged in each of these three areas are shared below.

Statistical Knowledge (SK)

Prospective teachers tended to stay away from formal, statistical language and seemed to prefer to describe things informally. When working with distributions, they often used terms and phrases such as clumped, separated out, and goes up and down to describe them. When trying to understand deviation, prospective teachers described variability in terms of bumpiness or being cluttered. And regarding the law of large numbers, some would describe the probability as "what you would expect," or as "what happened" rather than name them theoretical or empirical probabilities.

The use of informal language, by itself, is not a bad thing. The curriculum materials encourage an informal approach when introducing such topics to students. Many times, prospective teachers seemed to have a good understanding of the notions of center and spread and could describe a distribution based on those notions. However, the informal language that appeared in activities and tasks surrounding deviation and the law of large numbers could be indirectly pointing to another issue, namely a lack of confidence in these areas.

Other data show prospective teachers differed in their experiences and in their understanding, but that collectively, this group lacked statistical knowledge in these two areas of variability. Self-rated scores for comfort and understanding were low, assessments did not show a change in knowledge, and discussions revealed areas of statistical weakness. For example, in thinking about univariate or bivariate variability, prospective teachers rarely made any connection to distances from the mean or line of best-fit. Instead they discussed how “scattered” or “cluttered” the graphical display looked without providing evidence as to how they were judging the magnitude of scattered-ness or cluttered-ness. In thinking about the law of large numbers, there was clear evidence of a lack of knowledge. Analyses of whole group discussions showed that prospective teachers could think about the variability from sample to sample and the variability between empirical and theoretical probabilities when prompted by the instructor, but they were often incorrect on their own. Some were able to describe informally the idea that an increase in sample size results in a decrease in variability between empirical and theoretical probabilities. But in most cases, they did not seem to consider what that implies about smaller samples.

Technological Statistical Knowledge (TSK)

Throughout the study, prospective teachers learned technology skills with dynamic, statistical programs. Among other things, they learned how to create representations for univariate and bivariate data analysis (e.g. dot plot, histogram, box plot, scatter plot, least squares regression, residual plot). They also learned how to create probability simulations and collect and organize results from repeated samples. Based on prospective teachers’ interactions during online discussions (e.g. “green checks”) and submitted assignments, it is evident that they were comfortable with the skills mentioned above. They often made comments about their appreciation for the “visual” provided by the technology and made remarks like “cool” or “that’s what’s up” in the chat window about the technology or representations in general.

Prospective teachers were able to correctly describe actions relating to graphical representations they created with technology and could often describe the representation itself informally. However, when asked to make an explicit connection between the technology and statistical concepts or formulas, many of them struggled. For example, the small group discussion on standard deviation revealed a complete lack of understanding about standard deviation and a disconnect between that formula and what they had experienced with an activity using *Fathom* with a movable line. Only during the whole group discussion, when questions were scaffolded, were individuals who contributed able to correctly associate pieces of the formula with parts of the graphical display. The visual created in *Fathom*, which utilized dynamic features and linked representations, helped prospective teachers better understand concepts that had previously remained abstract.

Technological Pedagogical Statistical Knowledge (TPSK)

As mentioned above, prospective teachers were adept at performing technological skills but their abilities in relating results from technological displays to statistical formulas varied. In fact, their statistical knowledge in general varied from teacher to teacher and even from topic to topic of variability. Despite this, however, prospective teachers were encouraged by the instructor and by the curriculum materials to consider implications of teaching these topics using technology to students. As one might expect, this was difficult for them to do.

Prospective teachers were able to speak generally about the use of color or symbols overlayed on graphical displays, but they did not spend much time discussing these affordances for students. In fact, looking across all episodes of the 5-week study, there were four discussions that focused on TPSK at least 10% of the time. Two of those were small group and whole group discussions in Episode 2 which centered on the videocase of Chapter 2. Prospective teachers had video evidence of students working and copies of students' work to use to aid them in discussion. The other two discussions were from Episode 1 whole group and Episode 4 small group. These discussions focused on measures of center and residual plots respectively, two things that prospective teachers seemed comfortable with either from past experiences (measures of center) or followed more direct instruction (residual plot).

Four discussions did not discuss TPSK at all (Episode 3 small group, Episode 4 whole group, Episode 6 whole group, Episode 6 small group). These discussions focused on standard deviation, least squares regression, and the law of large numbers. This further confirms a lack of SK with these topics. Prospective teachers were unable to consider pedagogical issues because they were still attempting to understand the content itself.

Summary

This case reveals information about prospective teachers' interactions in a synchronous, online environment as they learned about teaching data analysis and probability with technology. Recall the community of inquiry framework (Garrison, Anderson, & Archer, 2000), with its social, cognitive, and teaching presences, from Chapter 2 (see Figure

1). In order to answer two research questions about discourse and prospective teachers' understanding of variability much data was analyzed that provided information about these three presences. In thinking about the entire online case holistically, however, it is difficult to summarize the presences individually. For example, online discourse was certainly affected by decisions made by the instructor and the content focus of the discussion. Likewise, the decisions made by the instructor were influenced by prospective teachers' understanding of content and the feedback she received from them. Therefore, while each of the presences in the case will be revisited below, it is the way in which they intersect with one another that best describes prospective teachers' discourse and understanding of content in the online class for this study. Brief recaps of each presence, analyses and relevant literature are provided.

Revisiting the Social Presence of the Online Case

The social presence described here is that in which prospective teachers participated. Interactions within *Elluminate* can take a number of forms and prospective teachers participated in a variety of ways. During whole group discussion, they spoke using a microphone and typed comments and questions in the chat window. They responded to the instructor with quick affirmations through the use of the "green check" and other emoticons. They also participated in surveys, controlled the instructor's computer mouse during demonstrations, and typed ideas on the interactive whiteboard. During small group discussion, prospective teachers moved into breakout rooms within the online class meetings in *Elluminate* and shared ideas by talking, chatting, and typing on the interactive whiteboard.

One noticeable part of the online discourse, in both small group and whole group settings, was the presence of simultaneous responses. As the discussion patterns for whole group discussions of each episode showed, there were certainly times when a “traditional” pattern of the instructor speaking and a participant speaking occurred. However, there were also many instances of simultaneous responses. Sometimes, the instructor would specifically ask for this type of response (e.g. put your idea in the chat window); other times the multiple responses were unsolicited. Another salient characteristic of the online discourse was the presence of pauses. Sometimes these pauses were thoughtful and it was apparent that prospective teachers were independently working to recreate a representation using the technology. Other times, however, the pauses seemed long and awkward. This was particularly noticeable in the small group discussion with focus group members.

McBrien et al. (2009) found that students used these features of the synchronous, online setting as points of personal engagement. Prospective teachers of this case seemed to also stay engaged during the class through the use of such features. Many of them commented on how they appreciated viewing live technology demonstrations and the opportunities to discuss issues related to content, technology, and pedagogy with one another. It seemed that the interactive nature of *Elluminate,* was especially appealing, something that other researchers have also found in their work (e.g. Cady & Rearden, 2009; Stephens & Mottet, 2008). One prospective teacher, a focus group member, even described working in *Elluminate* as being “hands-on.” How they worked online may not be as important as “what” they worked on. The next section includes information about prospective teachers' tendency to focus on SK and TSK and why TPSK discussions were hard to come by.

Revisiting the Cognitive Presence of the Online Case

Cognitive presence described here includes prior knowledge prospective teachers brought with them as well as new knowledge they acquired during the 5-week study. Because this study was centered in a technology methods course for prospective mathematics teachers, the content focus was really threefold. Prospective teachers learned or re-learned statistical/probabilistic content (SK) with technology (TSK) and were asked to consider whether the use of technology to teach that content might be beneficial or not (TPSK).

Admittedly, many of them had little experience with statistics and probability (SK). Three ideas related to variability were the content focus of this study: distributions, deviation, and the law of large numbers. Overall, prospective teachers were able to generally describe characteristics of distributions separately, but rarely made explicit connections with center and spread. Like Makar and Confrey's (2004) prospective teachers, most descriptions included informal language such as "clump," "up and down," and "scattered." In fact, the use of informal language was apparent throughout the study. This language was used appropriately in most cases and often mirrored that which was promoted in the curriculum text. From their language alone, it was difficult to make claims about prospective teachers' understanding of variability. Prospective teachers shared candidly, both in discussions and in follow up interviews (with focus group participants), their lack of confidence in statistics. There were multiple times when prospective teachers said "I'm not sure" and "it's complicated" when trying to express ideas related to standard deviation and least squares regression. Similar confusion was also apparent in discussions around the law of large numbers and no real improvement in prospective teachers' understanding in this area was seen. Several items of data pointed to a lack of statistical understanding throughout the study.

It is no surprise, then, that the majority of the discourse centered on statistical content despite frequent attempts to encourage prospective teachers to consider pedagogical implications of what they were learning. Discourse patterns of sharing ideas and affirmations without much justification may also point to an overall weak statistical content knowledge. Prospective teachers seemed to hesitate in building ideas off of one another and pushing one another to justify ideas throughout the study. The lack of justification was blatantly obvious during some small group discussions. Focus group members tended to share ideas willingly, but there was little evidence that some agreement about content ideas had been reached (e.g. Episode 6 - Small Group).

Prospective teachers gave no indication that technology skills were not learned during the study (TSK). Their work with the statistical content above was completed through the use of multiple technologies. Specifically, during this study prospective teachers worked with *TinkerPlots*, *Fathom*, *TI*-84 graphing calculators, *Microsoft Excel*, and *Probability Explorer*. Through affirmations during online class meetings (e.g. "green checks" and emoticons), follow-up interviews with focus group members, and assignments it was evident that prospective teachers were comfortable using the dynamic programs. Technological problems did occur during Episodes 5 and 6 when graphing calculators and *Probability Explorer* were used. This was due to the wide range of calculator models and compatibility problems for Macintosh users. Prospective teachers were able to work through these issues and correct problems prior to the final exam. Beyond simply developing technological skills, prospective teachers also provided evidence that they were using technology to develop their statistical understanding. The visual nature of representations created using dynamic programs was important in helping prospective teachers make connections to otherwise abstract formulas.

Because there was such a learning curve with new technologies and because prospective teachers were not comfortable with the ideas of variability described earlier, there was little evidence of focus on students (TPSK). Although many curriculum questions being discussed were identified as TPSK questions, discourse remained centered on statistical ideas for the most part. This confirms the notion of Lee and Hollebrands' (2011) TPSK framework, that is, that statistical knowledge is the foundation level and must exist before technological and pedagogical knowledge can be more fully developed. Times during the study when TPSK discussions were long-lasting, were times when prospective teachers were using samples of student work (Chapter 2 materials). This is in accord with previous research results that discussions in teacher education should be centered around content and student work (Cady & Rearden, 2009; Groth, 2007; Stephens & Hartmann, 2004).

Revisiting the Teaching Presence of the Online Case

The social, cognitive and teaching presence components did not exist in isolation. There were overlaps between how prospective teachers communicated and interacted with one another, the content of their discussions, and the subsequent knowledge they developed and shared. How the instructor worked together with the social and cognitive presences is important in thinking about implications of this research on future online endeavors in mathematics education.

Through analyses of lesson maps and timelines, it was clear that the instructor made purposeful decisions when considering the placement of whole group and small group discussions. This study does not intend to make claims about the appropriate ordering of such activities. Rather, data does suggest that the predictable nature of discussion activities in the online class was beneficial for prospective teachers. In their interviews, focus group members commented favorably on the structure of the class. One of them explained the structure in her own words and showed appreciation for designated times of independent and small group work in particular.

The use of small groups (or breakout rooms) in *Elluminate* was a welcomed surprise for prospective teachers. They spoke fondly of the ability to share ideas and "have a say." As their work showed (Stipek et al., 2001), prospective teachers should be engaged, allowed to try new things, and given the opportunity to collaborate and reflect with others. The whole group work which included, among other things, live demonstrations, use of emoticons and the chat window, allowed prospective teachers to do all of those things. Small group work held them accountable for it and allowed time for further discussion. In a breakout room the instructor could hear prospective teachers talking and read anything they were writing in the chat window or interactive whiteboard. The "noise" from other groups was entirely absent. However, while in one group she could see that other prospective teachers were talking and writing in their groups based on icons that turned yellow when a participant was actively using their microphone, chat window, or online writing tools. She could then move to other groups until she had visited breakout rooms for each group. The instructor's ability and frequent use of moving between small breakout rooms also provided "closer" contact with individual participants and allowed them to ask questions outside the whole group. Many prospective teachers took advantage of this opportunity.

As previously discussed, much discourse in the online class centered on statistical content. Interestingly, this was true for the majority of both the small group and whole group discussions. This means, despite a plan of using TPSK-level questions, the instructor made decisions during class to keep the focus on content. Knowing that time was a factor, she knew having a meaningful discussion about pedagogical issues without content knowledge would be difficult. However, this does not mean that pedagogical knowledge was not being built. If one agrees with von Glasersfeld (1984) and the constructivist philosophy, then prospective teachers in the online case were constructing a unique knowledge based on their own experiences and understanding. The fact that discourse was focused mostly on content meant that they were learning or re-learning statistical content while using technology. In turn, this implies that they saw first-hand what a “statistics lesson with technology” could look like. This should not be taken lightly, as many prospective teachers’ prior middle and high school classrooms looked very different than the one they were currently learning in. They had the opportunity to learn that teaching with technology is not just about using the technology. Rather, it changes the questions that are asked, the tasks that are posed, and the assessments that are written. If the old adage is true, that one teaches the way one is taught, then prospective teachers, re-learning concepts from statistics and probability in a technology methods course has its benefits.

CHAPTER 5: THE CASE OF TEACHING AND LEARNING IN A FACE-TO-FACE ENVIRONMENT

Introduction

This case begins by describing the face-to-face environment. An overview of the setting, class structure, and curriculum is provided in order to situate the contexts of episodes chosen for deeper analysis. The remainder of this case is divided into two large sections, one to address each of the research questions. First, recall this study considered how prospective mathematics teachers interacted with one another and with the instructor with curriculum focused on teaching data analysis and probability with technology. Six episodes were identified, one for each chapter in the text, for more detailed analysis. Each episode contained opportunities (whole group and small group discussions) for analyzing discourse surrounding identical questions presented in the online case. Follow-up interviews with three focus group participants provided further information on discourse and general attitudes about learning and working in the face-to-face environment. Discourse analysis for this case is divided by curriculum chapter. A summary of discourse trends related to form, purpose, topic, and TPSK are discussed.

Second, this study also aimed to understand how prospective teachers thought about variability, specifically related to describing distributions, deviation, and the law of large numbers. Episodes for each chapter also contained opportunities for analyzing understanding. Follow-up interviews, pre-/post-assessments, homework, and final exam tasks provided further information into how prospective teachers were thinking about variability. Understanding analysis for this case is divided by topic of variability (describing distributions, deviation, and law of large numbers) and sub-divided by data source (assessments, discussions, interviews). A short summary of trends related to prospective teachers' understanding for each topic of variability is included. Then, this chapter concludes with an overall summary of the face-to-face case.

Context and Overview

Twenty-five prospective teachers enrolled in the section of the course that served as the face-to-face group. The class met 1.25 hours, twice a week, for the duration of the five-week study. There were twenty PC desktop computers along three sides of the rectangular-shaped computer lab. Seven prospective teachers brought their personal laptops to each class and sat at tables in the center of the classroom (Figure 68). All focus group participants brought laptops. The figure below shows where they usually sat in relation to other prospective teachers, instructor, whiteboard, and computer projector screen.

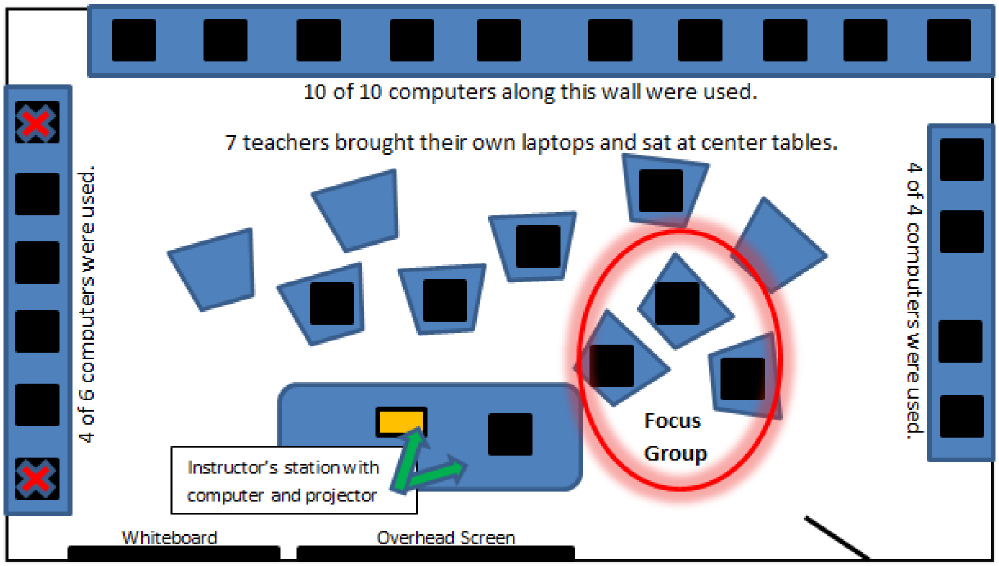


Figure 68. Diagram of face-to-face computer lab.

Each prospective teacher had access to a TI-83+/84 graphing calculator, Microsoft Excel, and all dynamic software programs used in the curriculum materials (*TinkerPlots*, *Fathom*, and *Probability Explorer*) either on the desktop computers or through the virtual computing laboratory (VCL) on laptops. In addition, each of them had purchased the textbook and brought it with them to class. The face-to-face curriculum implementation was a week behind that of the online class. The instructor gave careful attention to facilitating similar activities and discussions in both groups. An overview of content and implementation for each chapter of the curriculum text is provided below.

Chapter One

Working through activities from Chapter 1, participants revisited statistical ideas and concepts related to introductory data analysis and measures of center and spread through exploring a relevant data set while learning a new dynamic technology, *TinkerPlots*. Prospective teachers also used the idea of “typical” teacher salary from the data set as a springboard for a discussion regarding the potential benefits and drawbacks of approaching certain statistical topics informally. The instructor introduced the notion of Exploratory Data Analysis (EDA) that would be prevalent throughout the curriculum unit and provided demonstrations for some of the skills in *TinkerPlots* related to reading, organizing, graphing, and analyzing data.

Prospective teachers were instructed to try skills on their own and engage with the class during whole group discussions. In addition, small-group discussions occurred while the instructor moved from group to group. Most of Chapter 1 in the textbook was studied during class. Chapter 2 was assigned for homework.

With starting a new curriculum and learning a new technology, it was expected that much time would be needed by the instructor to introduce new skills and concepts. This was, in fact, the case with half of the time spent in Chapter 1 (129 minutes) being used by the instructor to introduce new content from the curriculum text or demonstrate a skill in *TinkerPlots*. Despite the large percentage, it does not mean that participants were not interacting during this time. They often responded to choral response questions from the instructor throughout the technology demonstrations. And, if technological problems arose, prospective teachers would often quietly solicit the help of another participant seated nearby. Whole group discussions occurred when no new content was being presented. This happened during approximately 26% of Chapter 1 instruction. Small group discussions occurred 19% of the time and prospective teachers were asked to work independently on small tasks 5% of the time. Figure 69 below summarizes the opportunities for interaction during instruction for Chapter 1.

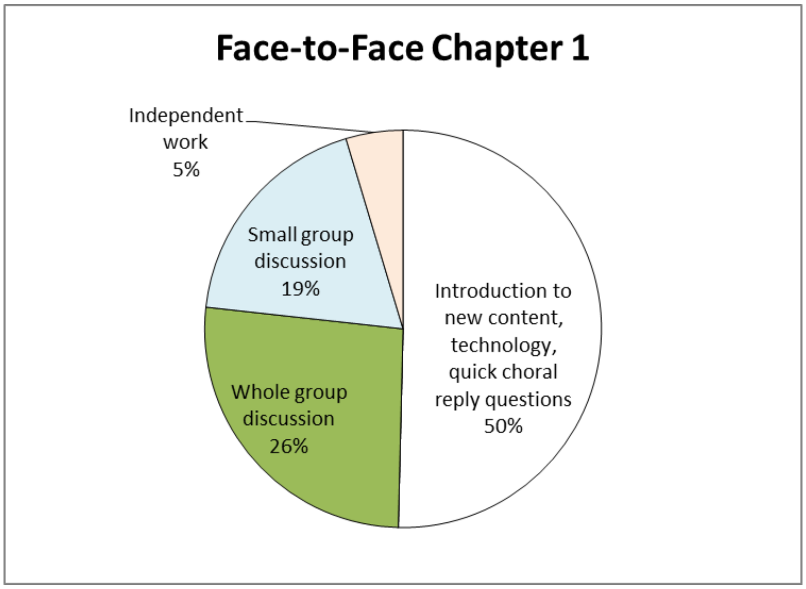


Figure 69. Opportunities for interaction in Chapter 1 of the face-to-face class.

The timeline below (Figure 70) shows opportunities for discourse in Chapter 1 as they occurred during the face-to-face class. The entire bar represents the total number of minutes, 129, spent studying Chapter 1 and the colored cells are proportional in length, depicting the proportion of time for each type of activity as it occurred during class time. Small markings above the timeline provide indicators of 20-minute increments of time. The small group and whole group discussions circled in the timeline indicate when, during the face-to-face class, Episode 1 occurred. The arrow indicates when, over the course of the chapter, one class meeting ended and the next one began two days later.

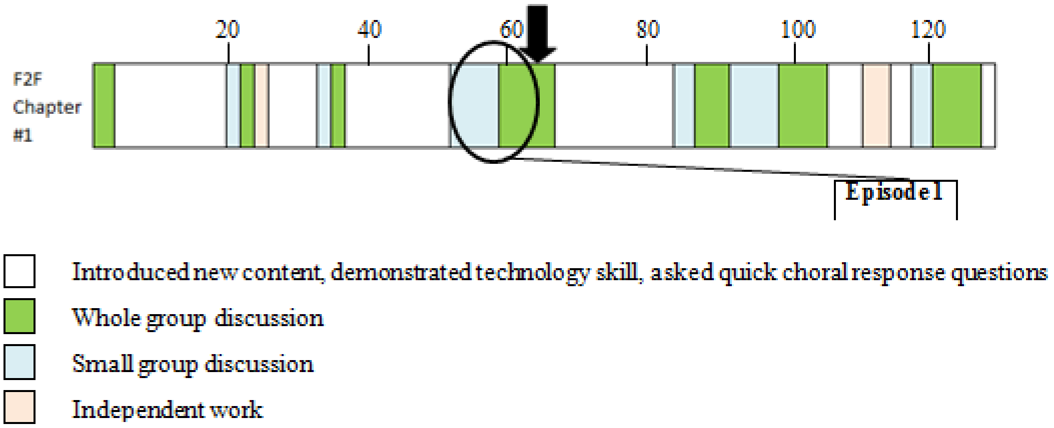


Figure 70. Timeline for Chapter 1 of the face-to-face class.

Setting of Episode One of the Face-to-Face Class. This episode contained one activity in Chapter 1 of the curriculum text where prospective teachers used measures of center to describe a distribution. To introduce *TinkerPlots*, data cards, default plot, bar graph of a qualitative attribute, and dot plot of a quantitative attribute were discussed in similar ways to the online class (Figure 71). In fact, beginning activities in the face-to-face class with

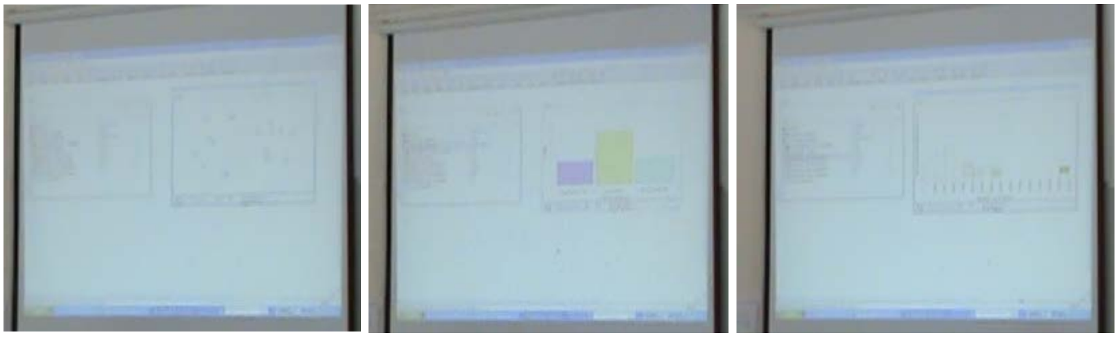


Figure 71. Face-to-Face demonstrations of introductory *TinkerPlots* activities.

TinkerPlots nearly mirrored those in the online class. Even the face-to-face demonstration of the divider tool to shade the top 50% and the middle 50% of the data involved a prospective teacher using the class-displayed computer to illustrate his work, just as the online demonstration did (Figure 72).

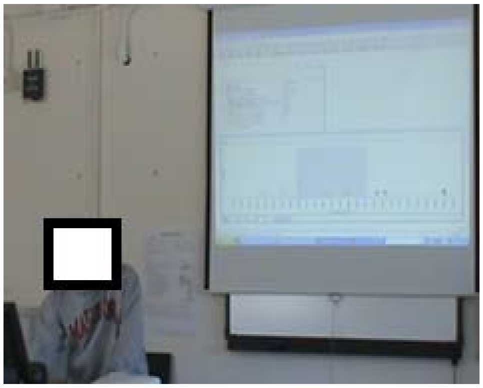


Figure 72. Prospective teacher demonstrating the use of the divider tool in the face-to-face class.

Episode 1 began with the instructor inviting prospective teachers to move into small groups to discuss measures of center of the teacher salary data. Like the online group, they were using a dot plot (Figure 73) along with statistical measures to describe a “typical” teacher salary. They also discussed how displaying measures on a graphical display in

TinkerPlots may help students understand these measures in relationship to each other and to the distribution of data. Members of small groups then shared ideas for a “typical” teacher salary in a follow-up whole group discussion.

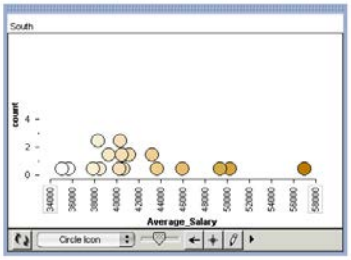


Figure 73. Dot plot of average teacher salary.

Chapter Two

The instructor revisited *TinkerPlots* skills previously learned in Chapter 1. Specifically, she reviewed how qualitative and quantitative data are displayed in *TinkerPlots* with static images of bar graphs, dot plots, box plots, and histograms. Activities and questions surrounding the videocase in Chapter 2 of the curriculum text were completed by prospective teachers outside of class time for homework. Prior to coming to class, participants reflected on their own use of technology and how they perceived the technology to be a help or hindrance to two middle school students who were viewed in a video clip, part of the Chapter 2 materials. At the beginning of the class meeting, approximately twenty minutes were spent discussing the task and videocase. Figure 74 below summarizes the opportunities for interaction during instruction for Chapter 2.

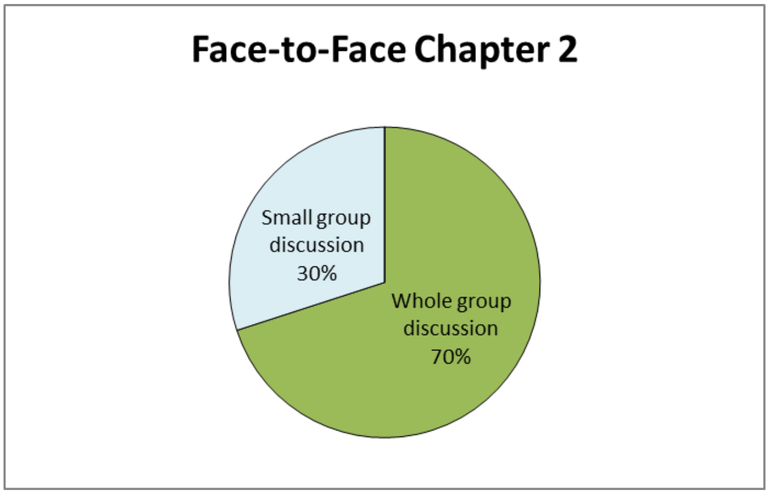


Figure 74. Opportunities for interaction in Chapter 2 of the face-to-face class.

The creation of a timeline for Chapter 2 was omitted since determining the amount of time prospective teachers spent outside of class was not possible. But, with regard to Episode 2, it is important to note the sequencing of discussions that occurred during class: 1) a short whole group discussion (2 minutes), 2) a small group discussion (6 minutes), and 3) a longer whole group discussion (12 minutes).

Setting of Episode Two of the Face-to-Face Class. Recall that prospective teachers were asked to specifically explore the graduation rates for public and private schools in the given set (see Figure 75, taken from one prospective teacher's assignment).

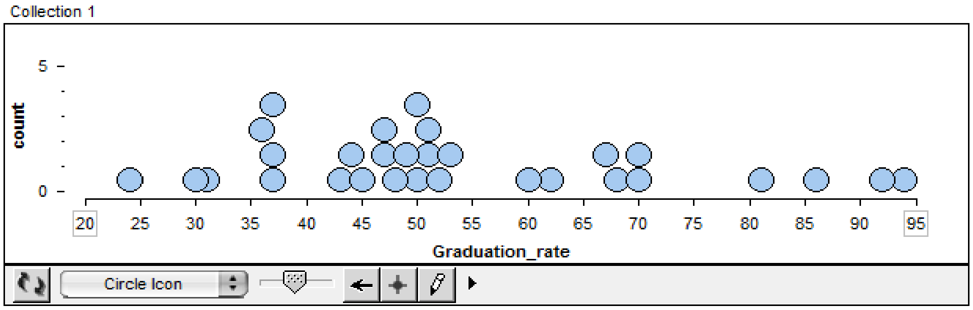


Figure 75. Prospective teacher's dot plot of graduation rates for public and private schools.

The first question in the task asked them to describe the distribution of graduation rates for NC schools. Prospective teachers were then encouraged to use various plot tools to investigate the spread and centers of the graduation rates for public and private schools and think about how these tools might be helpful for students. Figure 76 below show how one prospective teacher used various tools during her investigation.

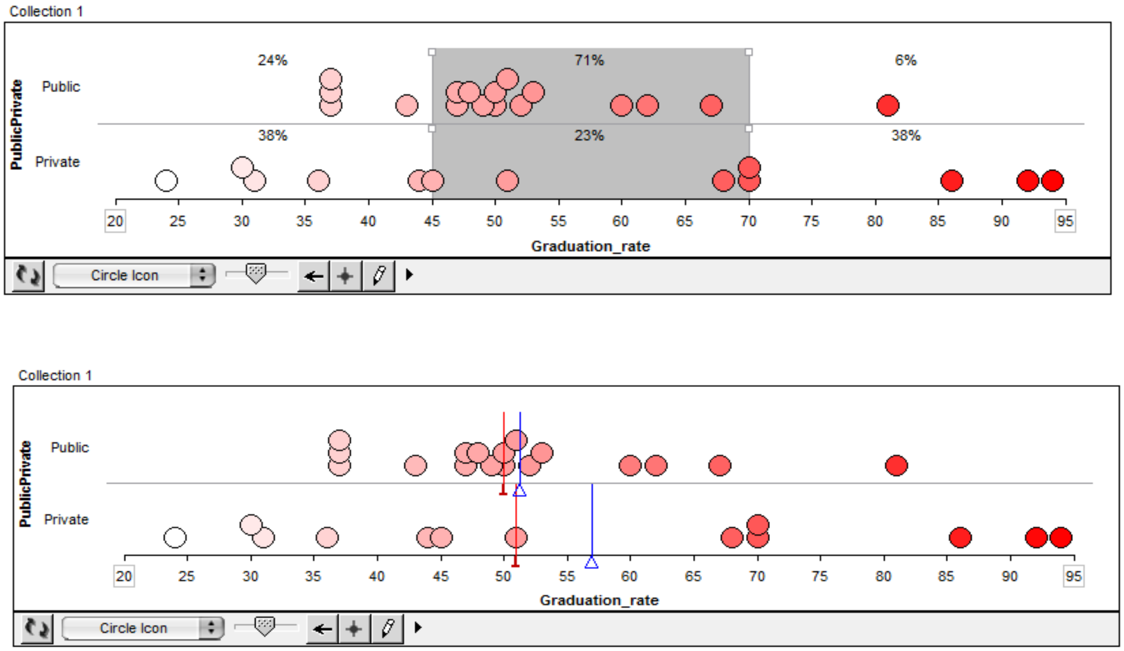


Figure 76. Prospective teacher's use of tools in *TinkerPlots*.

During the face-to-face class meeting, prospective teachers were given time in their small groups to discuss their work on the Chapter 2 assignment. They were encouraged to discuss Jordan’s and Kathy’s written work (Lee et al., 2010, pg. 39-40) and to use some questions from Chapter 2 Section 5 as a general guide. A whole group discussion followed during which the instructor allowed prospective teachers time to share some of the ideas discussed in small groups. She brought attention to evidence of students’ use of color in

TinkerPlots and reasons why the students may have included conflicting answers in their written work.

Chapter Three

Work with activities and tasks in Chapter 3 began immediately after the Chapter 2 discussion above. Similar to the online group, the face-to-face class began with information about some vehicles manufactured in 2006 and prospective teachers were asked to consider how Exploratory Data Analysis might be used with students. Many of the same statistical concepts related to measures of center and spread were discussed once again. However, with 2006 vehicles, prospective teachers explored data with the dynamic program *Fathom*.

Because a new technology was introduced in Chapter 3, approximately forty-seven percent of class time (111 minutes) spent in Chapter 3 was used by the instructor to introduce new content or demonstrate a skill in *Fathom*, as prospective teachers in the face-to-face group had only used the program once before. Twenty-six percent and twenty-three percent of class time for whole group and small group discussions, respectively, were spent discussing ideas surrounding the new technology, statistical content, and/or pedagogical issues that stemmed from activities in the text. If technological issues with *Fathom* appeared, prospective teachers would often quietly solicit the help of another participant seated nearby. Most of Chapter 3 in the textbook was studied during class. Figure 77 below summarizes the opportunities for interaction during instruction for Chapter 3.

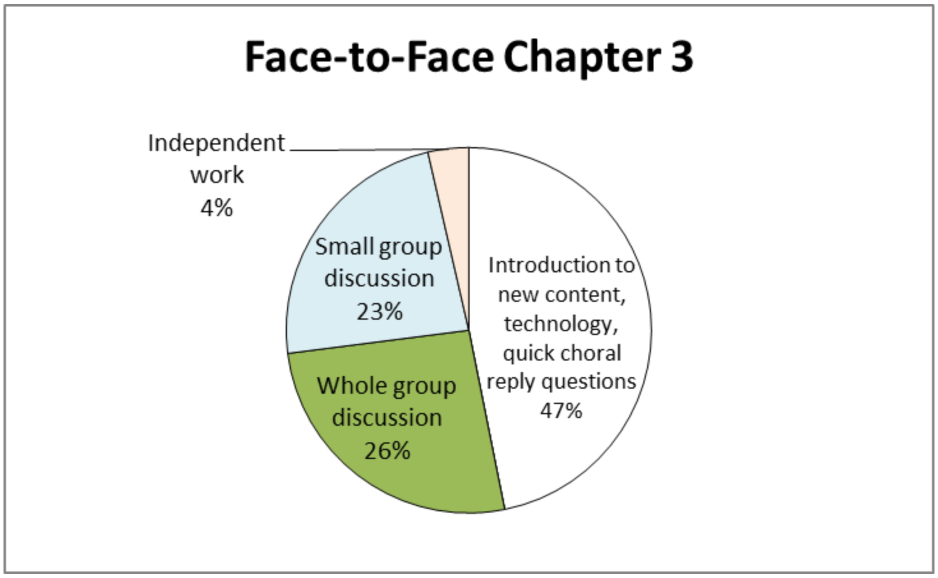


Figure 77. Opportunities for interaction in Chapter 3 of the face-to-face class.

The timeline below (Figure 78) depicts the sequence of activities during the 111 minutes spent studying Chapter 3. The arrow indicates when, over the course of the chapter, one class meeting ended and the next one began two days later.

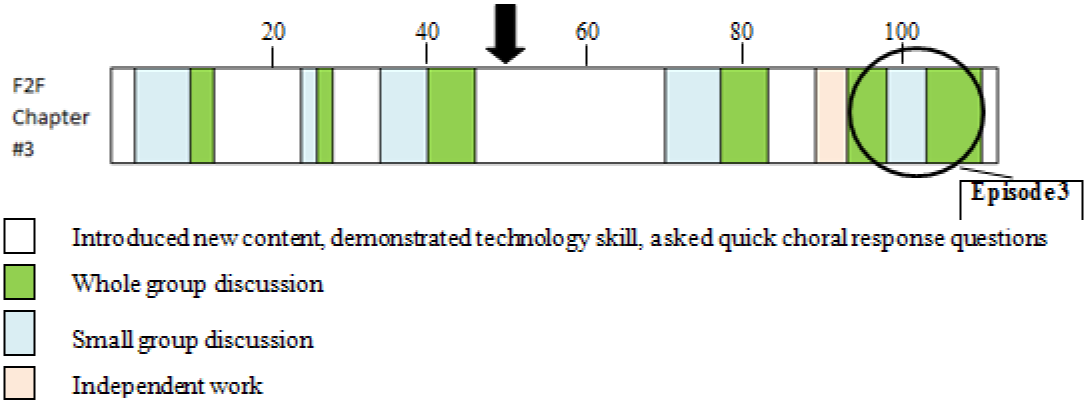
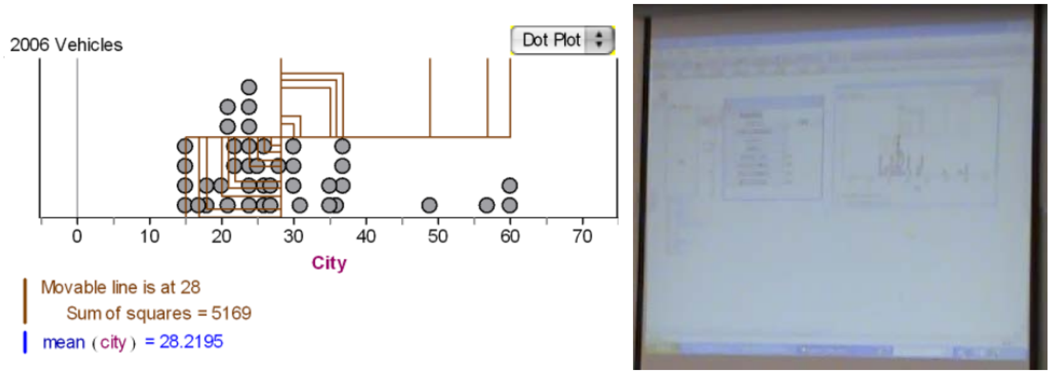


Figure 78. Timeline for Chapter 3 of the face-to-face class.

Setting of Episode Three for the Face-to-Face Class. At the beginning of this particular class meeting, the instructor reviewed univariate data analysis skills studied during the previous class. The question related to a “typical” value with three static images of graphical representations from *Fathom* (dot plot, box plot, and histogram) was projected. The instructor reminded the group to describe distributions with information related to center and spread. One prospective teacher, Cora, used a hand motion when describing a cluster of points. The instructor jocularly repeated the motion and said, “you can just claim that, the Cora cluster,” a phrase that prospective teachers in the class seemed to adopt and use later.

This episode contained discussion surrounding an activity from Chapter 3, Section 4, where prospective teachers were asked to explore standard deviation with data in *Fathom*. Specifically, prospective teachers created a dot plot and added a movable line. They were then prompted to add squares (Figure 79). The instructor noted the formula for standard deviation in the text and allowed prospective teachers time in small groups to discuss their successes and failures at interpreting the formula and the *Fathom* file simultaneously. A whole group discussion followed, in which the instructor tried to make explicit connections between the standard deviation formula and the dynamic representation in *Fathom* by asking prospective teachers to think about parts of the formula and how they might be explained by the representation seen below.



Movable line is at 28
Sum of squares = 5169
mean (city) = 28.2195

Figure 79. Movable line and squares with one attribute in *Fathom* projected in the face-to-face class.

Chapter Four

While Chapter 4 of the curriculum text built on previously learned skills in *Fathom*, recall that the statistical content of the text shifted as prospective teachers connected their understandings of univariate data analysis with ideas surrounding bivariate data analysis. Prospective teachers created a representation to illustrate how two quantitative attributes co-vary, similar to one pictured below (Figure 80), and considered the affordances and potential drawbacks of having the dynamic linkage of representations.

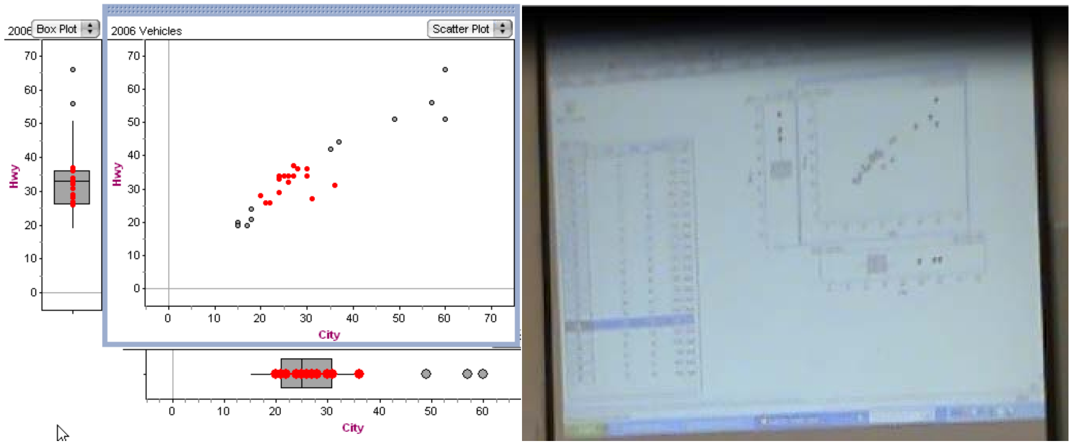


Figure 80. Building bivariate understanding in *Fathom* in the face-to-face class.

Prospective teachers also worked independently with a pre-created Fathom file (see Figure 24) from the text, which prompted them to adjust a slider so that the resulting scatter plot was similar to that from the City/Hwy data seen above, and used a summary table to determine the exact value of the correlation coefficient, *r,* for the City/Hwy data (see Figure 25). Prospective teachers then inserted a movable line in Fathom, and placed it in a location that "best represented the data." The instructor facilitated a lengthy group discussion about the equation for that movable line and how the slope and y-intercept should be interpreted given the context of the data. A misconception that the slope was always equal to the correlation coefficient was addressed with counterexamples.

Finally, the instructor engaged prospective teachers in a review of residuals and how to calculate those values. Prospective teachers were asked to insert squares into their graphical representation (Figure 81). The instructor said the squares were "very much connected to the ideas of squares with standard deviation" that the group had previously worked with." They were given time to try out this new skill and were asked to move their movable lines in order to minimize the sum of squares. Most of Sections 1 through 3 of Chapter 4 were studied during one class. Section 4 was assigned for homework and discussed the following class, along with Sections 5 and 6. Sections 7 and 8 of Chapter 4 were omitted.

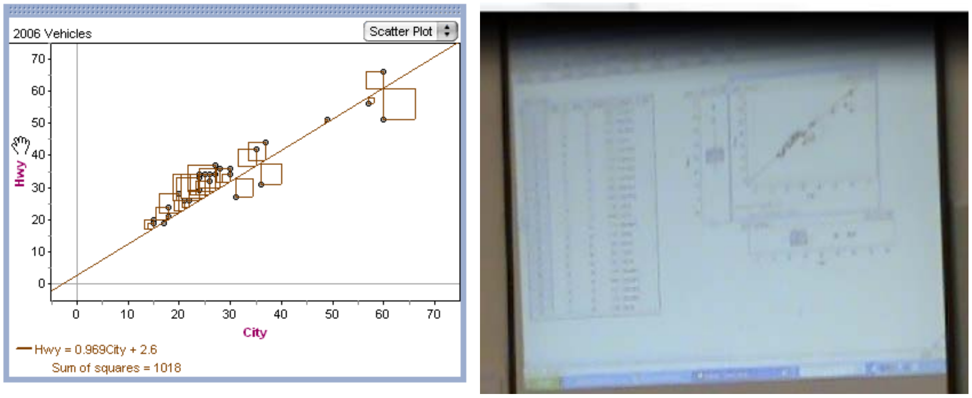


Figure 81. Movable line with squares in *Fathom* projected in the face-to-face class.

Partly because of the shift in content focus, thirty-two percent of class time (124 minutes) spent in Chapter 4 was used by the instructor to introduce new content or demonstrate a skill in *Fathom.* Forty percent and nineteen percent of class time for whole group and small group discussions, respectively, were spent discussing ideas surrounding the new technology, statistical content, and/or pedagogical issues that stemmed from activities in the text. Independent work accounted for nine percent of time spent in Chapter 4. If technological issues with *Fathom* appeared, prospective teachers would often quietly solicit the help of another participant seated nearby during this time.

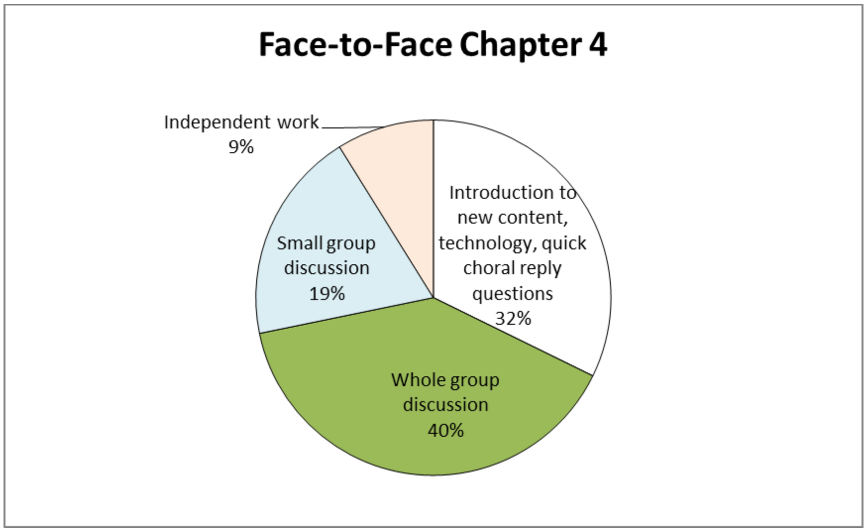


Figure 82. Opportunities for interaction during Chapter 4 of the face-to-face class.

The timeline below (Figure 83) indicates the sequential use of independent work, whole group discussions, and teacher-led instruction of new content. shows opportunities for discourse in Chapter 4 as they occurred during the face-to-face class. The whole group discussions in the episode that were analyzed spanned across two classes. Thus, the arrow indicates when prospective teachers were working on Chapter 4, Section 4 for homework and when one class ended and the next one began two days later.

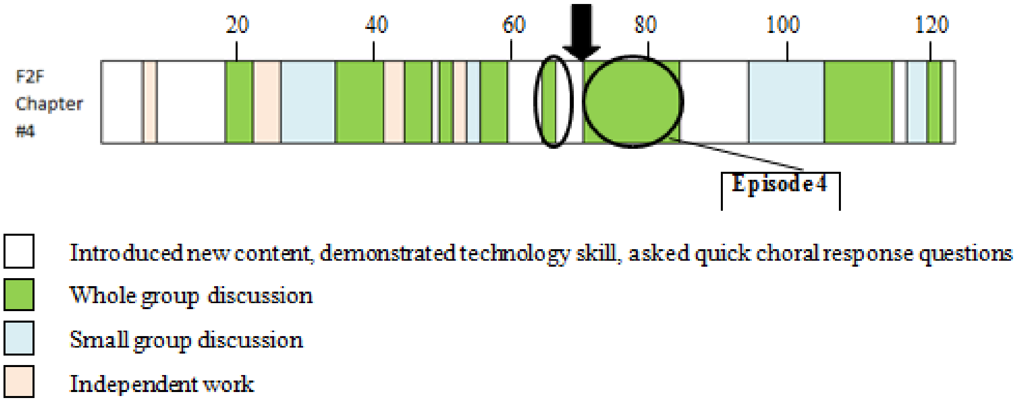
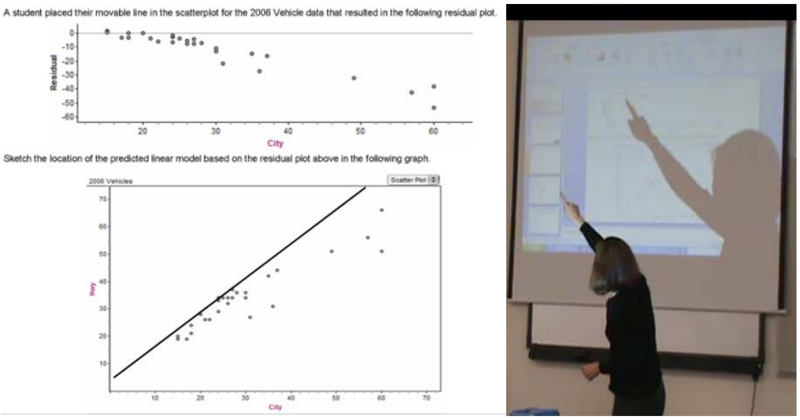


Figure 83. Timeline for Chapter 4 of the face-to-face class.

Setting of Episode Four for the Face-to-Face Class. This episode contained two activities, on two different days, in which prospective teachers were exploring sums of squares and residual plots with a movable line in *Fathom*. In the first activity, prospective teachers engaged in a whole group discussion centered on the sum of squares. Specifically, they were asked to move the movable line to minimize the sum and it became a contest to see who could find the smallest.

In the second activity, a whole group discussion surrounding Chapter 4, Section 4, which prospective teachers had completed for homework, was the focus of analysis. In that assignment, they were asked to interpret a residual plot and describe some of the conceptual difficulties students may have in interpreting and using the residual plot. While in the online class, one prospective teacher voluntarily shared his solution (see Figure 84 left), in the face-to-face class the instructor solicited ideas from prospective teachers but was in charge of the demonstration (see Figure 84 right).



A student placed their movable line in the scatterplot for the 2006 Vehicle data that resulted in the following residual plot.

Sketch the location of the predicted linear model based on the residual plot above in the following graph.

Figure 84. Interpretation of residual plots in the online (left) and face-to-face (right) classes.

Chapter Five

Beginning with activities in Chapter 5, face-to-face class discussions were focused on probability. The *TI*-84 graphing calculator and *Excel* were utilized in running simulations to begin to show effects of sample size on variability between samples and variability between empirical and theoretical probabilities. The instructor used *TI Smartview* during demonstrations and prospective teachers had access to a *TI-83+* or *TI-84* calculator during class.

Initially, one week (two 1.25 hour classes) was scheduled to complete activities in Chapter 5 in the face-to-face class. However, from prior experiences in teaching the course and from the knowledge that the online class had numerous technological issues with the graphing calculator that would not come up in the face-to-face class, the instructor knew she would not necessarily need that much time. Thus, she incorporated two 30-minute "working group" activities where prospective teachers in the face-to-face class collaborated on identifying objectives and brainstorming ideas for parts of a 4-phase lesson plan on data analysis or probability that utilized technology. Because working group discussions were not related to a specific activity from the curriculum text, but rather for an upcoming assignment, that time was not coded and is, therefore, not reflected in the following analysis.

Figure 85 provides information about the opportunities for discourse during the time spent in Chapter 5 (90 minutes). Instructor-directed activities (47%) and whole group discussions (43%) made up most of the class. Small group discussions (8%) and independent work (2%) were also part of class activities. Most tasks in Sections 1, 2, and 3 of Chapter 5, which rely heavily on the use of the graphing calculator, were completed during the first class of the week. A short homework assignment was given in which prospective teachers were asked to run several simulations of sample sizes 200 and 999 and enter their results in a Google Spreadsheet, which was displayed and used at the beginning of the next class. Section 4 of Chapter 5, which exclusively promotes the use of *Excel* for simulations, was also completed during that class. Technology issues with the graphing calculator and *Excel* were minimal.

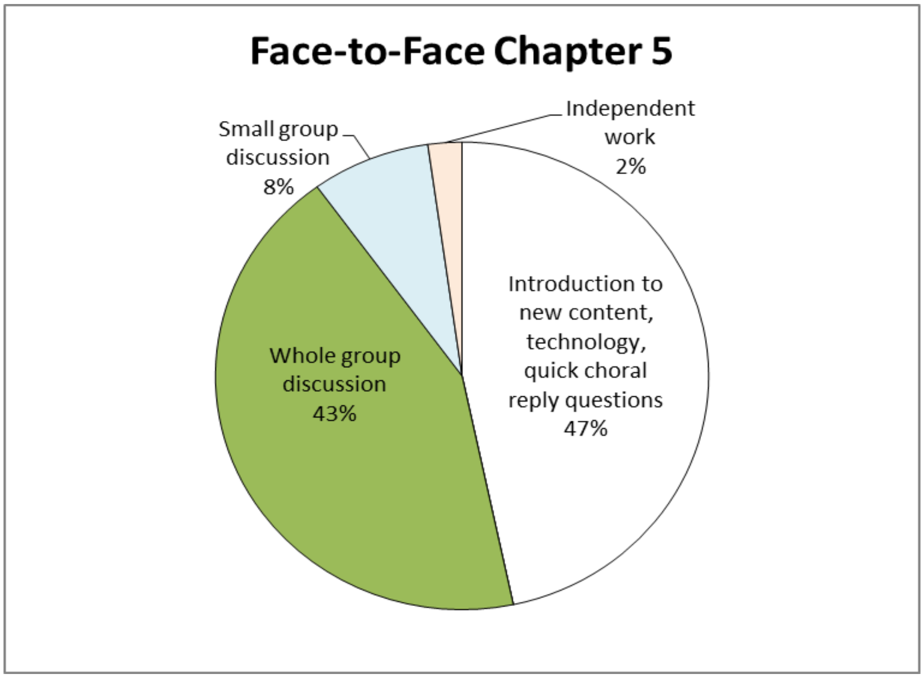


Figure 85. Opportunities for interaction in Chapter 5 of the face-to-face class.

The timeline below (Figure 86) illustrates the sequence of discusssions and introduction of new content in Chapter 5 as they occurred during the face-to-face class. Whole group discussions circled in the timeline indicate when Episode 5 occurred. The arrow represents when, during the chapter, one class ended and the next one began two days later.

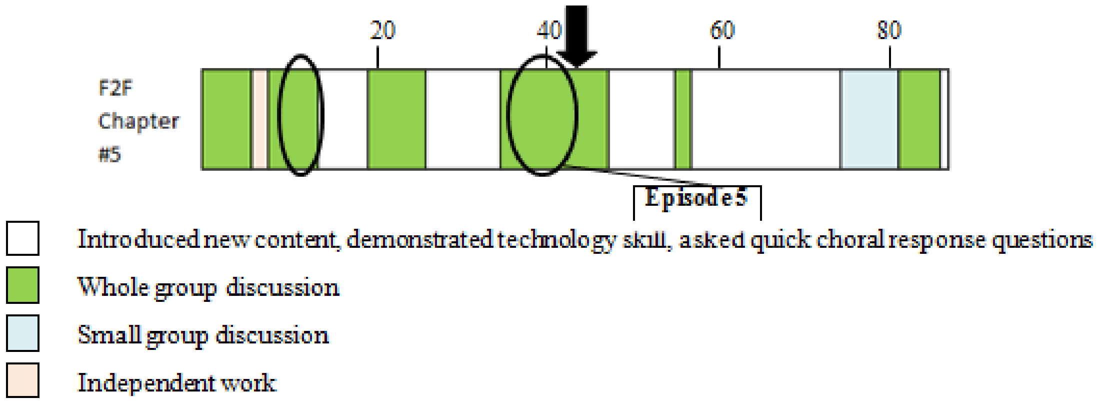
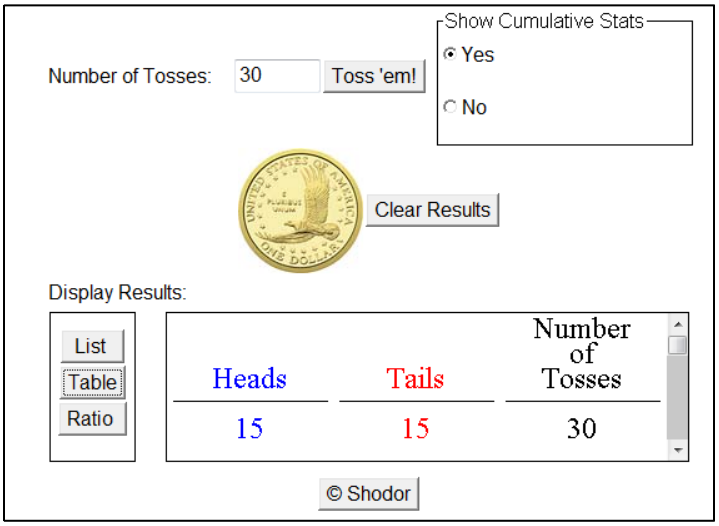


Figure 86. Timeline for Chapter 5 of the face-to-face class.

Setting of Episode Five for the Face-to-Face Class. This episode contained two activities that asked prospective teachers to begin thinking about difficulties students often have with probability. A quick review of deterministic and stochastic functions and a survey of prospective teachers' prior experiences with probability simulations were parts of the opening discussion. By asking for a quick show of hands, it was apparent that prospective teachers had mostly used coins in probability simulations and none had used real-world data in probability tasks.

Recall the activities in this chapter were focused on freshmen retention rate from some North Carolina colleges and universities (see Figure 34). When asked the probability of a randomly selected freshman returning at Chowan College, prospective teachers responded with values near 50%. With that in mind, it was decided that in a coin-toss simulation, "heads" would indicate the freshman returned and "tails" would indicate the freshman did not return. Prospective teachers used an online tool (Figure 87) to simulate a coin toss thirty times, find the proportion of freshmen returning, and indicate their value on a whiteboard at the front of the computer lab classroom.



Number of Tosses: 30

Show Cumulative Stats
● Yes
○ No

Heads	Tails	Number of Tosses
15	15	30

Figure 87. Coin toss simulation used in the face-to-face class (http://www.shodor.org/interactivate/activities/Coin/).

This episode began as prospective teachers were sharing results. As groups of participants came to the board, the instructor would ask, with regard to the distribution of proportions of freshmen returning, “what do we think is going to happen?” Like the online class, the instructor used a reference to the gray divider tool in *TinkerPlots* as a way to think about possible values, and asked prospective teachers to share where that gray box might need to be placed if the simulation was performed again.

The class continued with demonstrations about how to simulate this problem with the *TI*-84 graphing calculator. Finally, prospective teachers continued the problem of freshmen retention rate for Chowan College (50%) with a sample size of 500. Their results were recorded once again on the classroom whiteboard, only this time by the instructor (to save time) as prospective teachers reported their results aloud. This concluded Episode 5, but as a homework assignment the class was asked to perform five simulations of n=200 and n=999 and record their results in a pre-created Google spreadsheet. At the beginning of the next class, prospective teachers imported that data into *Fathom* so that differences and similarities between the distributions could be discussed (Figure 88).

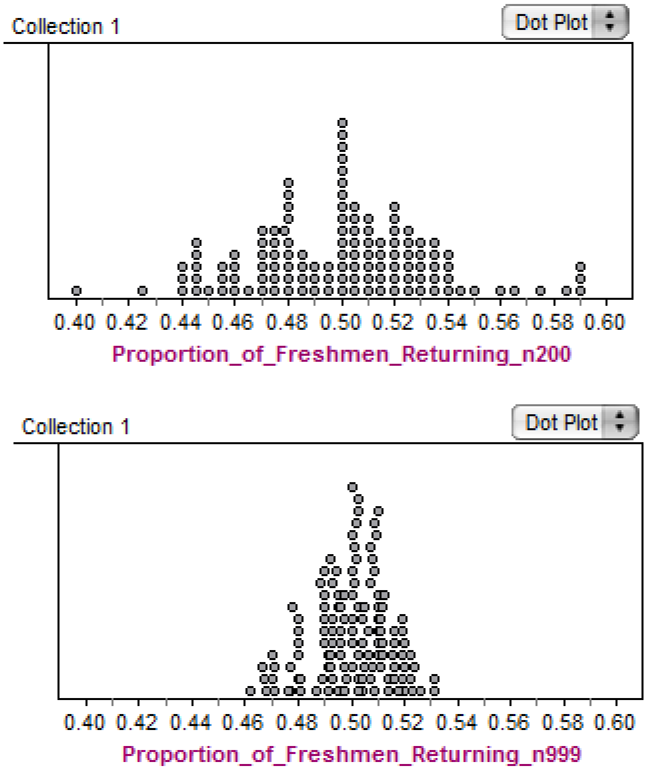


Figure 88. Dot plots created in *Fathom* with data collected by the face-to-face class.

Chapter Six

Prospective teachers' work with probability simulations continued with the programs *Probability Explorer*, *Fathom*, and the *TI-83+/84* graphing calculators. Once again, they had access to calculators during class. Because *Probability Explorer* is only compatible with PCs, prospective teachers who brought Macintosh laptops to class were allowed to work with someone else in the class. Participants used real-world data to influence the assumptions underlying each simulation. Specifically, birth data – the number of boy births versus the number of girl births in North Carolina in the year 2004 – were used in this lesson. Prospective teachers imported birth data from a website into *Fathom* and learned skills to take repeated samples in order to think about their distributions and whether or not certain outcomes were unusual. With *Probability Explorer,* prospective teachers used special tools to simulate births of two counties in North Carolina. Real-time changes assisted them with answering questions related to probability. Most of Chapter 6 was studied during class.

While the class had studied many features of *Fathom* in Chapters 3 and 4, many new skills for probability simulations were presented in Chapter 6. In addition, *Probability Explorer* was unfamiliar to prospective teachers as was using built-in functions on the *TI-83+/84* graphing calculators for the binomial formula. Hence, nearly half (51%) of the Chapter 6 total class time (151 minutes) was spent introducing new content and technology skills surrounding probability simulations. Technology problems with the graphing calculator and *Probability Explorer* were minimal. If technological issues with *Fathom* appeared, prospective teachers would often quietly solicit the help of another participant seated nearby. Other opportunities for discourse included seventeen percent for whole group discussions, seventeen percent for small group discussions, and fifteen percent for independent work.

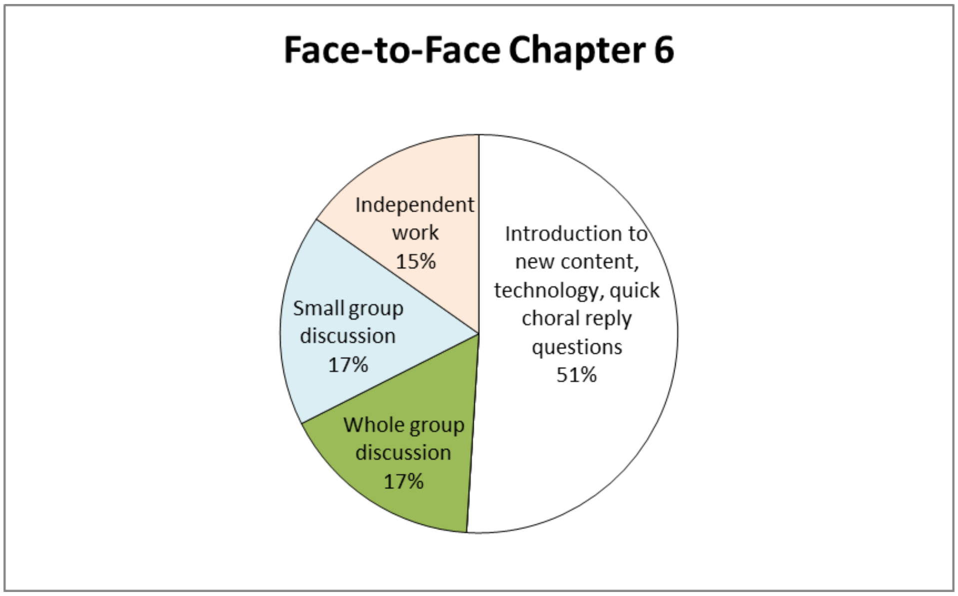


Figure 89. Opportunities for interaction in Chapter 6 of the face-to-face class.

The timeline below (Figure 90) indicates how whole group and some small group discussions were interspersed in the instruction on new content. The whole group discussion circled below occurs after prospective teachers have independently created a dot plot in *Fathom* based on real-world data imported from a website. The arrow represents when, during the chapter, one class ended and the next one began two days later.

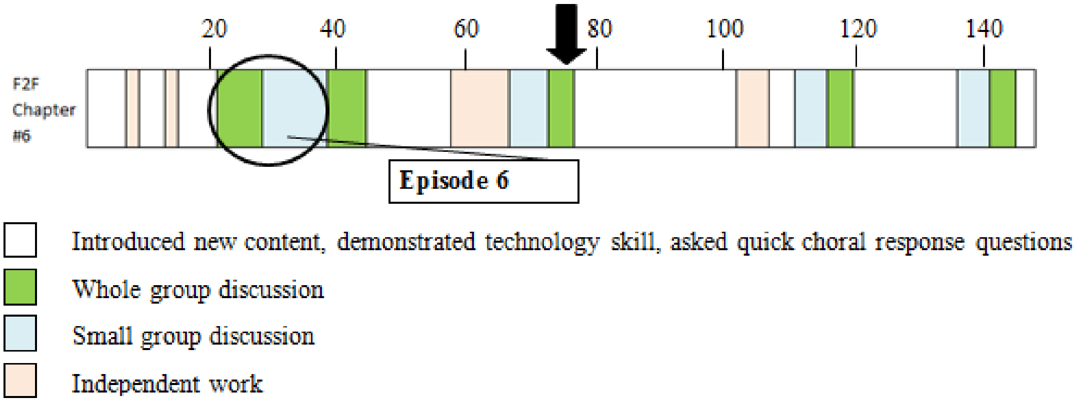
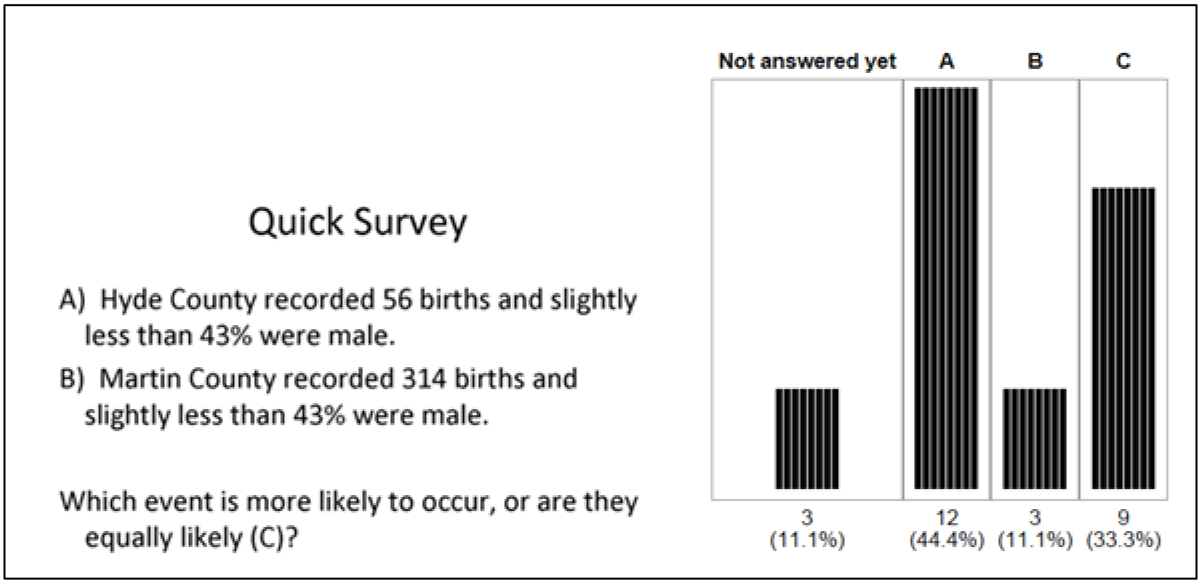


Figure 90. Timeline of Chapter 6 for the face-to-face class.

Setting of Episode Six for the Face-to-Face Class. This episode contained one activity regarding the proportion of male births in North Carolina. Based on real-world data, the probability of a live male birth is 0.51. Through different questions, prospective teachers were asked to consider the effects of sample size on distributions of proportions of male births. The technology used was *Fathom*. The instructor began by conducting a quick survey of the class. The question posed and the results are shown below in Figure 24. Note that 12 out of 25 prospective teachers in the face-to-face class had a correct choice initially.



Quick Survey

A) Hyde County recorded 56 births and slightly less than 43% were male.

B) Martin County recorded 314 births and slightly less than 43% were male.

Which event is more likely to occur, or are they equally likely (C)?

Figure 91. First Chapter 6 survey for the face-to-face class.

2004 birth data for each of the 100 counties in North Carolina was gathered from the internet and used to make a new collection in Fathom. Prospective teachers followed directions to create a new attribute called “PerctMale” to consider the proportion of males born in each county. Then, they were asked to create a dot plot of this new attribute (see Figure 40). This episode began with the instructor asking prospective teachers to examine the distribution of percent of male births. Specifically, she asked, “So do these data, do they seem to be clustered around 50 or some other value?” A whole group discussion followed as prospective teachers described how the cluster appeared to be slightly greater than 50.

Next, prospective teachers were instructed to add a vertical reference line and to overlay the “Total” attribute on the dot plot, which added more information to the plot through the use of color (see Figures 41 and 42). Purple represented the counties with the smallest populations while red represented the counties with the largest populations. Whole group discussion continued around this representation as prospective teachers considered the general trend in the spread of the data as the total number of births increased. At this point in the lesson, prospective teachers moved into their small groups to discuss several questions:

> “(1) Considering the mean percent of males and the spread of the data with counties with a large number of births (over 2000), what do you think is a reasonable estimate for the probability that a male is born? Justify your estimate.
> (2) Does assuming a 51% chance for males change your response to whether event A (43% with 56 births) or B (43% with 314 births) is more likely or if they are equally likely? Why or why not?
> (3) Why is it important in a probability task to have students state and understand the implications of assumptions about the likelihood of an outcome?” (Lee et al., 2010, pg. 136-137).

Discourse

While some details of the discourse that occurred during the five-week study were included in the overview and episode descriptions above, this section includes a much more descriptive and analytic account of the opportunities for interaction in the face-to-face class meetings. Based on analyses of lesson graphs and timelines of entire lessons, and transcripts of episodes from each chapter, this section contains findings about opportunities for discourse at the whole-group and small-group levels. A summary for each episode, of how prospective teachers chose to participate, is followed by additional information that was useful in explaining some of the number of occurrences stated. Data collected from individuals are also presented for triangulation to corroborate trends found from discussions which occurred during face-to-face classes.

Chapter One

Episode One – Small Group. Using a data set that included information about teacher salaries in the southern region of the United States, prospective teachers were asked to consider pros and cons of each of the measures of center being used to describe a “typical” teacher salary. Analysis of this small group discussion was difficult for two reasons. First, the discussion was only video recorded and due to other group discussions occurring simultaneously in the same room, it was difficult to hear what focus group members were saying. Because of this unfortunate circumstance, only some of the purposes and topics of their exchanges were discernable. Second, the focus group consisted of three prospective teachers. However, for this small group discussion, a fourth member of the class joined them. This may mean that the number of exchanges as well as the purposes and topics would have been different had she not participated with them.

Despite these issues, the number of exchanges is confidently recorded below. The table also includes some information about the direction, form, purpose, and topic of discourse during this small group discussion. Note that while each exchange has one direction and one form, it often contained more than one purpose and/or topic. Remember, too, that all questions asked, ideas shared, and affirmations were not captured by the video recording. This table, therefore, is not a summary of all that took place during this small group discussion.

Table 22. Face-to-Face Episode 1, small group discourse.

Episode 1, Small Group Discourse (57 exchanges, 6 min)		# of occurrences
Direction	Teacher-Small Group (T-SG)	57
Form	Talk	57
	Hand Gestures	4
	Points to Computer	1
Purpose	Asked a Question • Asked a new question	 6
	Answered a Question	5
	Shared an Idea or Concern	17
	Justified an Idea or Response	3
	Affirmed an Idea or Response	16
Topic	Class (logistical, curriculum text, etc.)	1
	Statistics • Statistical Knowledge	11 (6)
	Technology • Technological Statistical Knowledge	0 (0)
	Pedagogy • Technological Pedagogical Statistical Knowledge	0 (0)

As prospective teachers tried to craft arguments for using specific measures of center to describe a “typical” teacher salary, they tended to focus first on which measure would be most affected by “outliers” or “extreme values.” Sam even commented on how measures would change “if we removed DC” and gave a specific numeric value to where he wanted the case of DC moved. They also spoke about differences between a mathematical average and the multiple ways in which “average” is used in the English language. In fact, one participant even used her fingers to make quotations when saying the word “average.” Another interesting hand gesture was used when describing “large sections” of data. When describing a cluster of points, she used both hands with a horizontal in-and-out motion. In seeing this, another member of the group joined in with the same gesture. While the purpose and topic of group participants’ exchanges cannot be completely summarized, it did appear that the discussion focused mostly on discussed statistical content.

Episode One – Whole Group. Immediately following the small group discussion above, prospective teachers were invited to share ideas that were discussed regarding measures of center and a “typical” teacher salary. During the whole group discussion of Episode 1, twenty-two exchanges of communication occurred during approximately four and a half minutes. The table below provides an overview of the direction, form, purpose, and topic of discourse during this whole group discussion. Note that while each exchange has one direction, it often contained more than one form, purpose and/or topic.

Table 23. Face-to-Face Episode 1, whole group discourse.

Episode 1, Whole Group Discourse (22 exchanges, 4.5 min)		# of occurrences
Direction	Teacher-Whole Group (T-WG)	13
	Instructor-Whole Group (I-WG)	9
Form	Talk	22
	Hand Gestures	4
Purpose	Asked a Question	
	• Asked a new question	4
	• Restated a question from the text	2
	Answered a Question	3
	Shared an Idea or Concern	13
	Justified an Idea or Response	1
	Affirmed an Idea or Response	9

Table 23 Continued

Topic	Class (logistical, curriculum text, etc.)	1
	Statistics • Statistical Knowledge	16 (13)
	Technology • Technological Statistical Knowledge	5 (4)
	Pedagogy • Technological Pedagogical Statistical Knowledge	0 (0)

In this discussion, all questions were asked by the instructor. Hand gestures recorded in Table 23 were done exclusively by the instructor, since video camera did not capture prospective teachers in this discussion. She used hand motions when sharing techniques she had witnessed members of the class using in *TinkerPlots* during the previous small group discussion (e.g. showing percentages with the divider tool) and when she was describing variability, particularly when drawing attention to the “extreme values” in the data set. The one instance of justification came from a prospective teacher who provided statistical reasoning to another prospective teacher about why *TinkerPlots* was behaving a certain way with the percentages.

Nine different prospective teachers (36% of the whole group) along with the instructor participated in this discussion. As Figure 92 shows, the times when the instructor spoke (white) were often followed by a single reponse from a prospective teacher (purple). However, the discussion did begin with a sequential sharing of ideas from multiple prospective teachers.

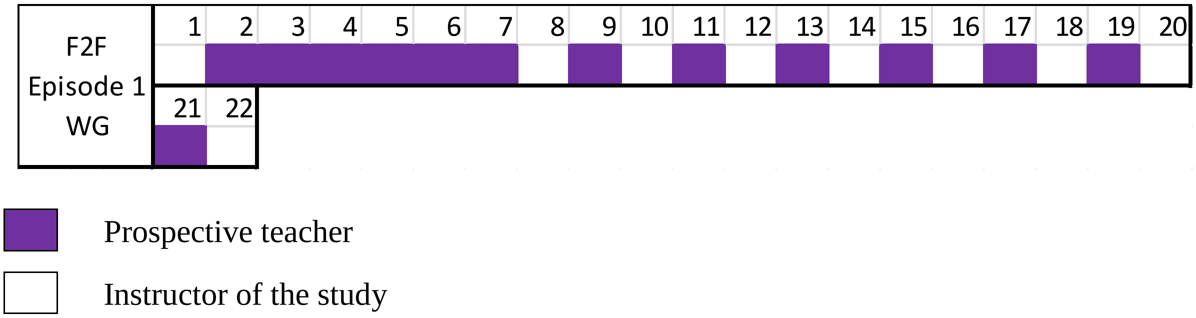


Figure 92. Face-to-Face Episode 1 whole group discussion pattern.

Chapter Two

Episode Two – Small Group. In small groups, prospective teachers reflected on their use of technology and how that was similar to and different from the work of Jordan and Kathy from the videocase. During the small group discussion of Episode 2, seventy-three exchanges of communication occurred during approximately five and a half minutes. The table below provides an overview of the direction, form, purpose, and topic of discourse during this small group discussion. Note that while each exchange has one direction, it sometimes contained more than one form, purpose, and/or topic.

Table 24. Face-to-Face Episode 2, small group discourse.

Episode 2, Small Group Discourse (73 exchanges, 5.5 min)		# of occurrences
Direction	Teacher-Small Group (T-SG)	73
Form	Talk	73
	Hand Gestures	3
	Points to Computer	1
Purpose	Asked a Question • Asked a new question • Restated a question from the text	 3 2
	Answered a Question	3
	Shared an Idea or Concern	37
	Justified an Idea or Response	3
	Affirmed an Idea or Response	31

Table 24 Continued

Topic	Class (logistical, curriculum text, etc.)	15
	Statistics • Statistical Knowledge	24 (0)
	Technology • Technological Statistical Knowledge	27 (6)
	Pedagogy • Technological Pedagogical Statistical Knowledge	29 (17)

The number of exchanges during this short small group discussion is telling. Focus group members were prepared to discuss the videocase and they were comfortable doing so. During the first part of their discussion, they tried to determine why Jordan and Kathy focused on a single attribute in their exploration. They often described, literally, how the middle school students completed the task; one prospective teacher in the group even acted out a mouse “clicking” as she said “she’s like the one in control of the computer and she’s just like clicking away and the girl’s just like ‘but, but, but’ and she’s like…” They did, however, think about pedagogical implications of the question that was posed to the students. After referencing the initial quotation of the task, Sam said, “It’s like by putting that statement in there, it’s like they restricted their minds… Like it would be interesting to see what would happen if they just crossed out that sentence and then just let them go.” Participants seemed to focus on TPSK issues often, but as the difference between TPSK and total number of exchanges shows (56) there were many more times when group members were affirming one another or just discussing pedagogical issues in general (e.g. one student seemed to be controlling the computer).

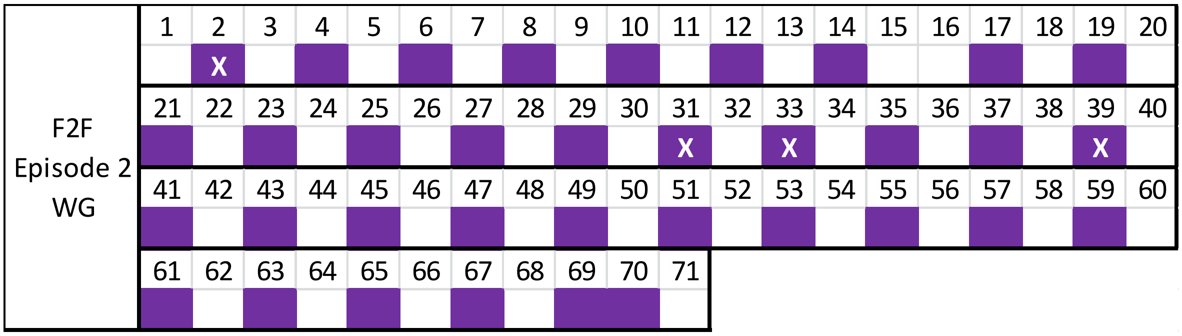
Episode Two – Whole Group. Immediately following the small group discussion above, prospective teachers shared ideas about Jordan and Kathy's statistical understanding and how *TinkerPlots* helped or hindered their work in the videocase. During the whole group discussion of Episode 2, seventy-one exchanges of communication occurred during approximately eleven and a half minutes. The table below provides an overview of the direction, form, purpose, and topic of discourse during this whole group discussion. Note that while each exchange has one direction, it sometimes contained more than one form, purpose, and/or topic.

Table 25. Face-to-Face Episode 2, whole group discourse.

Episode 2, Whole Group Discourse (71 exchanges, 11.5 min)		# of occurrences
Direction	Teacher-Whole Group (T-WG)	35
	Instructor-Whole Group (I-WG)	36
Form	Talk	71
	Hand Gestures	3
	Points to Computer	1
	Raised Hand	1
Purpose	Asked a Question • Asked a new question • Restated a question from the text	 17 1
	Answered a Question	13
	Shared an Idea or Concern	35
	Affirmed an Idea or Response	14
Topic	Class (logistical, curriculum text, etc.)	5
	Statistics • Statistical Knowledge	41 (1)
	Technology • Technological Statistical Knowledge	48 (10)
	Pedagogy • Technological Pedagogical Statistical Knowledge	43 (31)

Participants were comfortable sharing ideas from the previous small group discussions. The fact that Jordan and Kathy focused mostly on a single attribute continued to be a favorite point of discussion for this group of prospective teachers. They shared many reasons for the middle school students exploring the task the way they did. Prospective teachers also focused on color, a characteristic of *TinkerPlots* that they had personally enjoyed. When one shared that they believed the students "still had trouble getting the idea that one color was private and the other was public," the instructor responded with, "and what did they do that let you know that?" Several prospective teachers in the class provided evidence from the video, in unison almost, that would lead one to that conclusion. As expected, due to the nature of the chapter and videocase, much of the discussion had a TPSK focus. But, difficult to capture in this whole group discussion were forms other than talking by prospective teachers. Thus, all hand gesturing, pointing to computer, and raising hand reflected in the table above was done by the instructor.

Seventeen different prospective teachers (68% of the whole group) along with the instructor participated in this discussion. As Figure 93 shows, the times when the instructor spoke (white) were mostly followed by a single reponse from a prospective teacher (purple). Many times, prospective teachers were building on ideas from one another and the instructor was just affirming or acknowledging prospective teachers in between. Nevertheless, the speaking pattern is obvious. What this analysis fails to capture, however, were the times when multiple responses were shared simultaneously after the instructor asked a choral-response type question. These occurred at exchanges 2, 31, 33, and 39 in the discussion pattern below, and are denoted with an "X." Unlike the online setting, where simultaneous chats were all recorded, it was difficult to discern which prospective teachers participated at those times of the whole group discussion.



F2F Episode 2 WG	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
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	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
	🟪	⬜	🟪	⬜	🟪	⬜	🟪	⬜	🟪	⬜	🟪 X	⬜	🟪 X	⬜	🟪	⬜	🟪	⬜	🟪 X	⬜
	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
	🟪	⬜	🟪	⬜	🟪	⬜	🟪	⬜	🟪	⬜	🟪	⬜	🟪	⬜	🟪	⬜	🟪	⬜	🟪	⬜
	61	62	63	64	65	66	67	68	69	70	71									
	🟪	⬜	🟪	⬜	🟪	⬜	🟪	⬜	🟪	🟪	⬜									

🟪 Prospective teacher
X Multiple prospective teachers respond to a question simultaneously
⬜ Instructor of the study

Figure 93. Face-to-face Episode 2 whole group discussion pattern.

Chapter Three

Episode Three – Small Group. Using the 2006 vehicles file from the text, prospective teachers were asked to craft an argument about which engine type was best (using ideas of center and spread, including standard deviation) and then discuss how they might introduce standard deviation as a teacher (picture or formula first). During the small group discussion of Episode 3, thirty-nine exchanges of communication occurred during approximately six minutes. The table below provides an overview of the direction, form, purpose, and topic of discourse during this small group discussion. Note that while each exchange has one direction, it sometimes contained more than one form, purpose, and/or topic.

Table 26. Face-to-Face Episode 3, small group discourse.

Episode 3, Small Group Discourse (39 exchanges, 6 min)		# of occurrences
Direction	Teacher-Small Group (T-SG)	38
	Instructor-Whole Group (I-WG)	1
Form	Talk	39
	Hand Gestures	1
	Points to Computer	1
Purpose	Asked a Question • Asked a new question • Restated a question from the text	 5 1
	Answered a Question	3
	Shared an Idea or Concern	19
	Affirmed an Idea or Response	13
Topic	Class (logistical, curriculum text, etc.)	9
	Statistics • Statistical Knowledge	15 (1)
	Technology • Technological Statistical Knowledge	14 (11)
	Pedagogy • Technological Pedagogical Statistical Knowledge	5 (3)

Although the discussion included prompts that would require TSK and TPSK foci, the two focus group members present that day spent much time with unsolicited Exploratory Data Analysis. In other words, rather than using the statistical measures displayed in *Fathom*, these prospective teachers speculated why one engine type represented in the data was better than another. Perhaps this might explain the lower TPSK occurrences represented in the table above. They spoke freely about their own experiences. Ava said, “I have a Prius… and it does not get that good of gas mileage.” They also shared their potential bias and felt that students may be biased as well. The following exchanges show how these prospective teachers believed their bias was affecting their ability to craft an argument and that students would have the same difficulty.

> *Sam: It's hard to not like project what you already thought about engines before looking at this on the screen.*
> *Ava: Yeah, I feel students will be the same way because they're always hearing about hybrids…*
> *Sam: Yeah.*
> *Ava: …and how great they are.*
> *Sam: For sure.*
> *Ava: No one really talks about diesel engines.*
> *Sam: Yeah, they'll probably be pretty biased towards hybrids.*
> *Ava: Yeah.*

The dialogue above also shows a common pattern of sharing ideas and affirmations among prospective teachers. It also illustrates their attention to the importance of context in statistics and how one's experience with a context can influence your focus on a data set. At the end of the small group discussion, some lack of confidence was also revealed. When discussing whether, as a teacher, they would introduce the formula for standard deviation first or use a visual representation like the one presented in the textbook (Lee et al., 2010, pg. 62) or the one created during class using *Fathom*, Sam said, "I think the diagram first was like the way to go because that formula was just intimidating."

Episode Three – Whole Group. The instructor facilitated a whole group discussion to connect the dynamic representation in *Fathom* with the standard deviation formula and encourage prospective teachers to share how they might use a visual representation when teaching students. During the whole group discussion of Episode 3, fifty-eight exchanges of communication occurred during approximately seventeen and a half minutes. The table below provides an overview of the direction, form, purpose, and topic of discourse during this whole group discussion. Note that while each exchange has one direction, it often contained more than one form, purpose, and/or topic.

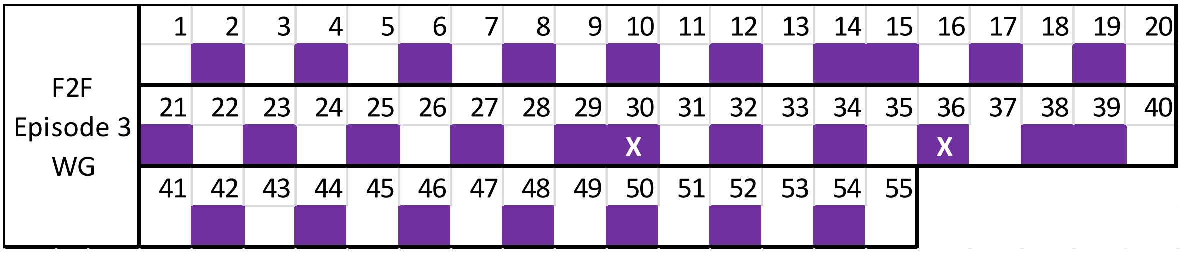
Table 27. Face-to-Face Episode 3, whole group discourse.

Episode 3, Whole Group Discourse (55 exchanges, 17.5 min)		# of occurrences
Direction	Teacher-Whole Group (T-WG)	28
	Instructor-Whole Group (I-WG)	27
Form	Talk	55
	Hand Gestures	9
	Points to Computer	2
Purpose	Asked a Question • Asked a new question • Restated a question from the text	 20 1
	Answered a Question	21
	Shared an Idea or Concern	21
	Justified an Idea or Response	1
	Affirmed an Idea or Response	12
Topic	Statistics Statistical Knowledge	43 (7)
	Technology Technological Statistical Knowledge	35 (25)
	Pedagogy • Technological Pedagogical Statistical Knowledge	14 (9)

Due to the nature of the video recording, it was impossible to capture forms other than talking by prospective teachers. Thus, all hand gesturing and pointing to computer represented in the table above was done by the instructor. Often they were directly related to the content or technology being discussed. For example, when prospective teachers were using a movable line with squares to better understand standard deviation, the instructor described the “little squares coming off of that” by using her fingers to make what seemed to be outlines of squares.

There were many questions asked during this discussion. But, only two of them were asked by prospective teachers and the content of those questions was technological. It was expected that much of the focus of this discussion would be on statistical knowledge as it was centered around minimizing the sum of squares and connecting the formula for standard deviation to the visual representation in *Fathom*. But, as soon as the instructor added a pedagogical focus, the group seemed to consistently be thinking about implications of technology on student learning. They again spoke candidly about their unfamiliarity with the standard deviation formula and understanding better with *Fathom*. Ellen Anne said, “Just being able to like see this and understand ok, you know, I have no idea. I couldn’t use that formula to save my life, but, just like looking at this helped me kind of, oh, well this is what it’s generally about.” Another prospective teacher, Sarah, shared her enjoyment with using technology. She jokingly said that her students would be instructed how to use the standard deviation formula prior to using *Fathom* because “if you learn the technology it’s more fun to do… they will get angry going back to the formula.”

Thirteen different prospective teachers (52% of the whole group) along with the instructor participated in this discussion. As Figure 94 shows, the times when the instructor spoke (white) were mostly followed by a single reponse from a prospective teacher (purple). Many times, prospective teachers were building on ideas from one another and the instructor was just affirming or acknowledging prospective teachers in between. There were times when multiple responses were shared simultaneously after the instructor asked a choral-response-type question. These occurred at exchanges 30 and 36 in the discussion pattern below, and are denoted with an “X.” It was difficult to discern which prospective teachers participated at those times of the whole group discussion.



F2F Episode 3 WG	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
		🟪		🟪		🟪		🟪		🟪		🟪		🟪	🟪		🟪		🟪	
	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
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	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55					
		🟪		🟪		🟪		🟪		🟪		🟪		🟪						

🟪 Prospective teacher
X Multiple prospective teachers respond to a question simultaneously

☐ Instructor of the study

Figure 94. Face-to-Face Episode 3 whole group discussion pattern.

Chapter Four

Episode Four – Whole Group. The implementation of the activities of this episode in the face-to-face class varied from that of the online class. Specifically, questions from Chapter 4 that were the focus of the small group discussion in the online group (e.g. residual plot and difficulties students may have) were used during a whole group discussion with the face-to-face group. This difference was not intentional. But, it means that the whole group discussion analyzed below is actually two separate whole group discussions compiled in one table, one three minute discussion and one ten minute discussion. Therefore, the table below contains two separate columns, one for each whole group discussion in this episode; it provides an overview of the direction, form, purpose, and topic of discourse during these whole group discussions.

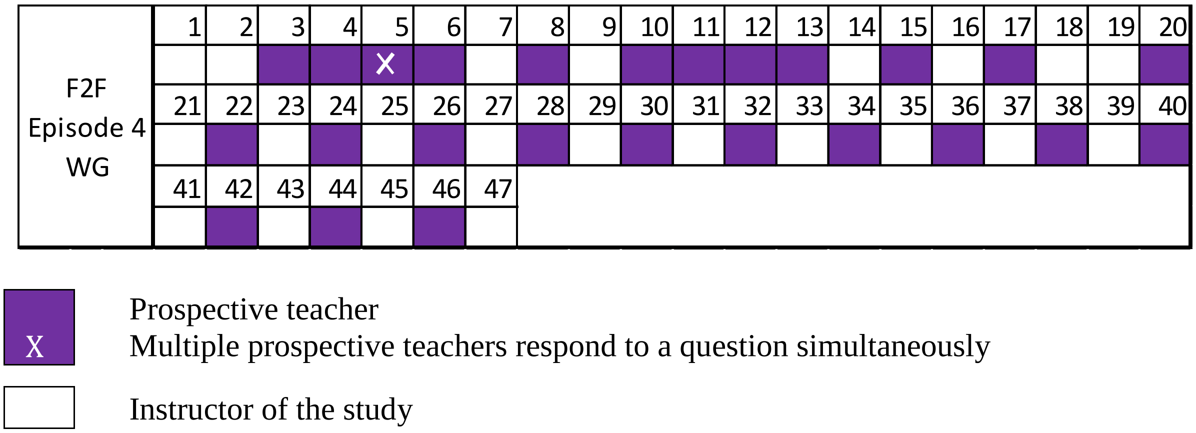
Table 28. Face-to-Face Episode 4, whole group discourse.

Episode 4, Whole Group Discourse (47 exchanges, 13 min)		# of occurrences WG #1	# of occurrences WG #2
Direction	Teacher-Whole Group (T-WG)	11	14
	Instructor-Whole Group (I-WG)	7	15
Form	Talk	18	29
	Hand Gestures	0	6
	Points to Computer	0	6
Purpose	Asked a Question • Asked a new question	5	11
	Answered a Question	6	9
	Shared an Idea or Concern	6	15
	Justified an Idea or Response	1	0
	Affirmed an Idea or Response	1	10
Topic	Class (logistical, curriculum text, etc.)	2	0
	Statistics • Statistical Knowledge	15 (0)	25 (5)
	Technology • Technological Statistical Knowledge	15 (15)	20 (10)
	Pedagogy • Technological Pedagogical Statistical Knowledge	0 (0)	10 (10)

Due to the nature of the video recording, recall that all hand gesturing and pointing to computer represented in the table above was done by the instructor. Also, most questions during this discussion were asked by the instructor. Only two questions were asked by prospective teachers. Once, when participants were trying to fit a movable line to the data, a prospective teacher asked another for the equation of his line. The second question from a prospective teacher was statistical in nature as he inquired about what a pattern in a residual plot might look like.

At least fourteen different prospective teachers (56% of the whole group) along with the instructor participated in this discussion. It was difficult to capture multiple responses that were shared simultaneously after the instructor asked a choral-response type question. This occurred at exchange #5 in the discussion pattern below. At other times during the discussion (#3, 4, 6, 17), prospective teachers were clearly speaking one at a time, but it was not always easy to discern who was speaking.

As Figure 95 shows, there were many times when the instructor spoke (white) that a single reponse from a prospective teacher (purple) followed. Many times, prospective teachers were building on ideas from one another and the instructor was just affirming or acknowledging prospective teachers in between. The two times (#3-6 and 10-13) when multiple responses were presented were times when the instructor asked prospective teachers "what's the smallest sum of squares that you were able to get with your movable line?" This created a spontaneous contest, almost, between participants to see who could get the smallest sum. Prospective teachers did not talk over one another, though. They reported their sums to the group one at a time. It was evident from the video, based on the amount of talking and the tone/laughter that could be heard, that this question generated much excitement and interest among prospective teachers.



F2F Episode 4 WG	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
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	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
	☐	■	☐	■	☐	■	☐	■	☐	■	☐	■	☐	■	☐	■	☐	■	☐	■
	41	42	43	44	45	46	47													
	☐	■	☐	■	☐	■	☐													

■ Prospective teacher
X Multiple prospective teachers respond to a question simultaneously

☐ Instructor of the study

Figure 95. Face-to-Face Episode 4 whole group discussion pattern.

Chapter Five

Episode Five – Whole Group. Prospective teachers, having just completed the coin simulation, were asked to anticipate an “acceptable” range if the simulations were run again. Discussion also focused on describing distributions of increasing size. During the whole group discussions of Episode 5, fifty-one exchanges of communication occurred during approximately ten and a half minutes. The table below provides an overview of the direction, form, purpose, and topic of discourse during this small group discussion. Note that while each exchange has one direction, it often contained more than one form, purpose, and/or topic.

Table 29. Face-to-Face Episode 5, whole group discourse.

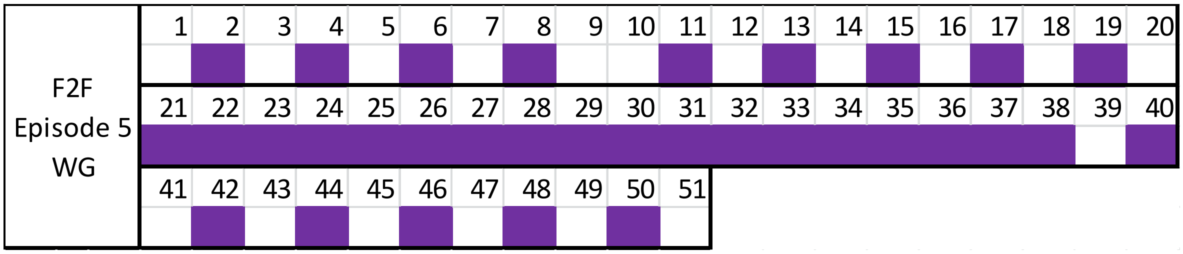
Episode 5, Whole Group Discourse (51 exchanges, 10.5 min)		# of occurrences
Direction	Teacher-Whole Group (T-WG)	33
	Instructor-Whole Group (I-WG)	18
Form	Talk	51
	Points to Computer	1

Table 29 Continued

Purpose	Asked a Question • Asked a new question	13
	Answered a Question	28
	Shared an Idea or Concern	13
	Justify	1
	Affirmed an Idea or Response	7
Topic	Class (logistical, curriculum text, etc.)	3
	Statistics • Statistical Knowledge	46 (43)
	Technology • Technological Statistical Knowledge	5 (4)
	Pedagogy • Technological Pedagogical Statistical Knowledge	0 (0)

All but one of the questions and affirmations came from the instructor. One prospective teacher asked a technology question, and another affirmed a simulation result the instructor wrote on the whiteboard. Prospective teachers certainly attended to statistical content throughout this discussion. The absence of TPSK focus revealed that the instructor made a decision during instruction not to address pedagogical issues at that time.

Twenty-one out of twenty-five prospective teachers were in class that day (84%). All of them participated in this whole group discussion. Figure 96 shows an interesting pattern in this episode. In the beginning, there were many times when the instructor spoke (white) that a single reponse from a prospective teacher (purple) followed. Many times, prospective teachers were building on ideas from one another and the instructor was just affirming or acknowledging prospective teachers in between. There was one obvious time during the lesson when responses were being solicited from each prospective teacher present at that time. At this point in the discussion, the instructor asked each participant to one-by-one share the result of his/her simulation in order to create a dot plot on the whiteboard (#17, #19, #21-38, #40). Thus, what was being shared was only a report of a simulation result, which was coded as “answering a question” since the instructor had asked for these results. Then, the intructor-participant speaking pattern resumed.



F2F Episode 5 WG	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	⬜	🟪	⬜	🟪	⬜	🟪	⬜	🟪	⬜	⬜	🟪	⬜	🟪	⬜	🟪	⬜	🟪	⬜	🟪	⬜
	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
	🟪	🟪	🟪	🟪	🟪	🟪	🟪	🟪	🟪	🟪	🟪	🟪	🟪	🟪	🟪	🟪	🟪	🟪	⬜	🟪
	41	42	43	44	45	46	47	48	49	50	51									
	⬜	🟪	⬜	🟪	⬜	🟪	⬜	🟪	⬜	🟪	⬜									

🟪 Prospective teacher

⬜ Instructor of the study

Figure 96. Face-to-Face Episode 5 whole group discussion pattern.

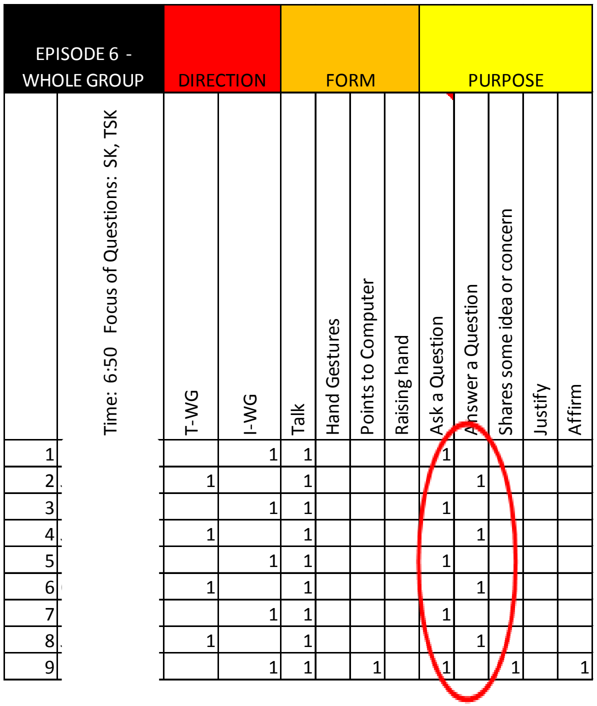
Chapter Six

Episode Six – Whole Group. Having completed the birth data survey task (for Hyde and Martin Counties), prospective teachers described the distribution of birth data for all 100 counties of NC emphasizing noticeable differences between smaller and larger counties. During the whole group discussion of Episode 6, twenty-nine exchanges of communication occurred during approximately seven minutes. The table below provides an overview of the direction, form, purpose, and topic of discourse during this small group discussion. Note that while each exchange has one direction, it often contained more than one form, purpose and/or topic.

Table 30. Face-to-Face Episode 6, whole group discourse.

Episode 6, Whole Group Discourse (29 exchanges, 7 min)		# of occurrences
Direction	Teacher-Whole Group (T-WG)	14
	Instructor-Whole Group (I-WG)	15
Form	Talk	29
	Points to Computer	4
Purpose	Asked a Question - Asked a new question	 14
	Answered a Question	12
	Shared an Idea or Concern	8
	Justified an Idea or Response	0
	Affirmed an Idea or Response	7
Topic	Statistics Statistical Knowledge	23 (7)
	Technology - Technological Statistical Knowledge	16 (14)
	Pedagogy - Technological Pedagogical Statistical Knowledge	2 (2)

All questions were asked by the instructor and all but one affirmation were given by the instructor. A cycle, which included the instructor asking a question and a prospective teacher answering the question, was evident from the beginning of the discussion. The circled portion of Figure 97 below shows how a portion of the transcript was coded and highlights the cycle described above.

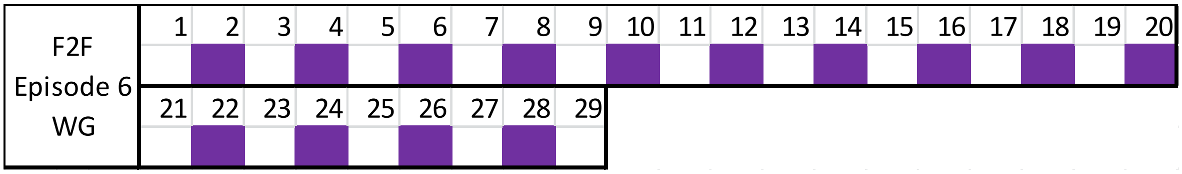


EPISODE 6 - WHOLE GROUP		DIRECTION		FORM				PURPOSE				
	Time: 6:50 Focus of Questions: SK, TSK	T-WG	I-WG	Talk	Hand Gestures	Points to Computer	Raising hand	Ask a Question	Answer a Question	Shares some idea or concern	Justify	Affirm
1			1	1				1				
2		1		1					1			
3			1	1				1				
4		1		1					1			
5			1	1				1				
6		1		1					1			
7			1	1				1				
8		1		1					1			
9			1	1		1		1		1		1

Figure 97. Partial coding for face-to-face Episode 6 small group.

There were only two instances of TPSK in this whole group discussion. This was not surprising, particularly in light of whole group discourse results from Episode 5 and knowing that the content focus in Episode 6 remained that of the law of large numbers. There was, however, a greater focus on TSK than in Episode 5. This is likely due to the use of a new program, *Probability Explorer*, and more advanced skills in *Fathom*.

At least ten prospective teachers (40% of the whole group) participated, along with the instructor, in this whole group discussion. Prospective teachers were clearly speaking one at a time, but it was not always easy to discern who was speaking. Nevertheless, the discussion pattern in Figure 98, illustrates the cycle mentioned above for the whole group discussion of this episode.



F2F Episode 6 WG	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	⬜	🟪	⬜	🟪	⬜	🟪	⬜	🟪	⬜	🟪	⬜	🟪	⬜	🟪	⬜	🟪	⬜	🟪	⬜	🟪
	21	22	23	24	25	26	27	28	29											
	⬜	🟪	⬜	🟪	⬜	🟪	⬜	🟪	⬜											

🟪 Prospective teacher

⬜ Instructor of the study

Figure 98. Face-to-face Episode 6 whole group discussion pattern.

Episode Six – Small Group. Immediately following the whole group discussion above, prospective teachers were asked to discuss a reasonable estimate for the probability of a male birth and the importance of assumptions in probability problems. During the small group discussion of Episode 6, forty-four exchanges of communication occurred during approximately nine minutes. The table below provides an overview of the direction, form, purpose, and topic of discourse during this small group discussion. Note that while each exchange has one direction, it often contained more than one form, purpose and/or topic.

Table 31. Face-to-Face Episode 6, small group discourse.

Episode 6, Small Group Discourse (44 exchanges, 9 min)		# of occurrences
Direction	Teacher-Small Group (T-SG)	44
Form	Talk	44
	Hand Gestures	4
Purpose	Asked a Question • Asked a new question • Restated a question from the text	 3 6
	Answered a Question	4
	Shared an Idea or Concern	30
	Affirmed an Idea or Response	15

Table 31 Continued

Topic	Class (logistical, curriculum text, etc.)	5
	Statistics • Statistical Knowledge	27 (17)
	Technology • Technological Statistical Knowledge	10 (6)
	Pedagogy • Technological Pedagogical Statistical Knowledge	5 (4)

One interesting finding was the mismatch between the number of questions asked by prospective teachers and the number of questions answered, which was lower. At the same time there were numerous exchanges where ideas were shared. Group members willingly shared their lack of confidence in answering questions from the text. When discussing the question regarding birth data and which result was more unusual (see Figure 91), Carrie said "I don't know. I say they are equally likely. But I really have no idea." Ava replied, "I don't know." The same type of conversation occurred at other places in the discussion as well (e.g. why are assumptions important). So, throughout the discussion there were ideas and concerns presented, but it was not obvious that each question had truly been answered. Perhaps this is one explanation for the small number of TPSK-focused exchanges represented in the table above.

Trends and Patterns in Discourse

For each chapter of the curriculum used in this study, implementation of the curriculum, and opportunities for discourse in the face-to-face class have been analyzed. This was done by analyzing episodes of whole group and small group (when possible) discussions within each chapter. It was difficult to compare themes and patterns across chapters since only a portion of the face-to-face class surrounding that chapter was studied closely. However, there are trends that are worth noting that certainly affected discourse in that environment. Some of those trends resulted from decisions made by the instructor in designing and managing the class. Others resulted from preferences and personalities of prospective teachers participating in the study and are presented as trends in purpose and topic.

Trends in Class Design and Management

Viewing timelines for all chapters (sans Chapter 2) simultaneously confirms that the placement of discussion activities seemed to be consistent. Whole group discussion time was interspersed throughout each lesson (Figure 99), and was kept to less than fifteen minutes in most instances. In addition, throughout the study, each time small group discussion occurred it was immediately followed by whole group discussion.

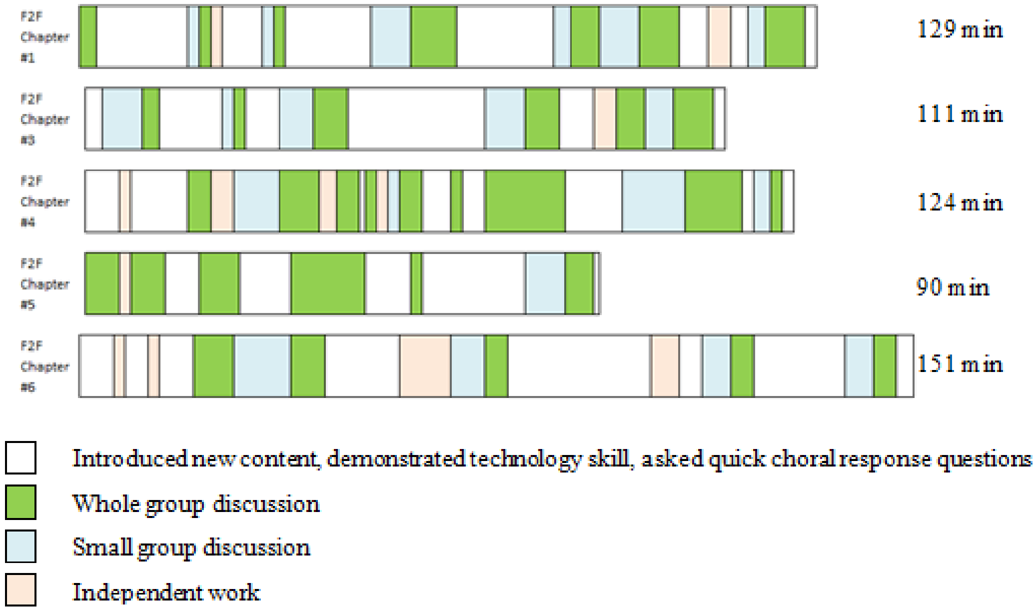


Figure 99. Comparison of chapters for the face-to-face class.

The role of the instructor, as facilitator, also surfaced during the whole group discussions for each chapter. In episodes corresponding to work in the first five chapters, where small group discussions preceded whole group discussions, the instructor worked to solicit responses from prospective teachers. The face-to-face class had little structure during such discussions, as anyone could voluntarily share at any time. The instructor would often refrain from commenting on the correct or incorrect nature of a prospective teacher's response but would rather leave it open for further discussion from the class at large. This resulted in a back-and-forth discussion pattern between the instructor and participants that was seen across episodes for the face-to-face class (e.g. Episode 6 whole group discussion, see Figure 98).

Often the overall structure of each class contained a cycle of demonstration (with prospective teachers trying the technology skill on their own simultaneously) followed by small group and whole group discussion. This cycle was effective for focus group members. When asked, in an interview, if the structure of the class was working for her, Ava replied "I think so, I like it." Later in the interview, she commented that she liked "all of the group discussions." Carrie also mentioned the discussions explicitly as something she enjoyed about the class. She shared that she liked that the class was "part lecture, part interactive." She went on to say, "I like how you do it on the overhead, or whatever that thing is called, and then we all have a little individual time and then a little bit of group time." Sam also mentioned interacting with others as one of his favorite parts about the class.

Trends in Form

Prospective teachers in the traditional, face-to-face setting interacted mostly through speaking. During whole group discussions, the instructor allowed participants to voluntarily share ideas. There was little structure as to the order prospective teachers should follow. Usually, prospective teachers would naturally take turns speaking and would not talk over one another. But, when the instructor asked choral-response type questions often many of them would respond simultaneously, making it difficult for the instructor to decipher what was said.

Ordinary hand gestures occurred as one might expect from someone talking to someone else. However, specific hand gestures and pointing to something projected on the screen were often directly related to the content or technology being discussed. For example, in the whole group discussion of Episode 3, when prospective teachers were using a movable line with squares to better understand standard deviation, the instructor described the "little squares coming off of that" by using her fingers to make what seemed to be outlines of squares. Another example in the whole group discussion on least squares regression in Episode 4 showed the instructor pointing to the screen – specifically to a point in the scatterplot in order to highlight the corresponding point in the residual plot. Unfortunately, during whole group discussions, the only hand gestures and pointing that was coded in the analysis were those the instructor used when she was standing in that area of the classroom.

Smiles, head nods, and other non-verbal forms were present as well. Unfortunately, those also could not be captured with the video. Other ways that prospective teachers communicated during whole group discussions were demonstrations at the instructor's computer and writing results on the whiteboard at the front of the classroom, though only one instance of each occurred in the episodes presented. The instructor also used the whiteboard on several occasions, but it was outside the window of what was captured in the videos of each class session.

During small group discussions, prospective teachers continued to communicate in similar ways. Fortunately, a second video camera was able to catch most of their interactions. Ordinary hand gestures along with specific hand gestures and pointing to something projected on their personal computers occurred. Again, specific hand gestures were often directly related to the content or technology being discussed. Several times, prospective teachers made a horizontal motion with both hands to aid in their description of variability (e.g. Episode 1 small group discussion).

One interesting form was a hand gesture that was named and used throughout the study. A group of points that appeared clumped together became known as a “Cora cluster,” a phrase initiated by the instructor and adopted by the face-to-face class after one prospective teacher (named Cora) described a cluster of points by moving her fingers and thumb as if she were grabbing points. This phrase was used in later small group and whole group discussions. Even in an interview, Carrie referred to a “Cora cluster” when she described how the residual plot showed the linear model was a good fit for that same group of points.

Trends in Purpose

Table 32 below shows the percentage of exchanges in which specific purposes were coded. Note that a single exchange was often coded as having multiple purposes. Thus, the percentages for each row (a small group or whole group discussion) may not necessarily add up to 100%. Looking across episodes for each chapter, there are several interesting results. First, seven discussions included times when ideas or concerns were shared more frequently than questions were answered. This corroborates the notion presented earlier that prospective teachers in the face-to-face class seemed to be comfortable sharing and building ideas off of one another. In other words, they did not simply answer the question. They added personal anecdotes and just seemed comfortable "bouncing ideas off of each other" (Carrie, Interview #2).

Table 32. Face-to-face exchange percentages of types of purpose.

	Ask a Question	Answer a Question	Share an Idea or Concern	Justify	Affirm
Episode 1 – SG	11%	9%	30%	5%	28%
Episode 1 – WG	27%	14%	59%	5%	41%
Episode 2 – SG	7%	4%	51%	4%	42%
Episode 2 – WG	25%	18%	49%	0%	20%
Episode 3 – SG	15%	8%	49%	0%	33%
Episode 3 – WG	38%	38%	38%	2%	22%
Episode 4 – WG #1	28%	33%	33%	6%	6%
Episode 4 – WG #2	38%	31%	52%	0%	34%
Episode 5 – WG	25%	55%	25%	2%	14%
Episode 6 – WG	48%	41%	28%	0%	24%
Episode 6 – SG	20%	9%	68%	0%	34%

Second, a much smaller percentage of the exchanges in small groups were "ask a question" than those in whole groups. This is most likely the influence of the instructor asking more questions in whole group settings. Finally, prospective teachers justified responses least often. Instead, they would often share ideas without returning to them in order to offer their reasoning behind them. And third, one surprising result in purpose was the number of affirmations. Most of the affirmations in whole group discussions came from the instructor (e.g. “Okay,” “Great, “That’s a good observation”). But, in small group discussions, prospective teachers would also affirm one another.

Trends in TPSK

Another trend that emerged from analysis of the six episodes was the attention given to statistics and probability in prospective teachers’ discussions. The following table lists the focus of questions in activities for each episode studied and the corresponding percent of interactions for statistical knowledge (SK), technological statistical knowledge (TSK), and technological pedagogical statistical knowledge (TPSK) associated with those questions. Again, row totals below may not add up to 100% if there were exchanges that had a non-TPSK focus (e.g. affirmation, comment or question about a technological issue).

Table 33. Face-to-face percent exchanges for focus of discourse (SK, TSK, TPSK).

	Focus of Questions	SK	TSK	TPSK
Episode 1 – SG	SK, TPSK	11%	0%	0%
Episode 1 – WG	SK, TPSK	59%	18%	0%
Episode 2 – SG	TPSK	0%	8%	23%
Episode 2 – WG	TPSK	1%	14%	44%
Episode 3 – SG	TSK, TPSK	3%	28%	8%
Episode 3 – WG	TSK, TPSK	13%	45%	16%
Episode 4 – WG #1	SK, TSK	0%	83%	0%
Episode 4 – WG #2	SK, TSK, TPSK	17%	34%	34%
Episode 5 – WG	SK, TSK	84%	8%	0%
Episode 6 – WG	SK, TSK	24%	48%	7%
Episode 6 – SG	SK, TPSK	39%	14%	9%

In three episodes, the topic of discourse was predominantly statistical in nature (discussions in episodes 1, 5, and 6). Also in three episodes, the topic of discourse was developing technological statistical knowledge (discussions in episodes 3, 4, and 6). Only twice was pedagogy a main focus. One occasion was in the episode surrounding Chapter 2, which included the videocase analysis. The second occasion occurred in the second whole group discussion in Episode 4 where prospective teachers described difficulties they thought students might have interpreting a residual plot.

Understanding of Variability

The second research question focuses on prospective teachers' understanding of variability. Before the face-to-face study began, a pre-assessment was given to obtain information on the twenty-five prospective teachers in this group. Two questions, one about their statistics background and one about their comfort level with statistics, revealed varied experiences with statistics and probability. Regarding the statistics background question, two prospective teachers, one of whom was a member of the face-to-face focus group, had not yet taken a statistics course in college and did not take one in high school. Eight prospective teachers listed having taken ST101, an introductory course at the university though one of them added that "it was really a joke and (she) didn't learn any stats." Five prospective teachers had taken an introductory statistics course at a community college or other university, some as long ago as twenty-five years. Ten prospective teachers (two of whom were members of the face-to-face focus group) reported they had taken a calculus-based statistics course in college.

The second background question on the pre-assessment asked prospective teachers to rate their comfort level on seven statistical topics (with 1 being very low or none, and 5 being high comfort). Because the background survey question was categorical in nature, viewing distributions of prospective teachers' self-assessments of comfort with each topic was helpful. The following figure shows a distribution of responses for each of the seven background questions. Each graphical display, created in *Fathom*, also includes a plot of mean (vertical blue line) and median (vertical green line) values.

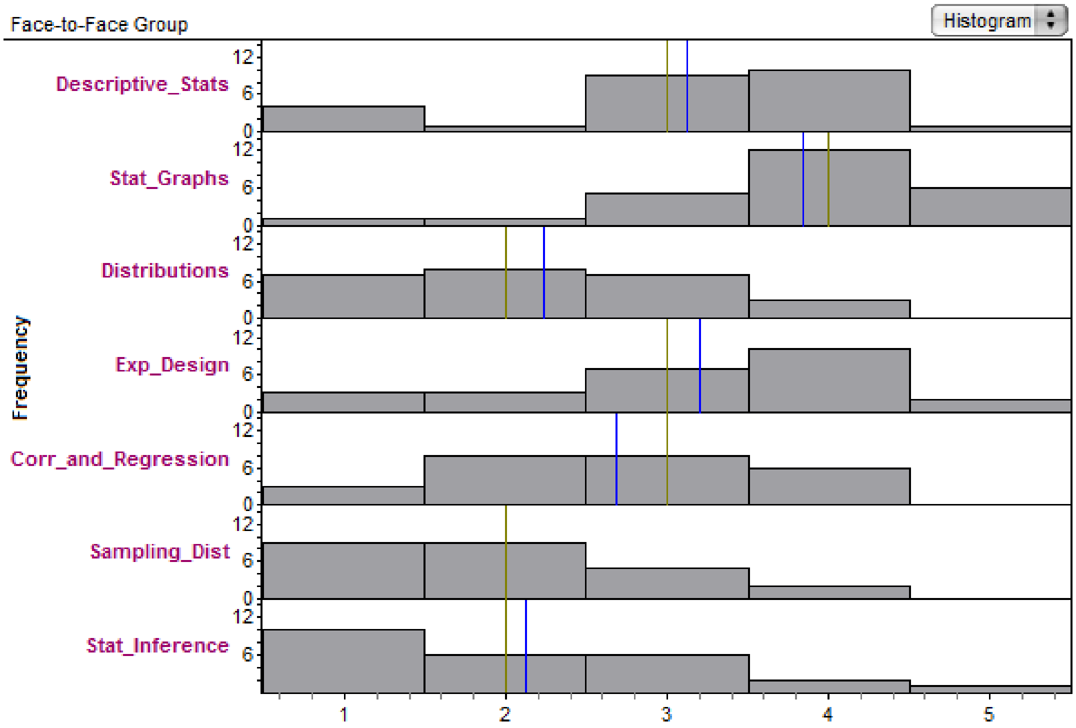


Figure 100. Face-to-face statistical background responses (1 low comfort, 5 high comfort).

As one might expect, many prospective teachers rated themselves most confident with descriptive statistics and statistical graphs. There were also many participants who rated their comfort level with experimental design as high. Seven participants used a score of five on one or more categories. This was somewhat unexpected from five of them, as they had only taken an introductory statistics course. Nevertheless, scores of five were rare across all seven categories. In three categories, distributions, sampling distributions, and statistical inference, no one claimed a high comfort level. The following table provides a summary of statistical topics along with the mean, median, and standard deviation of comfort level scores from twenty-five prospective teachers.

Table 34. Face-to-face pre-assessment summary of statistics comfort levels.

Face-to-Face Pre-Assessment (n=25) Statistics Comfort Level (1=low, 5=high)	*Mean*	*Median*	*Standard Deviation*
Descriptive Statistics (mean, standard deviation, z-score)	3.12	3	1.129
Statistical Graphs (histogram, boxplot, bar graph)	3.84	4	0.987
Distributions (normal, chi-square, probability density functions)	2.24	2	1.012
Experimental Design (surveys, blocking, bias, sampling methods)	3.2	3	1.155
Correlation and Regression (least squares, R^2, residuals, outliers)	2.68	3	0.988
Sampling Distributions (Central Limit Theorem)	2	2	0.957
Statistical Inference (t-tests, confidence intervals, chi-square tests, power, Type II error, ANOVA)	2.12	2	1.166

As with any self-rated data, there are surely some discrepancies between what was reported in the comfort level question and the statistics background. For example, one prospective teacher reported her only experience with statistics was the ST101 course mentioned earlier. In fact, she was the one who claimed her experience was “a joke.” Yet, she rated her comfort level of experimental design as being a 4. Perhaps the terms “surveys” and “bias,” which are words commonly used in today’s culture, in the description of experimental design misled her. She did not rate any other topics that high. Statistical graphs received a rating of 3 on her pre-assessment and all other topics received a rating of 1.

Other prospective teachers in this group may have been thinking about experimental design in a more informal way as well, as the group mean comfort level for that topic was greater than that of descriptive statistics, which was unexpected. Statistical graphs had the highest mean comfort level at 3.84. At any rate, it is important to point out that all other mean comfort levels were less than 3. In other words, this group of prospective teachers rated themselves with low to medium comfort levels with most statistical topics. Recall, from discourse analysis (see Table 33), that much face-to-face class time was spent discussing statistical content. Perhaps prospective teachers' lower comfort levels contributed to the need for statistical discussions.

This curriculum unit and study did not attempt to address content related to each of the seven topics listed in the pre-assessment background questions. Instead, the focus of this study was limited to prospective teachers' understanding of variability. Along with background questions, the pre/post-assessment also contained statistical content questions that intended to capture some of their understanding about variability. Using scoring rubrics (Madden, 2008), each test was graded. The maximum number of points prospective teachers could score on a single test was 31. Figure 101 below shows distributions for the pre/post-assessments in this face-to-face group. As a group, their performance on the post-test showed signs of improvement. A closer look at the gains, with a mean of 2.8, shows that most prospective teachers made some improvement in understanding of variability over the 5-week study (see Figure 102). Results from a Wilcoxon rank-sum test revealed that the improvement was significant (*P-value* = 0.006).

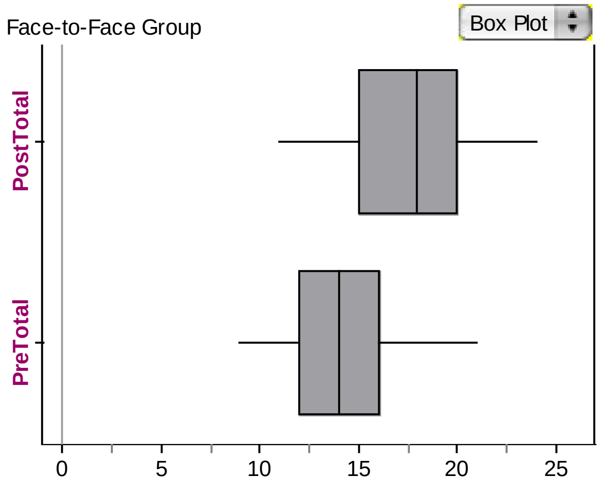
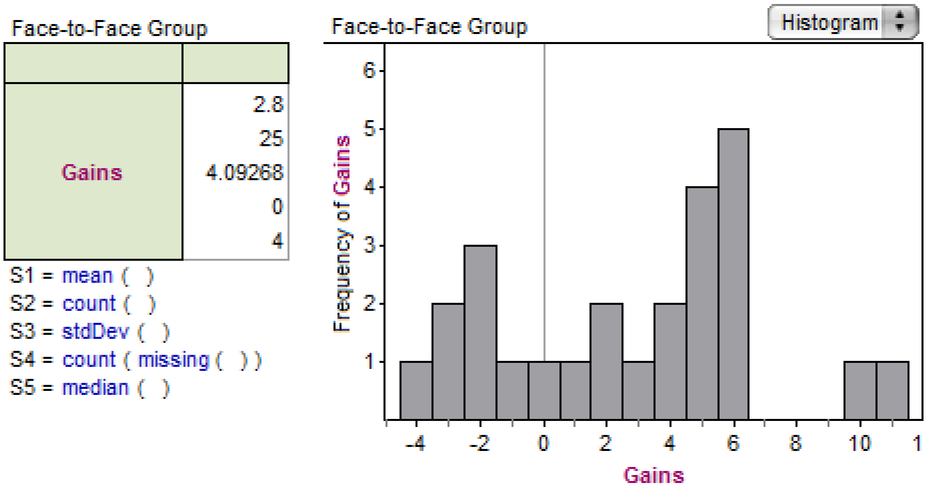


Figure 101. Box plots of the face-to-face group’s performance on the pre/post-assessments (out of possible 31 points).



Gains	2.8 25 4.09268 0 4

S1 = mean ( )
S2 = count ( )
S3 = stdDev ( )
S4 = count (missing ( ))
S5 = median ( )

Figure 102. Gains from assessment scores in the face-to-face group.

From the figure above, one can see that seven prospective teachers had negative gains from their pre-assessments to post-assessments. While it is not clear what caused them to do so, these seven prospective teachers provided less statistical information on the short answer questions focused on comparing distributions. Only one prospective teacher in this class showed no signs of change from pre- to post-assessment scores. This means that seventeen showed signs of some improvement. Also, it should be pointed out that there were two large, positive gains (10 and 11). The prospective teacher with a gain of 10 scored higher on over half of the questions on the post-assessment. Incidentally, she wrote "I've had very little experience with statistics in my education" on her pre-assessment background question. Her improvement should be considered a true gain. However, the prospective teacher with a gain of 11 did not respond to the last question of the pre-assessment; perhaps it was an oversight because that question was on a second page. That question was worth up to nine points. In terms of statistical significance, his oversight does not matter. In other words, if his pre-/post-assessment scores were removed, there would still be a significant difference (*P-value* = 0.011).

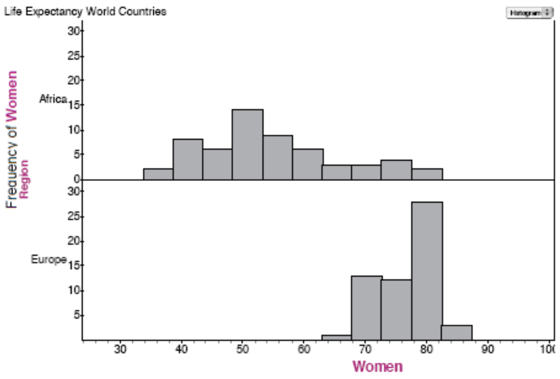
The pre/post-assessment was only one piece of data collected from each prospective teacher. The remainder of this section is organized by the three areas of variability: describing distributions, understanding deviation, and understanding the law of large numbers. For each section, analysis will include findings from assessments, discussions, and interviews.

Describing Distributions

The ability to correctly describe distributions plays a key role in understanding variability. There were multiple times over the 5-week study when prospective teachers were asked to describe distributions as part of a task they were completing. Therefore, there were multiple data sources associated with this statistical concept. Assessment data collected and analyzed included a pre/post-assessment question (n=25), a performance task (n=21), and a

Chapter 2 homework assignment (n=25). Discussion data from episodes described earlier include whole group and small group discussions from Episodes 1, 2, 5, and 6. Finally, data from follow-up interviews with focus group participants provided further evidence of prospective teachers' understanding of describing distributions. The following analyses are organized by data source.

Pre-/Post-Assessment. A short-answer question on the pre/post-assessment asked prospective teachers to describe similarities and differences of two distributions displayed as histograms (Figure 108). A scoring rubric was used to grade the question holistically (see Appendix C).



(Source: *Fathom2*; Taken from www.overpopulation.com which cites McDevitt, Thomas M., World population profile: 1998. Washington, DC: U.S. Government Printing Office, 1999)

Figure 103. Pre/post-assessment item for describing distributions (Madden, 2008, pg. 397).

While the group's overall post-assessment (mean=2.16, stdev=0.55) showed some improvement from their overall pre-assessment (mean=2.2, stdev=0.76), the shift in scores was minimal (see Figure 104). A Wilcoxon rank sum test (*P-value* = 0.931) confirmed the shift was not significant.

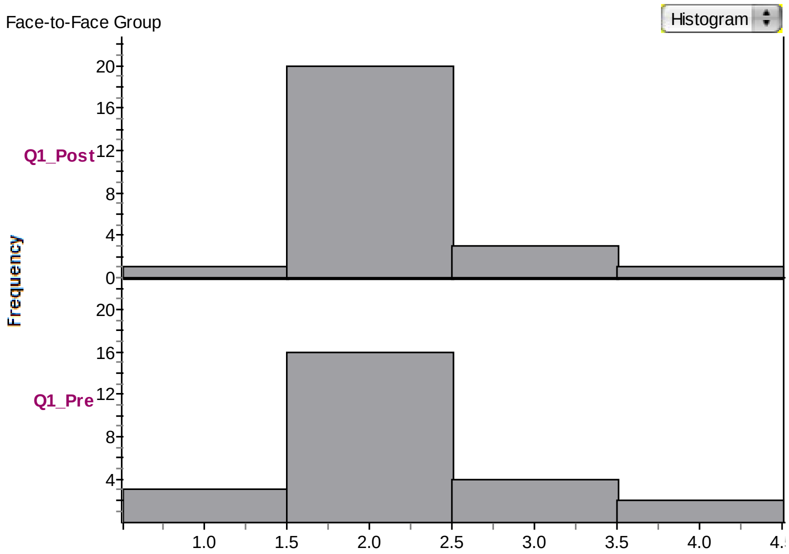
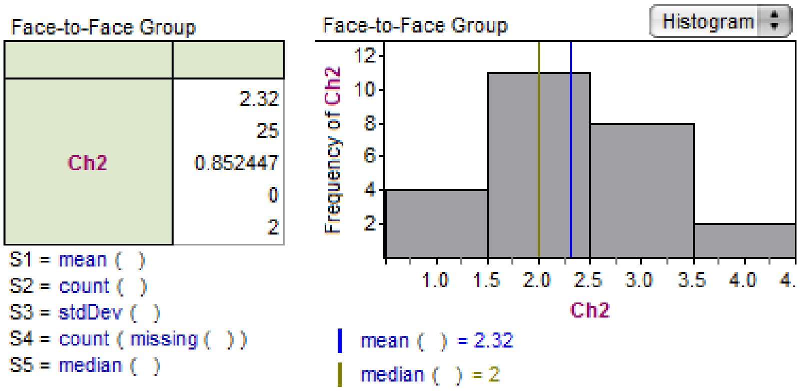


Figure 104. Face-to-face pre/post describing distribution assessment (4-point scale).

Most prospective teachers did not use statistical language to describe the similarities and differences between the two histograms. Rather, they kept references to statistical ideas on the descriptive level and used informal phrases such as “more spread out” and “less predictable” to explain differences. Many of them used references to a modal clump, though they did not label it as such. This is not surprising since the curriculum materials emphasize this informal approach. While some prospective teachers explicitly addressed center, spread, and shape in their post-assessment responses, most remained at the descriptive level and seemed comfortable describing distributions more informally.

Chapter 2 Homework. Twenty-five prospective teachers in the face-to-face class completed questions from Chapter 2 outside of class time. Two questions from the homework assignment were identified to provide additional information on prospective teachers’ understanding of distributions and their abilities to describe them. The questions were part of the initial task in Chapter 2 of describing the distribution of graduation rates and comparing graduation rates between public and private schools (see Figure 75). Prospective teachers' responses were graded holistically using the same scoring rubric used on the pre-/post-assessment above (a 4-point scale). The following figure shows the distributions of scores. Mean and median values are plotted and standard deviation is displayed in the summary table.



Face-to-Face Group

Ch2	2.32 25 0.852447 0 2

S1 = mean ( )
S2 = count ( )
S3 = stdDev ( )
S4 = count (missing ( ))
S5 = median ( )

mean ( ) = 2.32
median ( ) = 2

Figure 105. Face-to-face Chapter 2 assessment (4-point scale).

Like the pre-/post-assessment, this homework assignment showed that most prospective teachers did not use formal statistical language to describe a single distribution or the similarities and differences between the two distributions. Five prospective teachers' work included no references to formal, statistical ideas at all. Most teachers kept references to statistical ideas on the descriptive level and used informal phrases such as "more concentrated" and "tighter" to describe the spread. However, six prospective teachers included statistical language and seemed to intentionally address center, spread, and shape in their responses.

Performance Task. Twenty-one prospective teachers chose to complete a task related to describing distributions as part of their final exam. After showing competence in performing various skills in *TinkerPlots*, which included creating box plot representations (Figure 106), they were asked a pedagogical question to create a list of questions to help students compare distributions.

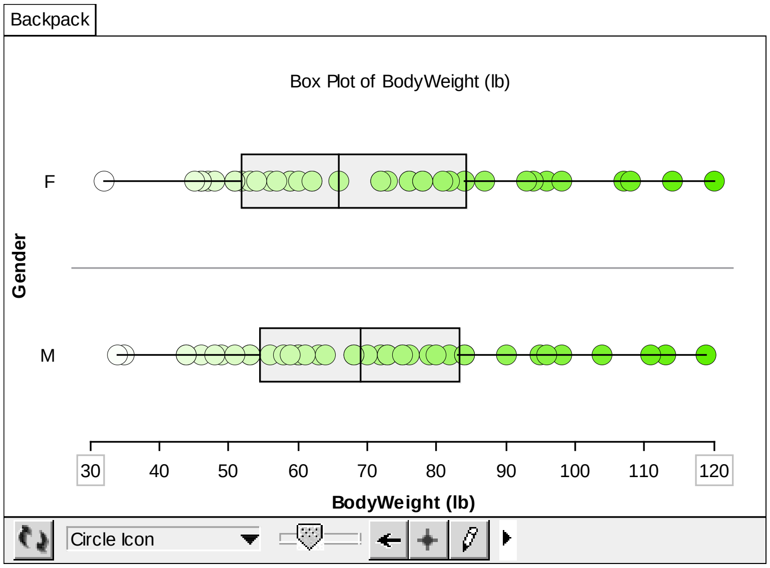


Figure 106. Sample performance task figure for comparing distributions (*Backpacks.tp* data from *TinkerPlots* resources).

A scoring rubric was used to grade their lists of questions (see Appendix H). Data showed a variety of competency levels in prospective teachers' ability to design questions for students related to describing distributions. Each row of Figure 107 below represents a prospective teacher who chose to complete this problem on the exam (n=21). Percentages for each row add up to 100% and provide information about how each teacher's overall score was determined. Each column represents the number of questions receiving a rubric score of 1, 2, or 3. A quick look across the columns below shows that only one (#18 below) prospective teacher's questions considered only one box plot or distribution. For the other twenty who did consider a comparison of distributions, more questions focused on a comparison of the mean than a comparison of spread. Thirteen teachers who chose to complete this question made it clear they were thinking about both center and spread with the questions they wrote for students. Only twelve prospective teachers in the face-to-face class (out of the 20 who used distributional comparisons) seemed to scaffold questions by getting students to consider one distribution at a time before focusing them on comparing centers and/or spreads.

Figure 107. Describing distributions performance task results for face-to-face class (3-point scale).

One thing the rubric failed to capture were the questions written that did not relate to distributional ideas at all. Three prospective teachers wrote questions that could not be captured in this analysis. They were more exploratory in nature, questions that would need to precede ones written for this task. These questions were coded “n/a.”

Discussion Analysis. Describing univariate distributions requires attending to its center, spread, and shape. Four of the six episodes (Episode 1, Episode 2, Episode 5, and Episode 6) for the face-to-face case contained small group and whole group discussions which provide some insight into prospective teachers’ ability to do one or all of those things. Specifically, in thinking about center, discussions revealed a tendency to consider the effects of higher and lower values on measures of central tendency. Prospective teachers in the face-to-face class also seemed to be more comfortable describing center in formal ways than when describing spread. But, there was minimal evidence that they were thinking about how technology might be used with students to assist them in developing a strong conceptual understanding of center and spread.

First, when thinking about measures of center, some prospective teachers considered the effects of higher and lower values on those measures. Recall that Episode 1 asked prospective teachers to share arguments for each of the measures of center (mean, median, mode, and midrange) to describe a “typical” value for average teacher salary. Though audio for the small group discussion in Episode 1 was poor, it was discerned that the second idea that Sam shared was that a measure of center was “not as affected by outliers.” In the middle of the discussion, he described the effect of moving the highest case (i.e. Washington, DC) to a higher numeric value. And at the end of the discussion, he wanted to focus on “everything except the two extreme endpoints.”

In the whole group discussion that followed in Episode 1, it was also evident that prospective teachers were considering effects of extreme values. In trying to craft an argument for the mean, one participant said “it usually shows where the majority of the data lies. But when you have outliers I mean it throws it off a little bit.” It appears that this prospective teacher may have been confusing the mean with the mode in the first part of her statement. However, in this data set, the mean was a value just to the right of the modal clump that had been described during the class prior to this discussion. She was trying to describe this shift from the modal clump due to the case of DC. Shortly afterwards, another prospective teacher shared a similar idea. Specifically, he said “I guess in this case [the median] is the most accurate. Because when you have outliers it kind of throws the mean off.” The notion of “throwing the mean off” is interesting. Did prospective teachers really believe that the mean was no longer accurate or were they trying to say that the mean was no longer an accurate representation of typical?

This attention to outliers and their effects occurred eight times over the episodes related to describing a distribution, including during small group discussions. It seemed to be a common practice to label any extremely high or low value an outlier. In Episode 5, for example, as prospective teachers were sharing simulation results for n=500, one prospective teacher said “mine’s an outlier” as he approached the whiteboard at the front of the room. While statistical outliers were determined a few times throughout the study, it was common to visually assess the distance from the case in question to “most” of the data to informally declare a case as having outlier status. After some discussion about the distribution looking or not looking like what was expected, the instructor said about the unusual value, “[he] was definitely sort of an outlier I would think. I haven’t done the 1.5 times the IQR but I think his might be an outlier.” She tried to reinforce that a visual assessment is not enough to determine if a case is a statistical outlier.

Interestingly, several times prospective teachers seemed to use a formal term (i.e. outlier) in their informal descriptions of the placement of “extreme” cases. The tendency to describe parts of a distribution informally was evident throughout the study. The prospective teacher’s reference to the “majority of the data” was not uncommon. A line-by-line analysis of transcripts of episodes, in which discussion focused on measures of center, revealed that informal language was used to describe the center of a distribution 41% of the time (or 16 out of 39). To describe center, they said things like “cluster” and “shifted to the right.” Prospective teachers also described the spread of the data in informal, descriptive ways (21 out of 25 times, or 84%). In the whole group discussion in Episode 5, for example, the instructor asked the class to share how a distribution may look differently if the sample size changed from n=500 to n=200 or n=999. Some responses included, “you get a bigger spread,” “approach 0.50 from both sides,” and “it looks a little stacked toward the left.”

The snapshot from Episode 5 above also corroborated the notion that prospective teachers rarely described a distribution in more than one way. Instead, many of their responses throughout the study included information about only one of center, shape, or spread. From the coding analysis of related episodes, it was determined that describing a distribution only by center, shape, or spread occurred approximately 71% of the time. By Chapter 6, questions from the text explicitly ask them to address center and spread. In the whole group discussion for Episode 6, when prospective teachers were examining the distribution of percent of male births (see Figure 40), the instructor asked, “What do you notice about the spread and where the data seems to cluster?” After a short time of no responses, the instructor said “Your dot plots are looking good. Do they seem to be clustered around 50 or some other value?” The omission of spread in the repeated question was not intentional. However, responses of “right around 50,” and “a little bit more” were given. Perhaps, for prospective teachers, center was more easily described.

Finally, analysis of episodes related to describing distributions showed that the number of times prospective teachers were thinking about how to use technology with students was minimal. Aside from Episode 2, which included the videocase, Table 33 also revealed a lower focus on TPSK in episodes related to describing distributions. A closer look at those instances of TPSK revealed that there was only one time (Episode 6 small group) when a prospective teacher explicitly mentioned how statistical software (e.g. *TinkerPlots*) might be helpful in forming conceptual understanding of measures of center. In that instance, Carrie described how the color was helpful for her in interpreting the birth data. She followed that with “I mean I think it, it might help more” when trying to answer whether the color gradient would help or hinder students’ reasoning. Other instances of TPSK were more descriptive (e.g. Episode 2 discussions) about what students were or would be doing. Prospective teachers seemed to be less attentive to addressing how the technology would affect students’ conceptual understanding. This is surprising since descriptive statistics and statistical graphs were topics they reported, on the pre-assessment, to be most comfortable with.

Interview Analysis. Interviews with focus group participants provided further evidence to support claims about prospective teachers' knowledge and understanding of distributions, and gave more insight into their developing TPSK. Interview data show that distributions are often described differently and with more informal language. In a follow-up interview, the instructor asked focus group participants to describe a distribution of a quantitative attribute (body length of cats) displayed as a dot plot in *TinkerPlots* (Figure 108 below). Carrie was unsure how to answer. She said, "Man, I don't know. Um, well I know it's not normal." The instructor asked her how she knew the distribution was not normal and she responded, "Well it doesn't look like normal. There's more right here and more right there but not in the middle. I don't really know anything about that kind of stuff."

The bimodal aspect of the distribution appeared to stand out to the other two focus group participants as well. In addition to focusing on the shape of the distribution, however, Ava and Sam also attended to the spread. When asked to describe the distribution, Ava answered "Um.. maybe like bimodal because it has the two big ones. And, I don't know, the range is about 10 that's about all I'd say." Responding to the same question, Sam said "There's a heavy concentration at 17 inches and 21 inches. And then it's pretty even distributed with the rest of it at the other points. It might be interesting to see a box plot of it." He then used tools in *TinkerPlots* to display a box plot and described the "heavy concentrations" as being at the beginning and end of the interquartile range. This showed he was attending to the spread of the data.

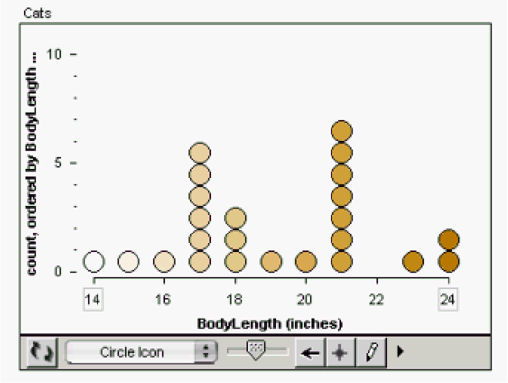


Figure 108. Cats *TinkerPlots* file used in interview #1.

In talking through how he would describe the box plot to students, Sam accurately described how the five-number summary points would be obtained and why certain pieces of the box plot were "bigger" than others. He said,

> "You could explain that the upper 50% of the interquartile range falls across a larger range of values than the lower 50%. So it's like the same number of points in each of these boxes (if we move them up there) but they're distributed over different ranges."

Despite his perceived confidence in this idea, Sam was noticeably disturbed after using a tool in *TinkerPlots* to show percentages. When describing the percentages displayed for the interquartile range for this data set, he said "In the middle, in the interquartile range you have 50%, but in the first half you have 12% and in the other half you have 38%." Figure 109 is a screen capture of Sam's computer work during his first interview. Sam used the divider tool to create the gray region in the center to represent the interquartile range. In his statement, he mistakenly calls the first half 12%, when he really meant the right whisker of the box plot. The time he spent exploring percentages with this tool, trying to get it to line up perfectly to be 25%, 50%, 25% provides an example of Sam developing his TSK. He was really trying hard to make a connection between what he knew *should* happen, and what really happened in *TinkerPlots*.

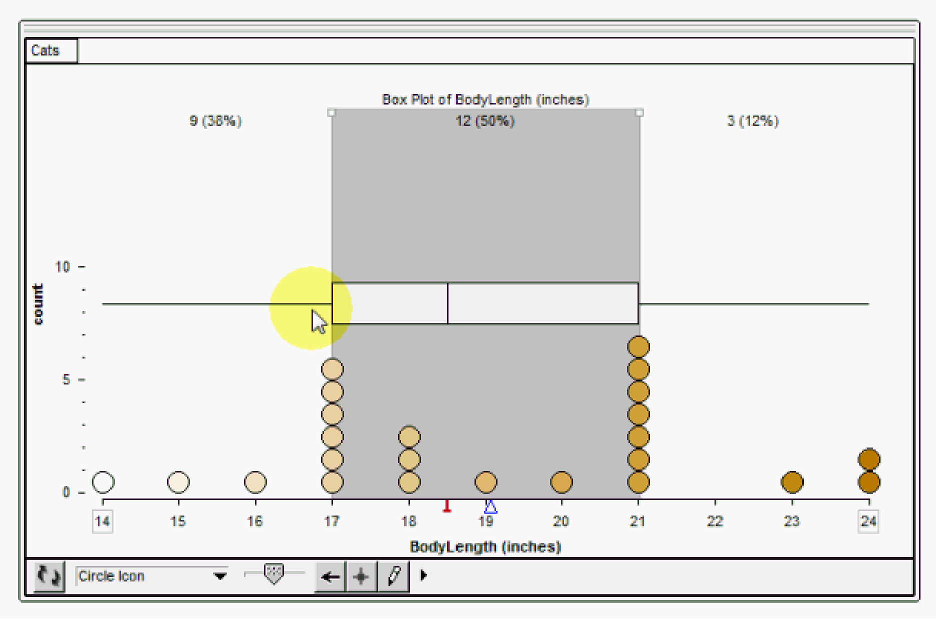


Figure 109. Screen capture of divider tool in Sam's first interview.

Time was spent during the interview discussing different ways that quartiles are calculated – something the curriculum materials addressed but was not part of the face-to-face instruction.

Summary of Understanding of Describing Distributions. Much data was collected and analyzed to determine how prospective teachers' thought about and described distributions. Each data source provided evidence which, when viewed collectively, gives a picture of prospective teachers' understanding during this study. Data show that prospective teachers often considered the effects of higher and lower values on measures of center. They would often explicitly state the highest and lowest values of data, sometimes inaccurately calling them outliers without determining whether or not those values were actually statistical outliers. They seemed astute at recognizing the effects “extreme” values had on the mean, median, and midrange.

Prospective teachers also seemed more comfortable describing distributions by center, rather than by spread or shape. In the whole group and small group discussions for episodes related to distributions, they tended to describe center less informally than they did with spread (41% vs. 84%). Results from the performance task also revealed a greater number of questions related to comparing centers of distributions (n=36) than comparing spreads of distributions (n=21). Prospective teachers’ comfort with center also became apparent in the questions they wrote for students.

Understanding Deviation

Curriculum materials used for this study allowed prospective teachers to think about deviation in a variety of contexts. Assessment data collected and analyzed from the pre/post-assessment question (n=25) provided insight into how prospective teachers were thinking about univariate deviation (e.g. deviation from a mean). A performance task (n=11) provided information about their understanding of bivariate deviation (e.g. deviation from a line of best-fit). Discussion data from Episodes 3 and 4 revealed how prospective teachers were thinking about both univariate and bivariate deviation, respectively. Finally, data from follow-up interviews with focus group participants provided further evidence of prospective teachers’ understanding of bivariate deviation. The following analyses are organized by data source.

Pre-/Post-Assessment. A short-answer question on the pre/post-assessment asked prospective teachers to decide which distribution had the largest variability (Figure 110). A scoring rubric was used to grade the question (see Appendix E).

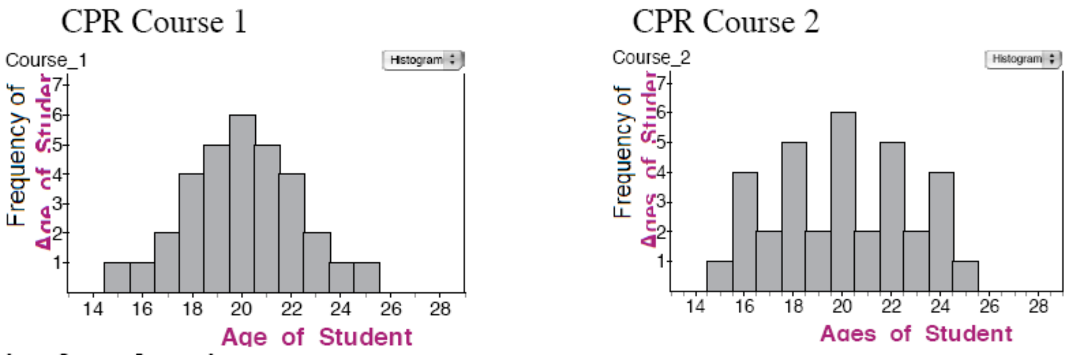


Figure 110. Pre/post-assessment item for understanding deviation (Madden, 2008, pg. 403).

Figure 111 shows the distributions of scores for this question on the pre- and post-assessments. There was some shift in the distribution of scores in the post-assessment (mean = 1.72, stdev = 0.792) and the pre-assessment mean score (mean = 1.52, stdev = 0.963). However, a Wilcoxon rank-sum test revealed that the shift was not significant (*P-value* = 0.136).

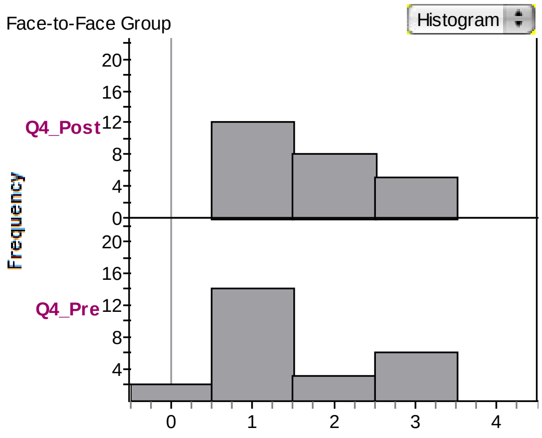


Figure 111. Face-to-face pre/post understanding deviation assessment (out of 4 points).

The zero score on the pre-assessment was not an accidental omission or oversight. That particular prospective teacher, for his response to this question, wrote “I have no idea.” His post-assessment response was “To be honest I can’t recall what variability is at the moment, but I chose this one (CPR Course 2) because it looks more random and less predictable.” Perhaps what he was attending to was the “up and down” nature of the second distribution. Like him, most prospective teachers in this face-to-face group were mostly thinking about variability as “bumpiness” and described the “up and down” shape of the second distribution in their answers. Some also included the lack of normal curve or bell curve shape as part of an explanation for greater variability in the second distribution. Only a few students seemed to focus on some measure of center and think about how values deviated from the center. When they did, they wrote things informally such as “not centered around one general area.” Overall, results from this assessment indicated prospective teachers did not seem to make a connection between variability and deviation in comparing two univariate distributions. And a couple of them mistakenly thought that since the means and ranges of the two distributions were approximately the same, variability would be the same as well.

Performance Task. Eleven prospective teachers chose to complete a task related to bivariate deviation as part of their final exam. After showing competence in performing various skills in *Fathom*, they answered a pedagogy question related to addressing students’ thinking about correlation. A scoring rubric was used to grade their responses (see Appendix I). Data showed a variety of competency levels in prospective teachers’ ability to design examples specifically addressing students’ understanding of correlation (see Figure 112).

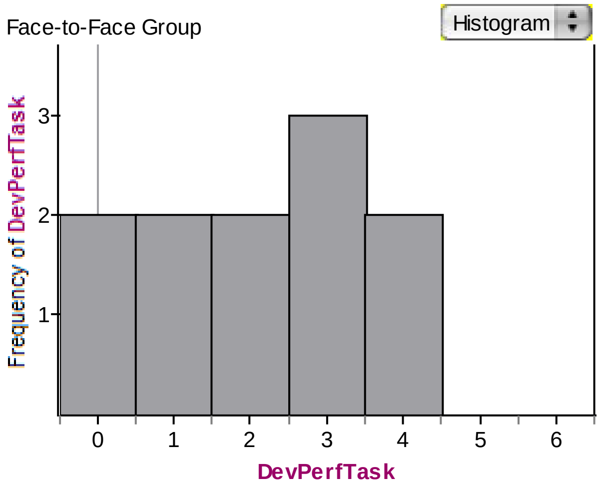


Figure 112. Face-to-face performance task assessment of deviation (6-point scale).

Two prospective teachers earned scores of 0 because they failed to address Jack's misconception at all. The lack of disagreement in their responses may potentially mean that they agreed with his ideas. Prospective teachers who earned scores of 1 or 2 failed to give good examples that might be used with these students. Instead, they were very general about how they might respond to students. For example, one prospective teacher wrote

> "Both students have at least some idea of what correlation is, but I would go ahead and further their understanding by showing them more examples of correlation. I would let Jack explain that 'the number for the correlation coefficient tells you how good a straight line fits the data'. I would make sure that they know correlation goes from 0 to +or- 1. Then I would let Jill tell the class that 'positive correlation means positive slope and vice versa'. Using the example given I would let them find the slope and then compare this to the correlation which they will find differ greatly."

Prospective teachers who score highest on this item gave specific examples that could help address Jack and Jill's ideas. One such response is below:

> "Jack definitely understands that a correlation coefficient of 0 means that there is no correlation, and that 1 means a perfect correlation. However, his idea that the slope would have to equal 1 is a bit off. An example that I would use to illustrate this point would be to create an x and y attribute, and enter in these cases:
>
> case 1: x=1, y=2

> case 2: x=2, y=4
> case 3: x=3, y=6
> case 4: x=4, y = 8
> case 5: x=5, y=10
>
> The least squares line for this example would be a perfect fit with a correlation of 1, yet the slope of the line itself would be 2. I would then enter this data set:
>
> case 1: x=1, y=10
> case 2: x=2, y=8
> case 3: x=3, y=6
> case 4: x=4, y = 4
> case 5: x=5, y=2
>
> This would have a correlation coefficient of -1, and a slope of negative 2."

These examples show that the prospective teacher has carefully selected two correlations and slopes for students to consider. They address the misconception that correlation and slope have to be equal. What was missing in her examples, and those from other prospective teachers, was a sequencing of questions that would help students build the understanding that correlation and slope are two distinct entities.

Discussion Analysis. There were two components of understanding deviation for this study. Like the pre/post-assessment, one component of understanding is at the univariate level. And, like the performance task, the other component is understanding deviation at the bivariate level. Two of the six episodes (Episode 3 and Episode 4) for the face-to-face case contained small group and whole group discussions, which provided some insight into prospective teachers' understanding of each of those components. The following paragraphs take a closer look at the episodes individually and the progression of deviation from univariate to bivariate contexts.

Recall that Episode 3 involved prospective teachers using *Fathom* and a movable line with squares (see Figure 79) to help conceptualize the standard deviation formula.

Prospective teachers appreciated the visual aspect technology provided. But, only twice did a prospective teacher in the face-to-face class explicitly explain how *Fathom* provided a good "visual." Instead, discussions seemed to remain at the TSK level (see Table 33). This means that prospective teachers were using the technology to do statistical work, but they were not sharing aloud how technology was helping them better understand standard deviation.

Perhaps a lack of confidence explains why some prospective teachers struggled to make a connection between the "visual" in *Fathom* and the formula for standard deviation. In the small group discussion for Episode 3, prospective teachers were asked to put on their "teacher hat" and think about whether they would show their students the formula for standard deviation first or some visual (either dynamic or static). Sam told other focus group members, "I think the diagram first was like the way to go because that formula was just intimidating." In the whole group discussion that followed, other prospective teachers revealed a similar trepidation with the formula. One said, "I have no idea, I couldn't use that formula to save my life, but just like looking at this [visual] helped me." Another added that the instructor's explicit connection between the visual and the formula was important for her. She said, "I liked how you took the formula and broke it down like that, like you started with, you know, each individual piece [of the formula]… standard deviation just means like the average distance from, from the middle, you know." By scaffolding the discussion with questions about "each piece" of the formula, the instructor was able to obtain correct responses for each of the "smaller" questions she asked about standard deviation. However, prospective teachers' comments revealed a lack of confidence with standard deviation, in general.

The lack of confidence displayed in discussions focused on univariate deviation was also evident during class centered on bivariate deviation. Before Episode 4 began, the instructor asked prospective teachers to rate their understanding of least squares regression along a continuum using the survey tool on the class *Moodle* page. Figure 113 below shows the self-assessment of some teachers in the face-to-face group.

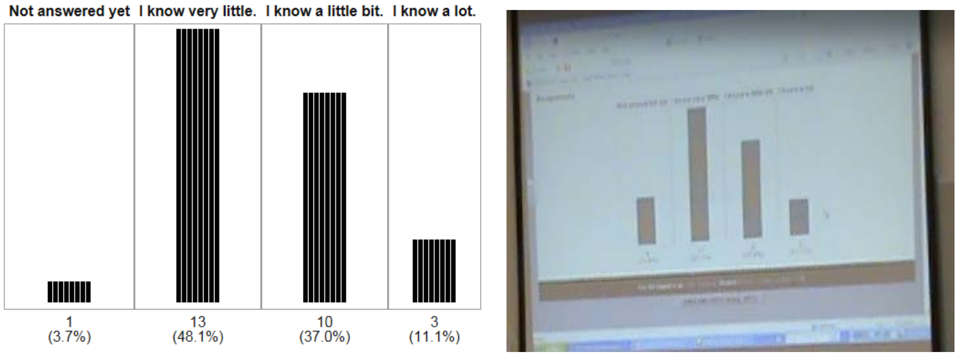


Figure 113. Face-to-face pre-survey of least squares regression understanding.

When deciding if a linear model is good, one looks at a number of things, including a residual plot. Analysis of discussions in Episode 4 show that prospective teachers were correctly identifying residuals and were considering different aspects of the residual plot in terms of the appropriateness of their linear model. One shared that “the value of the residual plot dictates how far away from the line it is, and whether it is above or below the line is based on if it’s positive or negative.” He seemed to correctly recognize how the scatterplot with the linear model and residual plot were generally connected.

Related to bivariate deviation, prospective teachers in the face-to-face group brought up several pedagogical issues. When asked what conceptual difficulties students might have with the residual plot and using representations such as the ones they created in *Fathom*, the following ideas were shared:

> *Corinne: "Um I said that they might have a hard time understanding why there are negative values in the residual plot because the linear model is in the first quadrant, so there aren't any negative values. So they might not understand why that there's negative values."*
>
> *Ava: "It may be confusing that highway is just not there anymore."*
>
> *Michael: "It might be hard to see why you know when you're moving the line in one scatter plot it actually moves the dots in the other one. They both kind of look like scatter plots but the one is measuring something completely different than the other. That might be hard to see."*

These comments show that prospective teachers were beginning to consider how students may have trouble with content and technological representations. However, the fact that the discussion never turned to how one might address those difficulties is significant and is similar to findings regarding TPSK discussions related to describing distributions. The overall focus on TPSK in Episodes 3 and 4 (12 out of 144 exchanges, or 27.2%) presented in Table 33 does not provide that detail. Looking at comments and responses for the two episodes collectively, teachers focused on TPSK twenty-two times (15%), TSK sixty-one times (42%), and SK sixteen times (11%). Perhaps the attention to statistical content and being allowed to "visualize" some ideas in a new way contributed to the collective shift in prospective teachers' self-assessment of understanding least squares regression seen in Figure 114.

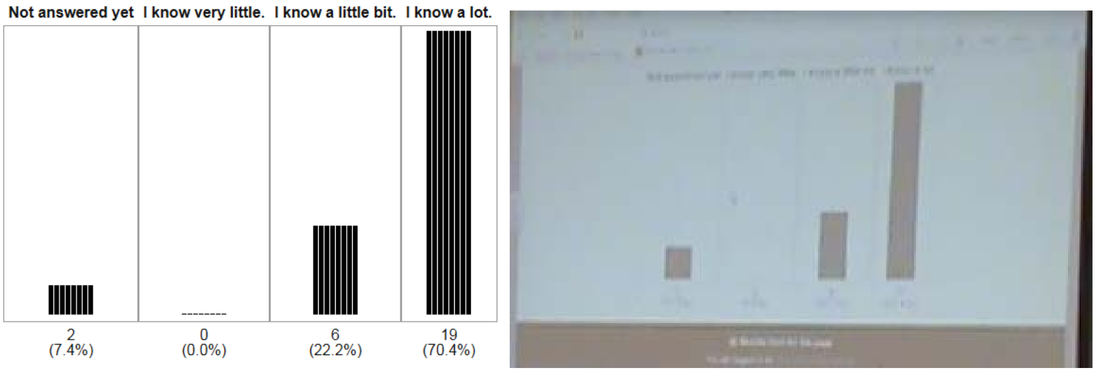


Figure 114. Face-to-face post-survey for least squares regression.

Interview Analysis. Interviews with focus group participants provided further evidence to support claims about prospective teachers' knowledge and understanding of deviation. In follow-up interviews, all focus group participants were asked to connect a residual plot with the corresponding linear model. Like in her first interview, Carrie began by openly sharing her lack of confidence with this content. She said, "Oh no, I don't think you want me explaining that to anybody." She went on to describe the residual plot this way:

> "It's how far away they are from the line. This line, what is it called? The least squares line. So the distance, like you want to, let me see, ok. I want to make sure I don't get it backwards – it's just so weird. Ok, so this line, you want to minimize the distance away from the residual line. I don't even know how to explain it."

Carrie tried to address that a residual plot shows how far away individual points are from the linear model, but she was unable to articulate that idea. Ava and Sam, however, seemed to divide the plot by thinking about residuals in terms of positive and negative values. Ava explained,

> "If you look at the top scatterplot, the points that are above the line are above the line in this plot and the ones that are below the line are below the line here. It's just showing how far away they are from the line. So it's almost like you've taken your scatterplot and made it horizontal."

Sam’s response was similar. He said, “So the distance of each dot from the x-axis on the residual plot is how far away it is vertically, you know above or below the line, depending on positive is above the least squares line and negative is below the least squares line.” He went on to describe the cluster of points in the bottom left corner of the scatter plot in terms of the residual plot. He referred to those points as a “Cora cluster,” a phrase initiated by the instructor and adopted by the face-to-face class after one prospective teacher (named Cora) described a cluster of points by moving her fingers and thumb as if she were grabbing points. Interestingly Carrie, later in her interview, also referred to a “Cora cluster” as she described how the residual plot showed the linear model was a good fit for that same group of points.

Interviews also gave more insight into focus group participants’ developing TPSK. They were asked to generate a question students could explore using a pre-created data set (bears.ftm from *Fathom*’s sample files) and describe the mathematics or statistics involved. Ava created a question that used one quantitative attribute and one qualitative attribute. She stated it would address content ideas such as “average, mean, mode, and median” and describes the graph as being bimodal with “a big jump here and a big jump here,” but admitted that she didn’t know if “her question was very good.”

Carrie said, “I don’t know I’m really bad at generating questions. I’m trying not to use correlation or something like that because that’s just going to make them look at one thing.” She took much more time exploring attributes in the data set before finally creating a question that would require their students to think about the correlation of two quantitative attributes. Carrie listed correlation coefficient, linear model, residuals, and comparing two different variables as important statistical ideas that would be addressed in her question. Sam also spent much time exploring the attributes before generating a question that would require students to focus on two quantitative attributes. In fact, he spent so much time exploring attributes in *Fathom* that the instructor asked him what he was thinking about. He said,

> "I was trying to see if any one of them was like really linear. Like I sorted months by ascending to see if these would also be, like, ascending in a nice manner. But, there not perfect obviously, because it's real data. It's not exactly what I was looking for. But that will be good too because sometimes it is good to have what you expect not happen."

He listed correlation, r and R^2, linear regression, and a least squares line as content goals for his question.

While all members of the focus group adequately provided a question to use with students, two interesting things stand out. First, despite having just completed the face-to-face class focused on scatterplots and least squares regression Ava reverted back to comparing two univariate distributions. This may be due to her focus on middle grades mathematics and thinking what she may do with her students. Second, all members of the focus group were able to demonstrate proficiency with using *Fathom*, but only one made explicit connections as to how and when they might utilize features of the dynamic environment when asked how *Fathom* might help or hinder students' thinking . At this point in the study (after four weeks), the lack of connection was surprising because similar questions were asked across chapters with the various technologies. Two prospective teachers only described how *Fathom* made things faster. The third focus group member commented that “there’s a movable line that you can mess around” and mentioned that having linked representations would be beneficial for students.

Summary of Understanding of Deviation. Each data source provided evidence, that, when viewed collectively, give a picture of prospective teachers’ understanding of univariate and bivariate deviation during this study. Data show that prospective teachers often thought about variability as shape and not deviation from a mean. With regard to univariate deviation, they used informal language such as “bumpiness” and “up and down” and rarely provided evidence that they were considering the distance from a mean.

The same was true for bivariate deviation. They seemed to have a strong understanding of the directions of a correlation (positive, negative, none) but lacked understanding about how correlation was a measure using each individual point and its difference from means. Prospective teachers, in general, came in to this study with little content knowledge of least squares regression. Much time was spent building the ideas of standard deviation and residuals with movable lines and squares in *Fathom*. Prospective teachers could accurately describe the actions made with technology, but struggled initially to make connections to formal statistical equations (e.g. standard deviation, regression line). Despite this, most of them seemed to appreciate the “visual” aspect of *Fathom* and became very engaged at points in the discussion where they were challenged to use the technological tool in a statistical way (e.g. find the smallest sum of squares). Several of them shared willingly how the technology, paired with an explicit connection to the more formula-based approach, was helpful for them.

Understanding the Law of Large Numbers

Understanding the law of large numbers requires one to understand the variability across samples and variability between theoretical and empirical probabilities, and the role that sample size has in affecting this variability. The number of opportunities for assessing prospective teachers' understanding of the law of large numbers was more limited than the other ideas of variability previously presented. Assessment data was collected and analyzed from the pre/post-assessment question (n=25) and a performance task (n=14). Data from whole group and small group discussions in Episodes 5 and 6 were also analyzed to better understand how prospective teachers were thinking about the law of large numbers. The following analyses are organized by data source.

Pre-/Post-Assessment. A multiple-choice question on the pre-/post-assessment asked prospective teachers the following question:

> "A certain town has two hospitals. In the larger hospital about 45 babies are born each day, and in the smaller hospital about 15 babies are born each day. As you know, about 50% of all babies are boys. However, the exact percentage varies from day to day. Sometimes it may be higher than 50%, sometimes lower. For a period of 1 year, each hospital recorded the days on which more than 60% of the babies born were boys. Which hospital do you think recorded more such days?
>
> A) The larger hospital
> B) The smaller hospital
> C) About the same number of days (within 5% of each other)
> D) Can't tell."

The matrix below (Figure 115) shows frequencies of responses for each answer choice. Thirteen prospective teachers correctly identified B, the smaller hospital, as the answer to the question on the pre-assessment. One of those thirteen changed his/her response to C, about the same number of days, in the post-assessment. Nine prospective teachers changed their incorrect pre-assessment responses to B on the post-assessment. A gain in number of correct responses (from n=13 to n=21) for this question was expected as the wording of this question was very similar to activities prospective teachers had completed in Episode 6.

Post – Assessment Responses

Pre – Assessment Responses

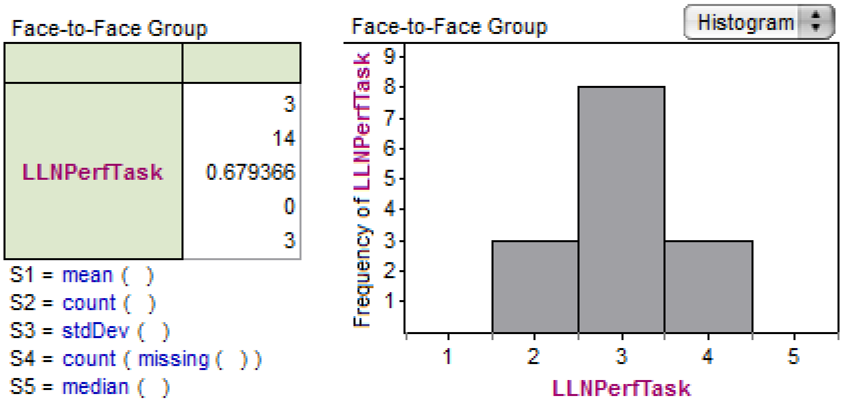
	A	**B**	**C**	**D**	**Total**
A		2	1		3
B		12	1		13
C	1	6	1		8
D		1			1
Total	1	21	3		25

Figure 115. Face-to-face pre/post-assessment for law of large numbers (correct response B).

Performance Task. Fourteen prospective teachers chose to complete a task related to probability and the law of large numbers as part of their final exam. After showing competence in performing various skills with *Probability Explorer*, they were presented with the following questions:

1) “Briefly describe how you would introduce the concept of the Law of Large Numbers to students.
2) Explain how you can use your spinner simulation to help students understand the similarities and differences between theoretical and empirical (experimental) probability.”

Tasks were graded with a 5-point scoring rubric (see Appendix J). The overall distribution of scores for this task (Figure 116) is symmetric with values ranging from 2 to 4. The fact that this exam question asked prospective teachers to use a technology simulation to answer the question meant that prospective teachers naturally received one point for the use of technology. Interestingly, however, four of them felt it was necessary to provide another example – a simulation with coins. Recall that in a “show-of-hands” survey before Chapter 5 began, the instructor asked prospective teachers about their experiences with probability simulations. Many hands went up indicating experience with using coins. Coins were also used to model the freshmen retention rate problem in Episode 5. Another commonality in prospective teachers’ responses for this performance task was the exclusion of discussion about variability one might expect from sample to sample. Only one discussed this explicitly in her response. Instead, most of them (71%) described how they wanted students to see in multiple simulations of different sizes that, as the sample size grew, the values became closer to the “desired result.” Thus, they seemed to be focused on the relationship between empirical and theoretical probabilities, which was explicitly stated in the task they were solving.



Face-to-Face Group

LLNPerfTask	3 14 0.679366 0 3

S1 = mean ( )
S2 = count ( )
S3 = stdDev ( )
S4 = count (missing ( ))
S5 = median ( )

Figure 116. Face-to-face performance task for law of large numbers (5-point scale).

Discussion Analysis. Two of the six episodes (Episode 5 and Episode 6) for the face-to-face case contained whole group and small group discussions that provided some insight into prospective teachers' understanding of variability as it related to the law of large numbers. Recall that Episode 5 began with a coin-toss simulation activity in which prospective teachers used an online tool to "flip" a coin thirty times. The instructor asked a small group to come to the whiteboard at the front of the computer lab and record their results, creating a dot plot. After that first group, she asked prospective teachers to anticipate "what do you expect is going to happen?" as more groups added data to the dot plot. One prospective teacher said "closer to the middle." After the second group recorded their results, she asked the question again, "what do we think is going to happen?" Another prospective teacher said "more variability."

It is interesting that these two comments were shared. The first prospective teacher seemed to be thinking about the variability between the theoretical and empirical probabilities decreasing. She expected more results near the "middle," or 0.5 on the number line. With the second prospective teacher's comment, it was unclear from the transcript of this episode if she was referring to the increased variability one would expect among results from only thirty trials or if she misunderstood the decreased variability one would expect between the theoretical and empirical probabilities from increasing sample sizes.

Later in that discussion, after prospective teachers had completed a simulation for n=500, the instructor engaged the class in anticipating what distributions might look like before actually modeling the problem with more simulations. The instructor asked, "how might this [distribution] look differently if we all did the simulation again and there were only 200 students at Chowan College?" One participant responded, "you might get a bigger spread because each point is worth more percentage. I mean the more, the more iterations you take the closer it's going to be to 0.50." This prospective teacher seemed to understand that a decreased sample size generally results in greater variability in results, though his description is awkward. The instructor then asked, "so if we had 999 students at Chowan and we did this, how would the distribution change?" One teacher answered, "it would approach 0.50 from both sides." His response also supports an understanding that as a sample size increases, variability among the simulation results will generally decrease.

While the comments from prospective teachers showed some understanding of the law of large numbers, Episode 6 revealed that not all had completely understood how sample size affected the variability between theoretical and empirical probabilities. In an opening survey (see Figure 91), less than half (44%) of the class answered the question correctly, even less than the 52% who answered the related pre-assessment question correctly. In the whole group discussion for Episode 6, when prospective teachers are examining the distribution of percent of male births, the instructor asked, "What do you notice about the spread and where the data seems to cluster?" After a short time of no responses, the instructor said "Your dot plots are looking good. Do they seem to be clustered around 50 or some other value?" The omission of spread in the repeated question was not intentional. However, responses of "right around 50," and "a little bit more" were given.

Simulations in *Fathom* provided prospective teachers opportunities to use color to help them relate sample size with the center and spread of the sampling distribution. This use of technology, along with other skills in *Fathom,* was not immediately effective for everyone.

In the small group discussion that followed, Carrie said, “Um I, it confused me when I first put the color up. I was like what is the color for, because the color and y axis right now are the same. So… if I didn’t have, like if I didn’t have the y-axis like that it would be more beneficial.” While it seems that the color of the plot created in *Fathom* hindered her understanding, it was really having both the color and the attribute on the y-axis the same that caused her confusion. Interestingly, it was the y-axis attribute that she wanted to remove from the plot to make it easier to understand. Perhaps, then, it was the color that helped her most in thinking about how population sizes were affecting the proportion of male births in that example.

Despite many positive aspects of the whole group and small group discussions, members of the focus group disagreed on the earlier survey. At the end of their discussion, they came back to the survey task and data regarding Hyde and Martin County births. The following transcript reveals some confusion they had in thinking about law of large numbers.

> *Carrie: Does assuming a 51 percent chance for males change your response to whether event A or B is more likely, or if they are equally likely? I don’t know. I say that they are equally likely. But I really have no idea.*
> *Ava: I don’t know.*
> *Carrie: I mean…it’s unusual that they’re both that low.*
> *Ava: I would put that A was more likely just because with the smaller sample size they could be anywhere.*
> *Carrie: That’s true, and that’s what she was saying earlier.*
> *Ava: But I don’t know why a 51 percent chance would make anything different.*
> *Carrie: Yeah, I don’t see how there’s like a correlation between the two. So…[mumbling] Okay, okay. I don’t know, that just confuses me.*
> *Ava: I think it would just make it farther from the…*
> *Carrie: The mean, yeah okay.*
> *Ava: I don’t know.*

Ava was correct in thinking a smaller sample size would result in an increase of variability between the empirical and theoretical probabilities, but the question about whether the assumption of 51% would change their answer stumped them. Carrie mistakenly calls the theoretical probability the mean. It is not clear whether she was remembering earlier discussions about describing distributions or if she thought the value she was referring to was the mean. Ava could not help her. Thus, a lack of understanding with the law of large numbers was revealed by both prospective teachers in this discussion.

A lack of confidence with the law of large numbers could be seen in topic summaries. For Episodes 5 and 6, discussion had a TPSK focus less than 5% of the time. This, coupled with the fact that there was a 19% focus on TSK, and a 54% focus on SK, reveals that prospective teachers outside the focus group were likely just as uncomfortable with this content as Carrie and Ava were during that discussion.

Summary of Understanding the Law of Large Numbers. Data collected revealed differences in prospective teachers' understanding of the law of large numbers. On the post-assessment question related to law of large numbers, 84% of prospective teachers answered correctly. This was a large increase from the 52% who answered correctly on the pre-assessment. This gain shows that curriculum activities and instruction had a positive impact on prospective teachers' understanding.

From performance task responses and discussions surrounding activities during class, it was also clear that some prospective teachers had a strong understanding that as a sample size grows, the variability between empirical and theoretical probabilities decreases. Often prospective teachers informally described how the results became "closer to the desired result." Other prospective teachers did not provide evidence that this concept was understood. In addition, few of them provided evidence of understanding variability one might see from sample to sample, depending on the size of sample.

The small group discussion also uncovered a lack of understanding with this idea. Focus group members had difficulty describing how the sample size affected an outcome and there was a great deal of confusion about the role of probabilistic assumptions. During their discussion, they were unable to help one another. This lack of statistical knowledge (SK) in the law of large numbers, may have caused few prospective teachers to consider pedagogical issues.

Trends in Understanding Variability

In this study, prospective teachers were asked to work through activities related to describing distributions, univariate and bivariate deviation, and the law of large numbers. At the same time, they were learning new technology skills with dynamic programs such as *TinkerPlots, Fathom,* and *Probability Explorer* and trying to think about pedagogical issues related to the content and to using technology with students. Much data was collected and prospective teachers' SK, TSK, and TPSK were analyzed. Common themes that emerged in each of these three areas are shared below.

Statistical Knowledge (SK)

Prospective teachers tended to stay away from formal, statistical language and seemed to prefer to describe things informally. When working with distributions, they often used terms and phrases such as "clustered" and "where the majority is" to describe them. The face-to-face group did, however, tend to use less informal language when describing center, as compared to language used when describing spread. When trying to understand deviation, prospective teachers described variability in terms like “up and down” or “less scattered.” And regarding the law of large numbers, some would describe the probability as “close to the desired result” rather than theoretical or empirical probability. The use of informal language, by itself, is not a bad thing. The curriculum materials encourage an informal approach when introducing such topics to students. Many times, prospective teachers seemed to have a good understanding of the notions of center and spread and could describe a distribution based on those notions.

Prospective teachers differed in their experiences. Yet, no one reported having a high comfort level for either correlation/regression (mean = 2.68, stdev = 0.988) or sampling distributions (mean = 2, stdev = 0.957). In addition, two in-class surveys (e.g. self-reported understanding of least squares regression, birth data survey) showed that as a group, statistical knowledge was similar (and low) initially. By the end of the curriculum, however, prospective teachers showed growth in content knowledge related to regression and the law of large numbers through surveys, post-assessment questions, and discussions.

With regard to deviation, prospective teachers rarely made any connection to distances from the mean or line of best-fit. Instead they discussed how “scattered” or “clustered” the graphical display looked without providing evidence as to how they were judging the magnitude of scattered-ness or cluttered-ness. Also, in analyses of whole group discussions related to the law of large numbers, prospective teachers often considered variability between empirical and theoretical probabilities but did not always explicitly provide evidence that they were also considering variability from sample to sample.

Technological Statistical Knowledge (TSK)

Throughout the study, prospective teachers learned technology skills with dynamic, statistical programs. Among other things, they learned how to create representations for univariate and bivariate data analysis (e.g. dot plot, histogram, box plot, scatter plot, least squares regression, residual plot). They also learned how to create probability simulations and collect and organize results from repeated samples. Based on prospective teachers' interactions during in-class discussions and submitted assignments, it is evident that they were comfortable with the skills mentioned above. In the interviews, focus group members also shared comments about their appreciation for the "visual" provided by the technology and spoke candidly about the technology helping them better understand the content.

Prospective teachers were able to correctly describe actions relating to graphical representations they created with technology and could often describe the representation itself informally. However, when asked to make an explicit connection between the technology and statistical formulas, many of them struggled. For example, the whole group discussion centered around standard deviation revealed a weak understanding about standard deviation and a disconnect between the formula and what they had experienced with an activity using *Fathom* with a movable line. By scaffolding questions, the instructor was able to provide an opportunity for prospective teachers to correctly associate pieces of the formula with parts of the technological graphical display. The visual created in *Fathom*, which utilized dynamic features and linked representations, helped prospective teachers better understand concepts that had previously remained abstract.

Technological Pedagogical Statistical Knowledge (TPSK)

As mentioned above, prospective teachers were adept at performing technological skills but their abilities in relating results from technological displays to statistical formulas varied. In fact, their statistical knowledge in general varied from teacher to teacher and even from topic to topic of variability. Despite this, however, prospective teachers were encouraged by the instructor and by the curriculum materials to consider implications of teaching these topics using technology to students. As one might expect, this was difficult for them to do.

Prospective teachers were able to speak generally about the use of color or symbols overlayed on graphical displays and the effects on their own learning, but did not spend much time discussing these affordances for students. In fact, over the course of the study, there were four discussions in the sampled episodes that were analyzed that focused on TPSK at least 10% of the time. Two of those were small group and whole group discussions in Episode 2 which centered on the videocase of Chapter 2. Prospective teachers had video evidence of students working and copies of students' work to use to aid them in discussion. The other two discussions were from Episode 3 whole group and Episode 4 whole group (discussion #2). These discussions focused on measures of center, standard deviation, and residual plots respectively, the content that prospective teachers seemed most comfortable with during this study.

Four discussions did not discuss TPSK at all (Episode 1 small group, Episode 1 whole group, Episode 4 whole group (discussion #1), Episode 5 whole group). The fact that Episode 1 did not contain TPSK was surprising as prospective teachers seemed confident with measures of center. The fact that SK and TSK foci were higher may be due to thinking about measures of center in a new way and the many new skills prospective teachers were learning with *TinkerPlots*. The first whole group discussion in Episode 4 did not have a pedagogical focus at all and the lack of TPSK in Episode 5 (coupled with the 84% SK focus, and a low pre-assessment score of only 52% correct) shows prospective teachers were perhaps addressing their own misconceptions about the law of large numbers.

Summary

This case reveals information about prospective teachers' interactions in a traditional, face-to-face environment as they learned about teaching data analysis and probability with technology. Recall the community of inquiry framework (Garrison, Anderson, & Archer, 2000), with its social, cognitive, and teaching presences, from Chapter 2 (see Figure 1). In order to answer two research questions about discourse and prospective teachers' understanding of variability, much data was analyzed that provided information about these three presences. In thinking about the entire face-to-face case holistically, however, it is difficult to summarize the presences individually. For example, face-to-face discourse was certainly affected by decisions made by the instructor and the content focus of the discussion. Likewise, the decisions made by the instructor were influenced by prospective teachers' understanding of content and the feedback she received from them. Therefore, while each of the presences in the case will be revisited below, it is the way in which they intersect with one another that best describes prospective teachers' discourse and understanding of content in the face-to-face class for this study. Brief recaps of each presence, analyses and relevant literature are provided.

Revisiting the Social Presence of the Face-to-Face Case

The social presence described here is that in which prospective teachers participated. During whole group discussion, prospective teachers asked questions, answered questions, and shared ideas orally. They participated in surveys (online through *Moodle* and more spontaneous “show-of-hands”), shared ideas at the instructor’s computer at the front of the classroom, and wrote ideas on the whiteboard. Also present in the whole group discussions were hand gestures. One interesting hand gesture was used by a prospective teacher named Cora when she was describing a clump of data points. The instructor acknowledged that gesture, moved her fingers and thumb as if she were grabbing points, and named it the “Cora cluster.” This phrase was used by prospective teachers in later small group and other whole group discussions, and even in an interview. In addition to hand gestures, the instructor pointed at the computer projection/screen several times across episodes (e.g. referring to a data point or reference line).

During small group discussions, each prospective teacher worked with two prospective teachers nearby. Each would continue having access to his/her computer, but the group would often move chairs closer together to create a more-defined group space (see focus group below).

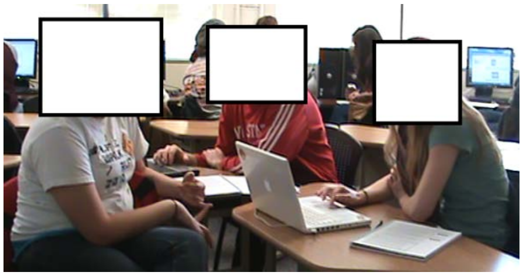


Figure 117. Physical arrangement of face-to-face focus group.

The instructor walked around during these discussions, listening in to groups' conversations, answering questions that came up and offering ideas if a group needed help. These interactions were not always captured on the video recordings of each class session. Also difficult to capture were affirmations from prospective teachers. They often responded to the instructor with quick affirmations, sometimes aloud, but most often through eye contact, smiles, and head-nodding. These affirmations were an important part of assessing the group's overall understanding for the instructor.

One noticeable part of the face-to-face discourse was a distinct pattern throughout the episodes of the instructor speaking and a prospective teacher following. This back-and-forth nature of the class made the discourse predictable for prospective teachers from week to week. They were comfortable with the routine and usually participated with choral-response type questions the instructor asked. The structure of the discussions was relaxed and without structure with regard to the sharing of ideas, particularly following a small group discussion.

According to focus group members, these discussions were a favorite component of the course. They seemed to appreciate the opportunity to "bounce ideas off of one another." As they considered questions related to SK, TSK, and TPSK, this became even more important to them. With a focus on content and students' thinking, discussions became more than just a venue to share disconnected ideas. During discussions, they worked through activities together and reflected on their own understanding. These practices of collaboration and reflection are ones teacher education research recommends (Cady & Rearden, 2009; Groth, 2007; Stephens & Hartmann, 2004; Stipek et al, 2001).

Revisiting the Cognitive Presence of the Face-to-Face Case

Cognitive presence described here includes prior knowledge prospective teachers bring with them as well as new knowledge they acquired during the 5-week study. Because this study was centered in a technology methods course for prospective mathematics teachers, the content focus was really threefold. Prospective teachers learned or re-learned statistical/probabilistic content (SK) with technology (TSK) and were asked to consider whether the use of technology to teach that content might be beneficial or not (TPSK).

Admittedly, few of them had much experience with statistics and probability (SK). Recall that three ideas related to variability were the content focus of this study: distributions, deviation, and the law of large numbers. Overall, prospective teachers were able to generally describe characteristics of distributions separately, but rarely made explicit connections with center and spread. Like Makar and Confrey's (2004) prospective teachers, most descriptions included informal language such as "clustered" and "up and down." In fact, the use of informal language was apparent throughout the study. This language was used appropriately in most cases and often mirrored that which was promoted in the curriculum text. From their language alone, it was difficult to make claims about prospective teachers' understanding of variability. Prospective teachers shared openly, both in discussions and in follow up interviews (with focus group participants), their lack of confidence in statistics. There were several times when prospective teachers said "I don't know" and "it's intimidating," particularly when trying to express ideas related to standard deviation and least squares regression. Similar confusion was also apparent in discussions around the law of large numbers, but improvements were seen in prospective teachers' understanding in this area. It is no surprise, then, that the majority of the discourse centered on statistical content and technological statistical content despite activities and tasks that encouraged prospective teachers to consider pedagogical implications of what they were learning.

Prospective teachers gave no indication that technology skills were not learned during the study (TSK). Their work with the statistical content above was completed through the use of multiple technologies. Specifically, during this study prospective teachers worked with *TinkerPlots*, *Fathom*, *TI*-84 graphing calculators, *Microsoft Excel*, and *Probability Explorer*. Through observing prospective teachers during the face-to-face classes, follow-up interviews with focus group members, and assignments it was evident that participants had developed a high comfort level in using the technologies.

Interestingly, over half (54%) of the episodes had a greater TSK focus. But, the fact that only one episode outside the Chapter 2 videocase (Episode 4 whole group (discussion #2)) had a strong TPSK focus revealed that prospective teachers were generally not attending to pedagogical issues without prompting from the instructor or the curriculum text. Because there was such a learning curve with new technologies and some ideas of variability described earlier, discourse remained centered on statistical ideas for the most part. This confirms the notion of Lee and Hollebrands' (2011) TPSK framework, that is, that statistical knowledge is the foundation level and must exist before technological and pedagogical knowledge can be more fully developed. Times during the study when TPSK discussions were long-lasting, were times when prospective teachers were using samples of student work (Chapter 2 materials). This complies with previous research results that discussions in teacher education should be centered on content and student work (Cady & Rearden, 2009; Groth, 2007; Stephens & Hartmann, 2004).

Revisiting the Teaching Presence of the Face-to-Face Case

The social, cognitive and teaching presence components did not exist in isolation. Instead, there were overlaps between how prospective teachers communicated and interacted with one another, and the content of their discussions and the subsequent knowledge they developed and shared. How the instructor worked together with the social and cognitive presences is important in thinking about implications of this research on future endeavors in mathematics education.

Through analyses of lesson maps and timelines, it was clear that the instructor made purposeful decisions when considering the placement of whole group and small group discussions. This study does not intend to make claims about the appropriate ordering of such activities. Rather, data does suggest that the predictable nature of discussion activities in the face-to-face class was beneficial for prospective teachers. In their interviews, focus group members commented favorably on the structure of the class.

The use of whole group and small group discussions, in particular, were a favorite and important part of the class. Focus group members spoke fondly of the ability to "bounce ideas off of one another" and to work with others, in general. As their work showed (Stipek et al., 2001), prospective teachers should be engaged, allowed to try new things, and given the opportunity to collaborate and reflect with others. The whole group work, which included, among other things, demonstrations and time for practicing new skills, allowed prospective teachers to do all of those things. Small group work held them accountable for it and allowed time for further discussion. The instructor’s movement around the room during whole group and small group discussions also provided more one-on-one contact with individual participants and allowed them to ask questions outside the whole group. This was an opportunity that many prospective teachers took advantage of.

As previously discussed, much discourse in the face-to-face class centered on statistical content or technological statistical content. Interestingly, this was true for the majority of both the small group and whole group discussions. This means, despite a plan of using TPSK-level questions, the instructor made decisions during class to keep the focus on content and how to solve statistical problems using technology. Knowing that time was a factor, she knew having a meaningful discussion about pedagogical issues without content knowledge would be difficult. However, this does not mean that pedagogical knowledge was not being built. If one agrees with von Glasersfeld (1984) and the constructivist philosophy, then prospective teachers in the face-to-face case were constructing a unique knowledge based on their own experiences and understanding. The fact that discourse was focused mostly on content meant that they were learning or re-learning statistical content while using technology. In turn, this implies that they saw first-hand what a “statistics lesson with technology” could look like. This should not be taken lightly, as many prospective teachers’ prior middle and high school classrooms looked very differently than the one they were currently learning in. They had the opportunity to engage with the technology just as a student would. Thus, they experienced for themselves the benefits and drawbacks of using certain technology tools to learn data analysis and probability.

CHAPTER 6: CROSS-CASE ANALYSIS OF ONLINE AND FACE-TO-FACE CASES

Introduction

In the previous two chapters, cases were presented for teaching and learning technology in online and face-to-face mathematics education methods courses. An identical curriculum was used with each group of prospective teachers (Lee et al., 2010), and the instructor for both groups was diligent about creating similar opportunities for discourse and facilitating activities to develop their understanding of variability. These efforts produced many similarities between the online and face-to-face groups. However, as one might expect, there were differences that emerged as well. This chapter takes a look at both cases simultaneously, presenting summaries of findings that highlight these similarities and differences.

Analyses below are divided into three large sections, one to address implementation of the curriculum and two for each of the research questions. First, a broad look at activities completed during class along with opportunities for discourse (number of minutes in class, percentage of time for whole/small group discussions) in the online and face-to-face environments was helpful in determining whether or not prospective teachers in each group had similar time with the content of this course. Second, recall this study considered how prospective mathematics teachers interacted with one another and with the instructor with curriculum focused on teaching data analysis and probability with technology. A cross-case comparison of discourse is organized into four smaller sections: (1) class management, (2) form, (3) purpose and (4) topic of discourse. Third, this study also aimed to understand how prospective teachers thought about variability, specifically related to describing distributions, deviation, and the law of large numbers. A cross-case comparison of prospective teachers' understanding is organized by topic of variability (e.g. describing distributions, deviation, and law of large numbers).

Curriculum Implementation

For this study, prospective teachers in a mathematics education methods course learned new technologies for teaching data analysis and probability. One section of the course met online synchronously once a week for approximately three hours. The second section of the course met in a traditional, face-to-face setting one hour and 15 minutes twice a week. Therefore, over the 5-week study, the online class met 5 times and the face-to-face class met 10 times. Activities and tasks presented in the curriculum (Lee et al., 2010) were written to develop teachers' statistical, technological, and pedagogical knowledge. The following table shows the sections in each chapter that were studied during class, assigned for homework, or omitted altogether. While some sections were assigned for homework, time was allotted at the beginning of the following class for discussion of those sections. Since the time spent outside of class was not recorded, and likely highly variable among prospective teachers, only class discussion time is reflected in the total time spent per chapter.

Table 35. Comparison of curriculum implementation.

Chapters and Sections from the Curriculum Text		**Online**	**Face-to-Face**
Chapter 1	Section 1	Class	Class
	Section 2	Class	Class
	Section 3	Class	Class
	Section 4	Class	Class
	Section 5	Omitted	Omitted
	Section 6	Omitted	Omitted
	Total Time for Chapter 1	**132 min**	**129 min**
Chapter 2	Section 1	Homework	Homework
	Section 2	Homework	Homework
	Section 3	Homework	Homework
	Section 4	Homework	Homework
	Section 5	Homework	Homework
	Total Time for Chapter 2	**21 min**	**20 min**
Chapter 3	Section 1	Class	Class
	Section 2	Class	Class
	Section 3	Class	Class
	Section 4	Homework	Class
	Total Time for Chapter 3	**123 min**	**111 min**
Chapter 4	Section 1	Class	Class
	Section 2	Class	Class
	Section 3	Class	Class
	Section 4	Class	Class
	Section 5	Class	Class
	Section 6	Class	Class
	Section 7	Omitted	Omitted
	Section 8	Omitted	Omitted
	Total Time for Chapter 4	**179 min**	**124 min**
Chapter 5	Section 1	Class	Class
	Section 2	Class	Class
	Section 3	Class	Class
	Section 4	Homework	Class
	Total Time for Chapter 5	**78 min**	**90 min**
Chapter 6	Section 1	Class	Class
	Section 2	Class	Class
	Section 3	Class	Class
	Section 4	Omitted	Class
	Total Time for Chapter 6	**137 min**	**151 min**

Differences in curriculum implementation were minimal and are highlighted above. Chapter 3 Section 4 focused on understanding spread of a distribution; it was in this section that the standard deviation formula appeared along with activities trying to connect that formula with the dynamic work in *Fathom*. Chapter 5 Section 4 focused on creating and running probability simulations in *Excel*. Both of these sections were completed during face-to-face classes, but had to be assigned for homework at the end of online classes due to time constraints. This was due to technological difficulties Macintosh users had with *Probability Explorer*'s compatibility. Similarly, activities in Chapter 6 took longer to complete in the online class. Thus, time expired before the last section, which used the graphing calculator and the binomial formula to compute a theoretical probability, could be completed.

The difference of total time spent with curriculum content over the study is apparent from the table above. Recall that the online class was one week ahead of the face-to-face class in terms of the activities and tasks they were completing. In addition, remember that the instructor conscientiously planned and structured face-to-face classes so that they would closely mirror corresponding online classes. The last two weeks in the face-to-face class were planned with activities from Chapters 5 and 6. The instructor knew that implementing activities in Chapter 5 similar to those that occurred in the online class would not necessarily take one week to complete in the face-to-face setting. However, so that class time was not wasted, the instructor incorporated two 30-minute “working group” activities where prospective teachers in the face-to-face class collaborated on identifying objectives and brainstorming ideas for parts of a 4-phase lesson plan on data analysis or probability that utilized technology. This difference of approximately one hour in total time spent with the curriculum during the class was purposeful.

Within each class meeting related to the chapters and sections above, there were obviously times that were more instructor-led, when new content was presented or new technology skills were demonstrated (see “Introducing new content” in Table 36). There were also many opportunities for whole group and small group discussions as well as times for independent work. The table below aids in comparing these opportunities from each learning environment.

Table 36. Comparison of opportunities of interaction by chapter.

Opportunities for Interaction by Chapter		**Online**	**Face-to-Face**
Chapter 1	Introducing new content	49%	50%
	Whole group discussion	28%	26%
	Small group discussion	8%	19%
	Independent work	0%	5%
	Technology issues	15%	0%
	Total Time for Chapter 1	**132 min**	**129 min**
Chapter 2	Whole group discussion	48%	70%
	Small group discussion	52%	30%
	Total Time for Chapter 2	**21 min**	**20 min**
Chapter 3	Introducing new content	34%	47%
	Whole group discussion	36%	26%
	Small group discussion	16%	23%
	Independent work	14%	4%
	Technology issues	0%	0%
	Total Time for Chapter 3	**123 min**	**111 min**
Chapter 4	Introducing new content	39%	32%
	Whole group discussion	35%	40%
	Small group discussion	14%	19%
	Independent work	12%	9%
	Technology issues	0%	0%
	Total Time for Chapter 4	**179 min**	**124 min**

Table 36 Continued

Chapter 5	Introducing new content	49%	47%
	Whole group discussion	27%	43%
	Small group discussion	0%	8%
	Independent work	2%	2%
	Technology issues	22%	0%
	Total Time for Chapter 5	**78 min**	**90 min**
Chapter 6	Introducing new content	39%	51%
	Whole group discussion	25%	17%
	Small group discussion	15%	17%
	Independent work	14%	15%
	Technology issues	7%	0%
	Total Time for Chapter 6	**137 min**	**151 min**
Total Time Working on Curriculum Content		**674 min = 11.2 hr**	**625 min = 10.4 hr**

The times when differences in opportunities for interaction were most noticeable were times when technological issues arose in the online environment (Chapters 1, 5, and 6). At the beginning of the online study there were numerous problems with *Elluminate*. Prospective teachers had issues with the program disconnecting, although the reason was not apparent. They also had difficulty participating due to microphone problems, late VCL reservations, and differences in graphing calculators. When these issues presented themselves, time spent in whole group and small group discussions seemed to be affected the most. The absence of noted technology problems in the online class during Chapters 3 and 4 and in the entire face-to-face class does not mean that issues were not present. Instead, previous analyses and the table above show that these issues did not become part of the discourse. Prospective teachers were able to resolve problems without involving the instructor. In the online class, this meant that prospective teachers were predominantly troubleshooting and correcting issues on their own. In the face-to-face class, prospective teachers would often quietly solicit the help of another participant seated nearby.

One noticeable difference in the table above is the total time spent with curriculum content in each environment. The timeline comparison below (Figure 118) shows one example (Chapter 4) of a sequence and types of discourse activities (independent work, whole group discussions, small group discussions, and teacher-led instruction of new content) from the online and face-to-face classes. The example shows that the sequence and types of discourse activities did not greatly vary between groups. Important to note, here, is the absence of technological issues in the online class (no yellow color). This shows that even when there were no technology problems to address, the online timeline is still longer, meaning that it simply took longer to do activities within a synchronous, online environment. This was true for Chapters 1-4. Chapters 5 and 6 were the exception as the face-to-face class spent more time with the curriculum content. However, Table 35, showed that in each of those chapters, the online group studied one less section than the face-to-face class.

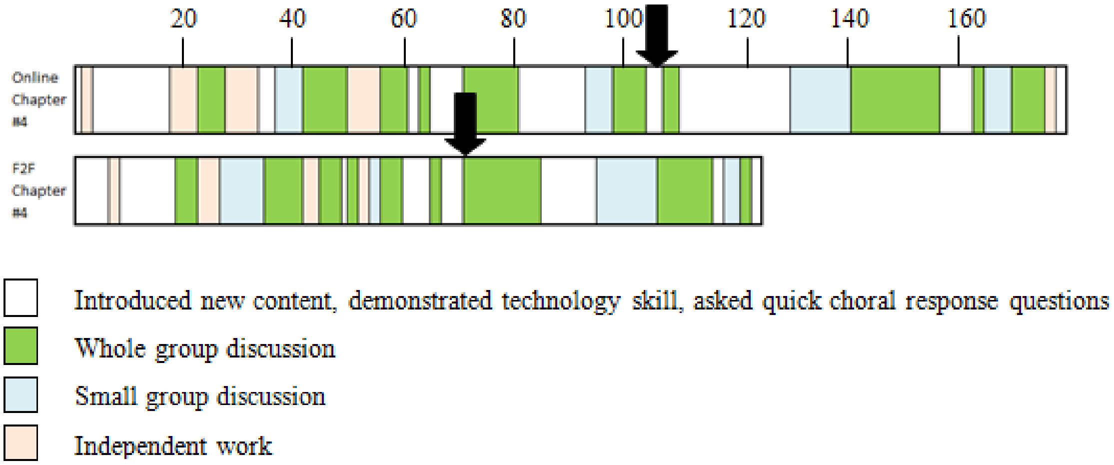


Figure 118. Timeline comparison of Chapter 4 in the online and face-to-face settings.

A second factor in the difference of total time spent with curriculum content over the study is not apparent from the table above, but was addressed in the preceding case analyses. Recall that the online class was one week ahead of the face-to-face class in terms of the activities and tasks they were completing. In addition, remember that the instructor conscientiously planned and structured face-to-face classes so that they would closely mirror corresponding online classes. The difference of approximately one hour in total time spent between the two groups was purposeful as the instructor implemented activities in the face-to-face class to help prospective teachers with an upcoming project.

Discourse

This study considered how prospective mathematics teachers interacted with one another and with the instructor with curriculum focused on teaching data analysis and probability with technology. Based on analyses of lesson graphs and timelines of entire lessons, and transcripts of episodes from each chapter, the two previous chapters contained findings about opportunities for discourse at the whole-group and small-group levels. In addition, data collected from focus-group interviews were also presented to corroborate trends found from discussions which occurred during the online and face-to-face classes. This cross-case analysis of discourse aims to present findings from each learning environment simultaneously. The comparison below is organized into four smaller sections: (1) class management, (2) form of discourse, (3) purpose of discourse and (4) topic of discourse.

Class Management

Within both of the online and face-to-face cases, it was shown that the placement of discussion activities seemed to be consistent. In other words, whole group discussion time was interspersed throughout each lesson, and was kept to less than fifteen minutes in most instances. In addition, across all episodes from each learning environment, small group discussions were immediately followed by whole group discussion. Independent work was introduced in both classes, seen in Episode 2 of the online class and in Episode 1 of the face-to-face class, so that prospective teachers had time to practice technology skills and/or troubleshoot technical errors that were occurring.

The technical difficulties in the online environment (either with *Elluminate* or one of the technologies being used in the curriculum), presented some differences in opportunities for discourse between the two classes. For example, Chapter 1 timelines below highlight the presence of technical issues (yellow) in the online environment. The time spent troubleshooting and addressing those problems resulted in less time for discussions, particularly with small groups (blue), for prospective teachers in the online class.

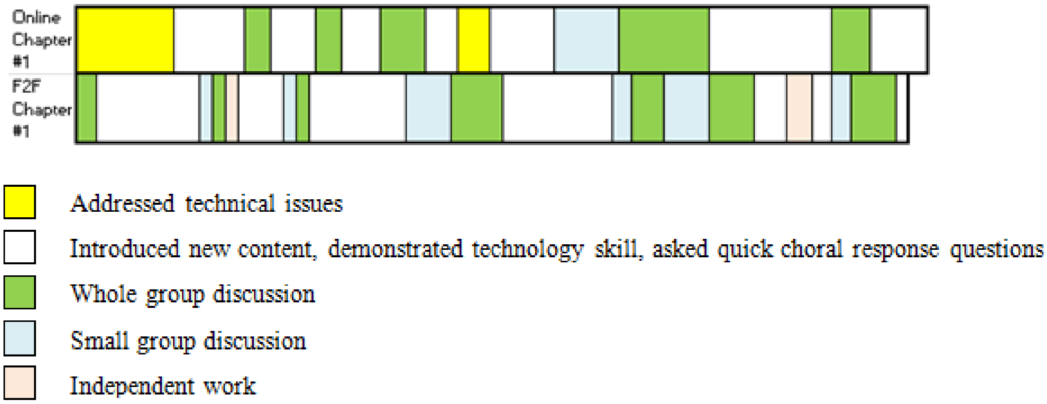


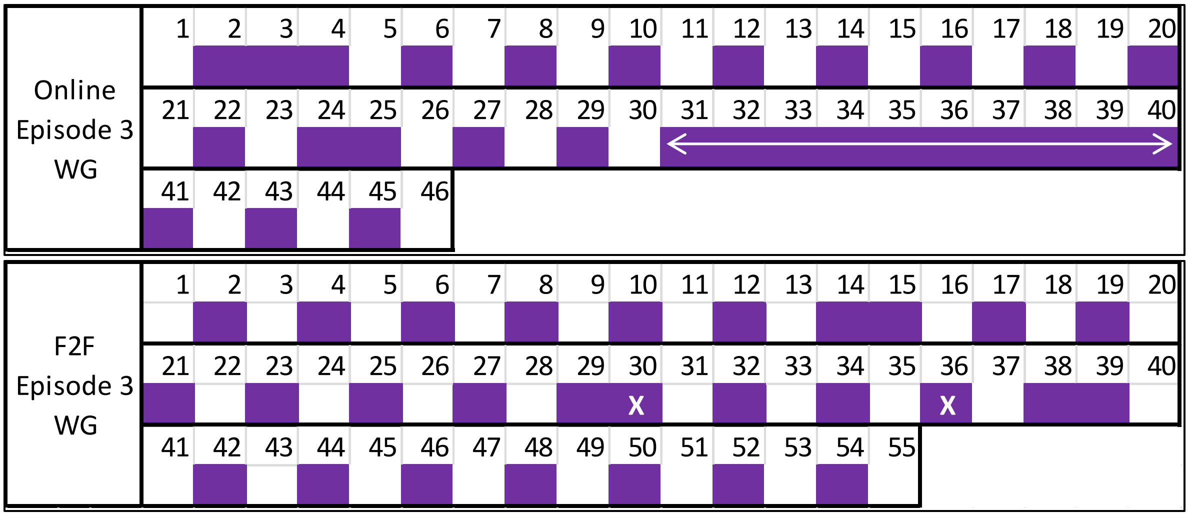
Figure 119. Timeline comparison of Chapter 1 in the online and face-to-face settings.

During small group discussions in both learning environments, the instructor's involvement was consistent as she enjoyed moving from group to group in order to listen in on prospective teachers' conversations about content, technology, and pedagogy. It was the way in which she moved that was different. In the face-to-face class, she would physically walk around the room in order to enter a small group's working space. In a small group, she could hear prospective teachers talking and watch how they were interacting with each other (e.g. hand gestures). With much talking going on in the classroom at one time, it was difficult to know what was going on in other groups until she moved closer to them.

In the online class, the instructor could also physically move from group to group by clicking on her name in the *Elluminate* participants' window and dragging it into a group's breakout room (denoted by a folder labeled Group 1, etc., in the participants' window). In a breakout room she could hear prospective teachers talking and read anything they were writing in the chat window or interactive whiteboard. The "noise" from other groups was entirely absent. However, while in one group she could see that other prospective teachers were talking and writing in their groups by looking at the *Elluminate* icons beside participants' names in the participants' window, which turned yellow when a participant was actively using their microphone, chat window, or online writing tools. She could then move to other groups until she had visited breakout rooms for each group.

The ways in which the instructor facilitated whole group discussions varied between settings as well. They also varied among types of whole group discussions. Consider, first, whole group discussions that followed small group discussions. In the online class, the instructor had to put procedures in place in order to minimize audio feedback within

Elluminate which resulted from multiple microphones being activated simultaneously. Groups were invited, in order, to share ideas. This certainly reduced the amount of audio problems, but added more structure to discussions. The face-to-face class was much less structured as anyone could voluntarily share at any time. This difference in structure carried over and could also be seen in whole group discussions that did not follow small group discussion. The face-to-face class remained more unstructured. Prospective teachers could respond at will and the instructor tended to acknowledge each idea they shared often by repeating the idea and building on it or asking another question. This resulted in a back-and-forth discussion pattern between the instructor and participants that was seen across episodes for the face-to-face class (Figure 120, bottom). Multiple, near simultaneous, responses from prospective teachers also occurred (e.g. see the “X” at #30 and #36), but it was difficult to discern everything that was shared. While the online discussions contained some times of back-and-forth discussion, there were many more times of asking a question and receiving multiple responses simultaneously (see the double-arrowed line in Figure 120, top). When these multiple responses occurred the instructor rarely acknowledged more than two or three teachers individually. Instead, she would make a general comment about the nature of responses she was reading in the chat window.



Online Episode 3 WG	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
		■	■	■		■		■		■		■		■		■		■		■
	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
		■		■	■		■		■		■ ↔	■ ↔	■ ↔	■ ↔	■ ↔	■ ↔	■ ↔	■ ↔	■ ↔	■ ↔
	41	42	43	44	45	46														
	■		■		■															

F2F Episode 3 WG	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
		■		■		■		■		■		■		■	■		■		■	
	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
	■		■		■		■		■	■ X		■		■		■ X		■	■	
	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55					
		■		■		■		■		■		■		■						

■ Prospective teachers
↔ Multiple prospective teachers in the online class using chat simultaneously
X Multiple prospective teachers in the face-to-face class talking simultaneously

🟨 Primary instructor of the online course

☐ Instructor of the study

Figure 120. Discussion pattern comparison of Chapter 3 in the online and face-to-face settings.

As the example above shows, there were similarities in discussion patterns between groups. Both learning environments had a considerable number of times when the instructors' question or comment was followed by a single comment by a prospective teachers (denoted by the purple and white pattern). There were also times when more than one prospective teacher would respond to a question, one after another. Still, at other points of the discussion a question by the instructor would elicit multiple, nearly simultaneous, responses from prospective teachers. These are denoted in the example above with a double-arrowed line and an X for the online and face-to-face classes, respectively. The nature of the online environment, with the use of the chat window, allowed all responses to be seen. Even though most chats during those times would be displayed within a few seconds, they were displayed in chronological order and thus, each exchange could be considered for the discussion pattern above. In contrast, during times when multiple responses were provided by participants in the face-to-face class, it was difficult to discern how many students actually provided a response and what those responses were. Therefore, the discussion pattern above only indicates the instances when multiple responses occurred.

Form of Discourse

As expected, the form of discourse varied greatly between the online and face-to-face groups. During whole group discussions within the online environment, prospective teachers participated in a variety of ways (use the microphone, type in the chat window, use an emoticon, draw/type on the interactive whiteboard, answer a survey, and assist with a technology demonstration). During times when someone spoke on behalf of a particular small group, he/she would always use the microphone. Interestingly, other prospective teachers rarely used the chat window during this time. Yet, when the instructor asked a choral-response type question, most participants seemed comfortable sharing an idea with a quick chat window response. Throughout the episodes analyzed in the five-week online study, emoticons and whiteboard tools within *Elluminate* were rarely accessed by prospective teachers unless directed by the instructor.

During small group discussions online, prospective teachers continued to interact in a variety of ways. Aside from the occasional faulty microphone equipment, members of the focus group seemed to strike a balance between microphone and chat window use, despite having different levels of comfort in communicating online with one another. The interactive whiteboard was used occasionally to record ideas during small group discussions, but members of the focus group, admittedly, found typing and editing comments on that whiteboard problematic. Regardless, having multiple ways of sharing ideas was also a feature of *Elluminate* that prospective teachers in the online group seemed to appreciate. The total usage of some of these features in *Elluminate* across the six online episodes provides an overall picture of what occurred during those times of class.

For the online class, across all discussions from the six episodes, the number of times the instructor or a prospective teacher used the microphone was 147. Compare that number to the 145 times the chat window was used, the 44 times interactive whiteboard tools were used, and the 26 times an emoticon was used. If non-microphone categories were merged one could compare 147 uses of microphone to 215 uses of other means of communications. Although prospective teachers rarely accessed emoticons and whiteboard tools unless directed by the instructor, the fact that other forms of communication outnumbered talking with the microphone is important. Granted, these totals only represent six episodes of a 5-week study, the differences are still noteworthy.

The number of ways prospective teachers shared ideas in the face-to-face class was smaller. In addition to speaking, they also wrote ideas on the whiteboard, completed surveys through *Moodle*, and shared ideas regarding a technology tool through demonstrations at the instructor’s computer station at the front of the classroom. While these means of sharing occurred several times over the course of the five-week study, they did not occur frequently (i.e. demonstration and writing on the whiteboard happened only once) during the episodes presented and were therefore not a part of analysis for the face-to-face case.

Ordinary hand gestures occurred as one might expect from someone talking to someone else. However, specific hand gestures and pointing to something projected on the screen were often directly related to the content or technology being discussed. For example, in the face-to-face whole group discussion of Episode 3 where prospective teachers were using a movable line with squares to better understand standard deviation, the instructor described the "little squares coming off of that" by using her fingers to make what seemed to be outlines of squares. Another example in the whole group discussion on least squares regression in Episode 4 showed the instructor pointing to the screen – specifically to a point in the scatterplot in order to highlight the corresponding point in the residual plot.

One interesting form was a hand gesture that was named and used throughout the study. A group of points that appeared clumped together became known as a "Cora cluster," a phrase initiated by the instructor and adopted by the face-to-face class after one prospective teacher (named Cora) described a cluster of points by moving her fingers and thumb as if she were grabbing points. This phrase was used in later small group and whole group discussions, as well as in an interview with a member of the face-to-face focus group. Smiles, head nods, and other non-verbal forms were also present throughout the face-to-class. Unfortunately, those also could not be captured with the video, and thus, could not be included in the analysis.

Aside from the physical differences in ways prospective teachers could communicate, another salient difference in discourse was presence of pauses within the online class. Sometimes these pauses were thoughtful and it was apparent that prospective teachers were independently working to recreate a representation using the technology. Other times, however, the pauses seemed long and awkward. This was particularly noticeable in the small group discussion with focus group members. Awkward pauses were not noticed in the face-to-face group. Certainly, there were quiet times when the instructor would give prospective teachers time to think about a question posed or to practice a new technology skill. But, these were natural breaks in discussion and did not seem long or awkward at all.

The many ways prospective teachers could stay engaged and interact with one another within the *Elluminate* environment (chat, emoticons, interactive whiteboard, and microphone) resulted in differences in the form of discourse between the online and face-to-face groups. While these forms were utilized, and many times appreciated, by prospective teachers, they sometimes produced awkward pauses, especially in small group discussions. Participants in the face-to-face class had less variety in the ways they could communicate with one another, but one advantage they had over their online counterparts was the ability to use hand gestures. Often, these gestures were used in descriptions of content and technology being used. There was even a time that such a gesture became an object of discussion in its own right ("Cora cluster").

Purpose of Discourse

The following table shows the time spent, number of exchanges, and percentages of purpose for each small group and whole group discussion in the six episodes described in the previous two chapters. Recall that a single exchange was often coded as having multiple purposes. Thus, the percentages for each column (a small group or whole group discussion) may not necessarily add up to 100%. Looking across episodes and across learning environments, there are similarities and differences worth noting.

Table 37. Comparison of episodes (time, number of exchanges, and purpose).

		Learning Enviornment	Episode 1		Episode 2		Episode 3		Episode 4		Episode 5	Episode 6	
			SG	WG	SG	WG	SG	WG	SG/WG1	WG/WG2	WG	WG	SG
	Time (min)	Online	9	7.5	9	9	4.5	14	4	8	7.5	7	8
		F-to-F	6	4.5	5.5	11.5	6	17.5	3	10	10.5	7	9
	Number of Exchanges	Online	32	20	18	50	8	46	12	44	31	35	23
		F-to-F	57	22	73	71	39	55	18	29	51	29	44
Purpose (% of exchanges)	Ask a Question	Online	28%	45%	0%	20%	25%	26%	17%	13%	6%	14%	26%
		F-to-F	11%	27%	7%	25%	15%	38%	28%	38%	25%	48%	20%
	Answer a Question	Online	19%	30%	17%	48%	25%	50%	8%	26%	58%	29%	26%
		F-to-F	9%	14%	4%	18%	8%	38%	33%	31%	55%	41%	9%
	Share an Idea or Concern	Online	34%	25%	44%	32%	38%	11%	17%	19%	19%	23%	22%
		F-to-F	30%	59%	51%	49%	49%	38%	33%	52%	25%	28%	68%
	Justify	Online	19%	15%	11%	6%	0%	4%	8%	2%	0%	3%	0%
		F-to-F	5%	5%	4%	0%	0%	2%	6%	0%	2%	0%	0%
	Affirm	Online	28%	15%	33%	16%	63%	39%	50%	49%	19%	43%	35%
		F-to-F	28%	41%	42%	20%	33%	22%	6%	34%	14%	24%	34%

First, the number of exchanges was greater in the face-to-face class in all but two discussions (Episode 4 WG/WG2 and Episode 6 WG). Often, in the case of small group discussions, those differences were even more noticeable. Perhaps the presence of pauses in the online class described above was a big factor in this difference. Prospective teachers' lower level of comfort working in the online environment likely contributed to that as well. The face-to-face environment is familiar. Therefore, discussing issues within a small group was easier and faster for prospective teachers in that setting.

Certainly, potential causes of differences in the number of exchanges should be considered when form is compared. Prospective teachers in the face-to-face class seemed to be comfortable sharing and building ideas off of one another. In other words, they did not simply answer the question. They added personal anecdotes and seemed comfortable "bouncing ideas off of each other." This was more difficult to do in the online environment, with the pauses and newness of the online learning environment. Regardless of the setting, however, prospective teachers, in both classroom settings, justified responses least of all. Instead they would often share ideas without offering their reasoning behind them.

One surprising result in purpose was the number of affirmations. Most of the affirmations in whole group discussions came from the instructor. But, in small group discussions, prospective teachers would also affirm one another. The lack of justification mentioned above, together with the abundance of affirmations is somewhat troubling. Prospective teachers in both groups seemed to enjoy the small group work, but often ideas were accepted without question, even when there were obvious discrepancies in responses (e.g. small group discussions for Episode 6 in both groups).

As the difference in the number of exchanges described above indicates, prospective teachers in the online class interacted with the instructor and with one another more frequently than those in the face-to-face class. There are likely several reasons for this, among them the presence of pauses and a lack of comfort communicating in the synchronous, online environment. Regardless of the setting, however, prospective teachers rarely justified ideas related to teaching statistics with technology. Instead, they often shared ideas and used affirmations sometimes without building off of one another's ideas, particularly in the online class.

Topic of Discourse

Prospective teachers very often attended to statistical and probabilistic content in their discussions. The following table lists the focus of questions in activities for each episode studied and the corresponding percent of interactions for statistical knowledge (SK), technological statistical knowledge (TSK), and technological pedagogical statistical knowledge (TPSK) associated with those questions. Again, recall column totals below may not add up to 100% if there were exchanges that had a non-TPSK focus (e.g. affirmations, comment or question about a technological issue).

Table 38. Comparison of episodes (topic).

		Learning Enviornment	Episode 1		Episode 2		Episode 3		Episode 4		Episode 5	Episode 6	
			SG	WG	SG	WG	SG	WG	SG/WG1	WG/WG2	WG	WG	SG
Focus (% of exchanges)	SK	Online	19%	15%	0%	2%	25%	41%	0%	28%	71%	43%	52%
		F-to-F	11%	59%	0%	1%	3%	13%	0%	17%	84%	24%	39%
	TSK	Online	19%	5%	0%	6%	25%	13%	30%	15%	16%	31%	4%
		F-to-F	0%	18%	8%	14%	28%	45%	83%	34%	8%	48%	14%
	TPSK	Online	9%	10%	17%	16%	0%	7%	10%	0%	3%	0%	0%
		F-to-F	0%	0%	23%	44%	8%	16%	0%	34%	0%	7%	9%

In episodes from both learning environments, the topic of discourse was predominantly statistical in nature. Very few discussions focused mostly on pedagogy (only once in the online class and only twice in the face-to-face class). Some discussions contained no evidence of TPSK at all. In the online class, the lack of TPSK was particularly noticeable in discussions centered on understanding residuals (Episode 4) and the law of large numbers (Episodes 5 and 6). In the face-to-face class, the lack of TPSK was also noticed in discussions related to the law of large numbers (Episode 5), as well as those focused on measures of center in Episode 1.

It is difficult to determine the exact cause of the small presence of TPSK during discussions. In discussions centered on the law of large numbers, it is likely that prospective teachers in both groups initially lacked statistical knowledge in that area. Pre-assessment data would confirm that and will be discussed in the upcoming section comparing participants' understanding of variability. It is also possible that a lower statistical content knowledge of deviation in the online class contributed to the lack of TPSK focus. In the face-to-face class, however, it is not clear from other data sources that prospective teachers' understanding of measures of center was weak. Therefore, the lack of focus on TPSK was related to something else, namely the context of the statistical problem. With data sets related to teacher salary and North Carolina colleges and universities, prospective teachers in the face-to-face class seemed to focus on attributes within those sets rather than discuss how students might work with that content topic or technology. One interesting finding among face-to-face episodes, however, was a TSK focus in three episodes. Those episodes were parts of discussions surrounding content in Chapters 3, 4, and 6, which, incidentally, all required the use of *Fathom*. The discourse percentages revealed that prospective teachers were focused on the new technology skills and representations they were using to learn or re-learn statistical concepts.

Summary of Discourse

During whole group and small group discussions, prospective teachers in both classes asked questions, answered questions, and shared ideas. The many ways prospective teachers could stay engaged and interact with one another within the *Elluminate* environment, however (e.g. chat, emoticons, interactive whiteboard, microphone), compared to a few ways in the face-to-face setting (e.g. talk, hand gestures, write on whiteboard), resulted in differences in discourse between the two groups. Prospective teachers participated in surveys, either within *Elluminate* for the online class or through *Moodle* and more spontaneous “show-of-hands” for the face-to-face class. They shared ideas using technology, either by controlling the instructor’s mouse within *Elluminate* or at the instructor’s computer at the front of the face-to-face classroom. They also wrote ideas on the whiteboard, either using tools and the interactive whiteboard within *Elluminate* or by walking to the classroom whiteboard of the face-to-face class.

In addition to the obvious physical differences in the ways that prospective teachers in each group produced discourse, there were also two noticeable differences in the way they communicated with the instructor and with one another. First, in the face-to-face class, participants used hand gestures when describing a technology representation and/or content, something that could not be done in the online setting for this study. Second, in the online class, there were often long pauses during discussions, particularly small group discussions that seemed awkward. These pauses were not noticed in the face-to-face class.

A back-and-forth discussion pattern between the instructor and participants was seen across episodes for the face-to-face class. Sometimes multiple, near simultaneous, responses from prospective teachers also occurred, but it was difficult to discern what was being shared. While the online discussions contained some times of back-and-forth discussion, there were many more times of asking a question and receiving multiple responses simultaneously due to the ease by which participants could quickly respond in the chat window.

During discussions, prospective teachers shared ideas and provided affirmations to one another. However, they rarely built upon previous comments. In addition, they did not often use justifications in their responses. These findings, along with the fact that most discussions focused on statistical content and did not consider pedagogical issues, likely point to a lack of confidence in the statistical ideas they were discussing. More about prospective teachers' understanding will be explored in the next section.

Understanding of Variability

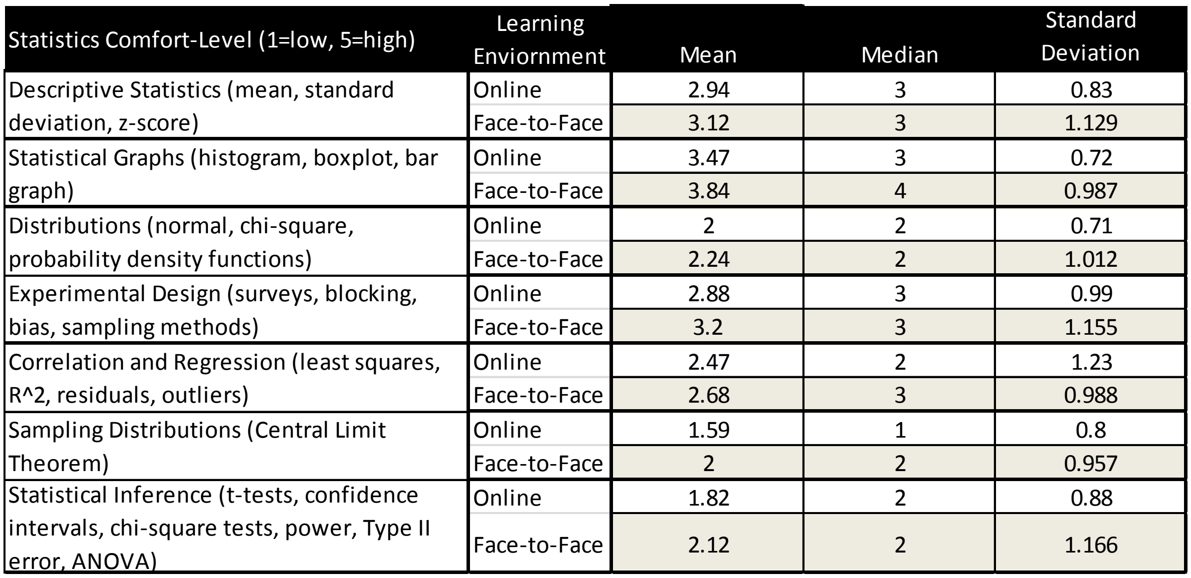
Before the study began, a pre-assessment was given to obtain information on prospective teachers in each group. Two questions, one about their statistics background and one about their comfort level with statistics, revealed varied experiences with statistics and probability. The table below shows similar numbers of students who reported having no statistics experience and having taken an introductory statistics course either at the university of this study or elsewhere. The largest difference fell in the number of students who reported having taken a calculus-based statistics course. A much larger percentage of the face-to-face class had taken an upper-level calculus-based statistics course.

Table 39. Comparison of prospective teachers' statistical background.

College Statistics Course	Online (n=17)	Face-to-Face (n=25)
Never had a statistics class	1 (6%)	3 (12%)
Introductory statistics course elsewhere (0-25 years ago)	5 (29%)	3 (12%)
Introductory statistics course	9 (53%)	12 (48%)
Calculus-based statistics course	2 (12%)	7 (28%)

Some differences between online and face-to-face prospective teachers' comfort levels with various statistical concepts were also noted from the pre-assessment. Table 40 below shows summary statistics for seven topics. As one might expect, many prospective teachers rated themselves most confident with descriptive statistics and statistical graphs. However, while a self-rated comfort level of 5 was rare from online prospective teachers on any topic listed, 28% of face-to-face participants used a score of five on one or more categories. But, as with any self-rated data, there were likely some discrepancies between what was reported in the comfort level question and the statistics background. For example, of the 28% from the face-to-face class that used a score of 5, 71% of them had only taken an introductory course. Prospective teachers in both groups claimed that a particular course was not rigorous; one even called it a “joke.”

Table 40. Comparison of prospective teachers’ comfort levels with topics of statistics.



Statistics Comfort-Level (1=low, 5=high)	Learning Enviornment	Mean	Median	Standard Deviation
Descriptive Statistics (mean, standard deviation, z-score)	Online	2.94	3	0.83
	Face-to-Face	3.12	3	1.129
Statistical Graphs (histogram, boxplot, bar graph)	Online	3.47	3	0.72
	Face-to-Face	3.84	4	0.987
Distributions (normal, chi-square, probability density functions)	Online	2	2	0.71
	Face-to-Face	2.24	2	1.012
Experimental Design (surveys, blocking, bias, sampling methods)	Online	2.88	3	0.99
	Face-to-Face	3.2	3	1.155
Correlation and Regression (least squares, R^2, residuals, outliers)	Online	2.47	2	1.23
	Face-to-Face	2.68	3	0.988
Sampling Distributions (Central Limit Theorem)	Online	1.59	1	0.8
	Face-to-Face	2	2	0.957
Statistical Inference (t-tests, confidence intervals, chi-square tests, power, Type II error, ANOVA)	Online	1.82	2	0.88
	Face-to-Face	2.12	2	1.166

The mean score for descriptive statistics in the face-to-face class was greater than three. In addition, the mean score for statistical graphs in both groups was greater than three. But, it is important here to point out that all other mean comfort level scores were less than 3. In other words, this group of prospective teachers rated themselves as having low to medium comfort levels with these statistical topics. While this study did not attempt to address content related to each of the seven topics listed in the pre-assessment background question, their self-assessments do provide some evidence that the groups were similar in their prior statistical experiences. Recall, from the discourse comparison, that many discussions were focused on statistical content. Prospective teachers' lower comfort levels may have contributed to the need for statistical discussions.

Along with background questions, the pre-/post-assessment also contained statistical content questions intended to capture prospective teachers' understanding of variability. Each group showed signs of improvement. The mean gains for the online and face-to-face groups were 1.76 and 2.8, respectively, but Wilcoxon rank-sum tests revealed that only the face-to-face group's improvement was significant (*P-value* = 0.006). Thus, it may appear that the members of the face-to-face class learned more and gained a better understanding of variability than the online group.

The pre/post-assessment was only one piece of data collected from each prospective teacher. Much data was collected to capture prospective teachers' understanding about variability (e.g. episode transcripts, interviews, assessments). This cross-case analysis aims to present findings from each learning environment simultaneously. The comparison below is organized into three smaller sections: (1) describing distributions, (2) understanding deviation, and (3) understanding the law of large numbers.

Describing Distributions

The ability to correctly describe distributions plays a key role in understanding variability. There were multiple times, over the course of the study, when prospective teachers were asked to describe distributions as part of a task they were completing. A pre-/post-assessment question and homework assignment showed that most prospective teachers in both groups did not use statistical language to describe the similarities and differences between the two histograms. Instead, they kept references to statistical ideas on the descriptive level and used informal phrases such as “more spread out” and “tighter” to explain differences. Many of them used references to a modal clump, though they did not label it as such. This is not surprising since the curriculum materials emphasize this informal approach. While some prospective teachers explicitly addressed center, spread, and shape in their post-assessment responses, most remained at the descriptive level and seemed comfortable describing distributions more informally.

On the final exam for the course, prospective teachers were allowed to choose three of five questions. Out of the five questions, three of them were related to statistical and probabilistic concepts discussed during the study. It is important then to note that 6 out of 17 participants in the online class chose a performance task related to describing distributions, compared to 21 out of 25 in the face-to-face class. This may be an indicator, in and of itself, of comfort level with *TinkerPlots* and with this notion of variability. At any rate, only about half of those prospective teachers who completed the task (50% in the online class and 61% in the face-to-face class), explicitly included ideas of both center and spread in their responses. This meant that approximately half of all prospective teachers in the study were only considering center and spread when writing questions for their future students.

Discussions throughout the study revealed that prospective teachers seemed comfortable describing distributions by center. One difference, however, between participants of each group, is the way in which they described center. Members of the face-to-face class tended to describe center less informally than the online class. However, their informal language regarding spread typically matched that of the online group. Discussions also highlighted that prospective teachers in both the online and face-to-face classes often considered the effects of higher and lower values on measures of center. They would often explicitly state the highest and lowest values of data, sometimes inaccurately calling them outliers without determining whether or not those values were actually statistical outliers. They seemed astute at recognizing the effects of “extreme” values on the mean, median, and midrange.

The trends above were also apparent during interviews with focus group members. Exploring the *TinkerPlots* cat data, prospective teachers from each class approached the task in similar ways. First, the bimodal shape of the distribution (see Figure 108) was something that was immediately included in their descriptions of the distribution, though most of them did not use the term bimodal. Prospective teachers in both groups seemed to consider the shape when trying to describe a “typical” body length of a cat. They also considered the higher and lower values in estimating a “typical” value. One prospective teacher even called the highest and lowest values outliers without doing anything beyond visual inspection.

Much data was collected and analyzed to determine how prospective teachers’ thought about and described distributions. Data show that prospective teachers often described distributions by addressing one of the following: center, spread, or shape. In addition, responses revealed a tendency to informally describe measures (e.g. clump, separated out, goes up and down). It was also evident that many prospective teachers considered the effects of higher and lower values on those values. They would often explicitly state the highest and lowest values of data, sometimes inaccurately calling them outliers without determining whether or not those values were actually statistical outliers.

Understanding Deviation

Curriculum materials used for this study allowed prospective teachers to consider deviation in a variety of contexts. Specifically, tasks and activities were completed throughout the class which required them to think about both univariate and bivariate deviation. First, a short-answer question on the pre-/post-assessment asked prospective teachers to decide which of two distributions had the largest variability (univariate deviation). Little improvement was seen in responses from both the online and face-to-face groups. Prospective teachers were mostly thinking about variability as “bumpiness” and described the “up and down” shape of the second distribution in their answers. Some also included the lack of normal curve or bell curve shape as part of an explanation for greater variability in the second distribution. Only a few prospective teachers seemed to focus on some measure of center and think about how values deviated from the center. When they did, they wrote things informally such as “Avg age will be approx. the same but in CPR2 the concentration in age will be all over” and “not centered around one area.” Overall, results from this assessment showed prospective teachers did not seem to make a connection between variability and deviation in comparing two univariate distributions. A couple of participants in the face-to-face class even mistakenly thought that since the means and ranges of the two distributions were approximately the same, variability would be the same as well.

Second, one of the five final exam questions was related to bivariate deviation. In that performance task, 6 out of 17 participants in the online class chose to complete the task, compared to 11 out of 25 in the face-to-face class. Again, this may be an indicator, in and of itself, of comfort level with *Fathom* and with this notion of variability. While this task was not designed to test total understanding of deviation, it did provide some insight into how prospective teachers were thinking about correlation and how they might address students' misconceptions. Some prospective teachers provided examples that indicated they were carefully selecting correlations and slopes for students to consider. They addressed the misconception that correlation and slope have to be equal. What was missing in many responses, however, was a sequencing of questions that would help their students build the understanding that correlation and slope are two distinct entities. In one instance, however, a prospective teacher in the online class showed an understanding that students' misconceptions stem from misunderstanding correlation overall. Instead of jumping to examples, she described an attempt to re-explain the underpinning ideas of correlation. She seemed to understand that correlation was measuring deviation from a best-fit line. Because she did not give specific examples as the directions asked, her rubric score for this task was low. But, she provided more evidence that pointed to a deeper understanding than those prospective teachers who gave clear examples and scored higher.

Focus group interviews as well as discussions throughout Chapters 3 and 4 provided opportunities to analyze prospective teachers' understanding of both univariate and bivariate deviation. Data show that prospective teachers often thought about variability as shape and not deviation from a mean. With regard to univariate deviation, they used informal language such as "cluttered" and "up and down" and rarely provided evidence that they were considering the distance from a mean. The same was true for bivariate deviation. They seemed to have a strong understanding of the directions of a correlation (positive, negative, none) but lacked understanding about how correlation was a measure using each individual point and its difference from means.

Pre-assessment questions at the beginning of the study and a self-assessment survey at the beginning of Chapter 4 showed that prospective teachers, in both groups, came in to this study with little content knowledge of least squares regression. Throughout episodes for Chapters 3 and 4, most of them seemed to appreciate the “visual” aspect of *Fathom* and discussed openly the differences between the dynamic movements they were learning and the formula-based, “plug-and-chug” methods they reported learning in their introductory statistics courses. However, in both classes, the instructor needed to assist most prospective teachers in making explicit connections to the underlying concept of deviation that was being visualized with tools like the movable lines and squared deviations (from a mean or a linear model).

In summary, data show that prospective teachers often thought about variability as shape and not deviation from a mean. With regard to univariate deviation, they used informal language such as “bumpiness” and “cluttered” and rarely provided evidence that they were considering the distance from a mean. The same was true for bivariate deviation. They seemed to have a strong understanding of the direction of a correlation (positive, negative, none) but often lacked understanding about how correlation was a measure using each individual point and its difference from means. Most prospective teachers did not offer explicit connections to the underlying concept of deviation that was being visualized with tools in *Fathom* like the movable lines and squared deviations (from a mean or a linear model). In other words, they struggled initially to make connections to formal statistical equations (e.g. standard deviation, regression line). Despite this, most of them seemed to appreciate the “visual” aspect of *Fathom* and discussed openly the differences between the dynamic movements they were learning and the formula-based, “plug-and-chug” methods they learned in their introductory statistics courses.

Understanding the Law of Large Numbers

Understanding the law of large numbers requires one to understand the variability across samples and variability between theoretical and empirical probabilities, and the role that sample size has in affecting this variability. The number of opportunities for assessing prospective teachers’ understanding of the law of large numbers was more limited than the other ideas of variability previously presented. This statistical concept may have been the most difficult for prospective teachers in both groups. And, interestingly, different data told different stories. While there were certainly signs of understanding, there were other data sources that brought that understanding into question.

From the beginning, the pre-assessment revealed a weak understanding with only 35% of the online class and 52% of the face-to-face class answering the related question correctly. The post-assessment that followed provided mixed results. Eighty-four percent of the face-to-face class answered the related question correctly, whereas only 47% of the online class answered the same question correctly. The lack of improvement in the online class was surprising as the wording of this question was very similar to activities prospective teachers had completed in Episode 6. Previous research has shown prospective teachers’ “ability to recognize sampling variability with respect to sample size” improved both using the curriculum for this course and otherwise (Lee & Lee, 2011, pg. 43). Therefore, simply engaging with the tasks and activities in Chapters 5 and 6 should have helped prospective teachers in both classes improve their understanding. While the face-to-face group outperformed the online group on this particular post-assessment question, other data showed more equivalent levels of lower understanding of the law of large numbers.

Throughout activities and tasks from Chapters 5 and 6, prospective teachers were provided multiple opportunities to explore this idea. Despite these efforts, small group discussions in Episode 6 for both groups showed confusion as focus group members failed to be able to justify their ideas or come to a collective agreement about problems related to the law of large numbers. In the online small group, prospective teachers shared their ideas individually, but never built upon previous ideas or questioned one another even when there was obvious disagreement. In the face-to-face small group, prospective teachers seemed to do a better job actually talking to one another and building on each other’s comments, but at several points in the discussion, they ended a conversation with “I don’t know” and moved on to another question. These examples provide further evidence of a lack of confidence around probability and the law of large numbers.

Another source of data was one of the five final exam questions, which was related to the law of large numbers. In that performance task, 11 out of 17 participants in the online class chose to complete that task, compared to 14 out of 25 in the face-to-face class. This was the only performance task where there was a greater percentage from the online class (65%) that chose to complete the task than the face-to-face class (56%). A cross-analysis of this task, however, is difficult due to the difference in the question between the groups. The online class exam question was missing the second part of the face-to-face class exam question, “*Explain how you can use your spinner simulation to help students understand similarities and differences between theoretical and empirical (experimental probability).”* While comparing rubric scores would not be meaningful, analyses did show similarities in the way prospective teachers described the law of large numbers. Often prospective teachers in both classes informally described how the results became “closer to the desired result,” or closer to a value that was “expected.” Thus, they seemed to be focused on the relationship between empirical and theoretical probabilities, which might be expected from the face-to-face class since it was explicitly stated in the task they were solving. Without theoretical probability being stated in the task given in the online class, however, it is worth noting that some of them (45%) explicitly referenced a theoretical probability. Strangely, however, several prospective teachers in the online group also explicitly used the term “odds” in their responses although statistical odds were not discussed in class or in the curriculum materials. It is quite possible, though, that they were using the term incorrectly, in place of probability. In either class, few of them provided evidence that they were thinking about the variability one might see from sample to sample, depending on the size of sample. However, the main conception of the law of large numbers is the idea of less variability with a larger sample size. Applying the law of large numbers to think of implications of variation across samples at different sizes may be a much deeper, more connected conception.

Data collected revealed differences in prospective teachers’ understanding of the law of large numbers. Written assessments showed a mixture of correct and incorrect responses. It was clear that some prospective teachers had a strong understanding that as a sample size grows, the variability between empirical and theoretical probabilities decreases, although they tended to describe this relationship informally (e.g. “closer to the desired result”). Other prospective teachers did not provide evidence that this concept was understood. Few prospective teachers provided evidence about their understanding of the variability one might see from sample to sample, depending on the size of sample.

The small group discussion also uncovered a lack of confidence with this idea. Focus group members in both groups had difficulty describing how the sample size affected an outcome and there was much confusion about the role of probabilistic assumptions. During their discussion, they were unable to help one another.

Summary

This cross-case analysis revealed information about prospective teachers’ interactions in a traditional, face-to-face environment and in a synchronous, online environment as they learned about teaching data analysis and probability with technology. Recall the community of inquiry framework (Garrison, Anderson, & Archer, 2000), with its social, cognitive, and teaching presences, from Chapter 2 (see Figure 1). In order to answer two research questions about discourse and prospective teachers’ understanding of variability, much data was analyzed that provided information about these three presences. In thinking about the entire online and face-to-face case holistically, however, it is difficult to summarize the presences individually. For example, discourse in each setting was certainly affected by decisions made by the instructor and the content focus of the discussion. Likewise, the decisions made by the instructor were influenced by prospective teachers’ understanding of content and the feedback she received from them. Therefore, while each of the presences in the case will be revisited below, it is the way in which they intersect with one another that best describes prospective teachers' discourse and understanding of content in the online and face-to-face classes for this study. Brief recaps of each presence, analyses and relevant literature are provided.

Revisiting the Social Presence

The social presence described here is the environment in which prospective teachers participated. Although certainly the instructor was an important part of the social presence, and some of her actions and decisions will be discussed in this section, a more thorough summary of the instructor's role will be presented in a later section. For prospective teachers in both the online and face-to-face groups, discussions were a favorite component of the course. They seemed to appreciate the opportunity to "bounce ideas off of one another" and to "have a say." As they considered questions related to SK, TSK, and TPSK, this became even more important to them. With a focus on content and students' thinking, discussions became more than just a venue to share disconnected ideas. During discussions, they worked through activities together and reflected on their own understanding. These practices of collaboration and reflection are ones teacher education research recommends (Cady & Rearden, 2009; Groth, 2007; Stephens & Hartmann, 2004; Stipek et al, 2001).

As expected, the social presences look very differently between the online and face-to-face environments. During whole group discussion, prospective teachers in both groups asked questions, answered questions, and shared ideas. The ways in which they did this, however, varied depending on the setting. In *Elluminate*, online participants spoke using a microphone and typed comments and questions in the chat window. They responded to the instructor with quick affirmations through the use of the "green check" and other emoticons.

They also participated in surveys, controlled the instructor's computer mouse during demonstrations, and typed ideas on the interactive whiteboard. McBrien et al. (2009) found that students used these features of the synchronous, online setting as points of personal engagement. Prospective teachers of the online case seemed to also stay engaged during the class through the use of such features. Many of them commented on how they appreciated viewing live technology demonstrations and the opportunities to discuss issues related to content, technology, and pedagogy with one another. It seemed that the interactive nature of *Elluminate*, was especially appealing, something that other researchers have also found in their work (e.g. Cady & Rearden, 2009; Stephens & Mottet, 2008).

Compare those forms of participation to that of the face-to-face environment. In the traditional setting, prospective teachers needed no equipment to speak. They often responded to the instructor through non-verbal forms of communication such as a head nod or smile. They participated in surveys (online through *Moodle* and more spontaneous "show-of-hands"), shared ideas at the instructor's computer at the front of the classroom, and wrote ideas on the whiteboard at the front of the classroom.

During small group discussions, prospective teachers again asked questions, answered questions, and shared ideas. But, again, the ways in which they did this depended on the class setting. In the online class, groups were predetermined. To "move" into groups, prospective teachers had to select their name in the participants' window and drag it to their group's breakout room. Once in the breakout room, members of the online focus group seemed to strike a balance between microphone use and the use of chat, despite having different levels of comfort in communicating online with one another. The interactive whiteboard was used occasionally to record ideas during small group discussions, but members of the focus group, admittedly, found typing and editing comments on that whiteboard problematic. During face-to-face small group discussion, each prospective teacher worked with two prospective teachers nearby. Each would continue having access to his/her laptop computer, but the participants would often move chairs closer together, creating a more-defined group space.

Aside from the physical differences in ways prospective teachers could communicate, two additional differences in discourse between the online and face-to-face classes are noteworthy. First, was the presence of pauses within the online class. Sometimes these pauses were thoughtful and it was apparent that prospective teachers were independently working to recreate a representation using the technology. Other times, however, the pauses seemed long and awkward. This was particularly noticeable in the small group discussion with focus group members.

Second, the differences in discussion patterns may be a key point in understanding the major differences in the two cases. The repeated back-and-forth pattern of the instructor and participants talking in the face-to-face class is in stark contrast to the multiple, simultaneous responses seen in the online class. The learning environment paired with the instructor’s facilitation decisions made before and during class are perhaps the underlying reasons for the difference. Out of necessity, the online class was more structured. Procedures were put in place to minimize audio feedback. Therefore, often during whole group discussions, small groups’ spokespersons spoke in turn. Other times, when the instructor asked a question for each prospective teacher to think about, they would offer ideas in the chat window, sometimes unsolicited. During these times, online prospective teachers may or may not have had their responses acknowledged by the instructor. Instead, she would scan all ideas and try to generalize comments she read in the chat window. It is also unknowable how much other class members read and thought about ideas or questions written in a chat window, particularly when many were written in succession and one would need to scroll back through the window to read contributions. Without an individual affirmation, prospective teachers may not have felt as engaged or connected with the instructor and with the class at large. On the other hand, the face-to-face discussions were much more relaxed and less-structured. Prospective teachers could respond at will and the instructor tended to acknowledge each idea they shared often by repeating the idea and building on it or asking another question. This may have made prospective teachers in that class feel like they were better "heard." This may have also affected whether prospective teachers shared ideas that built off of previous responses, something that was often missing in the online class.

Revisiting the Cognitive Presence

Cognitive presence includes prior knowledge prospective teachers brought with them, as well as new knowledge they acquired during the study. Because this study was centered in a technology methods course for prospective mathematics teachers, the content focus was really threefold. Prospective teachers learned or re-learned statistical/probabilistic content (SK) with technology (TSK) and were asked to consider whether the use of technology to teach that content might be beneficial or not (TPSK). The following paragraphs will summarize prospective teachers' knowledge in each of these areas.

Admittedly, few prospective teachers had much experience with statistics and probability (SK). Recall that three ideas related to variability were the content focus of this study: distributions, deviation, and the law of large numbers. Overall, prospective teachers were able to generally describe characteristics of distributions separately, but rarely made explicit connections with center and spread. Like Makar and Confrey's (2004) prospective teachers, most descriptions included informal language such as "clump," "scattered," "clustered," and "up and down." In fact, the use of informal language was apparent throughout the study. This language was used appropriately in most cases and often mirrored that which was promoted in the curriculum text. From their language alone, however, it was difficult to make claims about prospective teachers' understanding of variability.

Prospective teachers shared openly, both in discussions and in follow up interviews (with focus group participants), their lack of confidence in statistics. There were several times when prospective teachers said "I don't know," "it's complicated," and "it's intimidating," particularly when trying to express ideas related to standard deviation and least squares regression. Similar confusion was also apparent in discussions around the law of large numbers, and improvements in that area were questionable as several data pointed to a lack of statistical understanding throughout the study.

It is no surprise, then, that much of the discourse focused on statistical content and technological statistical content despite activities and tasks that encouraged prospective teachers to consider pedagogical implications of what they were learning. Discourse patterns of sharing ideas and affirmations with little justification may also point to an overall weak statistical content knowledge. Prospective teachers in the online group seemed to hesitate in building ideas off of one another and pushing one another to justify ideas throughout the study. Online focus group members tended to share ideas willingly, but there was little evidence that agreement about content ideas had been reached. In the face-to-face class, prospective teachers were better at building off of one another's ideas, but justification was still lacking in many episodes. In their small group, face-to-face participants continued to consider each other's comments and were often in agreement. However, in their discussion about the law of large numbers, when it was apparent that they had entered a territory they were less familiar with, they too did not push one another to justify or come to some agreement. Instead, they said, "I don't know," and moved on to the next question.

Prospective teachers gave no indication that technology skills were not learned during the study (TSK). Their work with the statistical content above was completed through the use of multiple technologies. Specifically, during this study prospective teachers worked with *TinkerPlots*, *Fathom*, *TI*-84 graphing calculators, *Microsoft Excel*, and *Probability Explorer*. Through observing prospective teachers during the face-to-face classes and affirmations during online class meetings (e.g. "green checks" and emoticons), as well as follow-up interviews with focus group members and assignments, it was evident that participants had developed a high comfort level in using the technologies.

The fact that SK and TSK were the focus of most discussions revealed that prospective teachers were generally not attending to pedagogical issues without prompting from the instructor or the curriculum text (TPSK). Perhaps it is because there was such a learning curve with new technologies and some ideas of variability described earlier, that discourse remained centered on statistical ideas for the most part. This confirms the notion of

Lee and Hollebrands' (2011) TPSK framework, that is, that statistical knowledge is the foundation level and must exist before technological and pedagogical knowledge can be more fully developed. Times during the study when TPSK discussions were long-lasting, were times when prospective teachers were using samples of student work (Chapter 2 materials). This complies with previous research results that discussions in teacher education should be centered on content and student work (Cady & Rearden, 2009; Groth, 2007; Stephens & Hartmann, 2004).

Revisiting the Teaching Presence

The social, cognitive and teaching presence components did not exist in isolation. Instead, there were overlaps between how prospective teachers communicated and interacted with one another, the content of their discussions, and the subsequent knowledge they developed and shared. As a result, just as Vlachopoulos and Cowan (2010) stated, the instructor had to adopt social, pedagogical, and intellectual roles. How the instructor worked together with the social and cognitive presences is particularly important in thinking about implications of this research on future online endeavors in mathematics education.

Through analyses of lesson maps and timelines, it was clear that the instructor made purposeful decisions when considering the placement of whole group and small group discussions. This study does not intend to make claims about the appropriate ordering of such activities. Rather, data does suggest that the predictable nature of discussion activities in the face-to-face and online classes was beneficial for prospective teachers. In their interviews, focus group members from both classes commented favorably on the structure of the class. One prospective teacher from the online class even explained the structure in her own words, and showed appreciation for designated times of independent and small group work in particular.

The use of small groups (or breakout rooms) in *Elluminate* was a welcomed surprise for prospective teachers. They spoke fondly of the ability to share ideas and “have a say.” As their work showed (Stipek et al., 2001), prospective teachers should be engaged, allowed to try new things, and given the opportunity to collaborate and reflect with others. The whole group work, which included, among other things, live demonstrations, use of emoticons and the chat window, allowed prospective teachers to do all of those things. Small group work held them accountable for it and allowed time for further discussion. In a breakout room the instructor could hear prospective teachers talking and read anything they were writing in the chat window or interactive whiteboard. The “noise” from other groups was entirely absent. However, while in one group she could see that other prospective teachers were talking and writing in their groups based on icons that turned yellow when a participant was actively using their microphone, chat window, or online writing tools. She could then move to other groups until she had visited breakout rooms for each group. The instructor’s ability and frequent use of moving between small breakout rooms also provided “closer” contact with individual participants and allowed them to ask questions outside the whole group. Many prospective teachers took advantage of this opportunity.

The use of whole group and small group discussions were also a favorite and important part of the face-to-face class. Focus group members spoke fondly of the ability to “bounce ideas off of one another” and to work with others, in general. Whole group work included, among other things, demonstrations and time for practicing new skills. Small group work held them accountable and allowed time for further discussion. The instructor's movement around the room during whole group and small group discussions also provided more one-on-one contact with individual participants and allowed them to ask questions outside the whole group.

As previously discussed, much discourse in both the online and face-to-face classes centered on statistical or technological statistical content. Interestingly, this was true for the majority of both the small group and whole group discussions. This means, despite a plan of using TPSK-level questions, the instructor made decisions during both classes to keep the focus on content. Knowing that time was a factor, she knew having a meaningful discussion about pedagogical issues without content knowledge would be difficult. However, this does not mean that pedagogical knowledge was not being built.

It is well-known that past traditional beliefs about instruction lead to traditional, classroom practices (Hiebert et al., 2003; Stipek et al., 2001). Many times, throughout this study, prospective teachers were learning or re-learning statistical content through activities that employed dynamic technology. In doing so, they often reflected on how they were taught and compared those methods to ones presented in the curriculum (Lee et al., 2010) and during class. Prospective teachers had the opportunity to see the use of technology modeled by their instructor, something Goodell and Yusko (2005) found to be critical for teachers. These opportunities, coupled with time for reflection, likely forced prospective teachers in this study to confront the traditional beliefs they may have had. In turn, pedagogical knowledge, particularly TPSK, was most likely being developed. The next and final chapter provides a deeper discussion of the findings to explicitly answer the research questions, make connections to literature, and provide implications.

CHAPTER 7: DISCUSSION

Introduction

Despite an increased focus over the past ten years on developing mathematics teachers' technology knowledge, some prospective teachers are still graduating and accepting teaching positions with little knowledge about how technology can and should be used in the mathematics classroom. Wanting to test whether an online technology methods course that prospective teachers anywhere could enroll in would be a valid option, the researcher designed a synchronous, online unit that closely compared to a face-to-face unit on teaching and learning data analysis and probability with technology. Curriculum materials (Lee et al., 2010) were used that aimed to develop prospective teachers' technological, pedagogical, and statistical knowledge simultaneously. While there were certainly technological undertones as new skills were introduced, activities and tasks encouraged prospective teachers to re-learn (and sometimes learn) statistical content using technology and consider implications of mathematics students using technology in a similar way.

One purpose of this study was to analyze discourse patterns and opportunities for interaction in two mathematics education methods classes, one face-to-face and one synchronous, online. With a growing interest in constructivism and an acceptance of the notion that knowledge is socially constructed and distributed (Putnam and Borko, 2000), this research looked closely at the ways in which prospective teachers interacted with one another in both settings. A second purpose of this study was to analyze prospective teachers' understanding of statistics. One well-known problem with research related to statistics education is that topics are undeniably intertwined (Chance, delMas, & Garfield, 1999). This makes studying a single statistical idea difficult. However, understanding variability has been identified as a key piece of statistical knowledge teachers must acquire (Ben-Zvi, 2004; Burgess, 2007). Fortunately, this is a topic that is prevalent throughout the curriculum text. Therefore, the ways in which prospective teachers discussed ideas related to variability, in particular, describing distributions, deviation, and the law of large numbers were closely analyzed. Multiple data sources allowed for a rich description of their understanding. Sometimes the ways in which prospective teachers discussed variability seemed to play a role in how their statistical knowledge for teaching was developed.

A two-fold purpose of this study resulted in two research questions. This chapter reviews those questions and uses data from the previous case and cross-case analyses to answer those questions. Explicit connections are made between findings and results to existing literature about discourse and understanding of variability. As with any study, there are limitations, and those are addressed. But, certainly there are implications of this study for mathematics education. Implications from results, along with recommendations for further research conclude the chapter.

Summary of Research Question 1 and Findings

What similarities and differences in discourse and opportunities for interaction exist between face-to-face and synchronous, online mathematics education courses?

Discourse

The idea that knowledge is constructed by an individual through interactions is not a new one (Bruner, 1966, 1986, 1990; von Glasersfeld, 1984, 1989; Wertsh, 1985). Even more recent research related to mathematics education describes discourse as being an important component of any learning experience (Clement, 1997; Groves & Doig, 2004; Picollo et al., 2008; Yackel et al., 1991). In this study, there were many opportunities to study discourse in both the online and face-to-face settings. Whole group and small group discussions were an integral part of class sessions. Episodes from each chapter were selected so that a more-concentrated focus on similarities and differences in discourse could be accomplished. Using a modification of Krussel et al.'s framework (2004), transcripts from those episodes were coded for direction, form, purpose, and topic of discourse. Codes used were similar to those used in previous research studies (Nandi et al., 2009; Topco & Ubuz, 2008). After carefully reviewing both the online and face-to-face cases, similarities resound in what prospective teachers said (or wrote) and what they talked (or wrote) about. But, as expected, it was the ways in which they spoke (or wrote) and interacted, in general, that was blatantly different between groups. To answer this first research question, these differences in form as well as similarities in purpose and topic will be thoroughly reviewed.

Differences in Form. As expected, the form of discourse varied greatly between the online and face-to-face groups. During synchronous discussions, members of the online class could participate in a variety of ways (use the microphone, type in the chat window, use an emoticon, draw/type on the interactive whiteboard, answer a survey, and assist with a technology demonstration). Prospective teachers seemed to appreciate the different ways ideas could be shared, something other researchers of synchronous environments have found as well (McBrien et al., 2009). Total usage of such features in *Elluminate,* across the six online episodes, provided an overall picture of what occurred during online class meetings.

For the online class, the number of times the instructor or a prospective teacher used the microphone was 147. Compare that number to the 145 times the chat window was used, the 44 times interactive whiteboard tools were used, and the 26 times an emoticon was used. If non-microphone categories are merged, one may compare 147 uses of microphone to 215 uses of other means of communications. Although prospective teachers rarely accessed emoticons and whiteboard tools unless directed by the instructor, the fact that other forms of communication outnumbered talking with the microphone is still noteworthy.

The number of ways prospective teachers shared ideas in the face-to-face class was smaller. In addition to speaking, they also wrote ideas on the whiteboard, completed surveys through *Moodle*, and shared ideas regarding a technology tool through demonstrations at the instructor's computer at the front of the classroom. While these means of sharing occurred several times over the course of the five-week study, they did not occur frequently (i.e. demonstration and writing on the whiteboard happened only once) during the episodes presented and were therefore not a large part of analysis for the face-to-face case.

Hand gestures, however, became noticeably present during whole group and small group discussions in the face-to-face class. Aside from ordinary hand gestures one might expect from someone talking to someone else, there were also content or technology-related gestures that enhanced ideas or elaborated responses that were being discussed. One episode in the face-to-face class highlighted a gesture initiated by a prospective teacher that became an "object of discussion" in its own right (Yackel & Cobb, 1996). After one prospective teacher (named Cora) described a cluster of points by moving her fingers and thumb as if she were grabbing points, the instructor named the gesture the "Cora cluster." This phrase and hand gesture was used in later small group and other whole group discussions, as well as in an interview with a member of the face-to-face focus group.

Similarities in Purpose and Topic. Just as obvious as differences in the forms of online and face-to-face discourse described above, were the similarities in purpose and topic of discourse between the two groups. Episodes from each of the online and face-to-face groups were coded for the following five purposes: 1) asking a question, 2) answering a question, 3) sharing an idea or concern, 4) justifying, and 5) affirming. Although there were some differences between episodes within a case or across cases, there were overwhelming similarities in prospective teachers' purposes in communicating with others. Their primary purpose during discussions was to share an idea. Often these ideas would be associated with a question that was posed, but would not directly answer that question. Prospective teachers seemed quite comfortable "bouncing ideas off of one another." One big difference, however, was that prospective teachers in the face-to-face class were better at not simply "bouncing" ideas but rather building ideas off of one another. The small group discussions from Episode 2 of each class provided an example of this difference of "talking at" versus "talking with" in the online and face-to-face classes, respectively. In their discussion about the videocase from Chapter 2, prospective teachers would use the microphone to make a claim about what they had anticipated middle school students might do and end by saying something like, "I'm going to write that on the whiteboard." In contrast, prospective teachers in the face-to-face class would share many ideas back and forth about what middle school students were doing before really coming to any consensus for the group.

Another prevalent purpose found in language used by the instructor and by prospective teachers in both groups was affirmation. Most of the affirmations in whole group discussions came from the instructor. But, in small group discussions, prospective teachers also affirmed one another. Sometimes a cycle of one prospective teacher sharing an idea and another one affirming that idea emerged. Prospective teachers in both classes were also similar in their lack of justification of ideas; they rarely provided explicit reasons for their ideas. Looking back, it may be that the lack of emphasis on justification during whole group discussions facilitated by the instructor influenced their behavior. A study by Webb and Nemer (2006) concluded that student behavior largely mirrored the discourse modeled by and the expectations communicated by teachers. Findings from the current study may support that. Prospective teachers also asked very few questions related to the curriculum content. As such, due to the abundance of affirmation with a lack of justification, it was difficult to discern if prospective teachers were really “thinking about their thinking,” something Piccolo et al. (2008) suggested teachers are able to do.

In addition to similarities in purpose, there were also similarities between groups in the topic of discourse. Prospective teachers very often attended to statistical and probabilistic content in their discussions. This does not mean that they were always carefully considering specific parts of a topic. They certainly tried to answer the questions given to them, but they did not always give evidence that they were thinking deeply about the statistical ideas. Perhaps a lack of confidence with statistical content was a contributing factor. This will be discussed further in the response to the second research question. Despite continual efforts from the curriculum, very few discussions focused mostly on pedagogy (only once in the six online episodes and only twice in the face-to-face episodes). This was concerning since scholars suggest that knowledge of how students think and learn is imperative knowledge for teachers to develop (Fennema & Franke, 1992; Van der Sandt, 2007). The lack of TPSK focus in both learning environments corroborates the notion that it is not the platform through which teachers discuss pedagogical issues related to using technology with students that is the biggest obstacle (Cady & Rearden, 2009), but rather the limited experiences prospective teachers bring to the discussions. The videocase in Chapter 2 was the main exception, as discussions in both settings were TPSK-heavy. Work in that chapter forced prospective teachers to consider their own use of technology, anticipate what middle school students might do, watch middle students working on a task using *TinkerPlots*, and analyze their written work. This confirms past research that pedagogical discussions in teacher education should be focused on content and students' thinking (Cady & Rearden, 2009; Groth, 2007; Stephens & Hartmann, 2004).

Opportunities for Interaction

As Sliva (2002) said about interactivity and discussion, "There must be time to talk" (pg. 80). Time to talk about statistical content, pedagogy, and technology was certainly an important part of the design of the online and face-to-face classes with the curriculum materials (Lee et al., 2010), and the time spent in activities varied within each class and across cases. Percentages of time allowed for whole group discussion in each chapter ranged from 17% to 70% in the face-to-face class and from 25% to 48% in the online class. Percentages of time allowed for small group discussion in each chapter went as high as 30% in the face-to-face class and 52% in the online class. There were multiple opportunities for discussion within a single class meeting in both groups. And, within in each of the online and face-to-face cases, it was shown that the placement of discussion activities seemed to be consistent. Whole group discussion time was interspersed throughout each lesson, and was kept to less than fifteen minutes in most instances. In each learning environment, small group discussions were immediately followed by whole group discussions in Episodes 1 through 5. The order of discussions was purposefully reversed for Episode 6 in each class. Technical difficulties in the online environment (either with *Elluminate* or one of the technologies being used in the curriculum), presented some differences in opportunities for discourse between the two classes. The time spent troubleshooting and addressing those problems occasionally resulted in large differences in overall time spent with that chapter material and discussions (e.g. online Episode 4). However, the difference in total time spent in whole group or small group discussions between groups across all six episodes (297 minutes in the online class and 306 minutes in the face-to-face class) was only nine minutes. Though the overall time prospective teachers engaged in discussions was similar in the groups there are additional similarities as well as differences in the ways whole group and small group discussions unfolded.

Similarities and Differences in Whole Group Discussions. Often whole group discussions asked prospective teachers to engage in and reflect on some statistical task using technology. This practice of “engaging in practical inquiry, trying new things, and reflecting in a collaborative setting” (Stipek et al., 2001) allowed prospective teachers to critically reflect on a common experience while sharing alternate viewpoints, which provided them opportunities to perhaps anticipate more about how their future students may approach a task differently and what they may do in response. This is so important to prospective teachers with little experience, especially with using technology to teach statistics and probability. The ways in which prospective teachers communicated during whole group discussion was greatly affected by the ways in which the instructor facilitated those discussions. Because the instructor consistently strived to create similar learning environments, there were many similarities in the ways discussions were facilitated. However, due to the physical separation of participants in the online class, special tools were used in *Elluminate* which made implementation between groups quite different. This section describes the social and analytic scaffolds that were put in place as well as the discussion patterns that evolved in each setting.

Nathan and Knuth (2003) found that whole groups were most effective when some scaffolds, social and analytic in particular, were present. In this study, the instructor purposefully used social scaffolds in each group. However, those purposeful acts took different forms. In the online episodes, the instructor asked prospective teachers to use tools within *Elluminate* such as surveys, “green checks,” emoticons, chats, and responses on the interactive whiteboard at various times to keep the level of engagement high. The tools in *Elluminate* enabled social scaffolding, or “eliciting contributions to whole class discussions from all students” (pg. 178). They allowed prospective teachers multiple points of personal engagement, as McBrien et al. (2009) described, and they allowed the instructor times for informal formative assessment. This was particularly important since, in the online environment, she could not see what they were working on with their personal computers.

Similar social scaffolding appeared in the face-to-face whole group discussions, though they looked undeniably different. There were no buttons to click on the computer or means to receive and view responses from everyone in the class like the emoticons and chat window of the online class. However, the instructor did elicit contributions by walking around and gathering information from prospective teachers at their computers (e.g. Face-to-Face Episode 3) and tried to hear responses from as many different participants as possible. There were also numerous times when eye contact, smiles, and head nods were affirmations and ways the instructor could informally receive formative assessment throughout class.

Analytic scaffolds also occurred when the instructor restated a contribution from a prospective teacher or highlighted part of a contribution in order to move the discussion in a certain direction (Nathan & Knuth, 2003). Analytic scaffolds certainly appeared in whole group discussions from both groups, but they were more spontaneous based on responses collected from prospective teachers at that time. While some scaffolding appeared in each episode, there was especially more analytic scaffolding in both the online and face-to-face classes during lessons related to Chapters 3 and 4, which were devoted to standard deviation and least squares regression, respectively. Prospective teachers in both groups seemed particularly interested in connecting the visual representations in *Fathom* to formulas some of them had used before. Most analytic scaffolding was through the use of spoken directions or questions by the instructor, although some questions included writing on the interactive whiteboard in the online group and with the dry-erase whiteboard at the front of the face-to-face class (e.g. Episode 3, whole group discussions).

The examples of scaffolding above show *what* the instructor was doing at certain times of whole group discussions. *How* the instructor provided the scaffolds and facilitated discussions, in general, varied between classes. In the online class, she had to institute procedures to minimize audio feedback within *Elluminate*, which resulted from multiple speakers. Groups were invited, in order, to share ideas. This certainly reduced the amount of audio problems, but added much more structure to discussions. The face-to-face class was much less structured as anyone could voluntarily share at any time. This difference in structure carried over and could also be seen in whole group discussions that did not follow small group discussion.

The face-to-face class remained less structured. Prospective teachers could respond at will and the instructor tended to acknowledge each idea they shared, often by repeating the idea and building on it or asking another question. This resulted in a back-and-forth discussion pattern between the instructor and participants that was prevalent across episodes for the face-to-face class (see Figure 120, bottom). Multiple simultaneous responses from prospective teachers also occurred, but it was difficult to discern exactly what was being shared.

While the online discussions contained some times of back-and-forth discussion, there were many more times of asking a question and receiving multiple responses simultaneously (see the double-arrowed line in Figure 120, top). When these multiple responses occurred the instructor rarely acknowledged more than two or three teachers individually. Instead, she would make a general comment about the nature of responses she was reading in the chat window. While it was possible for many prospective teachers to quickly share their ideas in the online class, it was not clear that each of these ideas were read by everyone and used to further the discussion. The patterns of engagement in whole group discussion in the face-to-face class allowed for more opportunities for the instructor and prospective teachers to listen to, reflect on, and build upon previous ideas.

Similarities and Differences in Small Group Discussions. There is much research that recommends the use of small group work and discussions for promoting effective discourse (Elbers, 2003; Webb, 1991; Webb & Nemer, 2006; Kazemi & Franke, 2004). Others have researched the unique benefits this type of work has on teachers (McCrory et al., 2008; Putnam & Borko, 2000). Of course, simply organizing prospective teachers into groups does not imply that social learning is taking place. Even online, small group social interaction is insufficient by itself (Larreamendy-Joerns & Leinhardt, 2006; Shea & Bidjerano, 2009). Therefore, in both the online and face-to-face classes, groups were given specific tasks related to content and/or pedagogy using materials from the textbook (Lee et al., 2010). Certainly there were technological undertones as prospective teachers learned new technology skills, but technology was not usually the focus of their discussions. The instructor intended for questions to push discussions into thinking about students' thinking and how the technology may benefit their conceptual understanding. However, like the whole group discussions, most conversations stayed centered on statistical content. At any rate, prospective teachers appreciated the opportunity to work closely with a small group in the class. Interviews from focus group participants of both classes revealed this was an important and favorite part of the class for them.

Similar to whole group discussions, *what* prospective teachers discussed in both learning environments was similar. *How* they participated in discussions and how the instructor interacted with them during those times varied between the face-to-face and online groups. During small group discussions in both learning environments, the instructor's involvement was consistent as she enjoyed moving from group to group in order to listen in on prospective teachers' conversations about content, technology, and pedagogy. It was the way in which she moved that was different. In the face-to-face class, she would physically walk around the room in order to enter a small group's working space. In a small group, she could hear prospective teachers talking and watch how they were interacting with each other (e.g. hand gestures). With much talking going on in the classroom at one time, it was difficult to know what was going on in other groups until she moved closer to them. In the online class, the instructor could also virtually move from group to group by clicking on her name in the *Elluminate* participants' window and dragging it into a group's breakout room (denoted by a folder labeled Group 1, etc., in the participants' window). In a breakout room she could hear prospective teachers talking and read anything they were writing in the chat window or interactive whiteboard. The "noise" from other groups was entirely absent. However, while in one group she could see that other prospective teachers were talking and writing in their groups by looking at the *Elluminate* icons beside participants name in the participants' window, which turned yellow when a participant was actively using their microphone, chat window, or online writing tools. She could then move to other groups until she had visited breakout rooms for each group.

Aside from the physical differences in ways prospective teachers and the instructor interacted during small group discussions, another salient difference in discourse was presence of pauses within the online class. Sometimes these pauses were thoughtful and it was apparent that prospective teachers were independently working to recreate a representation using the technology. Other times, however, the pauses seemed long and awkward. The difference in number of exchanges is telling (93 over 34.5 minutes for the online class and 213 over 26.5 minutes in the face-to-face class). Simply put, prospective teachers in the face-to-face class talked more. They did not always have a higher-quality discussion than their online counterparts (e.g. Episode 6, small group discussions about the law of large numbers), but their physical proximity seemed to make it easier for them to actually talk with one another and share ideas.

Summary of Research Question 2a and Findings

What is the nature of prospective teachers' understanding of variability and teaching concepts related to data analysis and probability with technology?

Among teacher educators, there is little dispute that teachers need to be mathematically proficient (Hiebert et al., 2003; NRC, 2001; Stipek et al., 2001; Usiskin, 2001). But, only a few studies have found and tested appropriate levels of content knowledge required for teaching (Ball et al., 2008). In statistics education, there are even fewer research efforts related to necessary content knowledge for teaching. However, there is now a growing body of literature that focuses on postsecondary students' understanding of statistics and probability (e.g. Chance, delMas, & Garfield, 2004; Garfield, 1995; Hammerman & Rubin, 2004; Heaton & Mickelson, 2002; Lee & Lee, 2011; Madden, 2008; Makar & Confrey, 2004; Reading & Shaughnessy, 2004). For this study, prospective teachers were engaged with curriculum materials related to data analysis and probability. Episodes selected across the six chapters were described in each of the online and face-to-face cases. These episodes specifically highlight prospective teachers' understanding of variability, in particular, describing distributions, deviation, and the law of large numbers. Assessment and interview data from each group provided additional information to use in describing their statistical understanding in these areas. From the analysis of the pre-/post-assessment, it was clear that prospective teachers in the face-to-face class made a significant shift (*P-value* = 0.006) in their ability to answer questions related to variability in a written assessment, while the gains made in the online class were not significant (*P-value* = 0.157). If that were the only form of assessment data, it may appear that the face-to-face group had gained a better understanding. However, that data does not tell the whole story. Therefore, findings from multiple data sources for each learning environment will be summarized and organized in three sub-sections below, one for each of the foci of variability mentioned above.

Describing Distributions

The ability to correctly describe distributions plays a key role in understanding variability, but using graphical displays to describe variation is generally not an easy task for novice students of statistics or their teachers (Jacobbe & Horton, 2010; Makar & Confrey, 2004). There were multiple times, over the course of the study, when prospective teachers in both groups did not use formal statistical language to describe the similarities and differences between distributions. Instead, they kept references to statistical ideas on the descriptive level and used informal phrases such as “clumped” and “tighter” to explain differences. This was not surprising since the curriculum materials emphasized this informal approach. And, their use of informal language was no surprise based on previous research either. Many have said this is a great start to describing characteristics of distributions (NCTM, 2000; Konold & Higgins, 2003; Bakker & Gravemeijer, 2004; Makar & Confrey, 2004).

Throughout the study, prospective teachers seemed comfortable describing distributions by center, although members of the face-to-face class tended to describe center less informally than the online class. Prospective teachers in both the online and face-to-face classes also considered the effects of higher and lower values on measures of center. They would often explicitly state the highest and lowest values of data, sometimes inaccurately calling them outliers without determining whether or not those values were actually statistical outliers. This attention to outliers was also seen by Madden (2008) in her study of prospective teachers' statistical reasoning with distributions. Participants in both groups of this study seemed astute, however, at recognizing the effects those "extreme" values had on the mean, median, and midrange. Thus, all data considered together indicate that prospective teachers' understanding about distributions is similar across groups.

Understanding Deviation

Curriculum materials used for this study allowed prospective teachers to consider deviation in a variety of contexts. Specifically, tasks and activities were completed throughout the class which required them to think about both univariate and bivariate deviation. First, regarding univariate deviation, prospective teachers thought about variability as "bumpiness" and described the "up and down" shape of the second distribution in their answers. Some also included the lack of normal curve or bell curve shape as part of an explanation for greater variability in a distribution. These ways of justifying univariate deviation were also seen in a study by Liu and delMas (2005). Their students described deviation with terms such as "bell-shaped" and "equally spread out" and did not consider variation about the mean (pg. 63). Slauson (2008) also tried to conceptualize how college students understood standard deviation. Her study showed that students did not think of standard deviation as an appropriate measure for variability. In this study, only a few prospective teachers seemed to focus on some measure of center and think about how values deviated from the center. When they did, they described the deviation informally.

Second, with regard to bivariate deviation, special attention was given to how prospective teachers thought about correlation and least squares regressions, including interpreting a residual plot. Although there was some evidence in both the face-to-face and online groups that prospective teachers had the misconception that correlation and slope have to be equal, most of them more accurately described correlation in terms of direction and strength and realized that the slope is an entirely different entity. They did not, however, seem to be thinking about how the correlation coefficient related to a line of best-fit. The lack of attention to a linear association was also noticed with teachers in Casey's (2010) study. Zieffler and Garfield (2009) also observed something similar as their "students realized that the absolute value of the correlation coefficient was related to the magnitude of the relationship, but did not relate that idea to the spread of scatter around the regression line" (pg. 9).

Prospective teachers, in both groups, came in to this study with little content knowledge of standard deviation and/or least squares regression. Throughout episodes for Chapters 3 and 4, most of them seemed to appreciate the "visual" aspect of *Fathom* and discussed openly the differences between the dynamic movements they were learning and the formula-based, "plug-and-chug" methods they reported learning in their introductory statistics courses. However, in both classes, the instructor needed to assist most prospective teachers in making explicit connections to the underlying concept of deviation that was being visualized with tools like the movable lines and squared deviations (from a mean or a linear model). In summary, the evidence shows very similar, and limited, conceptions of deviation in both groups in this study.

Understanding the Law of Large Numbers

Research has shown that prospective mathematics teachers have limited understanding of probability (Liu, 2005; Ives, 2009). In particular, prospective teachers often hold misconceptions about the law of large numbers (Carter & Capraro, 2005; Dinov, Christou, & Gould, 2009; Konold, 1995). Understanding the law of large numbers requires one to understand the variability across samples and variability between theoretical and empirical probabilities, and the role that sample size has in affecting this variability. This area of variability proved to be difficult for prospective teachers in this study as well.

Interestingly, different data sources told different stories. While there were certainly signs of understanding, there were data that would cause one to question that understanding. From the beginning, the pre-assessment revealed a weak understanding with only 35% of the online class and 52% in the face-to-face class answering the related question correctly. The post-assessment that followed provided mixed results. Eighty-four percent of the face-to-face class answered the related question correctly, whereas only 47% of the online class answered the same question correctly. The lack of improvement in the online class was disappointing as the wording of this question was very similar to activities prospective teachers had completed in Episode 6.

Throughout activities and tasks from Chapters 5 and 6, prospective teachers were provided multiple opportunities to explore this idea. Despite these efforts, small group discussions in Episode 6 for both groups showed confusion as focus group members failed to be able to justify their ideas or come to a collective agreement about problems related to the law of large numbers. In the online small group, prospective teachers shared their ideas individually, but never built upon previous ideas or questioned one another even when there was obvious disagreement. In the face-to-face small group, prospective teachers seemed to do a better job actually talking to one another and building on each other's comments, but at several points in the discussion, they ended a conversation with "I don't know" and moved on to another question. These examples provide further evidence of a lack of confidence and understanding around probability and the law of large numbers.

There were some similarities in the way prospective teachers described the law of large numbers. Often prospective teachers in both classes informally described how the results became "closer to the desired result," or closer to a value that was "expected." Thus, they seemed to be focused on the relationship between empirical and theoretical probabilities. Few of them, however, provided evidence regarding their understanding of the variability one might see from sample to sample, depending on the size of sample. However, as discussed in the cross-case analysis, applying the law of large numbers to think of implications of variation across samples at different sizes may be a much deeper, more connected conception.

Summary of Research Question 2b and Findings

What is the role of discourse in face-to-face and synchronous, online environments in developing this understanding among prospective mathematics teachers?

Recall that for the current study, an effort was made to anticipate important pieces of the educational experience denoted in the community of inquiry framework (Garrison, Anderson, & Archer, 2000). The result was a conceptual framework for studying discourse and how it affects knowledge related to teaching variability with technology. The instructor selected activities and facilitated discourse and interactions in order to encourage teachers to grow three types of knowledge simultaneously. This selection of activities and facilitation of the class was a critical component of the resulting discourse and interactions among prospective teachers and the knowledge they developed. As the teachers' knowledge (SK, TSK, and TPSK) grew, they developed a repertoire of content and technology skills for solving problems and acquired a growing awareness of how such tools may be used with students effectively. This, in turn, potentially affected how they communicated and interacted with their instructor and with one another. For the purpose of stage-setting, this answer includes a connection of responses to research questions 1 and 2a above to the community of inquiry framework. Then, findings from a close look into the intersection of the social and cognitive presences to answer the above research question will be presented.

Connection of Findings to the Community of Inquiry Framework

The community of inquiry framework presented in Chapter 2 (see Figure 1) attempts to understand the social, technological, and pedagogical processes that lead to collaborative knowledge construction (Garrison et al., 2000). The social, cognitive and teaching presence components that encompass this framework do not exist in isolation. However there were important similarities in each presence that are worth emphasizing once more.

As previously stated, the instructor worked diligently to facilitate the online and face-to-face classes as similarly as possible. Therefore, the overall structure of curriculum implementation was comparable. Identical curriculum materials (Lee et al., 2010) were used in each setting and opportunities for whole group and small group interactions were provided regularly for each class. In addition, the instructor strived to facilitate discussions so that prospective teachers were engaged with the technology and the statistical content at the same time, and to create a non-threatening environment where prospective teachers felt free to share ideas and ask questions. By no means, did this study show that the teaching presences were identical for face-to-face and online classes. However, the differences shown in responses to the first two research questions did not seem to have a large effect on prospective teachers' understanding of variability.

Furthermore, for the most part, there was also little difference in the ways prospective teachers described distributions, deviation, and the law of large numbers. As shown in the response to the second research question, prospective teachers often described these ideas of variability informally and in similar ways. Although there were certainly differences between groups on some assessments, only two findings raised questions about the similar nature of prospective teachers' understanding of variability. First, on the post-assessment, very few prospective teachers in the online class (47%) answered the law of large numbers question correctly, compared to prospective teachers in the face-to-face class (84%). This was particularly surprising as the question used an identical context to that used in the curriculum.

Second, while both groups showed some improvement from the pre- to post-assessment, only the face-to-face group had improvement which was statistically significant. With the teaching presence being comparable between groups, these differences in understanding could potentially be related to the discourse presence, which one knows from research question 1 looked differently between classes.

The Intersection of Social and Cognitive Presences

Supporting Discourse. To answer the research question above, one must consider the intersection between the social presence and the cognitive presence. One piece of that intersection is labeled “supporting discourse” (see Figure 1, Garrison et al., 2000), that is discourse which supported the cognitive presence. Placed outside of the teaching presence, supporting discourse for this study appeared in the form of small group discussions, occurring outside of the teaching presence. These interactions were important to prospective teachers and a favorite part of the class. However, as Koehler and Mishra (2005) reminded, in any learning environment, there are variations in the level of participation from prospective teachers and group functioning is not identical. This was true in comparing focus groups from each of the online and face-to-face classes of this study.

As the response to Research Question 1 stated, there were obvious physical differences in ways prospective teachers interacted during small group discussions. Prospective teachers in the online small group seemed to strike a balance between the use of the microphone and the use of the chat window during their discussions. But, one admitted during a small group exercise that talking to a computer felt “weird.” Their lack of experiences with a synchronous, online environment likely contributed to feeling uncomfortable sharing ideas in that setting. Once, members of the online group tried to record ideas on the interactive whiteboard of *Elluminate*, but found it cumbersome to use. Thus, most of their discussion time was spent, somewhat unsuccessfully, writing individual ideas on the group's whiteboard rather than on statistical or pedagogical content from the lesson.

The notion of simply sharing ideas was prevalent in both focus groups, but perhaps more so in the online environment. Online, often ideas would be shared through the interactive whiteboard or the chat window and members of the small group would rarely build off one another's ideas. In contrast, while the face-to-face group was not better at actually justifying responses or thinking about pedagogical implications for the content and technology being discussed, they did seem to acknowledge each other's ideas and keep the direction of the discussion moving forward. This points to evidence that shows prospective teachers in the face-to-face class were really listening and reflecting on each other's contributions.

Another noteworthy difference between the face-to-face and online small groups, as mentioned earlier, was the presence of pauses within the online small group. Sometimes these pauses were thoughtful and it was apparent that prospective teachers in the online focus group were independently working to recreate a representation using the technology. Other times, however, the pauses seemed long and awkward. The difference in number of exchanges is telling (93 over 34.5 minutes for the online small group work and 213 over 26.5 minutes in the face-to-face small group work). Simply put, prospective teachers in the face-to-face class talked more. They did not always have a higher-quality discussion than their online counterparts, but physical proximity seemed to make it easier for them to actually talk with one another and share ideas.

Educational Experience. To completely consider the intersection between the social presence and the cognitive presence, one must also attend to the intersection of the social, cognitive, and teaching presences. The intersection of all three circles in the framework is labeled "educational experience" (see Figure 1, Garrison et al, 2000) and for this study represents all activities which encouraged prospective teachers to be engaged with one another and with the content. One important factor to consider in this intersection is the difference in total time with curriculum content during class. The overall time difference can be misleading if one forgets that, for the online group, there were two sections of the text that had to be completed for homework and one section that was omitted altogether (see Table 35). When these sections were completed in the face-to-face class, there were most likely times when prospective teachers heard the instructor discussing ideas of variability, in particular, center and spread. Although, certainly sections completed for homework were discussed at the beginning of the following class meeting, there may have been parts of those conversations missed by the online participants.

Other factors to consider are the similarities and differences of whole group discussions between the online and face-to-face groups as presented in the response to Research Question 1. The examples of scaffolding showed *what* the instructor did during whole group discussions was similar. It was *how* the instructor provided the scaffolds and facilitated discussions, in general, that varied between classes. In short, findings showed that throughout the unit of study, the online class was much more structured than the face-to-face class. The more-relaxed setting of the face-to-face class resulted in a back-and-forth discussion pattern, between the instructor and participants that was prevalent across episodes. While the online discussions contained some times of back-and-forth discussion, there were many more times of asking a question and receiving multiple responses simultaneously that were only modestly synthesized and acknowledged by the instructor.

The fact that there were differences between the "educational experiences" of each learning environment is fitting. Certainly there were many similarities across groups and hopefully those have been presented in a way that clearly highlights the ways in which a synchronous, online environment can be utilized to mirror a face-to-face environment. However, there were undoubtedly differences. These differences in discourse did not necessarily point to one learning environment being better than the other, but rather just pointed to fact that they were different. Data show prospective teachers in this study walked away with similar knowledge about variability as the way they described center, spread, deviation, and the law of large numbers were comparable. But, consider once again, for example, the "Cora cluster" hand gesture that was present in the face-to-face class. The fact that prospective teachers in the face-to-face class used the "Cora cluster" phrase several times throughout the study shows that the small, spontaneous gesture was something they adopted and preferred to use. Such gestures could not be seen in the online setting. Certainly prospective teachers in both the online and face-to-face classes came away from the study with competence in identifying a cluster of points and describing a modal clump informally, but participants in the face-to-face class had a visual, a personal connection almost to recall when thinking about the center of a distribution. Without question, the "educational experience" between groups during that Episode was different. Results from this study do not point to one learning environment necessarily being better than the other. Findings show they were just simply different.

Limitations

This study was designed to characterize discourse and the knowledge about variability and teaching concepts related to data analysis and probability with technology, of prospective teachers in face-to-face and synchronous, online methods courses. Despite extreme care in the design and implementation of the study, there are always limitations. This study is no exception. One obvious limitation is that this study only looked at discourse and understanding of one episode per chapter. There were many factors that went into selecting the episodes and they were purposefully chosen to provide a representation of discussions throughout the study. However, having reviewed all recorded data, there were certainly important and interesting happenings in both classes that were not captured in an episode and thus, they were not part of formal analyses.

Data collection was also a limiting factor. A couple of recording issues surfaced during the study. In the face-to-face class, the videographer didn't zoom in appropriately during small group discussions in the first class meeting and the audio for that discussion was poor due to background noise. In the online class, there were multiple audio problems that disrupted the flow of the class. And, in the interviews one participant's responses were not fully captured. In all of these examples, evidence to support or reject claims about discourse and prospective teachers' understanding of variability may have been present and were unfortunately unavailable.

Another limitation with this research is related to the placement of this data analysis and probability unit within the course. Because the five-week study occurred toward the beginning/middle of the spring 2011 semester, the researcher used the first weeks of her face-to-face course to establish classroom norms in terms of discussion and interaction with other students. This was not possible in the class that would later meet online as her role in the beginning weeks of that class was one of an observer. Therefore, it is possible that members of the face-to-face class were more familiar with one another and the instructor than members of the online class at the beginning of the study. In addition, the online group may have unintentionally responded differently to the researcher since she was not listed as the primary instructor and because the primary instructor for the course participated in each online session.

The difference in meeting times for each class was also likely a limitation to this study. The online class met once a week for nearly three hours. The face-to-face class met twice a week for an hour and fifteen minutes. The online class had more time (approximately one hour) devoted to the curriculum materials than the face-to-face class, but a full week passed between class meetings. Thus, they were not hearing about data analysis and probability nor were they engaged in discussions and activities about the content of this study as often as prospective teachers in the face-to-face class.

In addition to limitations stemming from the design and implementation of the study, there may have also been limitations surrounding the researcher. Some may be skeptical that the researcher/instructor would be able to remove personal bias about teaching with technology and the use of synchronous, online platforms. This was anticipated prior to beginning the study, but because this was the first time an online learning environment was used in a mathematics education course at this particular university, the researcher believed the opportunity to test a synchronous, online setting with the technology methods course outweighed the cost of not doing the research. The other instructor for the course had limited knowledge of moderating an *Elluminate* session. Therefore, since the researcher had conducted a pilot study, it was determined by the researcher and her dissertation committee that eliminating the teacher “variable,” in order to really study similarities and differences between the learning environments, should be seen as a positive part of the design of the study.

Implications and Suggestions for Future Research

This study was set in a mathematics education technology methods course. Two classes, one face-to-face and one synchronous online, were studied during a unit where prospective teachers learned about teaching data analysis and probability with technology. In particular, discourse and prospective teachers’ understanding of variability were analyzed and compared. Thus, implications from this study as well as suggestions for future research fall into two main categories, mathematics teacher education and statistics education.

Mathematics Teacher Education

This study compared discourse and prospective teachers’ knowledge in face-to-face and online settings. While some implications apply to both settings, there are others with obvious connections to the online environment. First, a mathematics education technology methods course *can* be taught using a synchronous, online environment. Over the 5-week study, prospective teachers were able to develop competency in skills with *TinkerPlots,* *Fathom, Probability Explorer,* Microsoft *Excel,* and the *TI-83+/84* graphing calculators. During online discussions, it became apparent in their focus on content that prospective teachers were going through the tasks from the curriculum (Lee et al., 2010) and approaching content as a student. After viewing live demonstrations and having time for independent practice, technology was not the focus in their conversations and did not come up during discussions unless it was explicitly stated in the question(s). Thus, prospective teachers were able to learn technology skills in the synchronous, online environment.

Despite this positive finding, the online case described in Chapter 4 certainly revealed flaws in teaching and learning in this environment. On the teaching side, extreme care was taken to provide similar experiences (as best she could) for the face-to-face and online classes. This resulted in “educational experiences” that were different. Therefore, it is not apparent that similar experiences between environments are absolutely necessary. Findings from this study do not replace previous research which states that critical components of the online class are the same critical components of the face-to-face class (Gadanidis & Hoogland, 2002). However, more research work using synchronous, online environments such as *Elluminate* is needed to discern if the “best practices” of face-to-face instruction need to be emulated in the online setting. A second implication for all mathematics teacher educators, in any learning environment, stems from the advice of Sliva (2002), “There must be time to talk” (pg. 80). Findings from this study certainly add to existing research about interaction experiences for prospective teachers. Discussions were an important and favorite part of classes in both environments as it allowed prospective teachers the opportunity to reflect on a common experience and share viewpoints about how technology helped or hindered their conceptual understanding. However, as Feiman-Nemser (2001) reminds us, teacher discourse is not naturally productive. Teacher educators, therefore, need to foster norms of justification and making connections between technological representations and the statistical or mathematical concepts that are being visualized. The instructor assisted prospective teachers with the latter, particularly with standard deviation and least squares regression, but failed to establish classroom norms for argumentation and justification. As a result, justification was noticeably lacking among prospective teachers in both classes.

The intersection of the social and cognitive presences in each learning environment was described in the previous section of this chapter. What is unclear from this study and others, however, is if and when one presence should precede the other. For example, discussions in both the face-to-face and online classes of this study often revealed prospective teachers' informal understanding of center and spread. The researcher was left wondering if these informal descriptions were a byproduct of the way discussions were facilitated or prospective teachers' content knowledge. Perhaps it was a combination of both, but there is certainly much room for research in this area. Specifically, should prospective teachers be able to participate in quality discourse (whatever that might look like) before large cognitive gains can occur, or does there need to be a minimal amount of cognitive capacity (whatever that might look like) in order for quality discourse to occur?

Finally, a third implication for mathematics teacher education applies to developing prospective teachers' technological, pedagogical, and content knowledge simultaneously. Researchers are finding, however, while this approach is a good one, it does not necessarily imply that each knowledge can be built simultaneously from the "ground up." For this study, curriculum materials were implemented which introduced new technologies, revisited content related to data analysis and probability and asked prospective teachers to consider effects of using technological methods with their students. The TPSK framework (Lee & Hollebrands, 2011), which was an important part of the current work, is based on the notion that statistical knowledge is ground-level prerequisite knowledge that must exist before TPSK can be fully developed. Others have said the same thing, more generally (Hiebert et al., 2003; Simon, 1994) – that learning to teach mathematics requires that different types of knowledge be built on one another. At any rate, findings from this study highlight the benefits of developing prospective teachers' technological, pedagogical, and content knowledge simultaneously. Prospective teachers learned new technology skills while re-learning or learning content related to variability. Pedagogical issues were discussed throughout the study, although the times when it became the focus of conversation were times when their discussions centered on a videocase of middle school students' work or when directly prompted by the instructor. Certainly Cady and Rearden (2009) were correct in reporting that teachers' discussions should be centered on content and student thinking.

The use of student work in the videocase was most effective, likely due to prospective teachers' inexperience with working with students, in developing TPSK (Lee & Hollebrands, 2011). But, assessing prospective teachers' TPSK was difficult. After all, TPSK is not simply the sum of TK, PK, and SK. It is the unique knowledge constructed by the intersection. Thus, an area that continues to need much research is evaluation tools that closely align with TPACK (Mishra & Koehler, 2006) or TPSK frameworks.

Statistics Education

While the setting of this study was a mathematics education technology course, the content of the unit of study was data analysis and probability. Therefore there are implications and suggestions for further research which apply to statistics education. Specifically, this study aimed to understand prospective mathematics teachers' understanding of variability as it related to describing distributions, understanding deviation, and understanding the law of large numbers. Findings support existing research (Madden, 2008; Makar & Confrey, 2004) that prospective teachers describe variability in informal ways. In this study, however, the level of statistical understanding was not always clear from discourse alone. Data from multiple sources helped provide a richer description of how prospective teachers seemed to think about variability, particularly center and spread. While they seemed to have correct notions about center and spread of a distribution individually, they rarely made explicit connections between the two. Interestingly, prospective teachers seemed to make more connections when describing differences between empirical and theoretical probabilities. Thus, one suggestion for future research is a study in which center and spread are learned in a probabilistic context first.

This study also supports existing research about misconceptions related to the law of large numbers (Ives, 2009; Lee & Lee, 2011). Despite multiple tasks and discussions which emphasized effects of sample size on variability between samples and variability between empirical and theoretical probabilities, there were still mixed results in prospective teachers' understanding. More work is needed in developing, perhaps, a learning trajectory that includes critical ideas and concepts, along with explicit tasks and examples aligned to the trajectory, from mathematics and statistics that one must master before attending to the law of large numbers.

Final Thoughts and Lessons Learned

While my interest in this study stemmed from the absence of online technology methods courses for prospective teachers in mathematics education, I want to be clear that the face-to-face setting will always be my personal preference. I believe the positive ways an instructor may interact with his/her students in a face-to-face environment, getting to know them and respecting them as individuals and as developing teachers, should not be replaced. Having said that, I also believe there is a technology gap in what teachers experience in their degree and training programs that is not being addressed. Hoping that offering an online course would be a valid alternative for any prospective teacher, despite physical distances and limitations, I set out to show that it was possible. Happily, in this study, I have shown that teaching a technology methods course online is possible and can provide similar opportunities for prospective teachers as its face-to-face counterpart. I have not shown, however, that teaching and learning in the synchronous, online environment is flawless.

Based on my experiences, a completely online technology mathematics methods course would include several changes from plans used for the current study. First, one practice session would be required of all prospective teachers. This would ideally be a 30-minute session dedicated to testing audio equipment and just getting to know *Elluminate* and its many tools for communicating ideas. Breakout groups would be a part of this practice session so that prospective teachers were familiar with how to move into small groups and would expect small group discussions in upcoming class sessions. In this study, prospective teachers' initial comfort in using *Elluminate* was low. While they were provided with some information about what to expect (Appendix P), the practice session that was created for them was open over a few days and the instructor was not present. Therefore, most of them did not log in to *Elluminate* until our first class meeting. Second, for a completely online class, I would utilize the webcam tool in *Elluminate* for no other reason than to provide a face with the voice and to make the experience more personal.

Finally, perhaps the biggest change I would make in presenting new content would be the use of technology videos. Many parts of the online class of this study followed a predictable structure. Live demonstrations of a technology skill were followed by time for prospective teachers to try out the skill on their own, small group discussion and a follow up whole group discussion. This pattern became comfortable for prospective teachers as the weeks went by, but there were times when going back and forth from *Elluminate* to a particular technology program was likely cumbersome for them. Using technology videos would allow prospective teachers to view a technology skill demonstration as many times as they needed to, and would eliminate some of the repetitious technology directions that occurred online during this study.

REFERENCES

Alexander, M., Lignugaris-Kraft, B., & Forbush, D. (2007). Online mathematics methods course evaluation: Student outcomes, generalization, and pupil performance. *Teacher Education and Special Education, 30*(4), 199-216.

Baker, R. S., Corbett, A. T., & Koedinger, K. R. (2002). *The resilience of overgeneralization of knowledge about data representations.* Paper presented at the annual meeting of the American Educational Research Association, New Orleans, LA.

Bakker, A. (2004). Reasoning about shape as a pattern in variability. *Statistics Education Research Journal, 3*(2), 64-83.

Bakker, A., & Gravemeijer, K. P. E. (2004). Learning to reason about distribution. In D. Ben-Zvi & J. Garfield (Eds.), *The challenge of developing statistical literacy, reasoning, and thinking*. Boston: Kluwer Academic.

Ball, D. L., & Forzani, F. (2009). The work of teaching and the challenge for teacher education. *Journal of Teacher Education, 60*(5), 497-511.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389-407.

Ben-Zvi, D. (2004). Reasoning about variability in comparing distributions. *Statistics Education Research Journal, 3*, 42-63.

Bruner, J. S. (1966). *Toward a Theory of Instruction*. New York: W. W. Norton & Company.

Bruner, J. (1986). *Actual Minds, Possible Worlds*. Cambridge, MA: Harvard University Press.

Bruner, J. (1990). *Acts of Meaning*. Cambridge, MA: Harvard University Press.

Burgess, T. A. (2007). *Investigating the nature of teacher knowledge needed and used in teaching statistics.* Unpublished doctoral dissertation, Massey University, New Zealand.

Cady, J., & Rearden, K. (2009). Delivering online professional development in mathematics to rural educators. *Journal of Technology and Teacher Education, 17*(3), 281-298.

Casey, S. (2010). Subject matter knowledge for teaching statistical association. *Statistics Education Research Journal*, *9*(2), 50-68.

Cannings, T., & Stager, G. (1998). Online communities as a vehicle for developing secondary mathematics educators. *National Educating Computing Conference.* San Diego, CA.

Carter, T., & Capraro, R. M. (2005). Stochastic misconceptions of pre-service teachers. *Academic Exchange Quarterly, 9*, 105-111.

Chance, B., delMas, R., & Garfield, J. (2004). Reasoning about sampling distributions. In D. Ben-Zvi & J. Garfield (Eds.), *The challenge of developing statistical literacy, reasoning and thinking* (pp. 295-323). Boston: Kluwer Academic.

Chinnappan, M. (2006). Using the productive pedagogies framework to build a community of learners online in mathematics education. *Distance Education, 27*(3), 355-369.

Clement, L. (1997). If they're talking, they're learning? Teachers' interpretations of meaningful mathematical discourse. [serial online]. November 14, 1997; Retrieved from EBSCOhost.

Daniels, H. (2001). *Vygotsky and pedagogy*. New York: RoutledgeFalmer.

Denzin, N.K. (1978). *The Research Act: A Theoretical Introduction to Sociological Methods.* (2nd ed.). New York: McGraw-Hill.

Dinov, I. D., Christou, N., & Gould, R. (2009). Law of large numbers: The theory, applications and technology-based education. *Journal of Statistics Education, 17*(1).

Elbers, E. (2003). Classroom interaction as reflection: Learning and teaching mathematics in a community of inquiry. *Educational Studies in Mathematics, 54*(1), 77-99.

Engelbrecht, J., & Harding, A. (2005). Teaching undergraduate mathematics on the internet. *Educational Studies in Mathematics, 58*, 235-276.

Feiman-Nemser, S. (2001). From preparation to practice: Designing a continuum to strengthen and sustain teaching. *Teachers College Record, 103*(6), 1013-1055.

Fennema, E., & Franke, M.L. (1992). Teachers' knowledge and its impact. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147-164). New York: Simon & Shuster.

Friel, S. (1998). Comparing data sets: How do students interpret information displayed using box plots? In S. Berenson, K. Dawkins, M. Blanton, W. Coulombe, J. Kolb, K. Norwood, & L. Stiff (Eds.), *Proceedings of the Twentieth Annual Meeting, North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 365-370). Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education.

Fuglestad, A. B. (2007). Teaching and teachers' competence within ICT (Information and Communication Technology) in mathematics in a community of inquiry. In *International Group for the Psychology of Mathematics Education Proceedings*. Seoul, Korea, July 8- 13, 2007.

Gadanidis, G., & Hoogland, C. (2002). Mathematics teacher education online. In *World Conference on Educational Multimedia, Hypermedia and Telecommunications Proceedings.* Denver, Colorado, June 24-29, 2002.

Garcia, M., Sanchez, V., & Escudero, I. (2006). Learning through reflection in mathematics teacher education. *Educational Studies in Mathematics*, *64*, 1-17.

Garfield, J. B. (1995). How students learn statistics. *International Statistical Review, 63*, 25-34.

Garrison, D., Anderson, T., & Archer, W. (2000). Critical inquiry in a text-based environment: Computer conferencing in higher education. *The Internet and Higher Education, 2*, 1-19.

Garrison, D., Anderson, T., & Archer, W. (2010). The first decade of the community of inquiry framework: A retrospective. *The Internet and Higher Education, 13,* 5-9.

Ginsburg, A., Gray, T., & Levin, D. (2004). *Online professional development for mathematics teachers: A strategic analysis.* Prepared for the U.S. Department of Education, Planning and Evaluation Service by the American Institute for Research. Washington: D.C.

Goodell, J., & Yusko, B. (2005). Overcoming barriers to student participation in online discussions. *Contemporary Issues in Technology and Teacher Education*, *5*(1), 77-92.

Goos, M. E., & Bennison, A. (2008). Developing a communal identity as beginning teachers of mathematics: Emergence of an online community of practice. *Journal of Mathematics Teacher Education*, *11*, 41-60.

Groth, R. E. (2007). Case studies of mathematics teachers' learning in an online study group. *Contemporary Issues in Technology and Teacher Education*, *7*(1), 490-520.

Groves, S., & Doig, B. (2004). Progressive discourse in mathematics classes: The task of the teacher. In *International Group for the Psychology of Mathematics Education Proceedings*. Bergen, Norway, July 14-18, 2004.

Guri-Rosenblit, S. (2005). Distance education and e-learning: Not the same thing. *Higher Education*, *49*(4), 467-493.

Hammerman, J., & Rubin, A. (2004). Strategies for managing statistical complexity with new software tools. *Statistics Education Research Journal*, *3*(2), 17-41.

Harper, S. R., Schirack, S. O., Stohl, H. D., & Garofalo, J. (2001). Learning mathematics and developing pedagogy with technology: A reply to Browning and Klespis. Contemporary Issues in Technology and Teacher Education, [Online serial], 1(3). Retrieved from http://www.citejournal.org/vol1/iss3/currentissues/mathematics/article1.htm.

Heaton, R. M., & Mickelson, W. T. (2002). The learning and teaching of statistical investigation in teaching and teacher education. *Journal of Mathematics Teacher Education*, *5*(1), 35-59.

Hiebert, J., Morris, A. K., & Glass, B. (2003). Learning to learn to teach: An "experiment" model for teaching and teacher preparation in mathematics. *Journal of Mathematics Teacher Education, 6*, 201-222.

Hou, H. T., Chang, K. E., & Sung, Y. T. (2008). Analysis of problem-solving-based online asynchronous discussion pattern. *Educational Technology & Society, 11*(1), 17-28.

Hufferd-Ackles, K., Fuson, K. C., Sherin, M. G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education, 35*(2), 81-116.

Ives, S. E. (2009). *Learning to teach probability: Relationships among preservice teachers' beliefs and orientations, content knowledge, and pedagogical content knowledge of probability.* Unpublished doctoral dissertation, NC State University.

Jacobbe, T., & Horton, R. M. (2010). Elementary school teachers' comprehension of data displays. *Statistics Education Research Journal, 9*(1), 27-45.

Jacobs, J. K., Hollingsworth, H., & Givven, K. B. (2007). Video-based research made "easy": Methodological lessons learned from the TIMSS video studies. *Field Methods, 19*(3), 284-299.

Kazemi, E., & Franke, M. L. (2004). Teacher learning in mathematics: Using student work to promote collective inquiry. *Journal of Mathematics Teacher Education, 7*(3), 203-235. Key Curriculum Technologies. (2005). TinkerPlots (Version 1.0) [computer software]. Emeryville, CA.

Key Curriculum Technologies. (2007). Fathom (Version 2.1). Emeryville, CA.

Knuth, E., & Peressini, D. (2001). A theoretical framework for examining discourse in mathematics classrooms. *Focus on Learning Problems in Mathematics, 23*(2&3), 5-22.

Koehler, M. J., & Mishra, P. (2005). What happens when teachers design educational technology? The development of Technological Pedagogical Content Knowledge. *Journal of Educational Computing Research, 32(2), 131-152.*

Konold, C. (1995). Issues in assessing conceptual understanding in probability and statistics. *Journal of Statistics Education, 3*(1).

Konold, C., & Higgins, T. L. (2003). Reasoning about data. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Konold, C., Robinson, A., Khalil, K., Pollatsek, A. Well, A., Wing, R., & Mayr, S. (2002). Students' use of modal clumps to summarize data. In B. Phillips (Ed.), *Proceedings of the Sixth International Conference on Teaching Statistics: Developing a statistically literate society*, Cape Town, South Africa, July, 2002 [CD-ROM.] Voorburg, The Netherlands: International Statistical Institute. Retrieved from www.stat.auckland.ac.nz/~iase/publications/1/8b2_kono.pdf.

Krussel, L., Edwards, B., & Springer, G. T. (2004). The teachers' discourse moves: A framework for analyzing discourse in mathematics classrooms. *School Science and Mathematics, 104*(7), 307-312.

Larreamendy-Joerns, J., & Leinhardt, G. (2006). Going the distance with online education. *Review of Educational Research, 76*, 567-605.

Leavy, A. (2006) Using data comparison to support a focus on distribution: Examining preservice teachers' understanding of distribution when engaged in statistical inquiry. *Statistics Education Research Journal, 5*(2), 89-114.

Lee, H.S., & Hollebrands, K.F. (2011). Characterizing and developing teachers' knowledge for teaching statistics. In C. Batanero, G. Burrill, C. Reading, & A. Rossman (Eds.), *Teaching Statistics in School Mathematics - Challenges for Teaching and Teacher Education: A joint ICMI/IASE Study* (pp. 359-369), Springer.

Lee, H. S., Hollebrands, K. F., & Wilson, P. H. (2010). *Preparing to Teach Mathematics with Technology: An Integrated Approach to Data Analysis and Probability (1st Edition).* Kendall Hunt Publishing.

Lee, H. S., & Lee, J. T. (2011). Enhancing teachers' coordination of center and spread: A window into teacher education material development. *The Mathematics Educator, 21*(1), 33-47.

Leikin, R., & Zaslavsky, O. (1997). Facilitating student interactions in mathematics in a cooperative learning setting. *Journal for Research in Mathematics Education, 28*(3), 331-354.

Ling, L. H. (2007). Community of inquiry in an online undergraduate information technology course. *Journal of Information Technology Education, 6*, 153-168.

Liu, Y. (2005). *Teachers' understanding of probability and statistical inference and their implications for professional development.* Unpublished doctoral dissertation, Vanderbilt University.

Liu, Y., & delMas, R.C. (2005) Exploring students' conceptions of the standard deviation. *Statistics Education Research Journal*, *4*(1), 55-82.

Madden, S. R. (2008). *High school mathematics teachers' evolving understanding of comparing distributions*. Unpublished doctoral dissertation, Western Michigan University.

Makar, K. (2004). *Developing statistical inquiry: Prospective secondary mathematics and science teachers' investigations of equity and fairness through analysis of accountability data*. Unpublished Doctoral dissertation, University of Texas at Austin (USA). [Online: www.stat.auckland.ac.nz/~iase/publications/dissertations/dissertations.php]

Makar, K., & Canada, (2005). Preservice teachers' conceptions of variation. In Chick, H. L. & Vincent, J. L. (Eds.). *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 3, pp. 273-280. Melbourne: PME.

Makar, K., & Confrey, J. (2004). Secondary teachers' statistical reasoning in comparing two groups. In D. Ben-Zvi & J. Garfield (Eds.), *The challenge of developing statistical literacy, reasoning and thinking* (pp. 353-374). Boston: Kluwer Academic.

Maor, D. (2003). The teacher's role in developing interaction and reflection in an online learning community. *Educational Media International, 40*, 127-137.

McBrien, J. L., Jones, P., & Cheng, R. (2009). Virtual spaces: Employing a synchronous online classroom to facilitate student engagement in online learning. *International Review of Research in Open and Distance Learning, 10*(3), 1-17.

McCrory, R., Putnam, R., & Jansen, A. (2008). Interaction in online courses for teacher education: Subject matter and pedagogy. *Journal of Technology and Teacher Education, 16*(2), 155-180. Chesapeake, VA: AACE.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis.* Thousand Oaks, CA: Sage.

Mishra, P., & Koehler, M. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *The Teachers College Record, 108*(6), 1017-1054.

Nandi, D., Chang, S., & Balbo, S. (2009). A conceptual framework for assessing interaction quality in online discussion forums. In *Same places, different spaces. Proceedings ascilite Auckland 2009*. Retrieved from www.ascilite.org.au/conferences/auckland09/procs/nandi.pdf.

Nathan, M. J., & Knuth, E. J. (2003). A study of whole classroom mathematical discourse and teacher change. *Cognition and Instruction, 21*(2) 175-207.

National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics*. Reston, VA: The Author.

National Research Council. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Olive, J., & Leatham, K. (2000). Using technology as a learning tool is not enough. In M. Thomas & Auckland Institute of Technology (Eds.), *Proceedings of the International Conference on Technology in Mathematics Education* (pp. 244 – 251). Auckland, NZ, December 11-14, 2000.

Patton, M. (2002). *Qualitative research and evaluation methods* (3rd ed.). Newbury Park, CA: Sage Publications.

Pea, R. D. (1987). Cognitive technologies for mathematics education. In A. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 89-122). Hillsdale, NJ: Erlbaum.

Pfannkuch, M. (2006). Comparing box plot distributions: A teacher's reasoning. *Statistics Education Research Journal*, *5*(2), 27-45.

Picollo, D. L., Harbaugh, A., Carter, T. A., Capraro, M. M., & Capraro, R. M. (2008). Quality of instruction: Examining discourse in middle school mathematics instruction. *Journal of Advanced Academics, 19*(3), 376-410.

Putnam, R. & Borko, H. (2000). What do new views of knowledge and thinking have to say about research on teacher learning? *Educational Researcher, 29*(4), 4-13.

Quesada, A., Wheland, E., & Zachariah, S. (2001). A case study in professional development; Establishing an online mathematics community. *Ohio Journal of School Mathematics, 44*, 47-52.

Reading, C., & Shaughnessy, J. M. (2004). Reasoning about variation. In D. Ben-Zvi & J. Garfield (Eds.), *The challenge of developing statistical literacy, reasoning and thinking* (pp. 201-226). Boston: Kluwer Academic.

Reynolds, A. (1992). What is competent beginning teaching? A review of the literature. *Review of Educational Research*, *62*(1), 1-35.

Richardson, J., & Swan, K. (2003). Examining social presence in online courses in relation to students' perceived learning and satisfaction. *Journal for Asynchronous Learning Networks*, *7*(1), 68-88.

Rosales, J., Orrantia, J., & Vicente, S. (2008). Studying mathematics problem-solving classrooms: A comparison between the discourse of in-service teachers and student teachers. *European Journal of Psychology of Education*, *23*(3), 275-294.

Sfard, A. (2000). Steering (dis)course between metaphors and rigor: Using focal analysis to investigate an emergence of mathematical objects. *Journal for Research in Mathematics Education, 31*(3), 296-327.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Shea, P., & Bidjerano, T. (2009). Community of inquiry as a theoretical framework to foster "epistemic engagement" and "cognitive presence" in online education. *Computers and Education, 52,* 543-553.

Simon, J. L. (1994). What some puzzling problems teach about the theory of simulation and the use of resampling. *The American Statistician*, *48*(4), 290-293.

Sing, C. C., & Khine, M. S. (2006). An analysis of interaction and participation patterns in online community. *Educational Technology & Society, 9*(1), 250-261.

Slauson, L. V. (2008). *Students' conceptual understanding of variability.* Unpublished doctoral dissertation, The Ohio State University.

Sliva, J. (2002). Developing a mathematical community using an electronic discussion forum in an elementary mathematics methods course. *Contemporary Issues in Technology and Teacher Education* [Online serial], 2(1), 79-89.

Stephens, A., & Hartmann, C. (2002). Using an online discussion forum to engage secondary mathematics teachers in teaching with technology. *American Educational Research Association Conference.* New Orleans, LA.

Stephens, K. K., & Mottet, T. P. (2008). Interactivity in a web conference training context: effects on trainers and trainees. *Communication Education, 57*(1), 88-104.

Stipek, D. J., Givvin, K. B., Salmon, J. M., & MacGyvers, V. L. (2001). Teacher's beliefs And practices related to mathematics instruction. *Teacher and Teaching Education, 17*, 213-226.

Stohl, L. (2002). The Probability Explorer (Version 2.0) [computer software].

Topcu, A., & Ubuz, B. (2008). The effects of metacognitive knowledge on the pre-service teachers' participation in the asynchronous online forum. *Educational Technology and Society, 11*(3), 1-12.

Tudge, J., & Rogoff, B. (1989). Theoretical perspectives on the role of social interaction: Piaget and Vygotsky. In M. H. Bornstein and J. S. Bruner (Eds.), *Culture, Communication, and Cognition: Vygotskian Perspectives* (pp. 323-347). Cambridge: Cambridge University Press.

Tzur, R. (2001). Becoming a mathematics teacher-educator: Conceptualizing the terrain through self-reflective analysis. *Journal of Mathematics Teacher Education, 4*, 259-283.

Usiskin, Z. (2001). Teachers' mathematics: A collection of content deserving to be a field. *The Mathematics Educator, 6*(1), 85-97.

Van der Sandt, S. (2007). Research framework on mathematics teacher behaviour: Hoehler and Grouws' framework revisited. *Eurasia Journal of Mathematics, Science & Technology Education, 3*(4), 343-350.

Vlachopoulos, P., & Cowan, J. (2010). Reconceptualising moderation in asynchronous online discussions using grounded theory. *Distance Education, 31*(1), 23-36.

von Glasersfeld, E. (1984). Radical constructivism. In P. Watzlawick (Ed.), *The Invented Reality*. Cambridge, MA: Harvard University Press.

von Glasersfeld, E. (1989). Cognition, construction of knowledge, and teaching. *Synthese¸80*, 121-140.

Vygotsky, L. (1962). *Thought and language*. Cambridge, MS: MIT Press. Revised and edited by A. Kozulin, 2000.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge: Harvard University Press.

Watson, J. M., & Moritz, J. B. (2003). Fairness of dice: A longitudinal study of students' beliefs and strategies for making judgments. *Journal for Research in Mathematics Education 34*, 270-304.

Webb, N. M. (1991). Task-related verbal interaction and mathematics learning in small groups. *Journal for Research in Mathematics Education, 22*(5), 366-389.

Webb, N. M., Nemer, K. M, & Ing, M. (2006). Small-group reflections: Parallels between teacher discourse and student behavior in peer-directed groups. *The Journal of the Learning Sciences, 15*(1), 63-119.

Wertsch, J. V. (1985). *Vygotsky and the social formation of mind*. Cambridge: Harvard University Press.

White-Fredette, K. (2009/2010). Why not philosophy? Problematizing the philosophy of mathematics in a time of curriculum reform. *The Mathematics Educator, 19*(2), 21-31.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*(4), 458-477.

Yackel, E., Cobb, P., & Wood, T. (1991). Small-group interactions as a source of learning opportunities in second-grade mathematics. *Journal for Research in Mathematics Education, 22*(5), 390-408.

Zaslavsky, O., & Leikin, R. (2004). Professional development of mathematics teacher educators: Growth through practice. *Journal of Mathematics Teacher Education, 7,* 5-32.

Zbiek, R. M., Heid, K. M., Blume, G. M., & Dick, T. P. (2007). Research on technology in mathematics education: A perspective of constructs. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 1169-1207). Charlotte, NC: Information Age.

Zieffler, A. S., & Garfield, J. B. (2009). Modeling the growth of students' covariational reasoning during an introductory statistics course. *Statistics Education Research Journal*, *8*(1), 7-31.

APPENDICES

Appendix A. Questions identified to potentially capture prospective teachers' developing TPSK (Lee et al., 2010).

Chapter 1

Q30. Describe the benefits or drawbacks of having students use informal approaches to describing the middle or "typical" value in a distribution before teaching them formal techniques for computing mean, mode, midrange, or median.

Q42. Box plots are typically introduced in middle school. If you were teaching a unit in which one goal was to have students understand and use box plots, would you prefer to first teach the students to construct a box plot by hand before having them use technology to construct a box plot? Why?

Q43. How could students' understanding of distribution and box plots be affected by exploring data and constructing box plots with TinkerPlots?

Chapter 2

Q11. What difficulties do you anticipate students might encounter as they work on this task?

Q12. How might the color of an attribute help or hinder student thinking?

Q24. As a teacher, you are likely left wondering about some aspects Jordan's and Kathy's understandings of comparing distributions. Building from what you observed in the video and on their worksheets, what questions would you want to ask Jordan and Kathy in order to better understand their reasoning on this task and what they understand about comparing distributions?

Chapter 3

Q10. How can examining a distribution using three different linked graphical representations be a help or hindrance for students?

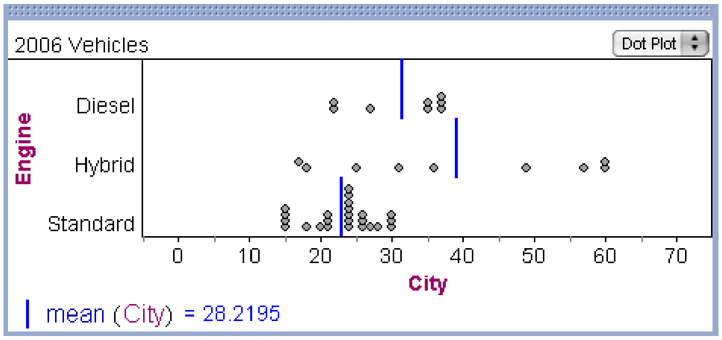
Q21. How can examining the statistical measures of mean and median along with the dot plot or box plot display of the distribution for each engine type assist students in reasoning about center and spread when comparing the three groups?

Q22. How could you use the data to help students understand why in each of the three box plots the whiskers are not the same length?

Q24. What are some of the key features of this vehicle data set that make it useful in helping students attend to important ideas of center and spread when comparing data sets?

Q32. Students are often introduced to standard deviation through formulas. What is a benefit of using a diagram such as the one in Figure 3.16, or squares on a movable vertical line in a Fathom plot, to help students conceptualize standard deviation as a measure that describes typical deviation from the mean?

Q33. What are the advantages or drawbacks of having students examine several distributions with the means indicated as in Q30 and asking them to predict magnitude of a standard deviation before using Fathom to compute the exact values? (Image below is from the referenced Q30.)



Chapter 4

Q26. Describe the benefits and drawbacks of building on what students already know about deviations from a mean with univariate data and standard deviation to find a linear model by minimizing the sum of squared residuals.

Q29. Describe some of the conceptual difficulties students may have in interpreting and using the residual plot. How will you help them understand the residual plot and its usefulness in analyzing a linear model?

Chapter 5

Q7. In question Q4, you were asked to anticipate results from repeated samples and then to compare the actual results to the anticipated results in Q5. How can the practice of anticipating results and comparing this to actual results help or hinder students' thinking about stochastic events?

Q19. Discuss why it might be beneficial to have students simulate the freshman retention problem for several samples of sample size 500, as well as sample sizes of 200 and 999.

Q34. The probability simulations in this lesson were used to make decisions that incorporated reasoning about variability and sample size—two important concepts for stochastic reasoning in the real world and in statistics. Describe the benefits and/or drawbacks to using probability simulations to collect empirical data, and using that data to introduce students to the importance of variability and sample size. For what statistical concepts could this approach provide a foundation?

Chapter 6

Q15. How can the color gradient used in the dot plot, box plot, or scatterplot help or hinder students' reasoning about the spread of the data and its relationship to sample size (total number of births in each county)?

Q23. If you had students in your class conduct the simulations as in **Q21**, how many samples of 56 and 314 trials would you want them to collect? How might you encourage them to organize the results (percent of male) of each sample? Explain your reasoning.

Q38. What are the potential strengths and weaknesses of using a simulation approach in your classroom to help students explore a probability problem such as the one posed in Q1? In your response, consider the use of computer simulations in a variety of classroom contexts, including one computer displayed with a projector, small groups or pairs working at a computer, and individuals working in a computer lab.

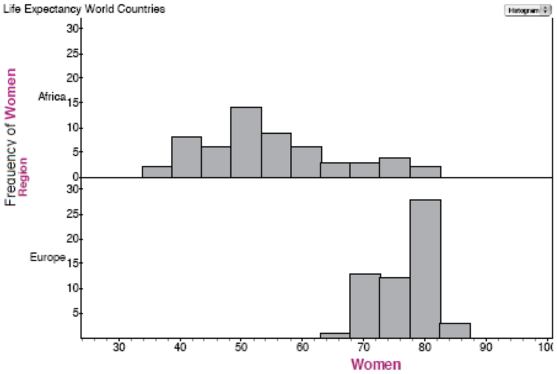
Appendix B. Pre-/Post-Assessment instrument (from Madden, 2008, pg. 386-391).

1. **Statistics Background.** Please describe the statistics coursework/experiences you have had. Also, have you taught a probability, statistics, or A.P. statistics course?

2. **Statistics Comfort level**. Please rate your level of comfort with each topic listed below by circling the level that best corresponds to a rating with **1 being very low/none** and **5 being high comfort**:

	low comfort		med comfort		high comfort
Descriptive statistics (mean, standard deviation, z-score)	**1**	**2**	**3**	**4**	**5**
Statistical Graphs (histogram, boxplot, bar graph)	**1**	**2**	**3**	**4**	**5**
Distributions (normal, chi-square, probability density functions)	**1**	**2**	**3**	**4**	**5**
Experimental Design (surveys, blocking, bias, sampling methods)	**1**	**2**	**3**	**4**	**5**
Correlation and Regression (least squares, r^2, residuals, outliers)	**1**	**2**	**3**	**4**	**5**
Sampling Distributions (Central Limit Theorem)	**1**	**2**	**3**	**4**	**5**
Statistical Inference (t-tests, confidence intervals, chi-square tests, power, Type II error, ANOVA)	**1**	**2**	**3**	**4**	**5**

3. The following distributions represent average life expectancies for women from Africa and Europe. These life expectancies are computed for various regions within each country.



(Source: *Fathom2*; Taken from www.overpopulation.com which cites McDevitt, Thomas M., World population profile: 1998. Washington, DC: U.S. Government Printing Office, 1999)

a) How are the distributions of life expectancies for African women and European women similar?

b) How are the distributions of life expectancies for African women and European women different?

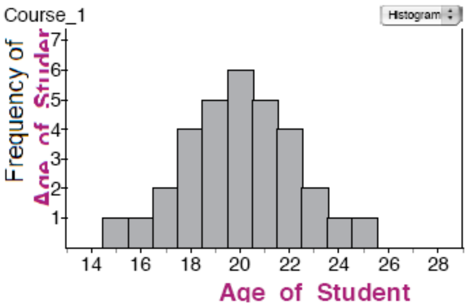
c) What can you say about the life expectancies of women from Africa and Europe? Please be specific.

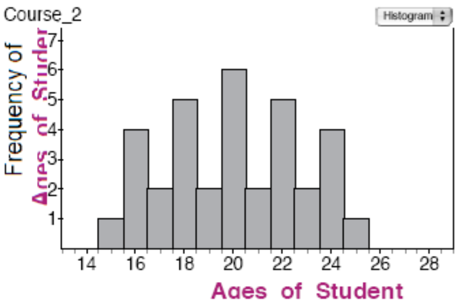
4. A certain town has two hospitals. In the larger hospital about 45 babies are born each day, and in the smaller hospital about 15 babies are born each day. As you know, about 50% of all babies are boys. However, the exact percentage varies from day to day. Sometimes it may be higher than 50%, sometimes lower. For a period of 1 year, each hospital recorded the days on which more than 60% of the babies born were boys. Which hospital do you think recorded more such days?

A) The larger hospital
B) The smaller hospital
C) About the same number of days (within 5% of each other)
D) Can't tell

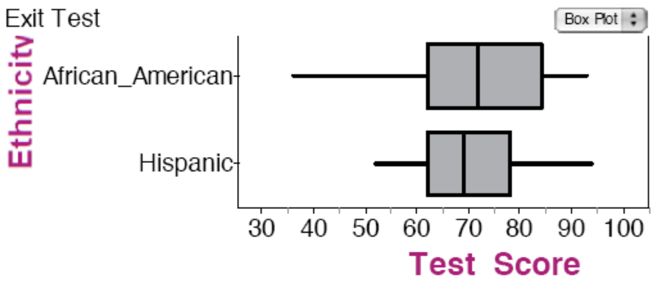
5. Given the average summer temperature in cities P and Q, explain briefly how you would decide which of the following two events is more unusual: a 90 degree summer day in city P or a 90 degree summer day in city Q.

6. Which distribution has the largest variability? Why?


CPR Course 1


CPR Course 2

7. The pair of boxplots below represents the performance of two groups of 11th grade students from an urban high school in Louisiana on a 2005 district-mandated test. The top boxplot describes the performance of 189 African-American students while the bottom boxplot represents the performance of the 46 Hispanic students in the school. For reporting purposes, the class is considered "low-performing" if less than 50% of the students in any subgroup pass the exam. A score of 70 is considered passing. Additional information is provided in the table.



Group	# of Students	Mean Test Score	Percent Passing Test
African American	189	72.0	62.1
Hispanic	46	71.5	49.1

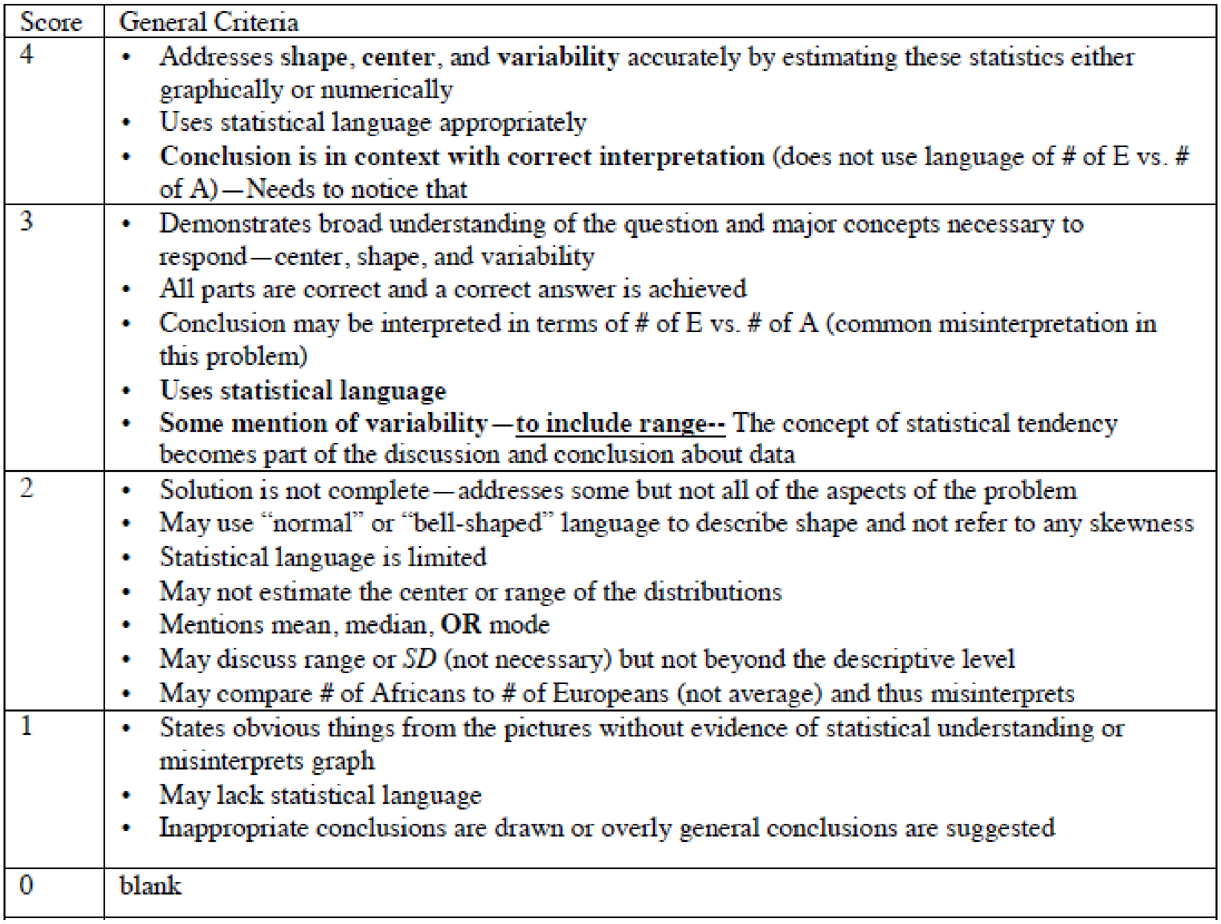
List three conclusions that would complete the following sentence: "By comparing the performance of Hispanic students with the performance of African-American students, I would draw the following conclusions…"

1)

2)

3)

Appendix C. Scoring rubric for grading question 3 of the pre-assessment (Madden, 2008, pg. 422).



Score	General Criteria
4	• Addresses **shape**, **center**, and **variability** accurately by estimating these statistics either graphically or numerically • Uses statistical language appropriately • **Conclusion is in context with correct interpretation** (does not use language of # of E vs. # of A)—Needs to notice that
3	• Demonstrates broad understanding of the question and major concepts necessary to respond—center, shape, and variability • All parts are correct and a correct answer is achieved • Conclusion may be interpreted in terms of # of E vs. # of A (common misinterpretation in this problem) • **Uses statistical language** • **Some mention of variability—to include range--** The concept of statistical tendency becomes part of the discussion and conclusion about data
2	• Solution is not complete—addresses some but not all of the aspects of the problem • May use "normal" or "bell-shaped" language to describe shape and not refer to any skewness • Statistical language is limited • May not estimate the center or range of the distributions • Mentions mean, median, **OR** mode • May discuss range or *SD* (not necessary) but not beyond the descriptive level • May compare # of Africans to # of Europeans (not average) and thus misinterprets
1	• States obvious things from the pictures without evidence of statistical understanding or misinterprets graph • May lack statistical language • Inappropriate conclusions are drawn or overly general conclusions are suggested
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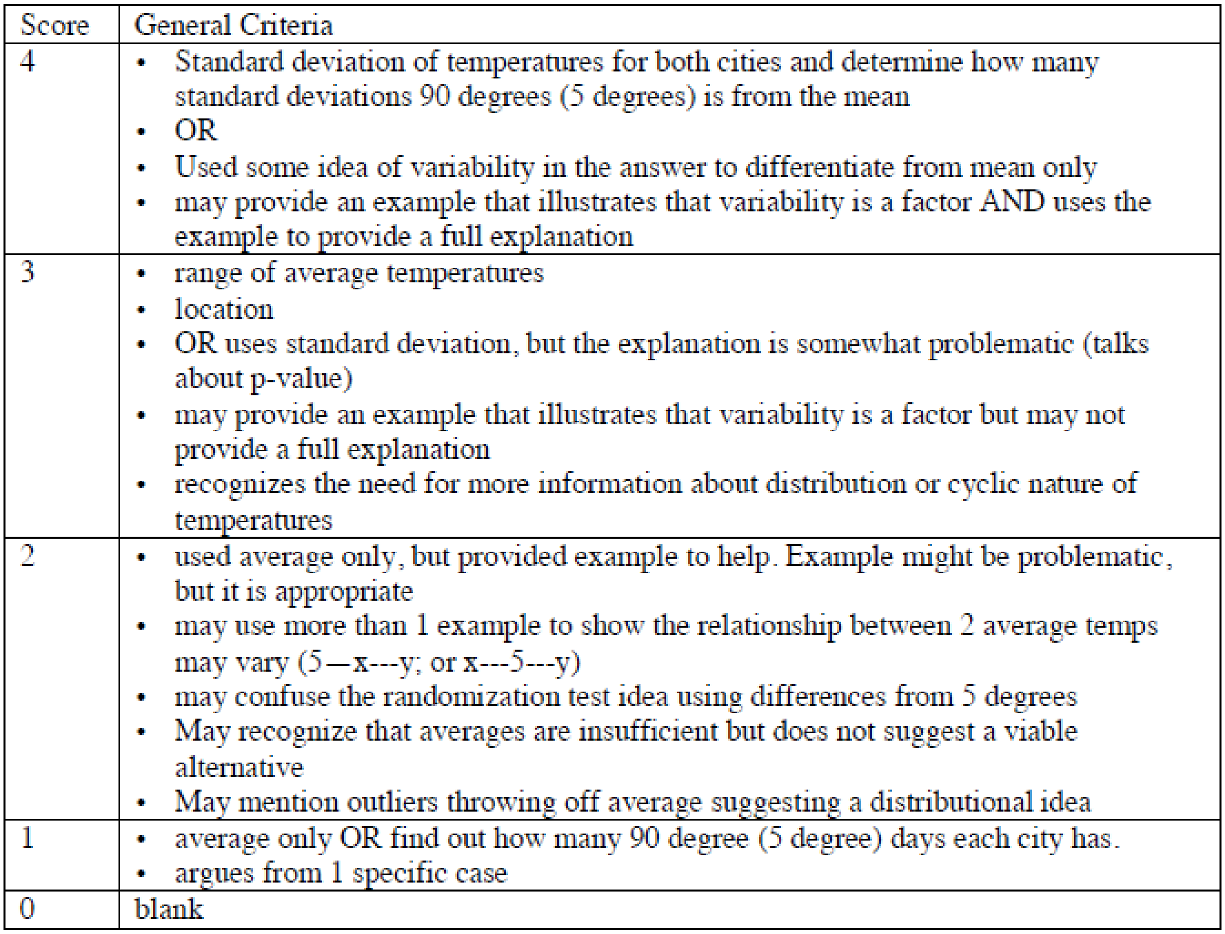
Appendix D. Scoring rubric for grading question 5 of the pre-assessment (Madden, 2008, pg. 423).



Score	General Criteria
4	• Standard deviation of temperatures for both cities and determine how many standard deviations 90 degrees (5 degrees) is from the mean • OR • Used some idea of variability in the answer to differentiate from mean only • may provide an example that illustrates that variability is a factor AND uses the example to provide a full explanation
3	• range of average temperatures • location • OR uses standard deviation, but the explanation is somewhat problematic (talks about p-value) • may provide an example that illustrates that variability is a factor but may not provide a full explanation • recognizes the need for more information about distribution or cyclic nature of temperatures
2	• used average only, but provided example to help. Example might be problematic, but it is appropriate • may use more than 1 example to show the relationship between 2 average temps may vary (5—x---y; or x---5---y) • may confuse the randomization test idea using differences from 5 degrees • May recognize that averages are insufficient but does not suggest a viable alternative • May mention outliers throwing off average suggesting a distributional idea
1	• average only OR find out how many 90 degree (5 degree) days each city has. • argues from 1 specific case
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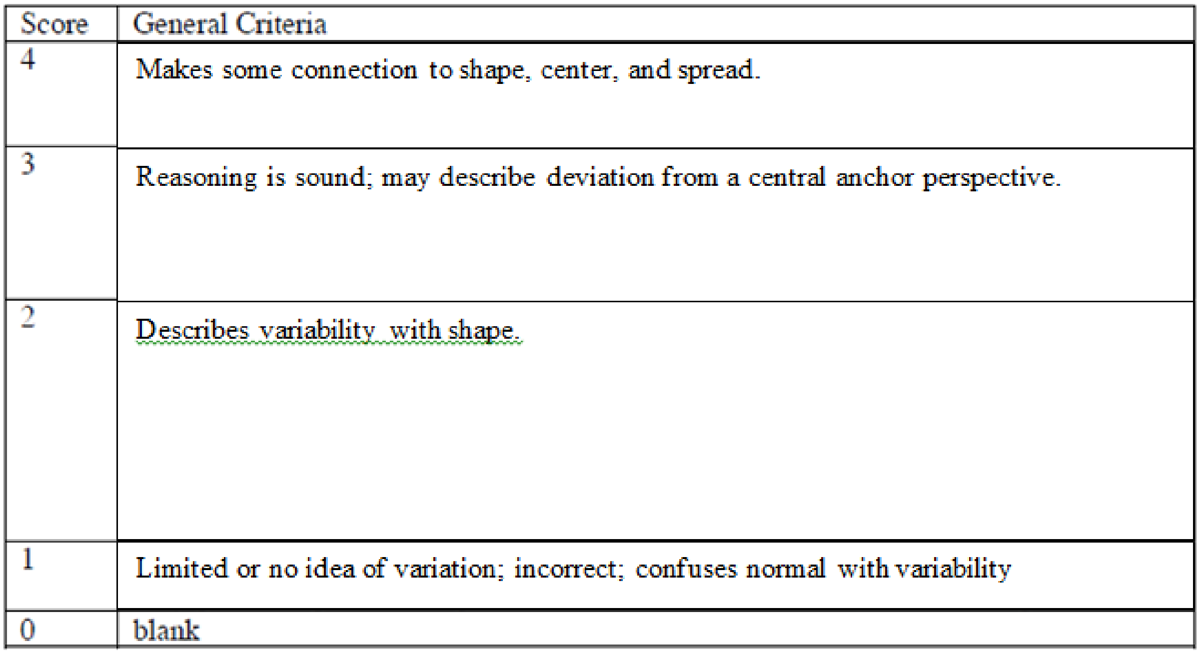
Appendix E. Scoring rubric for grading question 6 of the pre-assessment (modified from Madden, 2008, pg. 429).



Score	General Criteria
4	Makes some connection to shape, center, and spread.
3	Reasoning is sound; may describe deviation from a central anchor perspective.
2	Describes variability with shape.
1	Limited or no idea of variation; incorrect; confuses normal with variability
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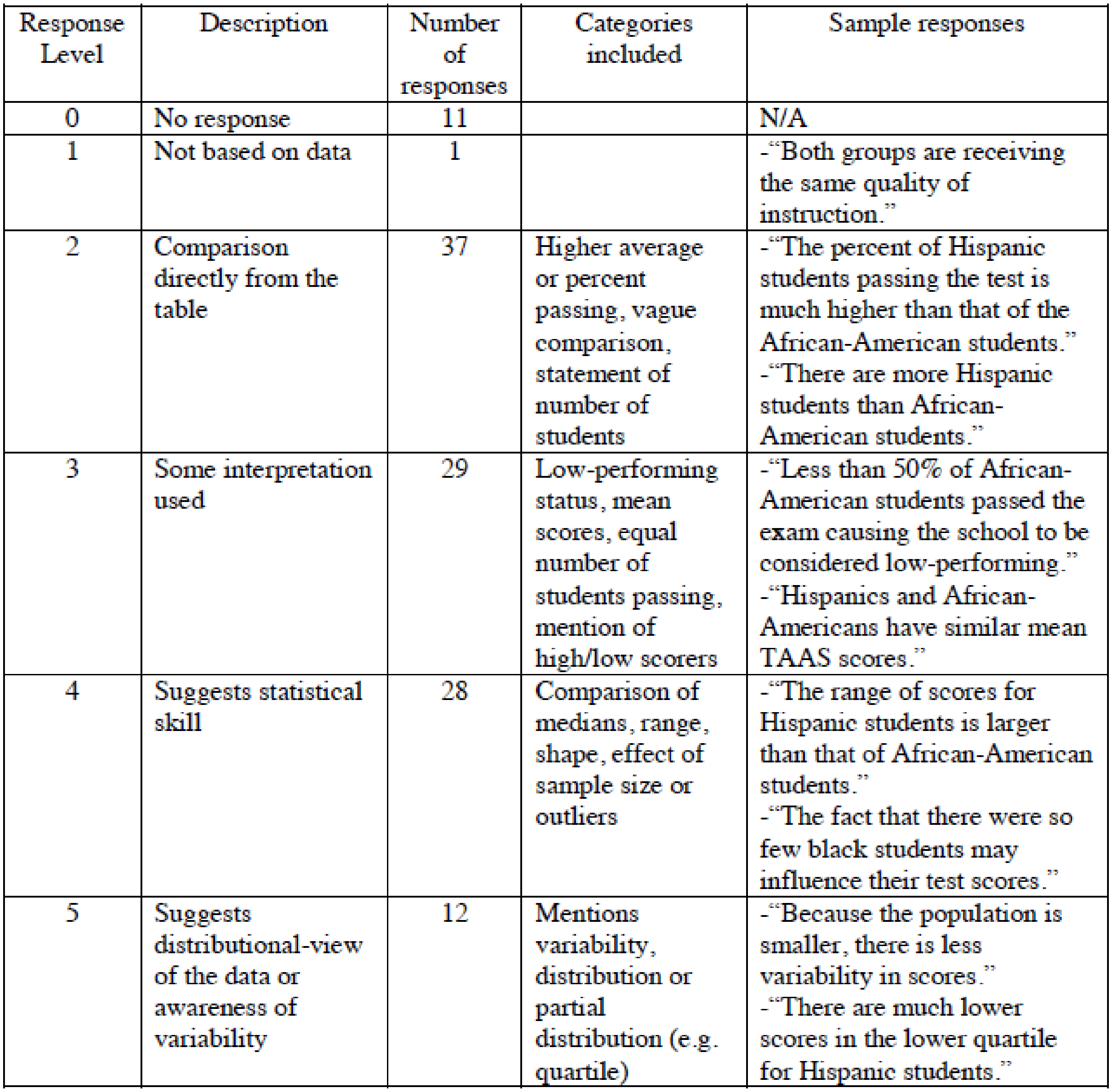
Appendix F. Scoring rubric for grading question 7 of the pre-assessment (modified from Madden, 2008, pg. 426).

“Makar’s (2004) scoring scheme (below) whereby each part, 1, 2, and 3, is scored on a 5-level scale” (Madden, 2008, pg. 426).



Response Level	Description	Number of responses	Categories included	Sample responses
0	No response	11		N/A
1	Not based on data	1		-“Both groups are receiving the same quality of instruction.”
2	Comparison directly from the table	37	Higher average or percent passing, vague comparison, statement of number of students	-“The percent of Hispanic students passing the test is much higher than that of the African-American students.” -“There are more Hispanic students than African-American students.”
3	Some interpretation used	29	Low-performing status, mean scores, equal number of students passing, mention of high/low scorers	-“Less than 50% of African-American students passed the exam causing the school to be considered low-performing.” -“Hispanics and African-Americans have similar mean TAAS scores.”
4	Suggests statistical skill	28	Comparison of medians, range, shape, effect of sample size or outliers	-“The range of scores for Hispanic students is larger than that of African-American students.” -“The fact that there were so few black students may influence their test scores.”
5	Suggests distributional-view of the data or awareness of variability	12	Mentions variability, distribution or partial distribution (e.g. quartile)	-“Because the population is smaller, there is less variability in scores.” -“There are much lower scores in the lower quartile for Hispanic students.”

Appendix G. Final exam performance tasks.

Highlighted questions below were analyzed during this study and provided more information about how prospective teachers described distribution, deviation, and the law of large numbers (in order of their appearance).

Task 1: Statistics – Using *TinkerPlots*

Technology Skills (8 points)

Go to our course Moodle page (under the May 6 date) and right-click and save the HeaviestBackpacks.tp file to the folder on your Desktop. Rename the file Task1. Perform the following steps in your file. Save your work often.

1. Create a vertically stacked dotplot of the BodyWeight attribute which also shows the Gender attribute for each data point. (3 pts)

2. Create a second graphical display to compare the dotplots of BodyWeight for Males and Females. (2 pts)

3. Create a third graphical display to compare boxplots of BodyWeight for Males and Females. Display the numerical values for the mean and median on the graph. (3 pts)

Pedagogy (4 points)

Open a new file in Microsoft Word. Name the file Task1Pedagogy and save it in your desktop folder. Answer the following questions in your file. Save your work often.

4. Consider the display that contains the boxplots for body weight by gender. Create a list of questions you would ask students help them compare these distributions. Be specific. (2 pts)

5. Suppose you want your students to do a little more Exploratory Data Analysis (EDA). Write two questions students might answer using this data set. Then, identify a content objective that would be a goal for students through answering each question. You may just state this objective informally and/or generally – no need to cite NCTM or NCSCOS. (2 pts)

Advanced Technology Skills (4 points)

Open your Task1.tp file. Perform the following steps in your file. Save your work often.

6. Create a fourth graphical display to compare boxplots of BodyWeight for the different Grades. Display the numerical values for the mean and median on the graph. Create a Duplicate Plot of this graph, and “Show Outliers.” Then, remove those outliers. (2 pts)
7. Compare the Mean and Median of the BodyWeight data in Grade 5 of the third plot with the Mean and Median of the BodyWeight data in Grade 5 of this fourth plot. In your Task1Pedagogy.doc file, explain how and why the Mean and/or Median changed or remained the same upon removal of the outlier(s). (2 pts)

Task 2: Statistics & Algebra – Using *Fathom* & *Excel*

Technology Skills (8 points)

Go to our course Moodle page (under the May 6 date) and right-click and save the NCTornado.ftm file to the folder on your Desktop. Rename the file Task2. Perform the following steps in your file. Save your work often.

1. Pull down a new graph into your workspace. Create a Scatter Plot with the Days_Since_April_16 and Customers_Without_Power attributes. (1 pt)
2. Insert a Least-Squares Line and a Residual Plot. (2 pts)
3. Use a Summary Table to find the correlation between Days_Since_April_16 and Customers_Without_Power. (2 pts)
4. Is the least squares line a "good" model for the relationship between Days_Since_April_16 and Customers_Without_Power? In a textbox, write your answer to this question. Provide any evidence in favor of or against using this model. (3 pts)

Pedagogy (4 points)

Open a new file in Microsoft Word. Name the file Task2Pedagogy and save it in your desktop folder. Answer the following questions in your file. Save your work often.

5. Two students are arguing about correlation and slope. Jack says that the number for the correlation coefficient tells you how good a straight line fits the data and that a 0 correlation with no pattern would be a horizontal line with slope 0, and that a correlation of 1 would be a perfect relationship so the slope must be 1. Jill disagrees and thinks that slope of the least squares line is only related to the correlation in that a positive correlation means a positive slope and a negative correlation means a negative slope.

   What examples might you use in class to capitalize on these two students' points of view and further develop their and the other students' understanding of correlation and the slope of a least squares line? Be specific. (2 pts)

6. Note the equation for the Least-Squares line in #2 above. How would you want your students to interpret the slope and y-intercept within the context of this problem? (2 pts)

Advanced Technology Skills (4 points)
Go to our course Moodle page (under the May 6 date) and right-click and save the DirtBike.xls file to the folder on your Desktop. Rename the file as Task2Advanced. Perform the following steps in your file. Save your work often.

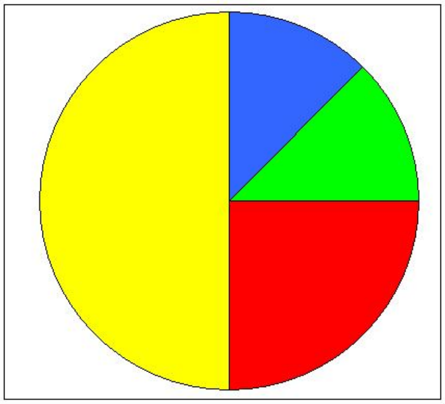
7. Create a scatter plot of your data and label the axes. Add a linear trendline (be sure to ask Excel to “Display equation on chart” and “Display R^2 on chart” as Options before clicking “Close”). (2 pts)

8. Use the linear trendline equation to create a new, third column labeled “Predicted Height Values.” Drag this new column of predicted values *down two cells beyond where the given data ends*. Answer the following questions in these two newly created rows: (2 pts)

    - Predict the height (inches) if the bike weighs 21.5 pounds.
    - Use Goal Seek to determine the weight (pounds) if the bike is 10.5 inches in height.

Task 3: Probability – Using *Probability Explorer* & *Excel*

Create two different ways to model this spinner in Probability Explorer. The yellow area is 2 times as large as the red area, the red area is 2 times as large as the green area, and the blue and green areas are equal.



1. Use the Design Your Own choice and the Weight Tool to create your first model. Take a screenshot showing the Weight Tool and insert the image in a Word document. (2 pts)
2. Run at least 1000 trials. Take a screenshot of your results (with both types of graphs and a table displayed) and insert into Word. Comment on how your empirical results compare to the theoretical model. Save this file with the trials as LASTNAMEModel1.pbe (1 pt)
3. Start a New Experiment and use the Bag of Marbles to create your second model. Take a screenshot showing the Marble Bag and insert the image in a Word document. (2 pts)
4. Run at least 1000 trials. Take a screenshot of your results (with both types of graphs and a table displayed) and insert into Word. Comment on how your empirical results compare to the theoretical model. Save this file with the trials as LASTNAMEModel2.pbe (1 pt)
5. In your Word document, comment on why your models are equivalent. (2 pts)

Save the Word file as LASTNAMESpinners.doc

Pedagogy (4 pts)

Open a new Word document and answer question 1. Save the Word file as LASTNAMEtask4pedagogy.doc

Briefly describe how you would use the simulation above to introduce the concept of the Law of Large number to students.

Advanced Skill (2 pts)

Use Excel to simulate spinning the spinner **100** times. Be sure to include both a frequency table (1 pt) and graph displaying the empirical frequencies and theoretical frequencies (1pt). Save the Excel file as LASTNAMEProb.xls

Appendix H. Scoring rubric for final exam performance task question one.

This particular rubric (modified from Makar, 2004/Madden, 2008, see Appendix E) allowed each question prospective teachers listed to be scored on a 3-point scale.

- A response of 1 meant that the only analysis of one box plot was required.

- A response of 2 meant that interpretation of two centers was required (e.g. comparing mean/median scores, comparing high/low values).

- A response of 3 suggested the interpretation of two spreads (e.g. comparison of range, shape, quartiles, or the effects of high/low values).

Appendix I. Scoring rubric for final exam performance task question two.

A 6-point scoring rubric was used to grade responses. The following table shows how points were awarded.

Recognition that Jack is incorrect.	1 point
Generally states a plan for addressing misconceptions.	1 point
Explicitly gives examples to address Jack and Jill's ideas: • Correlation of 0 (slope ≠ 0). • Correlation of 1 (slope ≠ 1). • Correlation of any positive value. • Correlation of any negative value.	1 point each

Appendix J. Scoring rubric for final exam performance task question three.

Tasks were graded with a 5-point scoring rubric. One point was awarded for each of the following:

(1) plan utilizes simulations

(2) simulations are performed with technology

(3) plan provides at least three examples of simulations involving varying sample sizes

(4) plan provides evidence of understanding that as the sample size increases, variability from sample to sample decreases

(5) plan provides evidence of understanding that as sample size increases, variability between theoretical and empirical probabilities decreases.

Appendix K. Protocol for first interview.

- Ask a few questions related to the setting/structure of the course. Ask if there have been technological issues they have had to resolve.

- Ask prospective teacher to open the Cats.tp file. Allow them to have a few minutes to explore the dataset. Then ask the following questions:

  1. Create a new plot of the body length attribute. Describe and demonstrate how you would want your own students to explore different attributes that may be related to body length.

  2. How would you help a student that seems to be only considering one attribute? What, exactly, do you want them to notice?

  3. Think about the term “average” in average body length. What are some misconceptions students bring about this concept? How might you use this data set to help them?

  4. Create a box plot. How would you interpret this box plot to students?

- Ask a couple of questions related to the structure of the course, in general.

  5. What is most helpful for you in learning content related to teaching data analysis with *TinkerPlots*?

  6. What would be useful for you, in terms of information or structure of the class, in learning content related to teaching data analysis with *TinkerPlots*?

Appendix L. Protocol for second interview.

- Ask a few questions related to the setting/structure of the course. Ask if there have been technological issues they have had to resolve.

- Ask student to open the Basketball.fm file. Allow them to have a few minutes to explore the dataset. Then ask the following questions:

1. Generate a question that involves a comparison of distributions that you would like your own students to explore. What mathematical/statistical idea(s) are involved?

2. Answer your own question. Is there any change to your previous answer about the mathematical/statistical idea(s) involved?
   If it does not come up, ask prospective teacher to create a residual plot. Ask how they would explain the residual plot to students?

3. How might *Fathom* help or hinder students' thinking about this question?

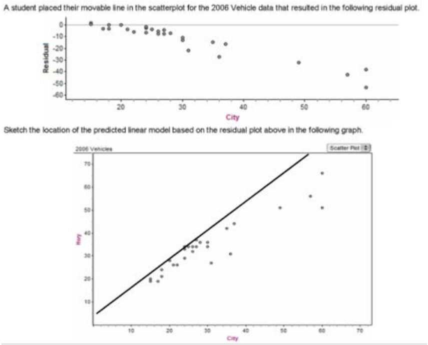
- Ask a couple of questions related to the structure of the course, in general.

4. What do you feel are some of the important things you have learned or started to consider about teaching data analysis with *Fathom*?

5. What would be useful for you, in terms of information or structure of the class, in learning content related to teaching data analysis with *Fathom*?

Appendix M. Example of transcript and coding (Online whole group, Chapter 4).

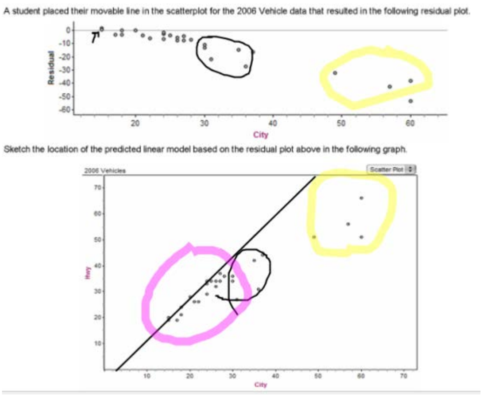
1 Instructor (mic): Where might that predicted linear model be based on the residual plot in the graph? If someone could kind of raise their hand, oh, we have somebody already. James has drawn a line (on the whiteboard). James, take over the mic and explain why you put it there.



A student placed their movable line in the scatterplot for the 2006 Vehicle data that resulted in the following residual plot.

Sketch the location of the predicted linear model based on the residual plot above in the following graph.

2 James (mic): Um, well the reason I put the line here, um is if you look at the residual line and you notice where, like you notice on the residual graph that the line is at zero. And, nearly all of the points are below that line, representing that they are a negative distance away from the residual line so I drew my line so that all of the points were below it representing a negative distance.
3 Instructor (chat): what does that mean – points at zero?
Instructor (mic): So I just put it in the chat window, but I want you to talk to this question James. At first you said this residual point here (draws an arrow on the whiteboard) is right at zero. What does that mean, if a point is right at zero?
4 Abby (chat): on the line
5 James (mic): if the point is on zero it means it is on your movable line. And, the further away that point is would be the further away from your movable line that point, the actual point is on the scatterplot.
6 Instructor (chat): [smiley face]
Instructor (mic): Good, so if it's on a zero, if your residual plot point is on zero, then that means your residual point lives on the movable line. If you have some residual points that are below the residual line, around the zero line (circles some points in the residual plot on the whiteboard), that means they have a negative residual which means they live below the movable line. So you may be thinking about points right in here –ish (circles points in the scatter plot below the movable line on the whiteboard), actually a little bit more to the right. And if we have points that are really below the zero line (circles points in the residual plot with the yellow marker tool), then that means we have points that are really below our movable line (circles corresponding points in the scatter plot with the yellow marker tool). And so, I think your estimation of where that line, that movable line might be, is good. Before we finish this discussion, any questions about what I've said or about what James explained?



A student placed their movable line in the scatterplot for the 2006 Vehicle data that resulted in the following residual plot.

Sketch the location of the predicted linear model based on the residual plot above in the following graph.

7 James draws pink circle on the whiteboard around some points in the scatterplot.
  James gives a smiley emoticon.
8 Roger (chat): I feel like the residual line is just the linear line placed horizontally without moving the axis.
9 Instructor (mic): Can you give me a green check or an emoticon that lets me know how you're feeling about this?
10-27 Green checks (15) and smiley face emoticons (3) [Abby gave both].
28 Instructor (mic): Next week when we have more time, I'd really like to hear from you about your experiences with residuals, if this is the first time you've seen this, um that sort of thing. Because if it's a new idea, there's a bigger learning curve in terms of just the mathematics behind it. But obviously, this movable line that's drawn here in the example, while it perfectly almost fits the residual plot, and that was the point, that movable line is not a very good predictor and that's what you're trying to show. If your residual plot has some sort of a pattern, where you might be able to predict where the next point in the residual plot would be generally, then you probably don't have the best model. If we moved the movable line so that it was a more balanced fit, where some points were above and below we would have a better looking residual plot that was more random. And, we'll talk more about that next time. What happens with the least squares line is that very thing. It fits it so that the residual plot is as random as it can be with some points above and some points below. Final thoughts or comments about this?
29 Debra (chat): I like playing with lines that move dots.
30 Allison (chat): me too

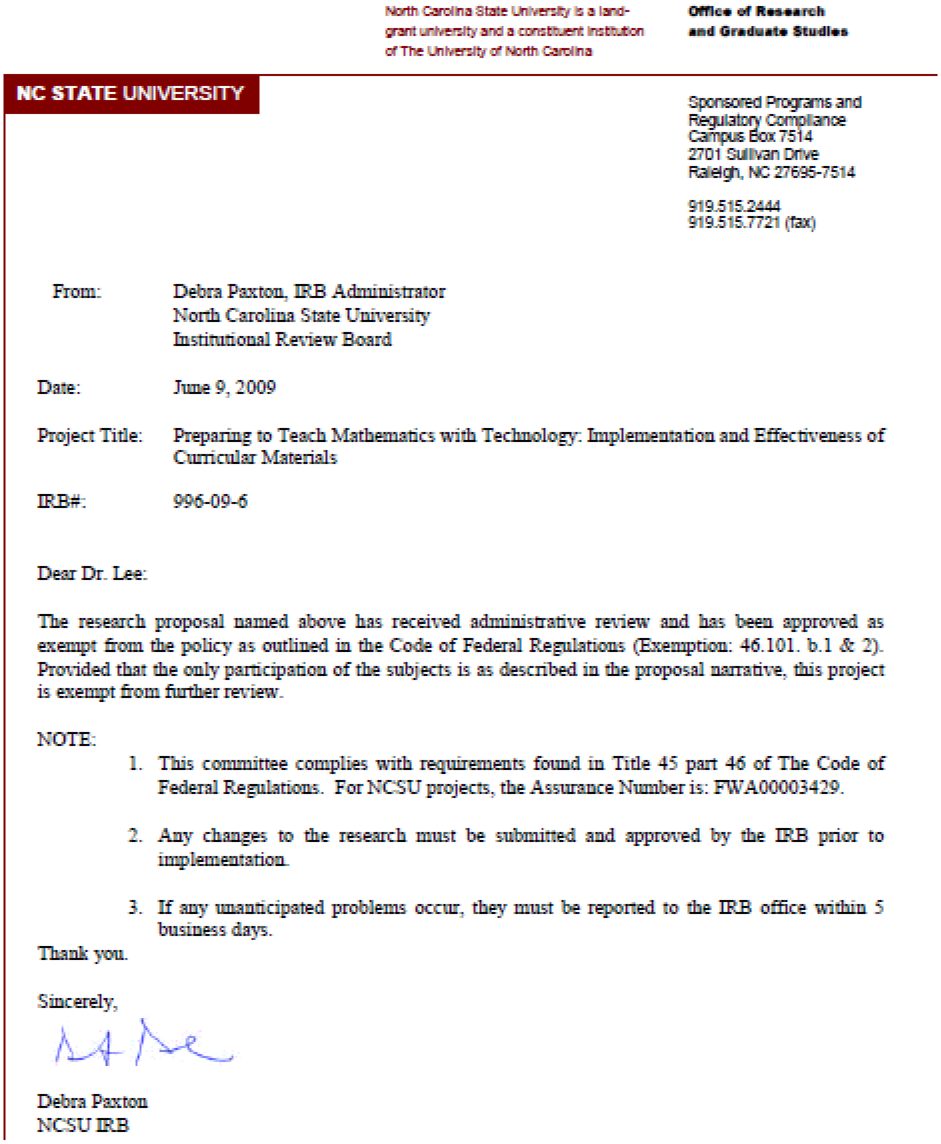
Continued on in the next class…

31 Instructor (mic): I just want to pause for a minute and ask if there are any, sort of leftover concerns or questions you had about residuals or residual plots that you'd like to ask before we move forward? Now would be a great time to ask those things. (Pause.) So, as a review, when we were talking about the residual plot, um I know we talked a lot about correlation as well, and if the correlation coefficient is close to one or negative one, then that means there's a pretty strong linear relationship. In terms of the residual plot, put in the chat window some

of the things that we look at to see if the residual plot was good. What are some of the things we look for to see if our linear model is a good one. Type your answers in the chat window.
32 Sue (chat): I'm sure I have questions I just don't know what they are.
33 Alice (chat): how linear it is
34 James (chat): Even spread of points above and below the residual line
35 Sally (chat): how far the point are from the line
36 Instructor (mic): It looks like a lot people are typing so let me give you a second to finish that and let me go back and sort of address some of these.
37 Martha (chat): for residual want some points above and below the line.
38 Alice (chat): compared to spread out points
39 Mitchell (chat): the points weren't way above or below the line
40 Instructor (mic): so our residual plot can be an indicator of how well our linear model fits our data. James reminds us that there is an even spread of points above and below the residual line. That’s a good sign, if we see that happening in our residual plot, that’s a good sign that our model, our linear model is good. It depends on how far off the line they are and how spread out they are, and Mitchell reminds us that the points weren’t way above or way below.
41 Thomas (chat): small residuals
42 Primary instructor (chat): no pattern in the residuals
43 Instructor (mic): So, we were looking for, as Thomas says, small residuals. So, good job you guys. So, with a residual plot, we’re looking for a plot that does not have a pattern – we do not want to see a pattern. We’re also looking for residuals that are small, close to zero, some above, some below.

EPISODE 4 - WHOLE GROUP		Direction				FORM								PURPOSE					TOPIC OF DISCOURSE				
	Time: 7:55 Focus of Questions: TPSK	T-SG	I-SG	T-WG	I-WG	Mic	Chat	Chats w/ symbols	Emoticons	Demo	Using whiteboard	Applause	Raising hand	Ask a Question	Answer a Question	Shares some idea	Justify	Affirm	Related to Class (lo	Online Equipment	Technology	Statistics	Pedagogy
1	Instructor				1	1					R			R,N								1	
2	James			1		1					R						1				1	1	
3	Instructor				1	1	1				1			N							1	1	
4	Abby			1			1								1							1	
5	James			1		1									1						1	1	
6	Instructor				1	1	1		SF		1			N		R		1			1	1	
7	James			1					SF		1					1					1	1	
8	Roger			1			1									1					1	1	
9	Instructor				1	1								N					1				
10	Abby			1					GC									1	1				
11	Alice			1					GC									1	1				
12	James			1					GC									1	1				
13	Kristy			1					GC									1	1				
14	Chase			1					GC									1	1				
15	Mitchell			1					GC									1	1				
16	George			1					GC									1	1				
17	Les			1					GC									1	1				
18	Primary Inst.			1					GC									1	1				
19	Ruby			1					GC									1	1				
20	Roger			1					GC									1	1				
21	Sally			1					GC									1	1				
22	Thomas			1					GC									1	1				
23	Peggy			1					GC									1	1				
24	Martha			1					GC									1	1				
25	Abby			1					SF									1	1				
26	Sue			1					SF									1	1				
27	Thomas			1					SF									1	1				
28	Instructor				1	1								N		1					1	1	
29	Sue			1			1									1					1	1	
30	Abby			1			1											1					
31	Instructor				1	1								N		1					1	1	
32	Sue			1			1									1			1				
33	Alice			1			1								1							1	
34	James			1			1								1							1	
35	Sally			1			1								1							1	
36	Instructor				1	1										1			1				
37	Martha			1			1								1							1	
38	Alice			1			1								1							1	
39	Mitchell			1			1								1							1	
40	Instructor				1	1										R						1	
41	Thomas			1			1								1							1	
42	Primary Inst.			1			1								1							1	
43	Instructor				1	1										R		1			1	1	

Appendix N. Institutional Review Board (IRB) approval.



North Carolina State University is a land-grant university and a constituent institution of The University of North Carolina

Office of Research and Graduate Studies

NC STATE UNIVERSITY

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Regulatory Compliance
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919.515.2444
919.515.7721 (fax)

From: Debra Paxton, IRB Administrator
North Carolina State University
Institutional Review Board

Date: June 9, 2009

Project Title: Preparing to Teach Mathematics with Technology: Implementation and Effectiveness of Curricular Materials

IRB#: 996-09-6

Dear Dr. Lee:

The research proposal named above has received administrative review and has been approved as exempt from the policy as outlined in the Code of Federal Regulations (Exemption: 46.101. b.1 & 2). Provided that the only participation of the subjects is as described in the proposal narrative, this project is exempt from further review.

NOTE:

1. This committee complies with requirements found in Title 45 part 46 of The Code of Federal Regulations. For NCSU projects, the Assurance Number is: FWA00003429.
2. Any changes to the research must be submitted and approved by the IRB prior to implementation.
3. If any unanticipated problems occur, they must be reported to the IRB office within 5 business days.

Thank you.

Sincerely,

Debra Paxton
NCSU IRB

Appendix O. Institutional Review Board (IRB) participant consent form.

Informed Consent Form: Prospective and Practicing Teachers

Preparing to Teach Mathematics using Technology: Implementation and Effectiveness of Curricular Materials, Dr. Hollylynne Lee, Principal Investigator

As a participant in a course using the *Preparing to Teach Mathematics with Technology* curriculum materials, you are invited to participate in a research project. The purpose of this project is to evaluate the effectiveness of the curriculum materials in preparing teachers to use technology tools to solve mathematics problems, to analyze students' understanding of mathematics using technology tools, and to prepare lessons using technology that foster students' conceptual understanding. **You will contribute to this research by completing regular class assignments and allowing researchers to have access to your work on these assignments, including any posts to discussion boards or synchronous chat rooms.** Some class sessions may be video recorded, including recording of class or work sessions completed in synchronous online environments. In addition, you may be asked to complete the following:

1. Pretest and Posttest that will assess your understanding of key mathematical ideas, technology skills, and pedagogy for teaching.
2. Surveys that ask for information about your use of the curriculum materials and beliefs about and self confidence in your abilities to teach mathematics with technology.

In addition, several participants may be asked to engage in a one-hour videotaped interview outside of class time that will involve similar tasks used in the curricular materials. This will allow researchers to gain a deeper understanding of your reasoning on such tasks. Some participants may also be asked to be observed teaching a lesson with technology in a grades 6-12 classroom. The observations will be recorded with field notes and audiotape. If approached for an interview or classroom visit, you have the option to not participate in that part of the study.

There will be no risk associated with your participation in the research study. Participation is voluntary. *Your grade in the course will not be affected by your decision to participate in the study.* The knowledge gained from your experiences will add to the knowledge base in mathematics education, especially with regard to how teachers learn how to teach mathematics with technology. The information derived from the class activities, assignments, tests, and surveys will be kept strictly confidential, with your name removed from the work. It will be stored securely in a locked file and will be made available only to the researchers unless you specifically give permission in writing to do so. No reference will be made to your name either in oral or written reports and transcripts that could link you individually to the study.

You are free to withdraw from the study at any time; however, you will still participate in all of the activities that are class requirements. If you have questions at any time, you may contact Dr. Hollylynne Lee at 919-513-3544. Her address is 502D Poe Hall Box 7801, NC State University. If you feel you have not been treated according to the descriptions in this form, or your rights as a participant in research have been violated during the course of this project, you may contact Deb Paxton, Regulatory Compliance Administrator, Box 7514, NCSU Campus (919/515-4514).

************************CONSENT***************************************

I have read and understood the above information. I have received a copy of this form. I agree to participate in this study.

Participant's signature ___________________________________ Date _____________

Investigator's signature Hollylynne S Lee_______ Date
______11/08/2010

Appendix P. Information on Elluminate provided to prospective teachers in the online group.

Summary of Data Analysis and Probability Unit
EMS480/580.002
Mrs. Tina Starling
ttstarli@ncsu.edu
Office Hour: T/Th 11:30-12:00 (Poe502E)
or other times by appointment

What you need to "attend" class:

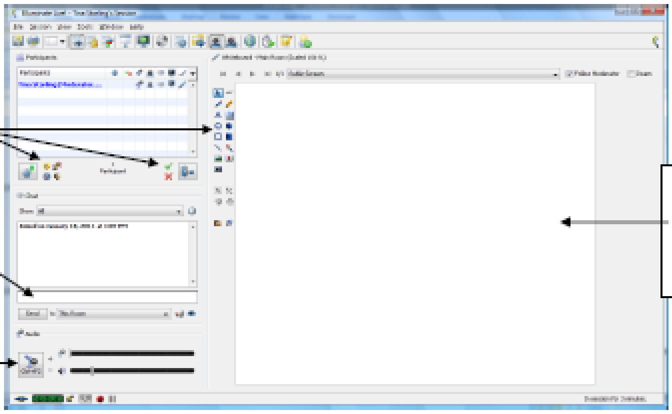
- Computer with access to the following programs: *TinkerPlots, Fathom, Probability Explorer, Microsoft Excel, Firefox internet browser*. *TinkerPlots* and *Fathom* are part of the software bundle for our class. You may be able to use the virtual computing lab for these as well. *Probability Explorer* is on the textbook CD.
- TI-83+/84 graphing calculator.
- Microphone and speakers/headset. I recommend a headset as the class will be highly interactive; headphones can help you hear and concentrate. Having said that, a public setting may not be ideal. ☺
- Preparing to Teach Mathematics with Technology textbook and accompanying CD.

What you'll need to do to "attend" your first class, February 3rd:

- You will receive an email invitation to join each *Elluminate* session. It will include a link which will open the *Elluminate* program and directions about two setup procedures:
  1) Audio Setup - In order to be able to talk and hear, you will likely need to setup your audio. Under the Tools menu, choose Audio, then Audio Setup Wizard. Follow the directions to setup your speakers/headphones and microphone and to adjust all sound levels.
  2) Connection Setup – *Elluminate* responds differently depending on the type of internet connection you are using. Under the Tools menu, choose Preferences, and find Session in the left column. Click on Connection, and make sure the correct connection speed is selected.

  *If you're using the same computer and equipment throughout the Data Analysis and Probability unit, you may only have to follow these procedures once.

- Once you have joined the *Elluminate* session, the following window will appear. This is your "classroom" for the Data Analysis and Probability unit. ☺ Your name will appear in the Participants window in the top left corner.



Tools you will use during class.

You may type a chat message here.

Click this button to turn your mic on/off.

PowerPoints, technology demonstrations, and more will appear here.