

**A KNOWLEDGE STRUCTURE FOR THE ARITHMETIC MEAN:  
RELATIONSHIPS BETWEEN STATISTICAL CONCEPTUALIZATIONS  
AND MATHEMATICAL CONCEPTS**

Mark A. Marnich, Ed.D.

University of Pittsburgh, 2008

This study examined cognitive relationships between the fair-share and center-of-balance conceptualizations of the arithmetic mean. It also hypothesized the use of these conceptualizations as blending spaces for the mathematical and statistical domains within a proposed knowledge structure for the arithmetic mean.

Twenty-nine undergraduate liberal arts students completed pre/post verbal protocols with written solutions to arithmetic mean problems. The problems emphasized either the fair-share or center-of-balance conceptualization, or mathematical concepts related to the arithmetic mean. The participants were divided into three groups: those that received fair-share instruction, those that received center-of-balance instruction, and a control group.

The data was analyzed using statistical methods, including contingency tables and ANCOVA, to investigate the effects fair-share and center-of-balance instruction had on knowledge of fair-share, center-of-balance, and mathematical concepts regarding the arithmetic mean. A qualitative analysis of the verbal protocols helped explain any statistically significant connection between the fair-share and center-of-balance conceptualizations, or between either conceptualization and mathematical concepts related to the arithmetic mean.

Analysis of the data indicated participants increased their knowledge of the fair-share conceptualization after receiving instruction that was focused on center-of-balance. Similarly, participants increased their knowledge of the center-of-balance conceptualization after receiving instruction that was focused on fair-share. In either case, the concept, ‘the sum of the deviations from the mean is zero,’ was used to transfer knowledge between the conceptualizations.

In addition, instruction in either the fair-share or center-of-balance conceptualization increased knowledge of the mathematical concepts related to the arithmetic mean. However, only specific mathematical concepts were impacted by each of the conceptualizations.

The results suggest that both the fair-share and center-of-balance conceptualizations are pertinent to pedagogical decisions regarding the arithmetic mean. Furthermore, the concept, ‘the sum of the deviations from the mean is zero,’ is a viable cognitive connection between the fair-share and center-of-balance conceptualizations.