This paper presents a study of current guidelines for training mathematics teachers in Colombia. Its particular focus is the teaching of probability. The structure of the Colombian educational system and the rules for becoming a secondary school teacher are described. The mathematical and pedagogical training required to teach probability is analyzed into eleven bachelor programs for mathematics teachers. Because these programs spend only a few hours on training about probability and how to teach probability, and because there is a lack of specific mathematical training required for entering the teaching profession, the analysis concludes that there is a general lack of specific pedagogical training for teaching probability. One implication is that programs to educate teachers should extend class time to enhance the competence of prospective teachers in the design of educational activities and in recognition of students’ difficulties, mistakes and misconceptions about probability.

INTRODUCTION

Research in various countries (Jones, Langrall, & Mooney, 2007; Batanero, Burrill, & Reading, 2011) has revealed failures in the training of mathematics teachers for teaching probability. As do other countries, Colombia includes probability in the curricula from primary school through high school, but this implies appropriate teacher training to prevent omission or the spread of misconceptions, biases or heuristics. Despite this, Colombia does not have any specific requirements in this area for public school teachers (described in second section).

The aim of this paper is to analyze the training about probability and how to teach probability offered by some of the Colombian universities which have undergraduate programs for prospective secondary school teachers. This analysis is based on two large components of the Mathematical Knowledge to Teaching (Ball, Thames, & Phelps, 2008), mathematical content knowledge and pedagogical content knowledge. To place this study in context, the educational system in Colombia is briefly described showing how probability fits into the secondary school curriculum. This is followed by a description of aspects of the competition used to select mathematics teacher for public schools. After that, characteristics of eleven undergraduate programs oriented to mathematics education are presented and analyzed. This paper concludes with suggestions for possible applications of these analyses.

PROBABILITY IN COLOMBIAN SECONDARY SCHOOL GUIDELINES

The structure of the Colombian educational system is preschool, elementary school (five years) and secondary school. Preschool lasts three years but is not obligatory. Elementary school covers children from six to ten years old and secondary school covers children from 11 to 14 years old; these nine years are obligatory and this period is called basic education. Some people leave school after the basic education, but others continue voluntarily in two more years of secondary education. They have two options in these final two years: academic, in order to prepare students for university, and technical which leads directly to employment.

The General Education Law (MEN - Ministerio de Educación Nacional de Colombia – National Ministry of Education of Colombia, 1994) does not allocate contents or methodological suggestions, but rather places this responsibility in the hands of each school according to the principle of autonomy. Schools include these definitions in documents called Institutional Educational Projects (Proyecto Educativo Institucional - PEI). Consequently, contents are flexible. As contents and their teaching are highly diversified, the Ministry of Education provides curricular guidelines and standards to build the institutional curriculum and to guarantee minimum quality.

Colombian standards (MEN, 2003) for teaching mathematics are organized around five main types of thinking: numerical thinking, spatial thinking, metric thinking, variational thinking and random thinking. Random thinking includes minimal amounts of probability and data systems.
In Gómez (2011) there is a detailed description of secondary education content. That study concludes that standards are very ambiguous for the 6th and 7th grades (11 to 12 years old) but are more precise in the other grades. Eighth and ninth grade (13 to 14 years old) curricula cover classical approach (e.g. tree diagrams, counting, and proportionality), but also covers the use of frequentist approach (e.g. experimentation), and relations between classical and frequentist approaches through modeling. In 10th and 11th grades (15 to 16 years old), learning is directed toward interpreting and applying basic inferential situations (e.g. experimental design in natural sciences and making inferences from a simple random sample). The subjective approach is implicit in some suggested contents such as conditional probability, independence and inference. Also, it is assumed that the intuitive approach was developed in elementary school.

In summary, the Colombian curriculum explicitly considers classical and frequentist approaches to probability while the subjective approach is only implicit. For good classroom performance a mathematics teacher should know these approaches and be able to broadly apply them to decision making problems including those in his/her own professional practice.

WHO CAN BECOME A MATHEMATICS TEACHER IN COLOMBIA

In Colombia, mathematics teacher are selected through public competitions open to university graduates. This competition consists of a test of basic competence (50% of score), a psycho-technical test (20%), an interview (10%) and an evaluation of the applicant’s curriculum vitae (20%). However, ASCOFÁDE’s (2006) report of specific test results for the competition for mathematics teachers in 2005 concludes that candidates who graduated in pure sciences had the best performances on this specific test. They were followed by those who graduated in mathematics education. Nevertheless, overall scores – rather than those for this specific test, were highest for candidates with teaching experience.

Mathematicians focus on mathematical knowledge as a pure science but do not have pedagogical training. Consequently they, and graduates other areas, should have to graduate teaching programs or equivalent training to develop and/or improve their teaching skills prior to working at a public school (MEN, 2005). This requirement would not apply to programs that begin with teaching credentials (Licenciaturas) because credential programs include pedagogical training that may be general or related to a specific field of knowledge. Specifically, undergraduate programs oriented toward training prospective mathematics teachers are called Teaching Credential in Mathematics (Licenciatura en Matemáticas) and Basic Education Teaching Credential with an emphasis in Mathematics (Licenciatura en Educación Básica con énfasis en Matemáticas). These two have different fields of application: the first is oriented toward secondary education (6th to 11th grades) and the second is oriented toward basic education (1st to 9th grades). This implies that deeper mathematical knowledge is required for a Teaching Credential in Mathematics than for a Basic Education Teaching Credential with an emphasis in Mathematics and also implies something similar for the pedagogical content of the two types of credentials. The following it is analyzed some Teaching Credential in Mathematics programs.

PROBABILITY IN UNDERGRADUATE PROGRAMS FOR MATHEMATICS TEACHERS

Undergraduate programs oriented toward training prospective secondary school mathematics teachers are very diverse because their contents are not fixed in the curricular guidelines provided by the Ministry of Education. This absence of fixed guidelines mirrors that for pre-university curricula. Nevertheless, some basic skills and competences are suggested as baselines for guaranteeing minimum standards of quality (MEN, 2010). Gómez (2011) describes eleven programs and focuses on probability and its teaching within those programs. The main results are the following:

In most universities, at least 40% of the curricula are topics in Mathematics but the depth of classes is highly variable. Only some universities, but not all, have a Mathematics Department and/or professors who hold PhDs who are qualified to teach specialized topics. In general, the curricula have few optional topics which usually include only one course about epistemology, one about the history of mathematics, one or two about the use of technology, and three thematic areas (Algebra and Arithmetic, Calculus, and Geometry). In some cases, there is a fourth thematic area about statistics or physics.
Probability’s meaning is covered from different points of view, with more emphasis on classical, frequentist and axiomatic approaches and only a few mentions of topics related to intuitive and subjective approaches. At the end, it is expected at the minimum that a student understand basic objects, probabilistic language, foundation theorems and principles, Bayes theorem, independence, random variables and distribution models (e.g. binomial and normal).

Teaching about the relation between probability and inference is scarce, as is teaching about inference (differences between probabilistic and non-probabilistic sampling, procedures with a simple random sample under normality, basic notions of sampling distributions and the Central Limit Theorem). Also, in some cases these contents are skipped because the intended content requires too much time.

Furthermore, the elements of probability which are taught are not taught to any depth. They are included in a course on descriptive and inferential statistics which uses textbooks suggested for students of the sciences and/or engineering. On the other hand, some universities suggest the use tools for data handling without mention specialized software. Three universities are highlighted for their stochastic components: one offers a course about probability only and has three optional courses on specific topics of statistics; another offers a course on random thinking, and the other includes the history of calculus and statistics.

Pedagogical training is also very diverse at those eleven universities. Courses about this topic account for 25% to 40% of curriculum and include primarily overall knowledge about psychology, curriculum, epistemology and the use of technology. The maximum number of courses offered about mathematical pedagogy is three while most of these programs offer only one. Specific pedagogical knowledge, including optional courses is absent in many universities.

Teaching practices required by MEN (2010) do not reach the minimum of one school year in most universities. The number of hours or credits (one credit=48 study hours) also vary greatly. For example, the oldest curricula offer only workshops, one university has a practical course for teaching each thematic area (arithmetic/algebra, calculus, geometry, statistics/probability) and another university includes three courses to confront students with some topics in real contexts.

Few universities have specific courses about statistical or probabilistic pedagogical knowledge. Only three include one for teaching statistics focusing on data handling which require previous knowledge of statistics and probability. The contents cover teaching models; curricular design following international guidelines; stochastic reasoning; some mistakes, obstacles and misconceptions; everything about representation systems, basic concepts of probability, location and dispersion measures.

CONCLUSIONS

Changes in school curricula make it desirable to include a focus on the teaching of for prospective teachers. This should include a change from excessive emphasis on the axiomatic approach to include other approaches that are used in secondary school (such as statistics applications), however this should not compromise the formal basis that supports the axiomatic approach. One result of this analysis is that the probability curriculum for prospective mathematics teachers in Colombia should be in line with the suggestions for improving probability literacy by Gal (2005). Those suggestions identify four of basic elements of probability-related knowledge: fundamental probabilistic ideas, understanding of probabilities, probabilistic language and applications to different contexts.

However, this knowledge about probability is not sufficient for teaching this topic because it lacks the depth required for solving some types of problems and required for understanding the basis of some common mistakes and misconceptions. With that level of knowledge there is no evidence that undergraduate students will progress from intuitive approach to the classical and frequentist approaches to obtain an overall meaning of probability, or if they will be able to help children do this. Moreover, since probabilistic pedagogical knowledge is scarce, prospective teachers from these undergraduate programs probably will not be able to predict and solve learning conflicts when children encounter them, such as those in other countries (Jones, Langrall, & Mooney, 2007; Batanero, Burril, & Reading, 2011; Díaz, Contreras, Batanero, & Roa, 2011). Specific courses about probability teaching are unusual, and only a few universities facilitate
contact between prospective teachers and real secondary students in a classroom in which they can teach probability or any other mathematical topic.

This analysis of curricula leads to some conjectural proposals about prospective teachers’ knowledge along the lines of the model of Ball, Thames and Phelps (2008): The probability common knowledge (CCK) of prospective teachers from these universities could be acceptable for teaching in schools. Specialized probabilistic knowledge (SCK) might not be sufficient, except for those who graduate from the four universities that offer specific courses about topics of mathematics teaching. Horizon knowledge is probably scarce, except for those who graduate from three universities that give more learning opportunities of this content area.

Probabilistic pedagogical knowledge is likely to be scarce among most prospective teachers who graduate from these eleven universities because there are few courses on mathematics teaching, and because there is an absence of practical classroom courses. Knowledge of content and teaching (KCT) could better than knowledge of content and students (KCS); both may be sufficient for teaching basic concepts but are not likely to be sufficient for teaching the contents needed in the higher grades. There is no evidence about teachers’ levels of knowledge of curriculum, even though epistemological guidelines include general competences about transversal principles and linking of different topics.

We hope this paper contributes to the improvement of the teaching of probability in universities and that it facilitates the work of future teachers in the classroom. Furthermore, we hope that teacher educators recognize that probability and its teaching should be included in the education of prospective teachers.

REFERENCES