EXPOSING YOUNG CHILDREN TO ACTIVITIES THAT DEVELOP EMERGENT INFERENTIAL PRACTICES IN STATISTICS

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Informal statistical inference has gained increasing recognition as an effective approach to teaching statistics. Distinct from descriptive statistics, inference provides learners with access to the power of statistics by giving them tools to make predictions beyond their data. International research in this area has focused on students from primary school through university. A series of teaching experiments introduced informal statistical inference to very young children (aged 5-6). Although making predictions was familiar as an everyday task, initial attempts revealed challenges to teaching informal inferential reasoning to young learners. Prior to conducting a statistical inquiry involving inference, activities were designed to generate a need for recording and organising data, the language of uncertainty and using data as evidence. Results suggest that the activities prior to inquiry likely supported students in their emerging inferential practices.

INTRODUCTION AND LITERATURE

Inference is at the very heart of statistics. Recent research in statistics education at the school level has placed new emphasis on informal statistical inference. One interpretation of informal statistical inference is as a claim with three key characteristics (Makar & Rubin, 2009):

- The claim (or generalisation) extends to “some wider universe” (Ben-Zvi, Gil, & Apel, 2007);
- Use of data as evidence for the claim (Fielding-Wells, 2013); and
- Articulation of the claim with non-deterministic (probabilistic) language to express uncertainty (Ben-Zvi, Aridor, Makar, & Bakker, 2012).

Most research in this area has targeted students from middle school to introductory tertiary level. Given that descriptive statistics is taught in middle school (around age 11-13), the challenge at the primary level is to embed the key processes of inference without formal descriptive statistics. Yet, there is promising evidence from researchers that inferential reasoning is productive in the primary years as a way to promote high quality statistical reasoning more broadly (English, 2012; Paparistodemou & Meletiou-Mavrotheris, 2008).

Our preliminary work suggested that young children were able to make predictions and express them with uncertainty. However, we saw that they often found it difficult to connect their predictions to data-based evidence. While we did not expect “data-based evidence” to have the same meaning for young children as it would with older students, we did hope to help them to see that they could use information they had collected about their class, for example, to make predictions about the class next door. In contrast, young children we had worked with often gave us anecdotes about individuals they knew in the class next door, personal stories, or information about their friends more generally to justify predictions beyond their data.

The aim of this paper is to investigate the potential to build emerging inferential practices in young children under seven years old. The purpose was not to ensure the children could fluently create informal statistical inferences, but rather engage them in experiences which may develop emergent practices in informal statistical inference. In addition, the study reported here sought to see if it was possible to focus the children on seeing the data as evidence, or if they were too young.

PARTICIPANTS AND APPROACH

The participants were 22 Australian children selected across five classes of Prep (first year of schooling, age 5) (Phase I) with the same children in Grade 1 six months later (Phase II). The children were selected as those with strong communication and investigation skills and above average performance in school. The teacher (and first author) was a specialist reading teacher and expert in inquiry-based learning in mathematics who had taught informal statistical inference to primary children for several years (e.g., see Makar & McPhee, 2009). At the time of the study, she

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was the reading teacher for the participants, who were accustomed to her mode of questioning and expectation to be reflective and responsive.

The study took place in two phases of three lessons each (60-90 minutes each). The aim of Phase I was to build skills that the students would need for making inferences, such as collecting and recording data, making predictions, and seeing collections as "aggregates". In the first lesson, prediction was discussed to scaffold the children's language (probabilistic language such as *maybe*, *possible*, and *likely*, and vocabulary such as *prediction*) through two activities: predicting from a story (Activity 1) and making predictions from simple data collection (Activity 2) (See excerpts at www.teacherstandards.aitsl.edu.au/Illustrations/ViewIOP/IOP00210/index.html). In the next two lessons, children made predictions from dice (Activity 3) and patterns (Activity 4). In Phase II of the study, students engaged with the inquiry question—*How big are our shoes?*—in three lessons.

In all lessons, students collaborated, shared progress, and participated in discussions to create a classroom culture that supported risk-taking, questioning, creative thinking and using evidence. The aim was to build their confidence to make an inference then check it without being worried about whether they were right or wrong; therefore, building trust in the classroom and being able to justify and share that information with the class was critical. Lessons were videotaped and annotated video logs were created to assist in selecting episodes that illustrated aspects of students' emerging informal inferential reasoning.

RESULTS

Our findings are given below. The key focus was on how the activities in Phase I supported students in Phase II to engage in aspects of practice aligned with informal statistical inference.

Phase I: Preliminary Activities

Children have quite a lot of experience using prediction in children's picture books, making predictions from the title and the cover. In Activity 1, they used knowledge of what they could see in the pictures or had heard so far in a story to predict what might happen in the rest of the story. Their knowledge of the word "predict" and prediction experiences were used to scaffold predicting in later data tasks and building a meta-language of prediction across the lessons.

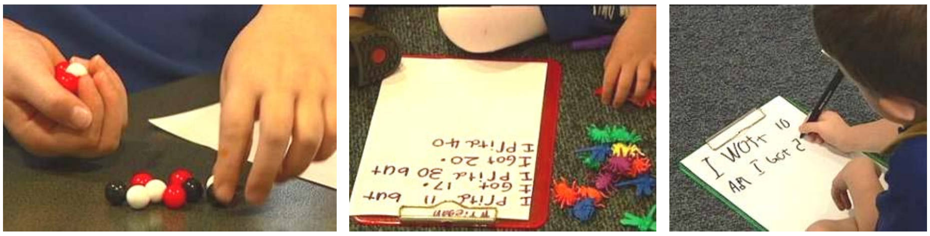

Figure 1: Children predicting and recording how many items they could pick up in their hand.

Following this activity, the children were asked to make predictions about the number of items (counters, blocks, marbles) they could pick up in their hand, then to count how many they actually picked up to test their prediction (Figure 1). By repeatedly predicting and checking their predictions, there was a desire for the children to use each cycle of predict-and-check to improve their next prediction. In this way, they would be using the data as evidence for their predictions. When they went to pick up items that were the same or different size or shape, the hope was that they would also adjust their predictions based on their previous experience of predict-and-check and considering the difference in size. This process would also help them to develop experiences in communicating their mathematical thinking. The children were accustomed to inventing recording methods to write down what they had done for their memory. When they shared their findings, some children reflected on the idea that the more they predicted, the more accurate they became.

> Liam: My first time helped me a lot. For the second time, I got really close to the, to the one, to the prediction that I wanted.

Not all children improved their predictions, suggesting that making predictions based on data as evidence is a challenging characteristic of informal statistical inference for children of this age. The activities in the next two lessons were focused particularly on this aspect of inference by building on their experiences recording and making predictions under uncertainty through exploring dice and spinners (Lesson 2, Activity 3) and (deterministic) predictions from patterns (Lesson 3, Activity 4). In Lesson 2, the children invented methods in pairs to record outcomes from a die or a spinner. The purpose was to give them experience recording random phenomenon, making sense of their recording and trying to predict what might happen if the experiment were repeated. The variability and uncertainty in this activity created a new challenge for children to make predictions. Children shared which numbers came up the most and the least then made predictions about whether these same numbers would come up most and least again.

DM: Will you get the same numbers?
Liz: I think we will have different numbers, because we had lots of different numbers [last time].
KM: What will happen this time?
Ravi: We'll get more of the numbers we got before.
KM: So how many times did you get a "one" before?
Kim: Umm (checking recording), zero.
KM: … Do you think you're going to get lots of ones [next time]?
Kim: Not really.

We wanted to raise students' awareness of uncertainty and variability: that if they conducted the experiment a second time, they would not get *exactly* the same outcome, but that they might see commonalities in getting *about* the same amount. Children shared their invented recording and reflected on whether their method made questions easy or difficult to answer, such as: "How many times did a 3 come up?", "Which number came up the most/least often?" In most cases, students recorded each roll as a narrative. One pair used tallies and this was adopted by a few children when they recognised it was easier to see quickly which outcome was least and most common through tallies than a narrative. However, their tallies did not always express frequency as one child used tallies to record each outcome (e.g., six tallies for a 6 on the die).

In Activity 4 (Lesson 3), students created patterns, a conventional activity in a mathematics lesson. However, students were not familiar with using patterns to make predictions such as how many "units" it would take to stretch to the end of the table or across three tables. Therefore it made a useful activity to scaffold prediction and focus on collections as aggregates. The activity was iterated several times so the children could see the structure of the prediction routine. Although patterns are deterministic, it was necessary for the children to use the pattern (data) in front of them to make a prediction about units ("repeats") that were not yet laid down.

DM: How many repeats do we have here? Not how many camels but how many repeats?
Class: Five!
DM: Can you show us where they are, Tina?
Tina: (Touches with finger as she counts) One, two, three, four, five.
DM: And that's your five. … So if there are five repeats here who would like to predict how many of those patterns we could fit to the end (of the table)?
Carly: Three.
DM: Do you mean three *more*? Because we already have five? (Carly nods.) So *eight* all together. Are you sure there would be eight?
Liz: Perhaps.
DM: Ooh, you used a good word. Would you say that word again?
Liz: Perhaps.
DM: Perhaps, yes, perhaps. That's a good word to use!

The initial question posed asked the class to count how many units were already in their pattern. This allowed them to both focus on the "evidence" they would need for their estimate, but also contrast the language used when they were certain, with the language needed to express uncertainty (Liz: "perhaps").

Across the three lessons in Prep (Phase I), by creating experiences that were often linked to activities they might be accustomed to in English (inferring in stories) and mathematics lessons (working with patterns, recording), the aim was to strengthen their inferential experiences to provide scaffolds for Phase II, making an informal statistical inference in an inquiry-based task.

Phase II: How Big Are Our Shoes?

Six months later, the same children (now in Grade 1) were brought together to conduct a statistical investigation to answer the inquiry question, *How big are our shoes?* In this investigation, the children collected shoe size data from each other and used their data to make inferences about the shoe sizes of children their age beyond their own class. In these three lessons, the authors were primarily interested in how the children relied on foundational concepts of informal statistical inference (Makar & Rubin, 2009) based on activities from Phase I. In particular, a focus in these lessons was on supporting the children to use the data they had collected as evidence for their predictions, rather than relying on anecdote or guessing.

A short discussion and activity reminded the children of work they undertook previously in Prep (e.g., making predictions from a story, recording outcomes, extending patterns through prediction). Children were then introduced to the inquiry question and asked to find out their classmates' shoe sizes, needing no prompting to record their data (Sam: "So you can remember"). The recording methods varied widely: Some children listed names and shoe sizes for each student, others drew a picture of a shoe(s) and listed the names and shoe sizes around it. One student chose to organise her information into "bins" while another created columns to record the name, shoe size and prediction of each shoe size (a practice that was appropriated by several others) to capture the idea that some students weren't sure of their shoe size.

As the teacher and researcher were walking around or when children shared their progress with the class, they asked questions such as "What is the biggest/smallest shoe size?" and "Which shoe size is the most common shoe size so far?" Although these were descriptive questions (describing the data they had), they helped students see the benefit of organising their data so that they could locate this information more efficiently. Once children had collected and shared their data, the teacher asked them to explain why more people were in the middle and fewer at the ends, and to predict what the shoe sizes for another class (same age) in their school might be like.

DM: Why are there seven people with [size] 13 and only one with a 10 and one with a 5[1]?
Liz: … Size 13 is the normal size for a Grade 1er to have?
DM: When you say "normal", what do you mean by "normal"?
Liz: Like, most people have that.
DM: … So what size do you think there would be the most of in [another class], Liz?
Liz: 12ish-13?

Liz's comments suggest that she was using the data she had collected to make her prediction about the most common sizes for another class. Although the most common size in their class was size 13—what Liz called the "normal" size for a Grade 1 student—her prediction went beyond that to include a slightly larger range and a bit of uncertainty.

In the final lesson, groups of children were given a set of small white cards with their class data (name, shoe size). They organised the cards using different strategies: Sorting cards into piles with the same (or similar) shoe size; sorting and stacking cards with the same shoe size (so all cards were visible); sorting, stacking and ordering cards according to shoe size; sorting, stacking and ordering cards with "gaps" left for values with no data; and forming patterns (e.g., into a rectangle, rainbow). As the teacher and researcher circulated among pairs, they asked probing questions, such as, "Which shoe size is the most popular?", "Which shoe size is the smallest/largest?", "Why is it useful to organise the data?" Students' responses included a range of ideas which suggested they had varying levels of understanding about (or ability to express) the utility of organising the data. Although the children were unable to explain *why* organising the data was useful, they were able to use their organisation to respond to questions about the data and justify their answers.

KM: Which size do you think is the most popular?
Tina: 13

KM: Why?
Tina: Because it's got the most.
KM: Can you show me?
Tina: (Counts how many cards are size 13.) There's 7.
DM: … I noticed you don't have anyone with 4. What happened to the 4?
Sarah: The numbers that we don't have we're just skipping.

Many pairs recognised that some groups had more productive systems of organisation and often adopted their methods because they appeared beneficial in answering these questions. The class gathered around one pair, Dan and Karl, so they could discuss how their data were organised.

DM: (to Dan and Karl) Who would like to explain what is going on? …
Dan: Well, we just sorted them in order. Karl is the smallest. Tina is the biggest. We just put them in order 10, 11, 12, 13, 1, 2, 3, 5. …
DM: I noticed you didn't have a shoe size 4 here. Can you tell me about that?
Dan: Cause, no one has a size 4.
DM: (To the class) Do you think if we went into another Year 1 class, that we might find some children in those classes with a size 4 shoe?
Class: Yes.
DM: Tell me about that. Tina?
Tina: Um, because. There could be because, because, um, because we don't know, but there could be another size 4 shoe. Cause I've seen another one in Year 1.
DM: So you think it's possible? Do you think there'd be a lot of people in other classes with a size 4?
Dan: I wouldn't say so.
Tina: I wouldn't say so. Not really, like maybe 1 or 2. Or maybe 3.
Karl: There wouldn't be like, five!

When Tina and Dan predicted the data from another class, their responses suggested that they were using the data they had for their prediction and that they recognised that their predictions were tentative ("maybe", "there could be" "we don't know, but" "I wouldn't say so").

In a class discussion, students were asked to make predictions about what shoe sizes could be in another class. Although some children thought the (distribution of) shoe sizes in the other class would be identical to theirs, most recognised there would be some differences. Even in less successful groups, children were comfortable *expressing* their predictions with uncertainty. Making predictions *from their data* was less consistent across groups; not all responses were relevant to the question as some students' comments were only about the context (e.g., explaining that they wore two pairs of socks because their shoes were too big; or stating that three names started with "J").

To test their predictions, students were given a set of yellow cards with names and shoe size of the other class. Most groups organised their data in some way, with more productive methods appearing as a stacked dot plot. When making predictions about shoe sizes in a neighbouring school, some relied on their data to make predictions while others continued to rely on personal beliefs (Carrie: "It's another school, so it might be any number"). This issue points to the continuing challenge of making informal statistical inferences at this age. However, the emergence of complex practices underpinning inference (such as comparing groups, Watson & Moritz, 1999) adopted by some students was encouraging.

DISCUSSION

Despite the challenges encountered, including inconsistencies in the children's extended responses, the excerpts above suggest that they showed an emergence of practices of informal statistical inference in determining shoe sizes of children their age. All of the children were able to demonstrate some aspect of informal statistical inference during the lessons. There was particular interest in the study to build students' awareness in using the data as evidence for their predictions, an area which in earlier work with young children we had found the most challenging. We saw encouraging results in many of the children's predictions, as illustrated in the excerpts above.

Informal statistical inference requires a number of skills and understandings that go beyond Makar and Rubin's (2009) three characteristics. An oversight in early work with young children, in retrospect, may have been an assumption that one could jump into an inferential context involving data and "teach" students to predict beyond the data (tick), express their predictions with uncertainty (tick) and use the data as evidence (tick).

The above study gives us greater insight into a possible process of working with young children to engage in informal statistical inference, through early valuing of foundational skills and understandings. A number of skills appeared to be important for supporting students to engage in emergent practices of informal statistical inference: Articulating or predicting from observations, recording and organising data using invented methods and working with aggregates and variability. In Lesson 1, the initial activities appeared to support students in beginning to recognise the utility of recording their data; and although they did make predictions, the data used as evidence was primarily experiential. Work with dice and patterns likely strengthened their skills and understandings in recording and making predictions from data, but there was still no sense that the data were distributed. Students' invented methods may have further assisted them in sense-making with data; recognising, for example, the value of organising their data to easily respond to the teachers' data reduction questions. By Phase II, students were fluent with the need to record, and developed a number of invented methods to collect and record the data on shoe sizes from their classmates. Questioning by the teacher or researcher again assisted most students in valuing the utility of organising the data.

We believe that the foundational practices above are achievable with young children and engaging in these practices in a low-stakes environment would foster progression towards developing informal statistical inference. Our aim in doing so is not to teach formal methods of hypothesis testing at an earlier age, but to develop children's power and positive perception of statistics, access to using and connecting statistics with familiar experiences, and creating a unifying perception and understanding of statistics.

ENDNOTE

[1] In Australia, young children's shoe sizes end at 13; older children and adult sizes begin with size 1.

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