CONTRIBUTIONS OF A HISTORICAL GAME PRESENTED THROUGH A STORY AS A SUPPORT TO THE TEACHING OF PROBABILITY IN ELEMENTARY SCHOOL IN BRAZIL

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The present study aimed to investigate the learning of probability with a group of students of the fourth year of elementary school in Brazil, taking into account a situation involving the concept of random events through the narrative used in a historical tale called: "Did you know that children played with the game of tiles a long time ago in France?" The Theory of Didactic Situations was used to evaluate the intervention activity that sought the development of competences and abilities related to probability. The results showed that the activities contributed to the teaching and learning of probability, and despite the students having some difficulties, their mistakes guided the teachers/researchers to think about different approaches to the appropriation of knowledge.

INTRODUCTION

We create stories; we tell our own stories; we create games; and we live them, as if their development, their plots, and their rules follow our own development in which we have to adapt to what is imposed on us. We can interpret stories according to our senses (Estés, 2005) and understand them according to our experiences, and this same relationship occurs in games (Freire & Scaglia, 2003). We consider history as a resource that can promote probabilistic work during the teaching and learning process. It becomes a resource that can provide multiple benefits to students and allows learning in a global and meaningful way for the child.

In addition, Vásquez and Alsina (2017) consider that to start teaching probability, we must start from situations of uncertainty in the child's environment because through them, the child will start to use their own probabilistic language in the face of impossible, probable, or certain events. In addition, the teacher must provide experiences that allow the child to observe phenomena with probability and through stimuli (questions, project works that start from a research question, observations/calculations, predictions, etc.), ask them to express possible directions.

To complement these ideas, Coutinho (2007) discussed the role of history in the choice of contexts to present the first probabilistic concepts in elementary school. To carry out an epistemological study, he described approaching chance in relation to the context in which it is inserted, considering the possible results of manipulations of a chance generator such as games of chance (manipulation of coins, dice, etc.).

Oliveira Júnior et al. (2017) considered arguments that reinforce the pedagogical and the questioning potential of the history of mathematics according to Miguel (1997), based on bibliographic research on the history of probability and on the development of class activities focused on basic probabilistic concepts. Considering situations that involve basic concepts of probability such as a sample space and its events, Oliveira Júnior and Barão (2021) investigated the probabilistic learning of a group of elementary school students through narrative used in a story and a historic game.

Associating games of chance with historical aspects, Viali (2008) recalls that the first recorded manifestation of games is the Tali (bone game) that was played with the talus, an animal bone with a shape similar to an irregular tetrahedron.

In this way, the use of history and games can enhance the learning of specific probabilistic content because students approach the content in a globalized way with other students and with content related to the emotional and affective environment. Approaching content in a contextualized way facilitates the students' contact with probabilistic situations outside the children's world.

Our objective was to investigate the learning of pupils in the fourth year of primary school in a public school in the state of São Paulo, Brazil through the Theory of Didactic Situations (TSD) (Brousseau,1978, 1986). We used a historical story and a historical game to propose situations that involved basic concepts of probability such as random events.

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Based on these points of view, the researchers opted for a theoretical-methodological perspective to develop activities for a school environment. The activities start with the presentation of a game through a historical account and involve probabilistic content that approximates daily activities carried out by students (spontaneously).

METHOD

As Chamorro (2005) points out, methodology based on TSD is one in which the child actively builds his own learning in collaboration with other people, including students and teachers. The teacher is part of the child's teaching–learning process but cannot provide ready-made knowledge. It is the child himself who, through the tools provided by the historical tale and guidance from the teacher, builds his learning. The teacher acts as a learning mediator during the students' engagement with the activity.

For Almouloud (2019), the TSD developed by Brousseau (1986) emphasizes social and historical dimensions in the acquisition of knowledge. The processes of knowledge acquisition are not viewed at the level of the subjects but at the level of the class. Knowledge acquisition must result from a process of adaptation of the subjects to the situations that the teacher has organized, and interactions with other students play an important role.

Several theoretical constructs have been developed in the context of TSD. In Figure 1, adapted from Almouloud (2019, p. 7), we present some of the constructs that underlie this theory.

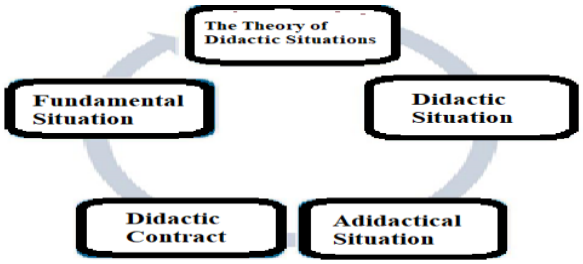

Figure 1. Some TSD constructs

The central object of TSD is the didactic situation that is defined according to Brousseau (1978) as the set of relationships established explicitly and/or implicitly between a student or group of students, a specific milieu (medium) that can contain instruments or objects, and an educational system for students to constitute knowledge.

According to Almouloud (2014), adidactic situations are an essential part of didactic situations, where the intention to teach is not revealed to the student, but is imagined, planned, and constructed (in the case of this work, the narrative of the historical game) to provide conditions favorable to the appropriation of new knowledge.

Brousseau (1996) recognizes four didactic situations: (a) an action situation dedicated to finding answers/results for a proposed problem without a need to explain theoretical arguments even if the theory was essential in the development of the activity; (b) a formulation situation consisting of the elaboration of statements supported by theoretical information for solutions to the proposed problem found by students; (c) a validation situation in which a proof mechanism is used to justify what has been stated through action and formulation, seeking to debate assertions and contesting or even rejecting propositions; and (d) an institutionalization situation that returns to didactics, that is, organization and synthesis of knowledge, elevating it to a status of knowledge that no longer depends on subjective and particular aspects.

We consider that students themselves are responsible for constructing their learning, that is, they must start from everyday situations familiar to them. Therefore, for the approach to be global, the methodology must be active and meaningful for students because students themselves must recognize the usefulness of the contents covered in their daily lives (Duch et al., 2001). Through active manipulation and experimentation, the teaching-learning process is carried out, and due to the young age of the children, we consider that it is the most appropriate way to carry out this process.

Codenames are used to preserve students' anonymity in presenting the results. Twenty-four Brazilian students participated in the research. These students were from a fourth-year elementary school

class in a municipal school in Santo André, São Paulo, and their codenames begin with the letter A and numbers: A1, A2, A3, ..., A24.

Before telling the story to students, we introduced the characters to the students so that they could identify themselves and, at the time of telling the story, they could create their own identity with these characters. The students were allowed to manipulate felt dolls made to represent the characters. We observed that students were curious about the story and began to interact with the characters.

According to the story, Dadito arrived at the house of twin girls Clara and Cora. He remembered the story of the Count of Buffon that his teacher had told in class, and he told it to his twin friends. We began with the presentation of the part of the story that dealt with aspects of the history of probability, and then we presented the felt puppet of the Count of Buffon.

According to the story of the Count of Buffon, a long time ago there lived a count who was a lawyer and a doctor who was very fond of the study of nature. He was known at the time as the Count of Buffon, at the court of Louis XV, King of France. The Count was appointed in 1733 as the founder of the King's Garden, today called the Garden of Plants located in Paris. It was said that one fine day while walking through the garden, the Count of Buffon suffered a fall and had to stand for a long time with his legs immobilized. While his legs were immobilized, to "pass the time," he observed the garden floor made of tiles and began to throw the rod with which he cleaned his pipe on the ground, noting that the rod sometimes crossed (or touched) the dividing lines, and sometimes not.

We told students that at that time the French liked games and that Count de Buffon created the tile game called "game de franc-carreau" that French children played a lot in the 18th century. The narration continued by demonstrating the game and explaining its rules, that is, a small coin was picked up and dropped randomly on a floor covered with tiles or square tiles. The game, according to Coutinho (2005), consists of throwing a coin on a square-shaped tile floor where the players bet on the final position for where the coin would fall. Thus, the tile would remain completely immobile on a single tile (position called “franc-carreau”) or over a joint between two tiles or multiple joints (not “franc-carreau”).

Remember that historically it was fashionable to place tiles on the floors of castles and gardens. In the story, after Dadito told the story of the Count of Buffon, he and the twins thought that these tiles could form a board and that they could easily build it. To replicate the setting in the classroom, boards consisting of black and white squares were built in some parts of the classroom for students to envision the tiled floors of the palaces of the Count of Buffon (Figure 2).

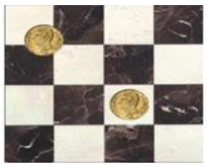

Figure 2. Board of the franc-carreau game indicating the possible outcomes of the random experiment to “flip the coin and observe where it will land”

After telling the story, we presented the following experiment: Toss the coin 20 times on the board and indicate the number of times it is franc-carreau (falling in the squares) and the number of times it is not franc-carreau (falling with some part on the edges of the squares).

We asked students to work in pairs to create a moment of interaction, but students were asked to identify their answers individually. We hoped to observe exchanges of knowledge between students as students searched for understanding of concepts that were not formalized. In the process, students should begin to consolidate their knowledge, that is, they should begin to form a group perception and the creation of motivation for the proposed activity while generating discussions to seek a solution to the problem.

Four boards were available in the classroom environment. We proposed that students carry out this random experiment—random because it is not possible in advance to determine if the coin will fall within the square or cross or touch the side of some square when tossed.

RESULTS

Table 1 displays students' recorded results for the coin tosses. The results were recorded for students to better understand the relationship between the number of times the coin fell within the square (franc-carreau) and the number of times the coin did not fall within the square (it was not franc-carreau) and to identify whether the 20 total coin tosses were recorded.

Table 1. Relationship between the number of times the coin fell within the square (franc-carreau) and the number of times the coin did not fall within the square (it was not franc-carreau)

n	Number of tosses that were franc-carreau (fell within the squares)	Number of tosses that were not franc-carreau (fell with some part on the edges of the squares)
1	16	4
2	18	2
3	15	5
4	15	5
5	16 times.	4 times.
6	11	9
7	17	3
8	[Did not answer]	[Did not answer]
9	17 times.	3
10	11 in the squares.	9
11	20 of moves gave franc-carreau.	None.
12	14	6
13	6	10
14	There were 20 times that gave franc-carreau.	There were 3 times that there was no franc-carreau.
15	3 times.	None.
16	14 times in franc-carreau.	6 times.
17	It was performed 12 times.	No - they were performed 8 times.
18	15 times.	5 times.
19	11 times.	11 times.
20	15. And yes, it fell inside the square.	5. And it was half and half.
21	16	5
22	20 times.	7
23	18	12
24	15	5

Engagement with the proposed activity provided a situation of action. Students dedicated themselves to finding answers to the proposed problems and acted in an operational way. We emphasize that we were not concerned with theoretical arguments even though theory was essential in the development of the activity.

Theoretical arguments start from the consideration that the coin was tossed 20 times. We expected that the sum of the number of tosses that were franc-carreau (Event 1) and those that were not franc-carreau (Event 2) compose the possibilities of the random experiment (the sample space—20 tosses, which we denote as S). In this case, these events can be considered mutually exclusive, that is, that the intersection between the sets is an empty set (Event 1 $\cap$ Event 2 = $\varnothing$) and that the union of the two events is the sample space itself (Event 1 $\cup$ Event 2 = S), Based on these concepts, we can consider that the events are complementary, that is, that "S - Event 1 = Event 2" or "S - Event 2 = Event 1."

Table 1 shows that 16 students, using different answers, were able to correctly answer the proposed question. We also highlight (in gray in Table 1) that some students incorrectly presented the individual events. Consequently, they did not apprehend the concepts that permeated the activity and that we highlighted in the previous paragraph. To reinforce students' knowledge or to develop

understanding of the proposed concepts for students who made mistakes, we next asked what was most likely to happen if students tossed a single coin on the board.

We identified correct responses from 23 students (95.83%) who participated in the activity. Only one of the students did not answer the question because he had to leave the classroom. In Table 1, we observed that only four students were unable to correctly answer the proposed question, the consequence of not performing the activity related to the experiment correctly. In this activity, students' responses indicated that there was a better perception of the proposed didactic concepts.

As a continuation of this activity, we also asked students to indicate what was most likely to happen after they threw the coin on the board. We indicate below the answers of 17 students that we consider correct.

A2: Most likely in the square.
A3: Fall further into the square.
A5: Most likely inside.
A6: Falling out of squares.
A9: It lands on some square.
A10: It was easier to get out, because my partner and I were far away [from the board].
A11: Falling into squares because they are bigger.
A12: Fall into the squares.
A13: Out.
A17: I thought I was going to fall in.
A18: To fall franc-carreau.
A19: Fall into the square.
A21: Dropped out.
A22: Falling into the square many times.
A23: Most likely to fall into the square.
A24: Drop in.

Regarding the students who we considered to have answered incorrectly: (a) two students indicated that it would be more likely that the coin would fall on the line or edge of the square; (b) two students indicated that it could be either in the square or in the lines that surround the squares, indicating that it would be 50% for each one; and (c) three students indicated answers that made it difficult to understand what they really wanted to express.

Here, we see that a formulation situation occurred. Students were able to elaborate ideas supported by the experiment according to the proposed problem. These statements were not characterized as justifications because students express them according to their interaction with the problem during the activity.

However, we can see that the activity brought an opportunity for students to express knowledge gained from the experiment and the provocation to think about which of the two events was more likely, i.e., where the coin would be more likely to fall. In addition, we started from the idea that the purpose of the activity was for students to indicate whether the coin would fall within the square or on the line that bounded the squares (within the board). Despite this indication, we consider the answers presented by students A13 and A21 to be correct, as they expanded the sample space, that is, they considered the possibility of the coin falling off the board and thus created a third event.

Finally, we emphasize that the students, for the most part, adequately answered the proposed questions, paid attention to what was presented, helped their classmates in the activities, and learned while playing.

DISCUSSION

The analysis of didactic situations through TSD provided favorable conditions for the researchers to develop, apply, monitor, and analyze an applied activity in which students were invited to build knowledge related to probabilistic content without direct interference from the researchers.

The researchers and students signed a didactic contract, the former as mediators and the latter as participants, committed to the appropriation of the knowledge proposed in the activities. For this, it was necessary that the study was attractive to the student and considered the students' reality.

We reinforce that the identification of errors was valuable, allowing the elaboration of new situations during the application of the activity, making the appropriation of knowledge more effective.

We note that it was possible to work on the probability content proposed by the National Common Curricular Base—BNCC (Ministério de Educación, 2018) in the early years of elementary school through a historical account and a game. It was noted that the students appropriated the developed history, perceived a historical fact, and managed to grasp the probabilistic concepts.

There is also a possibility that the same activities can be applied to students from a year prior to the one applied in this work, that is, the third year of elementary school, and even years after, bringing the analyzes and adjustments necessary for the appropriation of the concepts.

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