FACILITATORS' ORIENTATIONS TOWARDS LEARNING GOALS, PERCEIVED CHALLENGES, AND TEACHING RESOURCES FOR A PD COURSE ON CONDITIONAL PROBABILITY

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The data age has led to curriculum developments promoting the relevance of stochastics in mathematics education. This has resulted in an increased demand for teacher professional development (PD) courses, and thus in a rising number of facilitators for this purpose. Their expertise, however, remains under-researched, even though facilitators are the crucial factor in PD success. Our study aims at revealing how one aspect of facilitator expertise, orientation, is connected to other aspects: views on content goals, perceptions of possible challenges, and attitudes towards teaching resources. The explorations reveal differing orientations and learning goals with identical teaching resources for two facilitators who had also been working as a team for the re-design of the PD course for years.

INTRODUCTION

The relevance and accessibility of data are increasing in the modern and fast-evolving world. Thus, stochastics (statistics and probability calculation) has moved into the spotlight of mathematics education (Batanero et al., 2011) and led to several curriculum changes in Germany. Stochastics became mandatory, and this obligation forced teachers to cope with new and complex challenges in their teaching practice. Stochastics itself is challenging for various reasons (Burrill & Biehler, 2011): for example, when predicting future frequencies from probabilities, modeling must be taken seriously; the step from reality or real data to the world of mathematics involves an awareness of idealizations; and the necessary interpretation of results requires considering restrictions of the model. Therefore, the demand for teacher professional development (PD) for stochastics and qualification of facilitators is constantly growing, as facilitators are the agents of teacher PD.

In this paper, we focus on the part of a PD program that covers aspects of teaching conditional probability, connected to the concept of stochastic independence and Bayes' theorem. Some of the common mistakes, respectively misconceptions, of these topics include confusing condition and event, misinterpreting stochastic dependence as causality, and underestimating the relevance of the base rate P(A) for the calculation of the conditional probability P(A|B) (Bar-Hillel, 1983; Gigerenzer & Hoffrage, 1995; Kahneman & Tversky, 1973; McDowell & Jacobs, 2017). Teachers need substantial knowledge of common mistakes and misconceptions to choose appropriate teaching resources to address these challenges on the classroom level and to support students in reaching their respective learning goals. Facilitators should not only be experts at the classroom level but also at the PD level (e.g., Prediger, 2019; Prediger et al., 2019), where they are confronted with additional challenges while conducting teacher PDs. In addition to providing the necessary (pedagogical) content knowledge for teaching, facilitators must cope with participants' heterogeneity or pre-formed opinions on learning pathways and several other PD-specific aspects. Although facilitators and their expertise play an essential role for the impact and success of PD courses, there is still only little research on their orientation towards central learning goals, perceived challenges, and teaching resources (e.g., Prediger et al., 2019).

THEORETICAL CONSIDERATIONS

To illuminate the facilitator expertise necessary for leading a teacher PD, we draw on a framework for facilitator expertise (Prediger, 2019, adapting Bromme, 1992) that distinguishes among jobs, pedagogical tools, categories, and orientations. This framework helps to structure and describe what kind of challenges facilitators face during a specific situation (jobs), which thinking categories they activate, which kind of pedagogical tools they utilize for reaching their goals, and which orientations influence their choices. The thinking categories include several content-specific aspects ranging from the procedural and conceptual learning goals, the possible learning pathways, and the learning obstacles. The framework for facilitator expertise was aligned with analyzing actual teaching and learning situations, including the prepared recourses and decisions in the learning material. We are using this

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framework to investigate content goals and the pedagogical tools retrospectively during an interview immediately after the second day of the PD program.

Some of the underlying content goals of the second day of PD are to deliver fundamental knowledge of the definition for conditional probability, understanding the sense of this definition for calculating conditional probabilities (via the definition or by using Bayes' rule or other strategies) in a way suitable for learners. Additionally, the PD program supports apprehending the importance of the base rate P(A) for calculating the conditional probability P(A|B) and for determining probabilities with Bayes' rule. Misconceptions such as confusing condition and event, misinterpreting stochastic dependence as causality, or underestimating the relevance of the base rate are typical learning obstacles teachers are confronted with during their lessons. A combination of natural frequencies and an easily comprehensible representation might support learners in overcoming this learning obstacle and can foster their concept building (Gigerenzer, 2011; Wassner et al., 2007). An additional goal of the PD is to promote new and innovative teaching resources, such as the use of (ideal) simulations to obtain natural frequencies and to encourage active engagement with the data presented, which has been shown to "have the potential to make learning statistics easier" (Lane & Peres, 2006, p. 5). Simulations are introduced in the PD course as an adequate tool for calculating probabilities when learners' analytic means are insufficient and for strengthening the frequentist view on probability.

Overall, teachers need pedagogical tools to support them in introducing conditional probability and stochastic independence by utilizing appropriate tasks, activities, simulations, and visualizations, as well as considering misconceptions. These options, integrated within the PD course, share the characteristic that they intend to change participants' views on the importance of conceptual understanding rather than procedural skills in lessons. Some of them are the reflected use of absolute (natural) or relative frequencies, traditional tree diagrams with probabilities, (double) tree diagrams with absolute frequencies, 2 x 2 tables, linguistic scaffolding, hands-on experiments, digital simulations, data from digital simulations, ideal simulation (gaining natural frequencies by using artificial population sizes), authentic problems, and problems with artificial stories (see Binder et al., 2020, for an explanation of the different tools for visualization).

Participating teachers usually have pre-formed opinions about certain teaching approaches (e.g., simulations) or previous (positive or negative) experiences when teaching the same or similar content imparted in the course. They also have heterogeneous knowledge of the content and the introduced tools. Different concerns, misgivings, and opinions of the participants can be utilized as a pedagogical tool to enrich group discussions and to reflect upon the teaching options, led by the facilitator who can incorporate his or her own experience with the material into the discussion. Appropriately addressing the heterogeneity of the participants (also referring to their individual knowledge or their respective professional learning groups) and the resulting needs are other obstacles of which facilitators must be aware.

The content goals and the teaching resources can be connected in various ways with facilitators' more general (and less content-specific) orientations. We use them to reveal the underlying reasons for facilitators' interpretation and reaction to specific challenges. For example, an orientation to actively address misconceptions, such as a missing awareness of the impact of base rates, influences the choice of a PD activity or a teaching resource, such as using (ideal) simulations and double tree diagrams. Pursuing the goal to promote the use of simulations implies addressing their advantages in the PD by pointing out their usefulness for students and for concept building in activities or group discussions. While working as a team and undergoing the same qualification program, content goals, teaching resources, and orientations can still differ between two facilitators. We are interested in these differences and want to learn more about facilitators' views on these aspects. So, we aim to answer the following questions:

- RQ1: Which content goals and teaching resources are mentioned by the facilitators and how are they connected to each other?
- RQ2: What similarities and differences regarding the accentuated goals and teaching resources exist among the facilitator team?
- RQ3: What kind of orientations can be identified among the facilitator team?

We intend to carefully generalize our findings and identify potential facilitator types. Considering facilitators' views on PD courses can support the development and improvement of qualification programs by addressing their requirements.

CONTEXT OF THE PD COURSE AND COOPERATION

A group of experienced facilitators from a regional education administration cooperated with PD course designers on the re-development of an existing five-day course for stochastics at the upper secondary level in Germany over a period of three years. The revision was influenced by the experience of previous implementations, facilitators' expertise in conducting PD courses, their work as teachers, and new scientific results. The fundamental idea of the course was to promote concept formation via the use of simulations, digital tools, authentic examples, and real applications (Barzel & Biehler, 2017). In this paper, we concentrate on the day of PD on conditional probability, stochastic independence, and Bayes' theorem, with a focus on the use of natural frequencies, for example in simulations, and their use in double tree diagrams, which are regarded as an innovation in the German school context. A more traditional form of representation, 2 x 2 tables, is mentioned along the way. After undergoing several design-based research cycles, the facilitators moderated the program more than once in teams of two.

METHODOLOGY

A facilitator team consisting of "Mike" and "Will" (aliases) is in the spotlight of this paper. They are both male and well-experienced teachers with more than 16 years of service each. In addition, they have conducted PD programs, primarily for content other than stochastics, for more than 13 years.

Immediately after each day of PD, the facilitators were interviewed separately, following the same guideline consisting of questions regarding, for example, the PD learning goals (both from the facilitator's and the course designers' view), possible learning obstacles and how to overcome them, and the teaching resources offered by the PD course for the classroom level. The conversations were audio-recorded and later transcribed. The interviews were conducted in German and not translated into English for the analysis to avoid misinterpreting specific aspects during the translation process. Thus, only a few direct citations are included in the results. Mike's and Will's interviews lasted for around 70 minutes and had turns ranging in length between short comments of very few words and extensive elaborations of over 450 words. A deductive qualitative content analysis was conducted (Mayring, 2015) in order to dissect the relevant text passages and find answers to the research questions. A content-specific rating manual was used for the analysis, based on the intentions and fundamental ideas of the PD course. Emerging contradictory interpretations of facilitator statements were discussed among the authors until a consensus was found.

RESULTS

Facilitator Mike

During the interview, Mike reveals that promoting students' understanding and argumentation skills is an indication for better teaching (Mike's interview turns M_010, M_012, M_022), and thus he emphasizes the importance of this aspect for the PD (M_004, M_010, M_014, M_018, M_064). Mike connects the advancement of understanding in lessons with teachers' awareness of common misconceptions (M_010, M_018, M_111, M_121, M_127, M_165, M_171, M_175). "Students show many misconceptions, even with everyday relevance" (M_010), which he thinks are also present among the PD participants (M_107, M_167). Therefore, addressing misconceptions is just as necessary for the classroom as it is essential for the PD. Discovering ways of coping with typical misconceptions is a highly important PD goal for Mike (M_010, M_018).

The use of simulations, on the other hand, triggers questions about their usefulness for Mike (M_109, M_143). He is more interested in the results and not if the solution comes from a simulation or from a calculation (M_018). Mike primarily refers to hands-on experiments (M_014, M_024), thus indicating a certain reserve towards digital simulations. For Mike, digital simulations, which always require predetermining the number of overall experiments, can serve two purposes: (1) getting a grip on a complex context where the probabilistic model is unclear (M_018, M_145) and (2) creating data which can be used in visualizations such as tree diagrams or 2 x 2 tables (M_143). Mike immediately relativizes the second purpose because he sees no advantage over calculating the necessary probability via a 2 x 2 table with natural frequencies, unaware that these can indeed represent ideal simulations (M_143-145).

From Mike's point of view, another aim of a PD course is to present a range of teaching options (often in the form of tasks, M_052) teachers can choose from and not to present an ideal teaching approach that works perfectly in every classroom setting (M_127). Consequently, Mike refers most of the time to teaching resources for the classroom level: specific tasks and context that are authentic offer

different possibilities of use, and address common mistakes (M_010, M_012, M_127, M_128, M_145, M_157, M_173). Other pedagogical tools are visualizations such as double tree diagrams (M_022, M_149, M_151) and 2 x 2 tables (M_121, M_149, M_151). Both help students to connect absolute and relative frequencies, to bridge the transition from a tree diagram to the reversed tree diagram, and to experience the relevance of the base rate (M_145, M_147). Although the PD course emphasizes the use and advantages of double tree diagrams, Mike is still in favor of 2 x 2 tables (M_149) and would use double tree diagrams only subsequently (M_151).

Facilitator Will

Will, at the same time, focuses on different goals and teaching resources compared to Mike, even though the two have worked as a facilitator team for quite some time and were confronted with the same questions in the interview, covering the same PD material and the same day of PD. Promoting participants' content knowledge and understanding of central ideas are among Will's most important aims (Will's interview turns W_002-004, W_022, W_024). Furthermore, he intends to foster teachers' concept understanding regarding the connection between probability, expected value, and mean (W_014) or between statistics and probability calculations in general (W_012, W_022, W_268). According to Will, the PD course and its material support his goals and explicate these ideas (W_066, W_156; W_204-210, W_268). While fostering understanding and introducing (authentic) tasks, he sees participants' typical misconceptions as an additional element to deal with (W_022, W_026). From Will's point of view, participants are experienced with misconceptions or in general more reserved than students, when conclusions are concerned (W_332). Nevertheless, knowing the consequences of mixing condition and event is vital for hypothesis testing (W_326) and Will acknowledges the PD's effort to address typical mistakes and misconceptions (W_314).

For Will, simulations are a valuable pedagogical tool for lessons to develop ideas and to foster understanding of central ideas (W_008, W_024, W_058, W_068). Simulations and the resulting visualizations even allow for connecting conditional probability with, for example, the $1/\sqrt{n}$-law (W_060). Even though developing or using simulations is not really an issue during the second day of PD, delivering knowledge about simulations and illustrating them as a tool for the classroom is another important goal for Will (W_008, W_022). He lays the emphasis on demonstrating advantages and possible usage options during the PD, which he believes is sufficient for providing the necessary knowledge (W_039-042, W_226, W_282). Still, including more simulations in further PDs would be appreciated by Will due to their general advantages (W_042, W_048, W_114, W_148, W_150). Will seems to be well experienced in using simulations and has several ideas on how to implement them during school lessons—including which kind and sequence can be used beneficially (real-life experiments, GeoGebra, graphic calculator, other ready-to-use simulation programs from the Internet, W_042, W_058, W_060, W_110).

Presenting a range of teaching options as worthwhile, either for fostering concept understanding or for using simulations, is also an additional and relevant aim of PD for Will (W_006, W_270, W_282). This is actually the most significant advantage of the PD course for Will ("Many [teachers] leave [the PD course] with many new ideas, ideas for context, and examples, which can be used advantageously when planning lessons. I think this is the biggest advantage [of the PD course]", W_268). Here, Will focuses on the teachers and their jobs. First and foremost, he describes the teaching options as a tool for promoting participants' understanding, for introducing a new concept, and for connections among different topics (W_024, W_028, W_268). Nevertheless, including more and extended tasks that can be easily integrated into lesson plans is one of Will's ideas for improving the PD course on expected value (W_090-102, W_132). Thus, Will has taken into account not only that the teachers are participating in the PD to educate themselves, but also that they need to plan lessons.

Comparison Between Mike and Will

A comparison between Mike's and Will's individual results reveals several differences between their views on the PD goals, learning obstacles, and teaching resources. First of all, addressing misconceptions takes a different role for the two facilitators. Mike emphasizes fostering the awareness of common misconceptions. In contrast, Will addresses misconceptions of the participants only as typical learning obstacles he has to cope with and focuses more on teachers' conceptual understanding. For him, simulations are an adequate tool to promote conceptual understanding, and consistently he

wishes for more simulation activities during the PD. Mike is more reserved towards simulations and questions the usefulness and benefits of this tool. Both see advantages of the PD in providing a variety of appropriate tasks, context, and learning pathways. However, Mike and Will focus on different aspects. Whereas Mike thinks about improvements for students' learning pathways, Will has the participants' understanding and their jobs as a teacher in mind.

Additional similarities and differences can be identified in the data. Mike and Will are convinced that teaching options can be presented and discussed during the PD course, and that choosing and adapting the adequate task or method for a specific class and group of learners must be done by participants individually (M_107, M_127, W_296, W_300). The participants are often addressed as "colleagues" interested in the context and provided with material by Mike and Will (e.g., M_024, M_105, W_056, W_350). Mike sees himself teaching, not qualifying the PD participants (M_141). In contrast, Will is more interested in qualifying PD participants for planning lessons by providing good examples and necessary knowledge (W_268, W_282-284, W_300). He even considers himself as a mathematician; therefore, he has a clear focus on content knowledge, which he stresses is necessary for teaching (W_078, W_084), whereas Mike is more interested in the underlying didactics and misconceptions. Mike often refers to the decisions he has made or would make for his own teaching (M_097, M_145). Mike provides advice from his own experience and primarily focuses on advantages for the students (e.g., M_022, M_123). While reflecting on the PD course, Will's emphasis is on the participants and their needs for understanding the content and concepts or for overcoming misconceptions (e.g., Will_32, Will_84, Will_340).

CONCLUSION

Mike and Will reveal similarities regarding the general design of PD courses. They emphasize that teachers should be presented with a range of tasks and activities to choose from. An adaption of the material towards a specific classroom situation is necessary in their opinion, but neither of them addresses the necessary skills for this adaptation process during their reflections on the PD course. Mike and Will assume that the participants are capable of doing so on the basis of their previous experience as lesson planners. Still, the inputs and materials presented during PD should be easily adaptable for the classroom.

Even if Mike and Will were equally qualified during the cooperation and reflection on the same PD course, differences between their intended goals, perceived challenges, and use of teaching resources become apparent. Mike switches to reflections on the PD course and his work as a facilitator in the later parts of the interview (M_127 onwards) but retains his focus on his own suggested teaching and on teacher professional development only indirectly via the intended student learning. All in all, it becomes evident that Mike seems to focus on the improvements of student knowledge and skills (M_026, M_125), thus revealing a strong classroom-orientation. He sees the primary purpose of PD in teachers discussing and reflecting on concrete teaching situations, himself as primus inter pares—albeit acknowledging parallels between the PD course and a mathematics lesson. In these situations, Mike's expertise is based on his own suggested teaching and reveals an orientation towards teaching strategies that yielded satisfying learning results in the past, such as preferring 2 x 2 tables over double tree diagrams. An adequate qualification program for classroom-orientated facilitators who rely on their experience should include segments to familiarize themselves with the PD material. If these opportunities are missing, uncertainties in handling and implementing simulations in the classroom do not become clear, which may result in a more reserved recommendation for including simulations in lessons.

In comparison to Mike, Will has a more content-orientated view on the purpose of PD courses, which explains his choice and connection of goals, learning obstacles, and pedagogical tools. Fostering an understanding of central stochastic ideas and creating connections between the different topics is the key element of this PD program for Will and illustrates his expertise, which focuses on promoting participants' competencies and planning capabilities. Thus, his addressed learning goals are situated on the PD level, just as the various teaching tools which had originally been designed for the classroom level. A qualification can benefit facilitators like Will if it includes more elements focusing on the purpose of the PD course for classroom situations. This would support facilitators in recognizing the importance of actively addressing misconceptions in the PD not just as an additional learning obstacle but as a decisive step to foster conceptual understanding among participants and students.

Content-orientated and classroom-orientated facilitators can profit from possible synergies resulting from their divergent orientations and thinking categories. Via their strengths and interests, these types of facilitators can complement each other. They attach importance likewise to different aspects during the PD, which can benefit all participants and can result in an overall successful PD (at least from the facilitators' point of view, e.g., M_24, W_34-42). However, it is uncertain that these two facilitator types will always work together as a team. Therefore, certain aspects of the PD course ideas need to be conveyed successfully by each individual facilitator, to a certain extent. Otherwise, central PD ideas might be lost during an implementation. It remains to be researched if the facilitator types we have identified can be supported by Mike's and Will's actions during the PD course. Other facilitator teams might even reveal different aspects and thus paint a more differentiated picture of facilitators' views and types.

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