A FRAMEWORK FOR CAPTURING THE DEVELOPMENT OF STATISTICAL CONCEPTS

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Statistics education researchers have been challenged to consider the theory of inferentialism in understanding concept formation in students. A critique of inferentialism is that no method has been formulated to use the theory in practice. In this paper a Statistical Reasoning and Action framework is presented that appears to be capable of explicating the development of statistical concepts during learning. By following six 11-year-old's learning over several statistical modelling cycles using TinkerPlots, the framework was used to capture their interrogative cycles of noticing and wondering, giving and asking for reasons, and sanctioning and censuring, as well as oscillations between concretising language about actions and conceptualising language towards concept formation. Two teaching episodes at the beginning of a learning sequence are used to illustrate its use.

INTRODUCTION

Inferentialists understand learning takes place within individuals as they form and use a *web of reasons* that reflect both conceptual understanding and the mastering of a practice (Bakker & Derry, 2011; Brandom, 2000; Noorloos et al., 2017). Noorloos et al. (2017) argue that the theory of inferentialism (Brandom, 2000) may provide a way forward in understanding novices' concept formation. Bakker and Derry (2011) challenged statistics education researchers to consider inferentialism as a theoretical resource for epistemological reflection on student learning. A central tenet of inferentialism is the role language plays in one's social ability *to give* and *ask for reasons*. Brandom (2000) viewed these exchanges as a human desire for truth seeking that facilitated knowledge building, whereby concepts are created and used in terms of their reasoned and inferred connections. Inferentialism "explains the nature of language in terms of its role in reasoning where the meaning of words is explained in terms of their use in social practices" (Noorloos et al., 2017, p. 446). Claims or commitments require supporting reasoning and "what counts as valid reasoning, adequate judgment, or correct application of concepts depends on the norms being used in a particular practice" (Bakker & Derry, 2011, p. 12). For education in schools, the social norms of the classroom dictate what is inferred or questioned between individuals and hence inferentialism may explain the formation of concepts "in terms of the inferences that individuals make in the context of an intersubjective practice of acknowledging, attributing, and challenging one another's commitments" (Noorloos et al., 2017, p. 437). Furthermore, "for inferentialism, when students learn a concept, this means that they are learning how to use that concept in making inferences. They are mastering that part of the space of reasons in which the concept is embedded. These reasons and this space are not so much constructed as they are encountered and navigated in the course of classroom activity" (Noorloos et al., 2017, p. 450).

Radford's (2017) critique of inferentialism is that practical research methods have yet to be formulated to use the theory productively in education research practice. Currently there is no *applied* framework based on inferentialism that can examine students' conceptual development through analysis of their language, actions, and norms. Therefore, the development of a pragmatic framework could illuminate how students' statistical concepts and actions emerge and are employed when they are learning within a statistical modelling environment.

THE STATISTICAL REASONING AND ACTION FRAMEWORK

As part of a larger study that examined the *learning processes* involved in statistical modelling, the Statistical Reasoning and Action (SRA) framework was developed to explain and interpret *what* statistical concepts students were encountering and navigating when modelling, and to illustrate *how* this occurred. To elucidate how students might encounter and navigate concepts during modelling, inferential cycles (Figure 1) were constructed that drew on three methodological sources: (a) the theory of inferentialism (Brandom, 2000); (b) the pedagogical practice of noticing and wondering (Shaughnessy, 1997); and (c) the idea that reasoning coordinates knowledge and actions (Heusdens et al., 2018). The first and main source for the framework was the theory of inferentialism whereby the

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giving and asking for reasons encourages a response perhaps to have others agree (sanction), disagree (censure), or ask further questions, either to clarify or provide alternative reasoning. Speakers may reach a shared meaning or understanding, where agreed upon inferences become an accepted "truth" or *new* knowledge between discussants. The impetus for reasoning, however, depends on what students are perceiving in the statistics world. For example, students may notice characteristics of displays and then wonder about their meaning. Therefore, the pedagogical practice of *noticing* and *wondering* (Shaughnessy, 1997), which assists students to explain what they see and think, seemed to be an important second source to include in the framework. The third source was derived from the work of Heusdens et al. (2018), who, in their research in a vocational setting, fused an action-based theory of learning with inferentialism to demonstrate a continuous oscillation between actions and conceptual development. They theorised two sensitising language constructs from their data: (a) conceptualising, which is characterised as articulations about general concepts that promoted a movement toward understanding; and (b) concretising, which is characterised as articulations interpreting the situation about what is happening or what should be done that encouraged a movement towards action. Combined, these language constructs gave meaning to the situation or task and were used as evidence that concepts and actions were tied together during learning.

A conjecture is that the combination of the three methodologies will enable an examination of aspects of students' reasoning, actions, and norms, which will offer a comprehensive view of how students might develop their conceptual infrastructure. From these three sources, the SRA framework (Figure 1) was developed to explain how concept formation may take place when students are immersed in a statistical modelling environment.

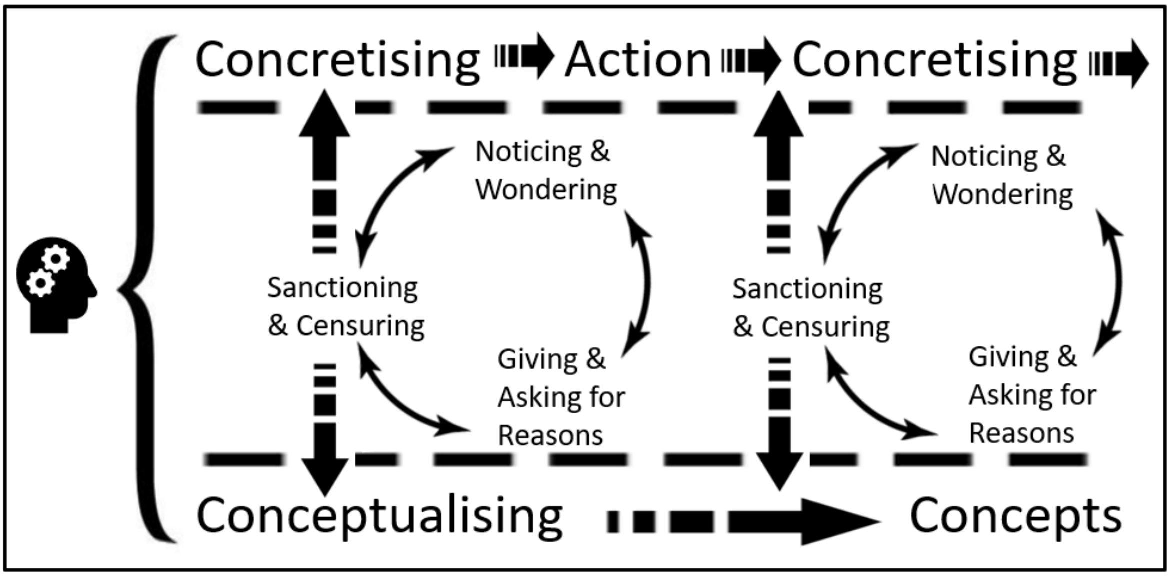

Figure 1. Inferential cycles towards concept formation (Patel, 2022, p. 75)

BACKGROUND TO RESEARCH AND PARTICIPANTS

This paper is set within a wider two-year study examining six students' and a teacher-researcher's actions, discourse, and norms (Patel, 2022; Patel & Pfannkuch, 2018). Design research (Bakker, 2018) was used to develop a learning sequence for 11-year-old novice students to model natural phenomena using the Sampler in *TinkerPlots* (Konold & Miller, 2015). The students had no prior experience of *TinkerPlots* or distributional ideas. They were considered to be high ability in their mid-socio-economic school. Student data (audio, screen captures, and artefacts) was micro-analysed to capture their interactions between each other and with the technology during a series of modelling tasks over 12 two-hour lessons. The SRA framework was developed using a grounded theory approach (Willig, 2013). It evolved from the back-and-forth dialogue and actions of these novice students between themselves and with the teacher-researcher as they engaged in building chance-based models to investigate a situation and was trialled using different modelling contexts.

ILLUSTRATIONS OF THE USE OF THE SRA FRAMEWORK

Using Konold and Kazak's (2008) context of repeated measures to initially explore variation in data, each member of the class of the six student participants in the study measured the *circumference* of a Moon Hopper with a measuring tape and the *diameter* of the Moon Hopper handle with electronic callipers. They recorded their measurements on Post-it notes. The researcher asked two students to place the Post–it notes of the Moon Hooper circumference in order, according to the measurement scale drawn on a whiteboard (Figure 2).

Figure 2. Arranging repeated measures of the Moon Hopper circumference

To illustrate the use of the SRA framework for how concepts may be seeded and take root, two episodes are given involving the Moon Hopper measurements. The first episode took place in the class of the six students, whereas the second episode presents a dialogue between the six students and the researcher, the first author.

First Episode

The students noticed that one of the Moon Hooper circumference measurements was extremely different from the others (Noticing). The researcher asked, "What's going on here?" (Asking for reasons). The students suggested that the measurement was an error or "mistake" (Giving a reason, Conceptualising), because the circumference of the Moon Hopper could not be 80 cm, as they could see all the other measures were between 112 cm and 115 cm (Giving a reason). The researcher used the language "dirty data" to describe the measurement error and asked, "Should we try and fix it, or discard it?" (Concretising language). Students in the class responded that it should be discarded, and the researcher removed the Post–it note from the board and tore it up (Action). This episode illustrates the inferential connections students may have made between concepts, such as measurement error, signal in data (central tendency), distance from signal, and error judgements. By using language and related actions and judgements for cleaning data, the "new" concept of "cleaning data" may be strengthened. The inferential concept formation cycle illustrated in Figure 1 is continued when the researcher asked the students, "What do you think the real circumference of the Moon Hopper is?" (Wondering). This question focused students' attention on features of the shape or centre of the distribution of measures shown by the Post–it notes (Noticing). The students agreed that the centre of the data, approximately 113.5 cm, was likely to be the "real" circumference according to the measurement data (Giving a reason and Conceptualising). Their reasoning was sanctioned by the researcher, and in this way an interrogative process between concretising (language toward actions) and conceptualising (language about concepts) is set in motion.

Second Episode

In preparation for building their first chance-based model in *TinkerPlots*, the six students in the study created plots of the Moon Hopper measures (Figure 3) and then were shown how to apply the average tools to represent the centre of the data. The mean is depicted by a blue triangle.

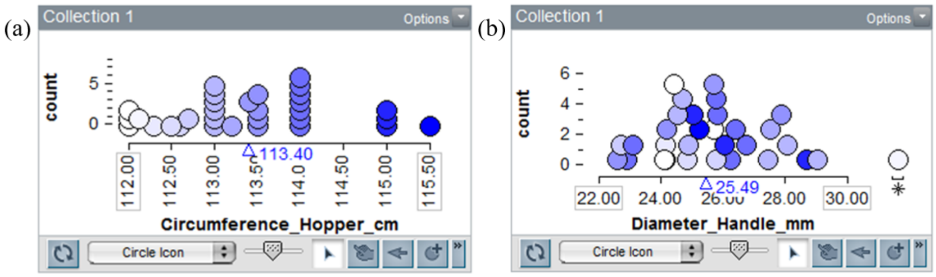

Figure 3. Plots of data sets in TinkerPlots: (a) Circumference of the Moon Hopper (80 cm measurement removed) (b) Diameter of Moon Hopper handle

After visualising plots of both data sets, the researcher began a discussion with the students to elicit possible *causes* for the variation in the distributions of measurements, similar to the repeated measures task in Konold and Kazak (2008). In line with Pratt's (2011) contention that students' intuitive causal reasoning should be harnessed in statistical modelling, the researcher asked the students to examine the distributions and think of causes that might underpin the variation observed.

R Can you comment on the shape of the graphs? [Wondering.]

Dan Pyramid. [Noticing and Conceptualising the shape of the data.]

R Are both plots similar in shape? [Wondering and Conceptualising shape.]

Mary Yeah, oh kind of. [Conceptualising shape.]

Dan There is also a missing gap after 114 (cm). [Noticing a gap in the Circumference data.]

R Can you suggest a reason why the handle (data) looks more like a pyramid or a triangle and the Hopper (circumference) has gaps? [Asking for a reason.]

Dan Cos, some people with the handle, sometimes they might have squeezed it, so you would get different results for that, and with the Hopper you could have it (measuring tape) lower or higher on the Hopper. So, it could be out of place. [Giving a reason and beginning to Conceptualise causes of variation.]

Nico Because it (tape) could be out of place. Because the Hopper is bigger, bigger data variation. [Sanctioning Dan's reasoning and conceptualising causes of variation.]

R What do you think is causing the bigger variation? [Asking for a reason.]

Ali The size of it, so there is more to measure. [Conceptualising cause.]

Leo Yeah, more room for error than that little device thing (electronic callipers) we had, you just stuck it on. Lots of different variables that could happen. [Sanctioning, Giving a reason and Conceptualising causes of error.]

Ali If they measure the moon hopper it (measuring tape) might have been down here or up there. [Giving a reason.]

R Right, so the angle might have been different. [Sanctioning and conceptualising a cause of variation.]

Ali Probably how tight they hold the tape around the hopper. [Giving a reason.]

Nico If you squeeze it, it might be crooked. [Giving a reason and concretising.]

Dan Yeah, angle means position. [Sanctioning, Conceptualising defining a cause of variation.]

The researcher clarified that position and angle could be thought of as two different causes; position was vertical displacement from circumference, whereas angle referred to tilt of measuring tape. She wrote on the board the possible causes of variation suggested by the students: position, angle, pressure, starting point. This action served to sanction and further conceptualise the students' reasoning about causes of variation. To prompt students to think about the numerical size of the factors causing the variation, the researcher then steered the discussion to possible values or sizes for each of the causal factors. The students then built their first chance-based model in *TinkerPlots*. In other words, their ideas became physical tools in an additive model that could be modified, trialled, tested, and critiqued.

Summary

In the two examples given, there is no example of censuring, however, in the process of building their chance-based "error" model of the situation, many instances of subtle and not-so-subtle censuring were observed between the students as they shared their reasoning, such as Ali stating, after noticing Leo attempting to calculate the range in the plot, "You can find out what it (the range) is exactly." The social norms that governed sanctioning and censuring also meant there was self-censuring, for example, as they built the chance spinners representing the variation, Leo said, "(We'll) do yours, as mine's not very accurate, I did millimetres and it's meant to be centimetres."

Note that the researcher did not explicitly ask for causes for the variation, rather she asked students to reason about the differences in the two distributions. The students intuitively drew on their prior experiences or actions measuring the Moon Hopper and offered valid contextualised reasons for the *causes of variation* in the data. These conversations seemed to elicit many connections between concepts that inferentialists term *webbing*. The reasons given were sanctioned in order to arrive at a shared understanding about several possible causes of variation. The reasoning was sanctioned, either by agreement or repetition, or expanding on the reasoning towards shared definitions and more robust inferences. The students used the measurement context and physical characteristics of the measurement tools and the Moon Hopper to *explain the variation in the distributions*, a key stage in the statistical modelling process.

Concepts, therefore, appear to form and grow through the interplay between concretising, which may or may not lead to actions (top broken arrow in Figure 1) and conceptualising, which may or may not lead to concept formation (bottom broken arrow in Figure 1). The interplay is an interrogative oscillating process that may strengthen webbing between concepts through noticing and wondering, the giving and asking for reasons, and the sanctioning and censuring of reasons. Thus, a conjecture is that the inferential cycles may provide a way of understanding how students' conceptual infrastructure may be formed.

CONCLUSION AND DISCUSSION

The Statistical Reasoning and Action (SRA) framework that was developed involved interrogative cycles of noticing and wondering (Shaughnessy, 1997), giving and asking for reasons, and sanctioning and censuring (Brandom, 2000), as well as oscillations between concretising language about actions and conceptualising language towards concept formation (Heusdens et al., 2018). As illustrated in the two episodes, the concepts encountered were navigated within interconnected interrogative cycles, that involved the students: acting on displays; drawing on contextual aspects of the task scenario to make sense of them; comparing them; and participating in an interrogative oscillating process between concretising and conceptualising that started to sow the seeds for concept formation. As Konold and Kazak (2008) stated, "seeing something new" is "synonymous with learning a new conception" and "what we know highly influences what we can see; what we see drives what we can come to know" (p. 30).

By applying the framework to the data collected in the larger study, it became apparent that the act of modelling is regulated by individuals shared reasoning and actions. What participants noticed and attended to are both in-the-moment and over time and influenced inferences, and therefore which concepts were strengthened, and which faded. Using the SRA framework as an analysis tool enhanced the researcher's awareness of the teacher's role in inducting and promoting the students' interrogative interplay of noticing and wondering, the giving and asking for reasons, and sanctioning and censuring. This interplay might encourage students to learn that articulations about their models are an inherently normative practice. Concept formation, however, appeared dependent on the authority of the researcher and what notions she may have privileged.

Nilsson et al. (2018, p. 460) identified that current frameworks in statistics education required a "stronger theoretical basis or treatment" of knowledge construction, and that frameworks needed to be grounded in learning theory. Furthermore, Bakker and Derry (2011) and Bakker et al. (2017) suggested inferentialism as potential theoretical resource for learning about how conceptual infrastructure might be formed. The SRA framework, based on inferentialism, seemed to have the capability of identifying students' emerging statistical concepts and examining how individuals coordinate their unique conceptions and actions through strengthening inferential webbing. As a result of creating and using the SRA framework to explicate students' reasoning over time, the findings could affirm Radford's (2017) view that the reformulation of inferentialism could offer "a fresh perspective on knowledge, concept formation, and learning that privileges inferential thinking" as well as offering "the ability to think in new ways about the question of task design and pedagogical action" (p. 505).

The SRA framework was trialled on data gathered from a small group of students over 12 two-hour lessons in a two-year period. The exploratory nature of the study means that the framework is in its infancy and needs to be extensively used in future research to understand whether inferentialism theory can indeed be used in practice.

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