PROMOTING CRITICAL CITIZENSHIP IN PROFESSIONAL DEVELOPMENT OF MATHEMATICS TEACHERS VIA STATISTICAL INVESTIGATIONS

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This proposal studies statistical investigations as tools to promote statistical thinking and critical citizenship in the process of professional development for mathematics teachers. A qualitative methodology with a critical dialectical approach was followed. The information was produced within a professional development for in-service teachers from a public school in the northwest of Colombia. A statistical investigation that addressed CO_2 emissions was implemented, and teachers of different teaching levels explored real data from the World Bank by using CODAP software. The units of analysis were the interactions of the teachers with their colleagues during the professional development. The main results exhibit evidence of statistical thinking and critical citizenship in mathematics teachers during the course of the empirical inquiry.

INTRODUCTION

Today's society is characterized by the availability of statistical information in multiple scenarios that everyday citizens must understand, use, and question for participation in their cultural and social environments. Nowadays, there are available indicators on environmental, social, and economic crises (in the sense understood by Skovsmose, 1999) such as global warming, poverty, and public health that should be understood by any citizen to respond to the challenges of society. The school environment has missed these critical scenarios as possibilities to promote students' statistical thinking and critical citizenship skills (Zapata-Cardona & Marrugo, 2019). For these scenarios to be taken advantage of, those who teach statistics need to be exposed to different teaching experiences in which they can explore statistics as a tool to empirically study and understand social crises while using the results to propose solutions. If teachers have experiences that challenge their statistical thinking and critical citizenship, they will be much more prepared, empowered, and confident to stimulate these skills in their students.

This proposal, framed in a qualitative paradigm with a critical-dialectical approach, uses the design of a statistical investigation based on a current and authentic context related to the emission of CO_2 for the professional development of in-service mathematics teachers who teach statistics. The purpose of the proposal is to show evidence of teachers' statistical thinking and critical citizenship when a statistical investigation is used as tool for professional development.

THEORETICAL FRAMEWORK

Statistical thinking has a close relation to the statistical problem-solving process (Bargagliotti et al., 2020). This involves making sense of data; understanding and explaining statistical processes; making interpretations based on data sets, graphical representations, and statistical summaries; creating, assessing, and using statistical models; and generating conclusions and inferences from the statistical process (Garfield, 2002; Garfield & Ben-Zvi, 2008) or making evidence-base decisions (Bargagliotti, et al., 2020). Much statistical thinking combines data and chance ideas with abilities such as explanation, inference, verification, and demonstration. Statistical thinking refers to the way people act and think during the process of empirical research (Campos, 2016), and it involves abilities such as generating questions, producing data, identifying and controlling variation, finding ways of representing data, and establishing relations and patterns. Statistical thinking is a complex multidimensional process that integrates the mastery of process and procedures as well as the construction and use of models to infer; this does not take place in isolation but within relevant contexts.

Critical citizenship is a kind of empowerment that allows subjects to challenge current structures and oppressive actions (Gutstein, 2006). The purpose of critical citizenship is to educate

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politically and socially aware individuals who can use statistics to build democracy and transform society (Campos, 2016; Martínez Castro, 2020; Weiland, 2019). A critical citizen is a sensitive person capable of thinking, understanding, researching, taking risks, revealing inequities in society, and contributing with their actions to a more just and equitable world (Giroux, 1997). According to Skovsmose (1999), being critical means focusing on a *critical situation,* identifying it, capturing it, understanding it, and reacting to it in order to transform it. Critical situations are tense scenarios—conflicts, dilemmas—that generate social inequalities, repression, or environmental imbalances such as poverty, malnutrition, garbage production, or unequal opportunities. Educating for critical citizenship consists of stimulating thought functions in people to act, understand, and transform their social, political, economic, or environmental reality that contribute to the creation of more democratic conditions for society (Zapata-Cardona, 2018).

METHOD

This study followed a qualitative paradigm with a critical-dialectical approach (Sánchez Gamboa, 1998) that explores the use of statistical investigations to understand and transform crucial contexts and to educate critical citizens. The critical-dialectical approaches have an emancipatory interest and conceive the participants as active subjects that transform themselves or their own reality. This approach seeks for the research process itself to help participants express contradictions, conflicts, and antagonistic interests. The approach is based on a fight for democracy and for democratic schools. The participants were nine in-service mathematics teachers (five from elementary school, four from middle school and high school) from the same public school in a city in northwestern Colombia. The teachers voluntarily joined the study because it was an activity of the periodic mathematics meetings that took place in the school to discuss issues related to the development of the area as well as an activity to frame professional development. The teachers worked on a statistical investigation that addressed a critical situation. Statistical investigations are ways of organizing teaching that simultaneously promote the statistical thinking and the social awareness that critical citizens require for their participation in the world (Zapata-Cardona, 2018). A statistical investigation as the main tool for the professional development seems to be coherent with the critical-dialectical approach that looks for participants' consciousness to confront conflicts and contradictions. The participants have never received any type of preparation about teaching statistics for citizenship; this was their first exposure to this format of professional development. The information was produced from the discussions held by the participants while they worked on a statistical investigation that empirically studied a critical situation related to the emission of CO_2 in the world.

As a warm up task, the teachers began by completing an online questionnaire that estimates the average CO_2 emission of each participant per year according to their consumption habits in housing, transportation, and food (see questionnaire). Then they used the Common Online Data Analysis Platform (CODAP) (The Concord Consortium, 2014) software to explore a database with the average CO_2 emission per capita per year (measured in tons) in different countries and regions of the world. Raw data from Climate Watch (2020) was downloaded from the World Bank website, organized by the researchers in a purposeful way, and given to the teachers ready to use in CODAP format. The database was organized in a rectangular array with 59 observations and 20 columns. In the columns, there was information about the year, the CO_2 emission of Colombia, and the CO_2 emission of several intentional aggrupation of countries (Europe and Central Asia, European Union, Poor countries, OECD countries, Latin-America and the Caribbean, etc.…). Because CODAP is visual and interactive software, the teachers did not require extra time to learn how to use it. The teachers received the link with the database already prepared, and they used their personal devises (or school devises previously arranged for them) for the multiple interactive explorations. Finally, the teachers established conclusions and reflections on their actions in order to mitigate CO_2 emissions. The actions and interactions of the teachers while working on the statistical investigations were audio and video recorded and transcribed verbatim to facilitate analysis. Transcripts and videos were independently read and watched several times by each researcher looking for those instances that suggested indications of statistical thinking and critical citizenship in the mathematics teachers' utterances. Those instances were presented and discussed in the research meetings, and those that better illustrate statistical thinking and critical citizenship were selected as examples.

RESULTS

The results highlight some evidence of statistical thinking and critical citizenship that was promoted during the professional development.

Statistical Thinking

The mathematics teachers used measures of spread and center to describe the behavior of a single group (e.g., Colombia, Europe and Central Asia countries, European Union countries) but also to propose comparisons between groups (e.g., Colombia and European Union countries, Colombia and OECD countries). In addition, they used bivariate strategies to make sense of the information in an effort to look for relations. The informed participation of citizens in society requires that they learn to think statistically, that is, to evaluate, understand and solve real problems using statistics (Campos, 2016). Using a dot plot (Figure 1a), the in-service teachers explored Colombia's annual tons of CO_2 emissions per capita over the last 59 years (from 1960 to 2018). The graph displayed in Figure 1a allowed them to get an idea of the behavior and distribution of the variable under study. Teachers then explored a similar graph that summarized measures of a variable that grouped the CO_2 emissions of countries in Europe and Central Asia (Figure 1b). They started with Europe and Central Asia because they associated them with a group of developed countries. The comparison with Colombia was inevitable because participants were Colombians. Teacher Dora (names in this report are pseudonyms to protect the identity of the participants under Colombian research law) used the upper limits of the range to account for the comparison: "Oh, God! [CO_2 emission for Europe and Central Asia] goes up to 13 points [tons] and we [Colombia] only 1.8 [tons]."

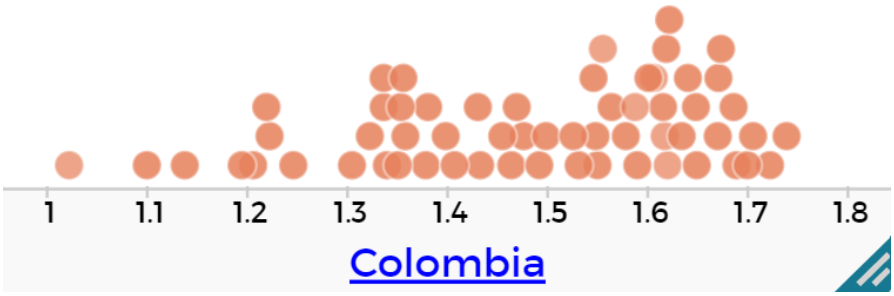

Figure 1a. Dot plot of CO_2 emissions in tons per capita per year in Colombia. Each point represents the average emission in a year from 1960 to 2018.

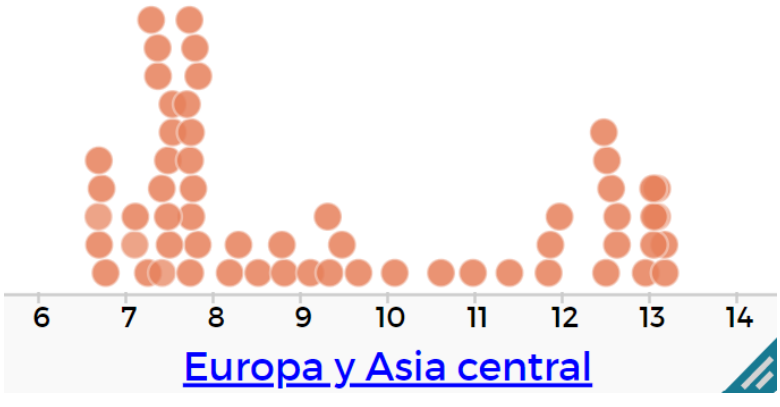

Figure 1b. Dot plot of CO_2 emissions in tons per capita per year in Europe and Central Asia. Each point represents the average emission in a year from 1960 to 2018.

Later, teacher Henry explored the measurement of the variable for the countries of the European Union (Figure 2) and also suggested a comparison with Colombia, but this time he focused on the arithmetic mean and made reference to the standard deviation: "The [average tons of CO_2 emissions]

of Colombia is 1.4 and that of the European Union is almost 8, the average." "[European Union] has a deviation of plus or minus 1.25 [tons]."

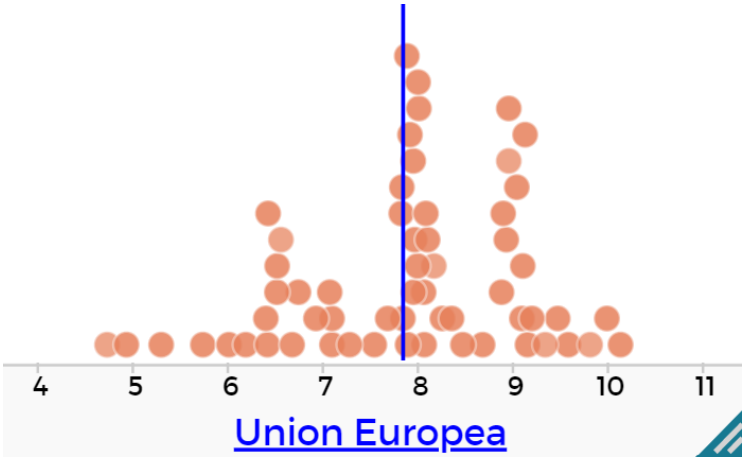

Figure 2. Dot plot of CO_2 emissions in tons per capita per year in the European Union. Each point represents the average emission in a year from 1960 to 2018.

Subsequently, the participants were suggested to focus on Colombia and identify the time of the highest CO_2 emission. To do this, the teachers explored a bivariate graph that combined the CO_2 emission per capita with the year of each measurement (Figure 3). The teachers identified that the year with the highest emission was 2016 and tried to give reasons to explain this behavior.

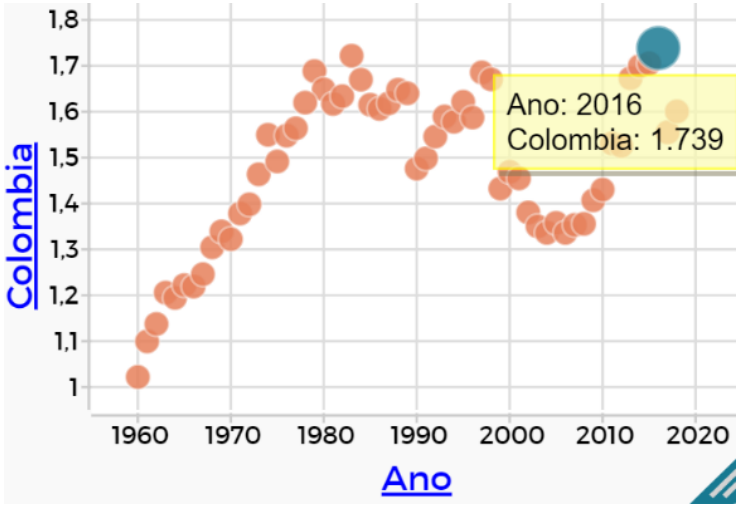

Figure 3. Scatter plot of CO_2 emissions in tons per capita per year in Colombia (vertical axis) Vs. year from 1960 to 2018 (horizontal axis)

The teacher Eduardo expressed "here […] people are getting many more vehicles. People buy two vehicles to skip the *restrictions from license plate*." (This is a vehicular restriction to reduce traffic congestion on the limited roads of main cities in Colombia. The local governments rotate restrictions for circulation of vehicles according to the last digit of their license plate.) The participant tried to attribute the CO_2 emission to the use of vehicles. This is an important observation, however. According to the Institute of Hydrology, Meteorology, and Environmental Studies (IDEAM, 2015), transport is considered within the energy category, which in Colombia represents 16% of CO_2 emissions. This means that other sources such as industrial processes, agriculture, or waste management, must also be considered.

Then, participants were suggested to explore the variable that grouped the countries of the Organization for Economic Cooperation and Development (OECD) (Figure 4) and raised the question:

why are the CO_2 emissions falling? Some participants offered their arguments: "There are already environmental treaties and those environmental conferences where the United States has not wanted to get involved" (Eduardo). "In the Netherlands […] they have a policy of having bicycle-only roads. […] And aside from that, the government is financing solar energy panels" (Dora). Eduardo and Dora's arguments offered a connection to the real world.

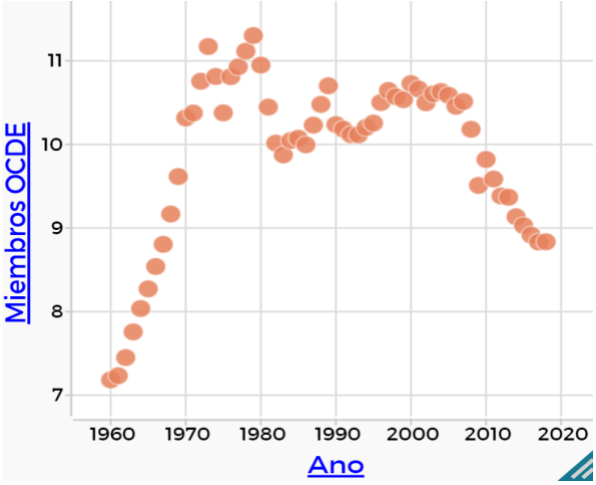

Figure 4. Scatter plot of CO_2 emissions in tons per capita per year in OECD countries (vertical axis) vs. year from 1960 to 2018 (horizontal axis)

Thinking statistically includes mastering concepts and procedures, building models, reasoning, inferring, or developing dispositions (Zapata-Cardona, 2018). In this sense, the participants engaged in tasks that allowed them to activate some of these skills.

Critical Citizenship

It is essential that people are able to critically interpret and analyze statistical information on conflicts and dilemmas from society in order to become critical citizens (Weiland, 2019). After studying the critical situation related to CO_2 emission with dynamic tools, the participants raised some reflections that suggest evidence of critical citizenship. The teacher Dora expressed that it is necessary to promote "policies that help raise awareness among people. [Real data] make us aware of what is happening." The teacher Angela, in a complementary sense, raised her reflection: "It is not the same that one is given the report than being able to see the statistics generated from the real data of the problem. […] [This task] is a form of awareness towards change."

The participants also proposed concrete actions, that from their role as members of a society, they can undertake to reduce CO_2 emissions. Some of them were: "Reduce energy consumption" (Eduardo), "Use means of transportation that generate less gases" (Dora), "Use natural light" (Dora), and "Reduce meat consumption" (Henry). At first sight, these ideas could either seem general or reflect conceptions that teachers had previously constructed, however, the strong connection with the data and with the critical situation suggest that the statistical investigation might have contributed to these reflections and to the potential actions. Critical citizenship is an intellectual tool that seeks for individuals to participate responsibly in society and promote transformative actions to face the conflicts and dilemmas they encounter (Zapata-Cardona & Marrugo, 2019). When participants use statistics to critically understand the world and propose transformative actions, there are indications that critical citizenship is taking place.

CONCLUSION

The results from this study provide evidence to establish that the implementation of statistical investigations as tools for teachers' professional development turns out to be favorable to promote statistical thinking. In-service teachers interpreted visual representations of data, used center and dispersion measurements to describe a single variable behavior, and used measurements to establish

comparisons between two groups. After carrying out the empirical investigation process, the in-service teachers proposed several ways to reduce CO_2 emissions. The participants' proposals are indications of awareness that was promoted when teachers were confronted with a critical situation to which they could not remain indifferent. This evidence suggests that statistical investigations, as tools for professional development, also promoted participants' critical citizenship.

The statistical investigations, as organizers of teaching, seem to be methodological strategies that help connect teachers' professional development processes with social dilemmas and from that point of view they seem to generate an affective connection. This study was conducted with a specific group of in-service teachers from a public school. It would be worthwhile to extend this type of experience in other contexts under other conditions to test whether the results are maintained over time.

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