

## STUDENTS' STATISTICAL MODELING ACTIVITIES USING TINKERPLOTS

Jennifer Noll, Kit Clement, Jason Dolor and Matthew Petersen  
 Technical Education Research Center  
 Portland State University  
 noll@pdx.edu

*Statistical modeling using technology is now an integral component of statistical thinking and identified as an important learning goal for K-12 and undergraduate statistics curriculum worldwide. Yet, the statistics education community has very little knowledge about the impact of such a focus on student learning. The goal of this research is to describe and compare the statistical modeling content knowledge of a small group of college students who received a reform-oriented curriculum focused on modeling and simulation using TinkerPlots™. The data reported here explores students' development of statistical models using TinkerPlots™ through their responses to questions set in the context of a statistical task. Implications for teaching and future research are also discussed.*

### INTRODUCTION

New powerful technologies make it possible to collect, organize, represent, manipulate, summarize, visualize and model overwhelming amounts of data. These technologies provide new ways to make sense of the world we live in (see Gould, 2010) and, for those who do the collecting, organizing, summarizing and visualizing, new ways to shape the perceptions of people who consume data. Data collected from everyday technology (smartphones, web, apps) are more available to the non-expert to use, manipulate, and interpret. With increased access to data come an increased responsibility to prepare every citizen to develop their ability to critically think about data information presented to them as well as how to manipulate data and make inferences using technology.

Nolan and Lang (2010) and Cobb (2007) have argued that technology is now a fundamental aspect to the practice of statistics; yet, many introductory statistics courses continue to emphasize procedures such as  $z$  and  $t$ -tests rather than focusing on modeling and simulation techniques using technology. Over the past decade some statisticians and statistics educators have been changing the approach to teaching introductory statistics courses. For instance, some statistics educators have focused on computer technologies that support students in developing methods for organizing and visualizing data (e.g., Laina & Wilkerson, 2017). Other statistics educators have developed new texts and curricula with a focus on modeling, randomization and computer simulations techniques (e.g., Garfield, delMas, & Zieffler, 2012; Lock et al., 2013; Morgan, 2011; Tintle, VanderStoep, Holmes, Quisenberry, & Swanson, 2011). Yet, statisticians and statistics educators (Cobb, 2007; Nolan & Lang, 2010; Shaughnessy, 2007) have argued that there needs to be more research into how students learn statistical modeling using technology.

The work presented here makes a contribution to statistics education research through studying the ways students created statistical models using TinkerPlots™ technology. The research questions that guided this investigation are: In an activity based curriculum that uses TinkerPlots™ technology, how do students create statistical models? What aspects of the statistical problem are important to them as they construct their models? What do these models mean to students?

### BACKGROUND

Models are fundamental to modern statistics, and are increasingly becoming a focus in statistics education. Yet, models are often defined differently in different fields. Models can be defined as conceptual systems that support sense making in a particular setting (Lesh and Doerr, 2003). Garfield and Ben-Zvi (2008) describe statisticians using models in two different ways: (1) “select or design and use appropriate *models* to simulate data to answer a research question”, and (2) “[f]it a statistical model to existing data” (p. 145, italics in original). In their description, a statistical model can be a random generator used to answer a statistical problem. We define student statistical models to be the models they create using the TinkerPlots™ sampler device (which

serves as a random generator) they create along with their descriptions of the device and what they think happens when the device is run.

The work presented here utilizes a slightly modified CATALST curriculum originally developed by Garfield, delMas and Zieffler (2012). The curriculum emphasizes chance models and simulation, methods for comparing two groups, and bootstrapping techniques. Core to the curriculum is a focus on conceptual understanding and critical thinking where the learner takes an active role in constructing knowledge through engagement in activities. New topics are introduced through model-eliciting activities (MEAs), based on the work of Lesh et. al (2000). According to Garfield et. al (2012), the MEAs were “designed to encourage students to build mathematical models in order to solve complex problems, as well as provide a means for educators to better understand students’ thinking” (p. 884). Following each MEA, there are several activities in each CATALST unit that guide students through key ideas raised in the MEA (e.g., randomness, chance/null model, informal inference based on a single population, p-value). We modified the activities to require students to do more of the model construction on their own so we could study how they develop statistical models. All activities were designed to engage students in model construction while working with peers where the instructor acts as a facilitator. Students are encouraged to test their models using new data sets and revise their models based on the results of their tests. After small groups of students have a chance to create, test, and revise a model, a whole class discussion ensues in which groups share their models and the class has an opportunity to question and critique different aspects of these models.

An important aspect of the curriculum design was the implementation of TinkerPlots™ technology. According to Garfield et. al (2012), TinkerPlots™ was chosen “because of the unique visual capabilities it has, allowing students to see the devices they select (e.g., sampler, spinner) and to easily use these models to simulate and collect data, which allows students to examine and evaluate distributions of statistics in order to draw statistical inferences” (p. 886). The unique modeling features of TinkerPlots™ correspond naturally to the designers’ guiding principal of modeling and supports active student learning as students work together to create TinkerPlots™ models, run simulations, and make statistical inferences. As such, TinkerPlots™ is a key feature of the course and used in order to achieve the pedagogical goals of having students develop models and conduct simulations.

## METHODS

The work presented here is part of a five-year study investigating student learning using the modified CATALST modeling and simulation curriculum. The authors implemented the CATALST curriculum materials (see Garfield, delMas, & Zieffler, 2012) using TinkerPlots™ software in a two-term undergraduate introductory statistics sequence in the Pacific Northwest of the United States. The course is a standard for non-mathematics/non-statistics majors. Many of the students in the introductory statistics sequence major in criminal justice, sociology, women’s studies, psychology, biology, business, etcetera. Data was collected in the first and second terms (10 weeks per term) over a three-year period providing a total of six classrooms of data as of February 2018. This paper focuses only on data from the NFL task given winter term 2016 from the first term of the sequence, see Figure 1, because for subsequent terms the task was altered. There were 58 students in the winter 2016 class, 7 students did not consent to have their work used as data in the study and 8 students did not complete the task (absent or dropped the course). Thus, there is a total of 43 students whose data is reported here.

The National (American) Football League (NFL) uses an overtime period to determine a winner for games that are tied at the end of regulation time. Between 1974 and 2009, the overtime period started with a coin flip to determine which team gets the ball first in overtime, and then the team that scores first wins. Data from the 1974 through 2009 seasons show that the coin flip winner won 240 out of the 428 (56%) games where a winner was determined in overtime. Research Question: Is there an advantage to the team that wins the coin flip? (Then students were asked to state the null hypothesis, create a null model in TinkerPlots™ and simulate under the assumption that there was no advantage).

Figure 1. First Version of NFL Task

## RESULTS

There was a lot of variability in the types of models students' constructed. Several of the model types outlined here are isomorphic and could effectively answer the research question posed in the NFL task. However, a primary point we wish to make is that students' choice for labeling the attributes and elements in their models and how they interpreted what happened when the model was run has important implications for how they made sense of these models. We identified four types of student generated models: Single Device – One Attribute – Equal Proportions; Single Device – One Attribute – Observed Sample Proportions; Single Device – Attends to Conditioning – Equal Proportions; Linked Device – Two Attributes – Equal Proportions, see Table 1. Fourteen out of 43 (33%) of these students' work was coded as Single Device – One Attribute – Equal Proportions. We further identified three sub-types to this category, depending on the attribute students focused on, *the coin flip*, *the winning team*, or it was *unclear* from their model and description. One student created single device model where they seemed to focus on the team that wins the coin flip using the observed sample proportions. Eighteen out of 43 (42%) of students created linked device models (two connected spinners) where one attribute focused on the coin flip winner and the other on the game winner and each device had equal probabilities for each outcome. We identified three sub-types to the linked device model where students focused labeling the elements as *team name* for both devices, the *team name* for one device and *win/lose* for the second device, or *win/lose* for both devices. Ten out of 43 (23%) students created a single device model that attended to both attributes through an acknowledgement of conditioning by looking at the team that wins the game given that the team already won the coin flip. The students that created this type of model used equal probabilities for the two outcomes. Table 1 shows the summary counts for each model type. In the sections that follow we share more detail about these model types and student thinking.

Table 1. Counts for Four Model Types

Model Types	Counts
Single Device – One Attribute – Equal Proportions	14
Focus on coin flip	6
Focus on the team that wins the game	3
Unclear	5
Single Device – One Attribute – Observed Sample Proportions	1
Linked Device – Two Attributes – Equal Proportions	18
Focus on teams for coin flip and game	1
Focus on teams for coin flip and win for game	1
Focus on win for coin flip and win for game	16
Single Device – Attends to Conditioning– Equal Proportions	10
Total	43

Students who constructed Single Device – One Attribute – Equal Proportions and Observed Sample Proportions models needed the most development in their understanding of statistical modeling and experienced the greatest challenges in answering the research question posed in the NFL task. The primary reason these students' conceptions seemed least developed is because their models showed evidence that these students were only able to make sense of and attended to one aspect of the context, either the coin flip or the winning team, and in some cases could not clearly articulate which of these aspects their models focused on. Figure 2 contains models A-E that reveal different challenges students encountered as they made sense of the model. For example, Models A-C are unclear as to whether the winner of the coin flip or the winner of the game was being modeled, with model C trying to suggest both but in an unclear way. Models D and E show the coin flip winner is being modeled. From some of the written work we could see that some students focused on modeling the team that won the game, but they did not show evidence that they were assuming that this team had already won the coin flip. Other students

focused on modeling the coin flip winner, but it was not clear that they considered who won the overtime games. These students disregarded information in the problem without an explanation as to why. In many cases, it was not clear from their descriptions what they were modeling – the winner of the coin flip, the winner of the game, or the winner of the game given the winning team had already won the coin flip. Thus, despite the fact that the Single Device – One Attribute – Equal Proportions can answer the research question, many students who initially created this model did not really understand what they were modeling nor could they justify the reason for their model choice.

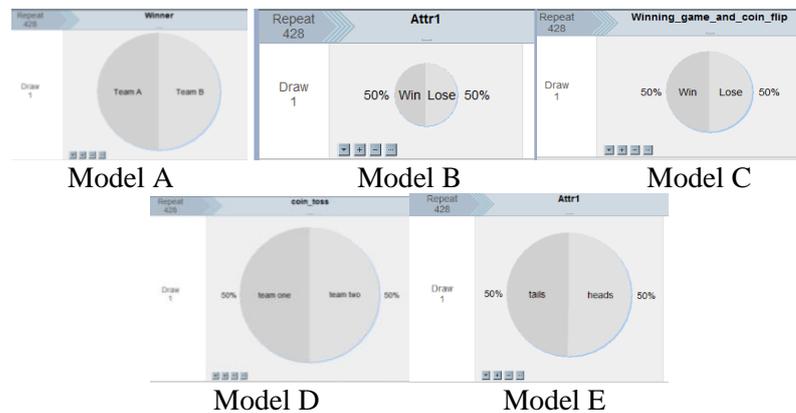


Figure 2. Single Device Models – One Attribute – Equal Proportions

Students who used the proportion from the observed sample data had the added challenge that they were not assuming the null model of no advantage, see Figure 3. These students either did not pay attention to attribute labels or were challenged to find a solution to the research question because their interpretation of their attribute and element labels led to confusion about what data to collect.



Figure 3. Single Device Model – One Attribute – Equal Proportions

Linked Device – Two Attributes – Equal Proportion models provided evidence that some students considered all aspects of the NFL task, the coin flip and the winner of the game. There were three versions of these linked device models, see Figures 4-6. While a statistician would see these models as isomorphic and valid for answering the research question, for our students, developing their conceptions of statistical models, these models were not the same and provided different affordances and challenges. For example, the model in Figure 4 reveals that students focused on labeling *the team* as the elements in their device. The team labels keep track of which team wins the coin flip and which team wins the game. This choice of what to attend to provided affordances as students progressed toward the creation of a sampling distribution. The team labels helped students see what they wanted to collect data on – the counts for TeamA,TeamA and TeamB,TeamB because each of these columns represent the winner of the coin flip, winning the game. Figure 5 shows a student generated model focused on *teams* as the labeled element for coin flip and *win/lose* for game. This labeling provided students similar affordances as the previous linked device model, allowing them to see two columns – Team A won the coin flip and won the game (Team A, won) or Team B won the coin flip and won the game (Team B, won), both necessary to answer the research question.



Figure 4. Linked Device – Two Attributes – Team Labels



Figure 5. Linked Device – Two Attributes – Team and Win/Lose Labels

Unfortunately most (16 out of 18) of the students who created a linked device focused on labeling the elements as *win/lose* for both the coin flip and the game. The lack of team information in the data created major challenges for students when it came time to collect data to create their sampling distributions. Figure 6 shows an example of this and a corresponding dot plot, which displays outcome data as combinations of win/lose coin flip and win/lose the game. Since students were looking for when the coin flip winner won the game, many would only examine the “Won,Won” section, as this is literally when the coin flip winner won the game. However, this perspective is limiting students from seeing that “Lost,Lost” is the same, as the coin flip loser losing the game and; thus, is equivalent to the coin flip winner winning the game. It is easier to see this perspective in the previous linked devices modeling scenario, as “Team A, win” and “Team B, win” or “Team A, Team A” and “Team B, Team B”. These are equivalent scenarios from different perspectives; however, the model in Figure 6 does not represent different teams’ perspectives as easily.



Figure 6. Linked Device – Two Attributes – Won/Lost Labels

Finally, Figure 7 shows a student generated single device model that looks much like Model B in Figure 2. However, the meaning imparted by students and what it says about their understanding of the context problem is very different. Ten students were classified as creating one of the models shown in Figure 7 and provided written clarification that they were modeling the attribute of coin flip winner and whether that coin flip winner won or lost the game. The labeling of the attribute and elements in Figure 7 helped us see that the student was considering the coin flip winner. Here’s an example of how one student articulated this model in writing: “*Researchers should assume that the players who win the coin flip have a 50/50 chance of winning*”. The assumption of having won the coin flip is important particularly because if the conditioning had been different they could not simply ignore this aspect and still get an appropriate answer to the research question.



Figure 7. Single Device – Attends to Conditioning – Equal Proportions

CONCLUSION

The CATALST curriculum coupled with TinkerPlots™ technology impacted the way these students thought about statistics. Students appeared to work through statistical problems visually, using TinkerPlots™ sampler device. The models they created and the explanations they gave yielded information about what was important to them. For example, student choices for attribute

and element labels changed the story students told about the NFL problem because these choices resulted in labels in their dot plots that they interpreted in a particular way, subsequently impacting their decisions on what data to collect and ultimately their inferences. Students who created linked devices, focused on labeling elements as *win/lose* versus *team names* had very different experiences with what data they collected (e.g., win/win and not lose/lose versus teamA-wins and teamB-wins) and the conclusion to their story. Other students, possibly to keep the problem simple, ignored one of the attributes. For example, some students simply modeled the coin flip winner. It is possible that these students were influenced by a previous activity where they were actually asked to model a coin flip, but it still shows evidence that in this new problem they ignored a large part of the problem context and this impacted the conclusions they could draw. The statistical models students created in TinkerPlots™ acted like personal stories for them and two seemingly isomorphic models may not be isomorphic from the students' perspectives. Thus, we argue that seemingly small aspects of the sampler (e.g., labels for attributes and elements) and students reasoning as they run these models carry different affordances or challenges for students in the modeling process. This work suggests important implications both for task design (how we can design tasks to leverage these different model types and build discussion around these models into the classroom) and pedagogically for teachers. Teachers need to know how to ask the right follow up questions to better understand student thinking and push students' developing conceptions.

## REFERENCES

- Cobb, G. W. (2007). The Introductory Statistics Course: A Ptolemaic Curriculum. *Technology Innovations in Statistics Education*, 1(1). Retrieved from <http://escholarship.org/uc/item/6hb3k0nz>.
- Garfield, J. B., & Ben-Zvi, D. (2008). *Developing students' statistical reasoning: Connecting research and teaching practice*. Springer.
- Garfield, J., delMas, R., and Zieffler, A. (2012). Developing Statistical Modelers and Thinkers in an Introductory, Tertiary-level Statistics Course. *ZDM – The International Journal on Mathematics Education*, 44(7), 883–898.
- Gould, R. (2010). Statistics and the Modern Student. *International Statistical Review*, 78(2), 297–315.
- Konold, C., and Miller, C. (2015). TinkerPlots™ Version 2.2 [computer software], Learn Troop. Available at <http://www.tinkerplots.com/>.
- Laina, V., & Wilkerson, M. (2017). Modeling data by visualizing it. *Proceedings from the International Forum on Statistical Reasoning, Thinking and Literacy*, New Zealand.
- Lesh, R.A., & Doerr, H. M. (2003). Foundations of a models and modeling perspective on mathematics teaching, learning, and problem solving. In R.A. Lesh & H. M. Doerr (Eds.), *Beyond Constructivism: Models and Modeling Perspectives on Mathematics Problem Solving, Learning, and Teaching* (pp. 3–34). Mahwah, NJ: Lawrence Erlbaum Associates.
- Lesh, R.A., Hoover, M., Hole, B., Kelly, A., & Post, T. (2000). Principles for Developing Thought-Revealing Activities for Students and Teachers. In *Research Design in Mathematics and Science Education* (pp. 591–646). Mahwah, NJ: Lawrence Erlbaum Associates.
- Lock, R.H., Lock, P.F., Lock Morgan, K., Lock, E.F., and Lock, D.F. (2013). *Statistics: Unlocking the Power of Data* (1<sup>st</sup> ed.), Hoboken, NJ: John Wiley & Sons.
- Morgan, K. L. (2011). Using Simulation Methods to Introduce Inference. Consortium for the Advancement of Undergraduate Statistics Education (CAUSE) webinar [online]. Available at <http://www.causeweb.org/webinar/teaching/>.
- Nolan, D., & Lang, D. T. (2010). Computing in the statistics curricula. *The American Statistician*, 64(2), 97–107.
- Shaughnessy, J. M. (2007). Research on Statistics Learning and Reasoning. In F. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 957–1009). Greenwich, CT: Information Age Publishing, Inc. and NCTM.
- Tintle, N., Topliff, K., VanderStoep, J., Holmes, V., and Swanson, T. (2012). Retention of Statistical Concepts in a Preliminary Randomization-Based Introductory Statistics Curriculum. *Statistics Education Research Journal*, 11(1), 21–40. Retrieved from [http://iase-web.org/documents/SERJ/SERJ11%281%29\\_Tintle.pdf](http://iase-web.org/documents/SERJ/SERJ11%281%29_Tintle.pdf).