USE OF THE FOCUSING FRAMEWORK FOR CHARACTERIZING STUDENTS' FOCI OF ATTENTION WHEN REASONING ABOUT DATA DISTRIBUTIONS

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This research highlights our use of the focusing framework (Lobato et al., 2013) as a new lens for exploring students' actual foci of attention and ways of reasoning in the context of statistics instruction intended to orient their focus and reasoning to specific ideas. We present results of our analysis of ideas expressed by ninth-grade students as they participated in an instructional sequence involving the use of TinkerPlots software to organize univariate data samples and designed to orient students' attention to variability when making an inference about the sampled population. Students' written responses and surrounding classroom discussions analyzed in terms of the focusing framework highlight and explain the uniformity and diversity of their reasoning when comparing distributions.

INTRODUCTION

Mathematics and statistics instruction is often driven by teachers' reflective efforts to orient students' focus of attention to specific ideas considered essential to a particular target understanding or way of thinking. However, taking a reflective stance on the teaching-learning process can orient us to possible contrasts between a teacher's *intended* focus of attention for students and students' *own* or *actual* foci of attention within particular instructional contexts. In this paper we explore potential contrasts between students' foci of attention for students *intended* by the designed instructional environment and what students actually notice as salient and their *own* foci of attention as they participate in such environments. We do so by employing the *focusing framework* (Lobato, Hohensee & Rhodehamel, 2013) as a lens for looking at ninth-grade students' ideas and reasoning that emerged in their participation in instruction designed to provoke them to reason about data distributions and variability as a basis for making an inference about a sampled population. Our paper is structured around an example drawn from classroom discussions and students' written responses to tasks framed in terms of students' noticing and focusing phenomena (Lobato *et al*., 2013). Our example illustrates the usefulness of the focusing framework for illuminating both students' thinking and foci of attention, and features of the instructional environment that may work in coordination as sources for orienting such foci.

EPISTEMOLOGICAL STANCE

We view the concept of data distribution as a fuzzy and ambiguous construct, in that conceiving of a distribution can admit and invite a diversity of perspectives and ways of organizing and construing data to arrive at some conclusion. It is its fuzziness and multifaceted-ness that make distributions both interesting from an educational research perspective, and problematic to learn and teach in practice (Lehrer, Kim & Jones, 2011). We use the wording "conceiving of a distribution" deliberately to convey our underlying epistemological stance regarding the concept: although graphically represented (univariate) data sets can be seen to hold the potential to attract an observer's attention to certain perceptual features, we see distribution not so much as something to *perceive*, but more as an object-construct that a person must *conceptualize*. This distinction underscores the importance of shedding light on students' tendencies regarding their focus of attention in relation to their engagement with instruction designed to orient their attention to specific features, to shape their construal of such features and to provoke their reflections about them. Our view that conceptualizing a distribution is shaped by perceptual and conceptual aspects that one selectively attends to among a variety of possibilities is highly coherent with Lobato *et al.*'s view of learners being called to "work with particular mathematical features or regularities when multiple sources of information compete for one's attention" (Lobato *et al.*, 2013, p. 809). This kinship makes their focusing framework well-suited for characterizing students' foci of

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attention as they participate in activities designed to orient their attention and thinking to particular aspects around data distributions.

THE FOCUSING FRAMEWORK

Inspired by work in cognitive science and applied anthropological linguistics (Gibson, 1989; Goodwin, 1994), the focusing framework views student *noticing* (i.e., their focus of attention) as socially situated and emerging out of the interplay among features of the learning environment within which they interact. In this perspective, student noticing is a complex phenomenon that is distributed across individual cognitions, social interactions, material resources and norms of classroom participation. The focusing framework provides a useful perspective for addressing the following questions: i) what aspects or ideas do students notice and focus on as they engage with tasks designed to orient their attention and thinking to particular aspects and ideas?
ii) how might the interplay among class participation and the use of material and discursive resources shape and support the emergence and evolution of students' "centers of focus" within instructional contexts? Lobato *et al*. (2013) define their focusing framework in terms of 4 constructs that help account for how "centers of focus" emerge for students in socially situated and organized interactions within learning environments. "Center of focus" refers to specific aspects, properties, regularities or conceptual objects that students notice as salient and that they focus on during a given period of time.

Focusing interactions

Focusing interactions are defined as a particular set of classroom discourse practices that can serve to orient students' attention to specific features of the classroom social environment, and that help account for how what students' notice is socially organized. Three central types of focusing interactions are identified in the framework:
(1) *Highlighting* is any observable operation upon external phenomena or representations, such as labeling and annotating, that make particular features prominent and thus function to potentially orient and shape others' perceptions. Although Lobato *et al.* (2013) refer only to perceptual features of the environment when defining highlighting, we see highlighting as also holding the potential to orient or shape others' *conceptions* of a particular phenomenon under consideration within a given instructional interaction.
(2) *Coding* is defined as " the use of a category of meaning by professionals as a lens through which to view events" (Lobato *et al*., 2013, p. 824). In our interpretation, coding is distinguished from highlighting in that it need not refer to observable operations upon external phenomena. Instead coding is akin to a *framing* of something—a feature or an event under discussion—that can be communicated and shared in an effort to convey a way of seeing, or thinking about, that something.
(3) *Renaming* refers to changing the name of a previously defined construct by "using a category of meaning from mathematical practice" (Lobato *et al*., 2013, p. 824).

Mathematical tasks

Mathematical tasks are seen as the media and situations or contexts within which focusing interactions can arise and evolve. They may entail features specifically designed to influence and shape what students notice and attend to, both in the moment and in enduring ways. Such features can be diverse; they can include prompts to create specific objects, to perform specific actions on them, to attend to specific features of them, to use a specific tool intended to provoke a certain framing or way of seeing something (coding) and prompts to reflect on such features and to communicate one's reflections to others. Thus, in our view, mathematical tasks are a key component of instructional environments designed to provoke and promote the emergence and development of particular intended focusing interactions among students. In this regard we see tasks as fecund media, potentially propitious for the emergence and development of such interactions. It is in this sense that we view mathematical tasks as *affording* students' noticing, learning, and development of certain ideas. We would add that we do not take the learner's agency out of consideration when affirming or describing how mathematical tasks "help account" for the emergence of students' centers of focus.

Nature of mathematical activity

Drawing on Cobb and Yackel's (1996) constructs of classroom social norms and socio-mathematical norms, the nature of mathematical activity refers to the "global character of discourse practices that regulate who is allowed to talk and what types of contributions they can make" (Lobato *et al*., 2013, p. 814). This includes the norms governing participation in classroom interactions that can contribute to the emergence of centers of focus. Examples include general expectations regarding the teacher's and students' actions, such as whether students are expected to communicate and share their thinking with others, whether the teacher is expected to provide the mathematical content and the degree to which the teacher is expected to guide the interactions. All of these are seen to potentially influence the centers of focus that can emerge for students. Lobato *et al*. (2013) characterize these norms as global in order to contrast them with focusing interactions, which are seen as "specific discursive moves of teachers or students that serve to direct the attention of others to particular features mathematically" (p. 814).

As emphasized by Lobato et *al*. (2013) the focusing framework is seen as an *interactional system* in that no single component accounts for a phenomenon. Instead the components are seen as interacting in coordination to occasion the emergence of particular centers of focus and to support or drive their evolution across time and in tandem with the progression of a sequence of instructional tasks with which students engage.

THE FOCUSING FRAMEWORK IN ACTION: AN ILLUSTRATIVE EXAMPLE

The context

Our example consists of excerpts from a classroom discussion and student work that emerged in the context of their engagement with an instructional task sequence adapted from Konold and Miller (2012). The sequence was designed to provoke students to compare distributions of the lengths of two types of fish (a genetically modified version of a species and a "normal" version) in a random sample of 43 fish drawn from a pond, with the eventual aim of supporting their ability to make a data-based claim about which of the two types of fish generally tends to grow longer. Students were prompted to use *TinkerPlots* software (Konold & Miller, 2011) to organize the data sample. Figure 1 displays the graph produced by several pairs of students; it formed the basis of the discussion excerpt which revolves around the following question, deliberately open so as to admit diverse foci of attention: *Write down some things you noticed about this sample of fish*.

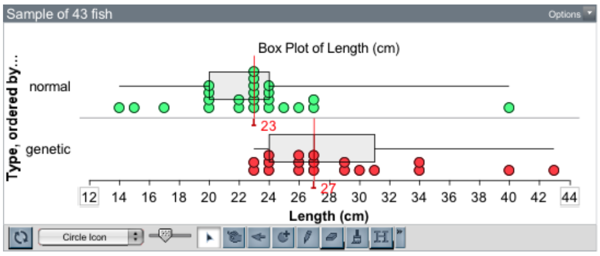

Figure 1. A separated dot plot of the sample of 43 fish lengths overlaid with boxplots and displaying the median length of each sub-group within the sample.

Excerpt 1

The discussion excerpt below illustrates a sequence of interactions between the teacher (denoted as "T") and two students that emerged out of the general expectation that students share their ways of thinking and responses to task questions—an expectation instituted throughout the lessons leading up to this excerpt, and that we view as part of the nature of the mathematical activity in the class. The excerpt codes utterances within an interaction in terms of concepts from the focusing framework as described above. In the first part of the excerpt (lines 1-12) we see student S20 volunteering what he noticed in the activity of organizing the data set with *TinkerPlots*; his first center of focus was evidently on the minimum values of the two sub-collections of fish lengths (lines 2-4), suggesting that he might not have been focused on the two collections as distributions per se, nor on comparing them qualitatively. The teacher subsequently engaged in a series of highlighting moves (lines 5-9) that oriented S20's attention and appears to have contributed to a refinement and sharpening of S20's focus on the numerical difference between the minimum lengths of the two types of fish as a second center of focus (line 10), thus signalling a shift in focus to a more quantitative comparison.

Speaker	*Utterance*	*Focusing framework codes*
1. T:	Can anybody volunteer two things they noticed? S20?	Nature of math activity: soliciting students' ideas (what did you notice?)
2. S20:	Um, that the shortest genetic fish is longer.	Center of focus 1 (CF1)
3. T:	Please listen up.	Nature of math activity: expectation that students listen to ideas shared by others
4. S20:	The shortest genetic fish is longer than the shortest normal fish.	CF1: qualitative comparison of minimal values
5. T:	So here's, where's the shortest genetic fish, right here? [points to displayed graph]	Focusing interaction: Highlighting by the teacher
6. S20:	Yeah.	
7. T:	Is longer than what?	Focusing interaction: Highlighting and request for clarification by the teacher
8. S20:	The shortest normal fish.	
9. T:	So you're comparing these two. [points to extreme values on displayed graph]	Focusing interaction: Highlighting clarification by the teacher
10. S20:	Yeah, about nine centimeters.	CF2: quantitative comparison of minimal values
11. T:	Okay, and they differ by about nine centimeters. They differ by about nine centimeters, right? Okay, good.	Focusing interaction: highlighting reformulation by the teacher
12. S20:	Yeah.	

In the second part of the excerpt (see lines 13-29 below) the same nature of mathematical activity solicits a second student, S7, to share what he noticed about the data set as a result of having organized it with *TinkerPlots*. We see a third center of focus (CF3) in S7's attention to the two distributions of lengths at a rough and global level, what he referred to as "eyeballing" (line 18). This formed the basis of S7's initial impression that the genetic fish were not discernibly longer than the regular fish, a conclusion we can view as an interpretation consistent with coding (Gibson, 1994). S7's subsequent utterance then revealed that his work in *TinkerPlots* had oriented him to view the data sets in terms of ordered quarters, indicating that he had evidently employed *TinkerPlots*'s boxplot tool in a way that shaped his conception of the data set—comparing corresponding quarters of lengths in the two distributions— and the conclusion he ultimately drew from it. This part of the excerpt also illustrates a series of highlighting moves (such as labeling) on the part of the teacher that seemed to re-orient and help clarify S7's center of focus; they served to re-iterate S7's initial conclusion based on his first center of focus, then to re-orient S7's attention to the graph so as to refine his explanation that he was comparing the shortest 50% of lengths of normal fish with the shortest 25% of lengths of genetically altered fish (thus a fourth center of focus, CF4, is made evident). This last center of focus was the basis of his conclusion that the shortest 50% of normal fish lengths were smaller than the shortest 25% of genetically altered fish lengths.

Speaker	*Utterance*	*Focusing framework codes*
13. T:	Anybody else notice anything? S7?	Nature of math activity: soliciting other students' ideas
14. S7	I noticed that the genetic fish were not greatly	CF3 (initiated)

	enhanced compared to-	
15. T:	Please, shush, listen to S7.	Nature of math activity: expectation that students listen to ideas shared by others
16. S7:	Compared to the normal fish.	
17. T:	Say that again, I'm sorry.	
18. S7:	I noticed that the genetic fish were not greatly enhanced compared to the normal fish, but that was just eyeballing it.	CF3: global qualitative comparison, concluding that genetically altered fish are not much longer than normal fish
19. S7:	Then I realized that the box plot shows that the first fifty percent of the [normal] fish were shorter than the first twenty-five percent of the genetic fish.	CF4: refined comparison emerging from use of boxplot to highlight quarter structure of the data sub-collections so as to compare them. Focusing interaction: Student proposes a way of looking at data based on interpreting boxplot diagram (a form of coding)
20. T:	So just by eyeballing it you couldn't tell whether they were a lot bigger?	Focusing interaction: Highlighting in terms of re-voicing/re-emphasizing by the teacher
21. S7:	No.	
22. T:	So you used the box plot. And what did that tell you, again?	Focusing interaction: Highlighting in terms of re-emphasizing and questioning by the teacher
23. S7:	That the first twenty-five percent of the normal fish were shorter than the first twenty-five of the genetic fish.	CF4: reiteration by the student
24. T:	First twenty-five percent of the normal fish, that's this area right here? [points at first quarter of distribution on displayed graph]	Focusing interaction: Highlighting in terms of emphasizing/clarifying by the teacher (a form of labelling or annotating)
25. S7:	Oh, no, fifty percent.	Clarification for student as consequence of previous highlighting by the teacher
26. T:	Fifty percent? What about them?	
29. S7:	They're shorter than the first twenty-five.	CF4: reiteration by the student

Excerpt 2

We also analyzed students' written responses to the question above (*Write down some things you noticed about this sample of fish*), and to the follow-up question: *Based on this sample, do the genetically engineered fish in the pond tend to grow longer than the normal fish in the pond? Support your conclusion by referring to your graph.* Responses to these questions were analyzed to identify salient foci of attention among students, the diversity of such foci, and instances in which their focus differed substantially from that intended in instruction. We found that all students (*n*=23) compared the lengths of the two sub-groups of fish in the sample, and all concluded that the genetically engineered fish tended to grow longer than the normal fish. However, responses varied with regard to the sharpness and specificity of students' centers of focus. For example, 15 students showed evidence of having a global and somewhat qualitative focus of attention regarding the data sample, consistent with S7's "eyeballing" perspective seen in line 18 of the discussion transcript above. The following student response exemplifies this level of focus:

S1: "I noticed that the normal fish are generally smaller than the genetic fish".

On the other hand, 13 students showed evidence of having much sharper and specific centers of focus, some of which entailed the coordination of various components of distribution. The following student's response exemplifies the most elaborate such foci:

S9: "On average, the genetically engineered fish are 5.820 cm longer than the normal fish. The median of the genetic fish is always higher as well. There appears to be more variance in the normal fish, but this is mostly due to one strange 40 cm data point; the rest vary less than their engineered counterparts".

Regarding centers of focus that diverged substantially from that targeted in the instructional tasks, 9 students focused their attention almost exclusively on the unequal numbers of two types of fish in the sample, as though that was the most salient feature to them. The response below exemplifies this focus:

S10: "[…] So there were more fish taken from the normal than the genetic ones. There are 24 normal fish, & 19 genetic fish […]".

CONCLUDING REMARKS

The focusing framework (Lobato *et al*., 2013) offers a perspective wherein students' centers of focus—features and ideas they attend to as salient—are seen to emerge not only in individual thought but also within classroom interactions centered around tasks designed to orient students' attention and to provoke their reflection on what they attend to. This framework stands to enhance analyses of data generated within instructional contexts by providing a window into *focusing interactions*—a dimension that is not accessible through purely individualistic analyses of students' written responses to task questions. In the example presented here the focusing framework enabled us to capture both the consistency and the diversity of students' reasoning when comparing two data distributions by considering this dimension in conjunction with individual analyses of students' written responses. Moreover, being an interactional perspective, the focusing framework views students' focus of attention not as being necessarily fixed and immutable, but rather as dynamic and subject to evolution—it can be shaped and refined by students' interactions with tools, such as *TinkerPlots* box plots, and by their classroom interactions with teachers in which the coordination of focusing interactions are seen to occasion the emergence of particular centers of focus and to drive their evolution across such interactions.

REFERENCES

Cobb, P., & Yackel, E. (1996). Constructivist, emergent, and sociocultural perspectives in the context of developmental research. *Educational Psychologist, 31*, 175–190.

Gibson, J. J. (1989). *The ecological approach to visual perception*. Hillsdale, NJ: Earlbaum.

Goodwin, C. (1994). Professional vision. *American Anthropologist*, *96*(3), 606-633.

Konold, C. & Miller, C. (2012). Fish-Length Distributions. Available online at *http://www.tinkerplots.com/activities/data-analysis-and-modeling-activities.*

Konold, C., & Miller, C. D. (2011). *TinkerPlots*™ (v.2.0). Computer program. Emeryville, CA: Key Curriculum Press.

Lehrer, R., Kim, MJ., & Jones, R. S. (2011). Developing conceptions of statistics by designing measures of distribution. *ZDM*, *43*(5), 723-736.

Lobato. J., Hohensee, C., & Rhodehamel, B. (2013). Students' mathematical noticing. *Journal For Research in Mathematics Education*, *44*(5), 809-850.