WAYS TO STRENGTHEN THE STATISTICAL LITERACY, REASONING AND THINKING IN THE MATHEMATICS TEACHERS TRAINING

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A growing public and political consensus, translated into an increase in fundamentals and curricular changes worldwide, has highlighted the importance of people being statistically literate, valuing their ability to reason and think using evidence-based, trustworthy arguments. These changes have affected a number of educational actors, including mathematics teachers, who have experienced these variations and are faced with the obligation to teach statistics in the school, even though they are self-aware that they are not well prepared to teach statistics or deal with difficulties of their students. Consequently, in this work we identify several recommendations present in the research literature about the aspects where to pay attention to strengthen the initial formation of teachers of mathematics and promote in it literacy, reasoning and statistical thinking.

BACKGROUND

At present, the learning of the statistics and probabilities has been valued all over the world as a basic component to unroll ourselves effectively in the information society, where we must make decisions in situations of uncertainty and participate in an informed way in the public and political debate (Gal, 2002). In this way, in an environment where we become daily consumers of data, increasingly used to add credibility to the numerical information in the media, political and economic decisions, being statistically educated, reasoning and thinking people are valued as an elementary necessity to improve the quality of life, since with these competences it is expected to develop our capacity to monitor and promote social justice (Ben-Zvi & Makar, 2016).

This situation has been considered by several countries throughout the world, which have carried out a reform movement around statistical education in the school, promoting their teaching from the initial elementary levels with basic ideas of discipline and finishing the educational trajectory with aspects of statistical inference (Batanero & Borovknic, 2016). These changes were introduced through various initiatives, for example, in the United States through the documents Principles and Standards for School Mathematics (PSSM, 2000) and the Guidelines for Assessments and Instruction in Statistical Education (GAISE, 2005a and b), it is mentioned that for more than 30 years under the name of *data and chance* it has been incorporated into statistics and probabilities in mathematics K-12 programs. In turn, this trend has been followed in Spain through the Royal Decrees 126/2014 (MECD, 2014) and 1105/2014 (MECD, 2015) which establish the minimum teachings of primary and secondary education respectively, where it has been incorporated to statistics and probabilities from the first year of primary school to the last year of secondary school.

Although these updates have driven much research in the field of statistical education, to continue it is necessary to stop on the way in which the three main ideas of this work will be understood, namely, literacy, reasoning and statistical thinking. In this sense, we go back to the context of the fifth version of the International Conference on the Teaching of Statistics (ICOTS-5) held in 1998 in Singapore, where a group of researchers identified three trends to address their studies, which unfortunately had not referred to the same aspects, so they organized for the following year the first International Research Forum on Statistical Reasoning, Thinking and Literacy (SRTL), where they propose to clarify a differentiation of these terms. As a result of the first two versions of the SRTL, Garfield, delMas & Chance (2003) together with Ben-Zvi & Garfield (2004), made a differentiation between these three notions, associating the following characteristics to each one: (1*) Statistical Literacy* considers the ability to understand and react to statistical and probabilistic information in everyday contexts; (2*) Statistical Reasoning* analyzes from a cognitive perspective the way in which people give meaning to ideas and statistical information; and (3) *Statistical Thinking* refers to the understanding of the overall "statistical process", from asking questions, collecting data, choosing the analysis and the ability to communicate, evaluate and draw conclusions.

THE PROBLEM

Garfield & Ben-Zvi (2007) claim that there seems to be a consensus among educators and researchers about statistics and their teaching, insofar "*literacy, reasoning and statistical thinking are the most important goals for the learning of statistics, at present these are not fully achieved by the students*" (p. 3). Therefore, considering that the success of the new programs and curricular guidelines depends strongly on those responsible for implementing them, we address the problem of how to strengthen their initial training, since they are the ones who fulfill the role of adapting and interpreting the new requirements according to the characteristics and conditions of its students. In this sense, for Ben-Zvi & Makar (2016) the statistical education reform has a pending task in terms of teacher training, since "*many statistics courses at the university level still teach the same progression of content and emphasize the same development of skills and procedures*" (p. 3), where it is necessary to incorporate reflection processes on the nature, role and purpose of the discipline and not only to focus on the procedures but on the promotion of skills related to the professional tasks of the statistician, which are closely related to the stages of the PPDAC research cycle (Wild & Pfannkuch, 1999).

WAYS TO STRENGTHEN TEACHER TRAINING IN STATISTICS

To train a teacher of mathematics that is up to the current requirements for the teaching of statistics, it is necessary that part of its training process incorporate at least one subject dedicated solely to statistical education. For this reason, we take part of the results presented by Pfannkuch & Ben-Zvi (2011), who describe the learning experiences that teachers need to develop their ability to think and reason statistically, establishing five basic principles in the instructional design of this course, which we will complement with the present works in Batanero, Burrill & Reading (2011), identifying two additional aspects that enrich the curricular development of a possible subject of statistics addressed to future teachers of mathematics.

(1) Develop an understanding of key statistical concepts

From the perspective of the disciplinary content, the authors emphasize that this subject must pursue the development of the understanding of the key statistical concepts and their interconnections. In this sense, Burrill & Biehler (2011) based on a review of various theoretical approaches and program of stochastics (European term to refer both to statistics and probability, Shaughnessy, 1992) implemented in several countries, propose seven topics as Fundamentals: data, variability (different sources of variation or dispersion), distribution, representation, association and bivariate correlation, probability and sampling and statistical inference. Therefore, teacher educators in this area should be able to involve future teachers in activities that develop each of the mentioned notions along with their relationships, and to promote their interest in reviewing specific investigations that analyze the role of teacher in teaching them.

In this sense, Pfannkuch & Ben-Zvi (2011) and Batanero, Díaz, Contreras & Roa (2013) identify in the literature a series of relevant elements around the ideas mentioned above, some that deserve to be collected below.

- *Data:* Although this concept is part of the heart of statistics, because it is the raw material that allows us to know the real world and base our decisions on the evidence that we can analyze, it is necessary to understand and/or face the situation to generate them in concrete investigations. To carry out this task, it is necessary to foster the ability to identify how to measure some desired attribute of a problem and how to design data collection methods that avoid making measurement errors, aspects that are not common in traditional statistics subjects.
- *Variation or Statistical Variability:* Given the variable nature of the data in statistics, this is one of the characteristics that differentiate it from the determinism of mathematics (Burrill & Biehler, 2011; Batanero & Borovcnik, 2016). Skills such as understanding, explaining and quantifying variability are fundamental in the resolution of statistical problems and in the decision-making process (GAISE, 2005a). For this reason, teachers must be able to learn to deal

with the different sources of variation (random, measurement, sampling, etc.) or dispersion (the range, standard deviation, etc.) and incorporate them into the design of their lessons. In addition, they must face the challenge of interpreting and teaching the different measures used in the discipline to quantify the variation.

- *Distribution:* To develop the understanding of this idea, it is necessary to learn to reason from the distributions in several senses: (1) a set of data should be explored as an entity (a distribution) rather than as individual and isolated cases; (2) graphically visualize the shape, symmetry with respect to the center and variation of the data distribution; and (3) distributions can be formed from individual data sets (frequency distribution), sets of possible values of a random variable (probability distribution) or summary statistics (sample distributions).
- *Patterns or Representations:* The ability to *"look for patterns in the data is similar to trying to unlock the hidden history within them*" (Pfannkuch & Ben-Zvi, 2011, p. 326) and to achieve this, it is necessary to know different ways of representing the data (for example, graphically, tabularly, in diagrams, etc.), since with each of them you can know different aspects within the history of the observations. In addition, for the interpretation of these patterns, it is also necessary to know the context of the problem in order to notice, decode, evaluate and judge the inherent messages in the representations.
- *Association and Correlation:* According to Batanero et al. (2013), while in the functional dependency to each value of a X variable (independent) corresponds a single value of another Y (dependent), in the study of the association or correlation to each value of X corresponds a distribution of values of Y, so this concept extends the functional dependency. The usual measures for the quantification of the degree of linear association between two variables are the covariance and the correlation coefficient, however, with respect to the second of these statistics, the literature has identified a series of problems with its interpretation, since sometimes its result can suggest the presence of a statistically significant relationship between two variables that have no meaning or do not have any linear relationship, which is known as "spurious correlation" (Lahura, 2003), that is, its value does not provide information about causality between the variables X and Y.
- *Probability:* School statistics programs incorporate three usual approaches to probability: classical, frequentist and subjective, which are only one part of the abstract concept behind probabilities. In this sense, Batanero & Borovcnik (2016) highlight the connection between statistics and probabilities under a frequentist approach to introduce statistical inference, and through the Bayes theorem the connection between the subjective approach to probability is possible (when the frequentist approach does not apply) and statistical data by updating the judgments of probabilities a priori and making them "more objective".
- *Sampling and Statistical Inference:* Pfannkuch & Ben-Zvi (2011) consider inference as one of the great ideas of statistics since within it most of the work is to take a random sample and use it to estimate or make decisions about some parameter or characteristic of the population or process with which you are working. In addition, as in the previous cases, several fundamental concepts surround and sustain it, for example, the sample type and its respective sampling distribution, sampling, population, distribution, confidence intervals and hypothesis contrasts.

(2) Develop the ability to explore and learn from data

In the context of statistics, data is considered as the raw material that allows us to explore the real world and base our decisions on the evidence that we can analyze from them, therefore, it is a requirement in the courses of this subject that real and motivating data are used in the process of learning statistics (Makar & Fielding-Wells, 2011; Chick & Pierce, 2012). In this way, it is recommended that future teachers experience the full cycle of empirical research (PPDAC) through activities that allow them to build the concepts that support statistical thinking and serve as an example in their future professional practice. In other words, they should be able to pose their own questions about the data, collect new information about their context and present their findings through statistically valid inferences and conclusions.

(3) Develop statistical argumentation

This idea is closely related to the topic mentioned in the previous point, since it is necessary to develop statistical argumentation skills throughout the PPDAC cycle, where in its final stage the conclusions must be accompanied by persuasive arguments based on the results of the analysis of the data according to the interest of the problematic situation. In this sense, Ben-Zvi & Makar (2016) highlight the new trend in research related to statistical argumentation, where issues such as linguistic and discursive aspects in the learning of the discipline, the ability to apply knowledge outside of the school and the connection between critical thinking and statistical reasoning are analyzed. Therefore, as for teacher educators, a course aimed at future teachers should develop in them an assessment of the statistical argumentation in the class, where the discourse should be based on giving a foundation about the procedures chosen in the study of the phenomenon in question and in the students' commitment to share their ideas and results with each other.

(4) Change in the evaluative paradigm

Highlighting the importance of evaluation to measure the effectiveness of a teaching process, it is natural to propose other evaluation methods in this new discipline teaching and learning paradigm. In this way, the use of evaluation is advised as a means to provide feedback to the students' learning process, instead of considering it only as a summative measure of achievement. Therefore, future teachers need to be instructed with alternative methods of evaluation, which allow them to collect training information to guide student learning, for example, the use of statistical projects with which the research process could be completely evaluated (Garfield & Ben-Zvi, 2009; Batanero & Díaz, 2011).

(5) Understand the reasoning of students

Within the series of tasks that a teacher should perform in their professional work, when planning teaching lessons they should, among other things, be clear about the purpose of the session and the prior knowledge that students should put into play. However, Pfannkuch & Ben-Zvi (2011) suggest complementing this design through the review of literature related to student conceptions, such as research about the Models of Cognitive Development developed by Jones et al. (2004), understood as a theory that suggests different levels or patterns of growth in statistical reasoning, as a result of the effects of maturity or interactions in structured learning environments or not, to begin to appreciate what would be the expected reasoning of their students and what their intuitions may be, prior to the implementation of the class. Without this knowledge, teachers would continue with the designed program and use resources that do not take into account the way their students think. Therefore, teacher educators should make future teachers aware of the tools that are available in the literature on how they can develop understanding of specific topics in their students and motivate them to use them to design their classes.

(6) Use of technological resources

Technological advances have been a source of change in several aspects of current life, and statistical education has not been the exception, since computers and new technologies completed the "*data revolution*" in this field (Ben-Zvi & Garfield, 2008). Moreover, within the teaching recommendations established by Ben-Zvi & Garfield (2004), it is important to highlight the use of computational tools to automate statistical procedures, an idea they maintain over time, assuring years later that it is necessary to challenge traditional forms of memorization and replace them with forms of active learning, which are usually improved by the use of technologies (Ben-Zvi & Makar, 2016). In this way, teacher educators should appreciate the contribution that computer tools offer by allowing them to learn about modeling, construct models to describe the behavior of data and generate simulations to explore in different possible scenarios (Batanero, Burrill & Reading, 2011). That is to say, in this subject future teachers must know and consider simulation as a tool that allows to visualize a phenomenon of the real world through a mathematical model instead of a model through the data.

(7) Differences between mathematical and statistical thinking

As a result of the tightness between statistics and mathematics, by the formal structures that define them, this discipline has been located within the same curriculum in the school, which means that, within the initial training of future mathematics teachers, these become aware that statistical thinking is something different from mathematics, both being essential in modern society. According to delMas (2004), several aspects of mathematical and statistical reasoning are similar, however, the objectives pursued by the tasks of each discipline can produce different sources of error in the reasoning used. For example, *"in mathematics a counterexample refutes a conjecture, while in statistics a counterexample (a particular case) does not refute a theory of group tendency"* (Pfannkuch & Wild, 2004, p.35).

Finally, delMas (2004) mentions the role of the context within the instruction in both disciplines, however, the practice of statistics depends on the context of the data and in the practice of mathematics, it tends to be used as example or application. Therefore, these differences should be an active source of reflection in a statistics course for future teachers, so that they can improve statistics teaching in a similar way to what they do with mathematics (Batanero, Burrill & Reading, 2011), since that methodology does not adapt to the nature of the discipline.

DISCUSSION

In this study, we have identified the main orientations suggested in the specialized literature to promote the development of literacy, reasoning and statistical thinking in the training of mathematics teachers. In this sense, we have organized around seven topics the aspects in which teacher educators should guide the teaching of statistics and probability to be in tune with the current demands in relation to the discipline. In this way, according to Batanero (2002) the main objective is not to convert future citizens into "*amateur statisticians"*, since the reasonable and efficient application of statistics for problem solving requires a broad knowledge of this subject and it is a competence of professional statisticians. It is not about training in the calculation and graphic representation, since nowadays computers solve this problem. The purpose is to provide statistical literacy as a continuous process that aims to develop statistical thinking. For this reason, future teachers need specific support to help students acquire the ideas related to stochastics, value and recognize the complementary characteristics between mathematical and statistical knowledge, learn to use technology optimally and guide their teaching to students with different skills and previous knowledge (Batanero & Borovcnik, 2016), aspects that should be considered in the teacher's training process, which is expected to be provided through this work.

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