ON THE USE OF KEY WORDS AND VISUAL REPRESENTATIONS IN SOLVING PROBABILITY PROBLEMS

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This study aimed to demonstrate the use of keywords and visual representations in solving probability problems. The research design was descriptive qualitative using the phenomenographic approach, involving fifteen third year undergraduate Mathematics education majors enrolled in Elementary Probability Theory. A 7-item pretest and a parallel posttest consisting of typical probability problems were administered and interviews using the Newman Error Analysis prompts were conducted after each test. Instruction focused on the use of keywords and visual representations. Pretest results showed students committing low-level comprehension errors. Higher-level errors were observed in the posttest with more appropriate and predominant use of keywords and visual representations. The results of the study look promising and using keywords and visual representations as a pedagogical approach in teaching probability is recommended.

INTRODUCTION

Probability problem solving can be quite difficult for students because “people have natural misconceptions about probabilistic concepts” (Garfield and Ahlgreen (1988), Konold (1989) in Corter & Zahner, 2007), more so in the Philippines where English is not the first language. In recognition of this perceived difficulty, articles have been written recommending how to teach concepts in probability.

The main concern that has been raised, though, is whether the in-service and pre-service elementary and secondary school teachers are prepared to teach this subject matter. Batanero, Godino & Roa (2004) pointed out that, in general, in-service teachers are not adequately trained to teach it. Batanero and Diaz (2012) identified specific issues regarding the training and preparation of teachers to teach probability. Their opinion is that correct and adequate preparation of the teachers, as well as their belief that probability is important for their students to learn, contribute to the effective teaching of probability. However, as Reston (2012) pointed out, many pre-service programs in teacher education do not provide adequate training for this.

This study looked into the possibility of addressing this gap by exploring how the use of key words and visual representations could help improve the pre-service teachers’ conceptual understanding of probability through problem solving. Specifically, this study aimed to determine if there was improvement in the level of performance, use of key words and visual representations, and levels of error at the end of the school term.

METHODS

The study used the descriptive qualitative method utilizing the phenomenographic approach, designed “to answer questions about thinking and learning” (Marton, 1986; in Ornek, 2008) in order to probe how pre-service teachers experienced understanding and constructed new knowledge when they were taught the concepts of probability.

Fifteen third year students of the Bachelor of Secondary Education (major in Mathematics) program of the College of Education in a higher education institution in Bacolod City who enrolled in Elementary Probability Theory were included in the study. All students were female.

Three instruments were utilized. The first two were the teacher-made (or modified) pretest and posttest containing 7 questions each on classical and complementary(#1), and relative frequency(#2) probabilities, addition rules for mutually(#3) and not mutually exclusive events (#4), multiplication rules for independent (#5) and dependent events (#7), and conditional probability (#6). Most questions were of the typical variant. The pretest and posttest items were constructed so that they were generally isomorphic and were randomly arranged. These were subjected to content and face validity. The third instrument contained the Newman’s Error Analysis or NEA (Newman, 1977, in Ellerton and Clements, 1996) prompts and rubric. These were used in the individual pretest

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and posttest interviews for error analysis, to determine the level at which a student's problem solving process had broken down for each problem.

In addition, the use of key words and visual representations (Venn diagrams, tables, pictures, trees, patterns) were given emphasis in the instruction and were ascertained from the test papers and the individual interviews. The following key words were emphasized, together with their visual representations and the formulas for solving the required probabilities:

- $A \cup B$ ⇒ (either) A or B will happen
- $A \cap B$ ⇒ (both) A and B will happen
- A' ⇒ A will not happen
- $A - B$ ⇒ A will happen but B will not happen; only A will happen
- $A \oplus B$ ⇒ either A or B will happen, but not both
- $(A \cup B)'$ ⇒ neither A nor B will happen
- B/A ⇒ B will happen, given that (or, if) A has happened

Data analysis included the comparison of the error profiles for both pretest and posttest based on the NEA rubric. These were considered to be a good measure of the improvement in student performance. Pretest and posttest interviews using the NEA prompts provided additional insight into the solving process of the student and the tools that were used, like keywords, which were not captured from test papers. More importantly, the interviews provided a glimpse into the knowledge gaps that could have existed in the minds of the students.

RESULTS AND DISCUSSION

Pretest results showed that the pre-service teachers had very little knowledge of probability at the beginning of the semester. Except for the most basic question (Item 1) wherein 7 students got the correct answer, mostly intuitively, the rest of the questions were considered difficult, with only one correct answer each for questions 2, 3 and 6. Posttest results, however, showed increase in the number of students getting correct answers, except in Item 7, where only one student got the answer correctly. All got item 1 correctly, while items 2 to 6 got from 4 to 7 correct answers.

There was minimal use of either key words or diagrams/trees in the pretest items. However, posttest results and interviews using the Newman prompts showed increased application (Figure 1).

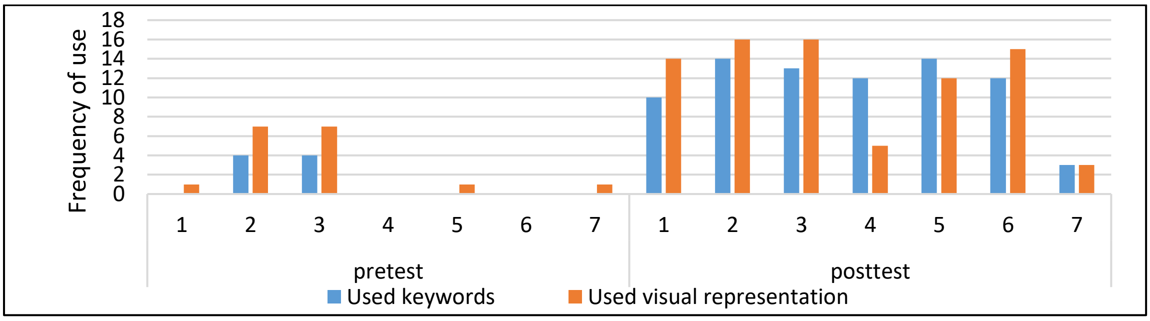

Figure 1. Comparison of the frequency of the use of keywords and visual representations between the pretest and posttest

Table 1 compares the levels of error committed by the students per item per type of test (pretest vs. posttest), based on the NEA. The items were presented according to increasing levels of difficulty. No student committed a reading error in both pretest and posttest, but the interviews and written results showed that the common error in the pretest was *comprehension*. Because the NEA error levels are hierarchical, those who committed higher level errors were those who were able to hurdle lower levels. For example, less students committed *transformation* error because many already had difficulty with *comprehension*. More students made mistakes as the level of difficulty of the test items increased.

Considering the common errors in the pretest, improvements were generally observed for all items in the posttest. All surpassed the *comprehension* error level, except for Item 7 (Multiplicative Rule for Dependent Events); with 26.7% of the students still having that difficulty.

Process errors tended to dominate from Items 2 to 5, but as the level of difficulty increased, so did the number of students committing the lower-level *transformation* error. This could imply that the students knew what they were supposed to do, but they could not identify the procedure or process that they needed to apply in order to successfully solve the problem.

Table 1. Comparison of errors committed between the pretest and posttest, by item

	Level of Error															
Test Item	Reading		Compre-hension		Transfor-mation		Process		Encoding		No error		Careless Error (X)		Total	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
1	0	0	4	0	3	0	0	1	1	0	7	14	0	0	15	15
2	0	0	7	0	6	2	1	5	0	1	1	7	0	0	15	15
3	0	0	7	0	6	2	1	7	0	0	1	6	0	0	15	15
4	0	0	7	0	8	5	0	5	0	0	0	5	0	0	15	15
5	0	0	10	0	5	6	0	5	0	0	0	3	0	1	15	15
6	0	0	12	0	1	7	1	1	0	0	0	7	1	0	15	15
7	0	0	14	4	1	9	0	1	0	0	0	1	0	0	15	15
Total	0	0	61	4	30	31	3	25	1	1	10	43	1	1		

The results presented in Table 1 agree with the study of Ellerton and Clements (1996) that different questions produced quite different error patterns. The authors further indicated that the high percentage of *comprehension* and *transformation* errors in studies using the Newman procedure in different contexts has provided "unambiguous evidence of the importance of language in the development of mathematical concepts." This raises the difficult issue of what educators can do to improve a learner's comprehension of mathematical text or his ability to identify an appropriate sequence of operations that will solve a given problem.

Figure 2 illustrates how a student's solution to a conditional probability problem showed progression in the posttest. In the pretest, the student simply used classical probability in solving the problem without considering the given information and did not attempt to use representation.

Her posttest solution involved formal definitions of the events and a presentation of the given information in tabular form. The student explained her solution thus: *"…because I made a table, Miss, it is easier to answer. So the given is female, so I based my solution on the part of the*

POSTTEST: A school surveyed the favorite snacks of 1000 of its students. Of the 625 male students who were surveyed, 200 liked ice cream, 125 preferred pizza, while 300 chose hamburger. Among those surveyed, 300 liked ice cream, and 450 chose hamburgers. A student is selected at random from the group. If it is known that the student is female, find the probability that she likes pizza.

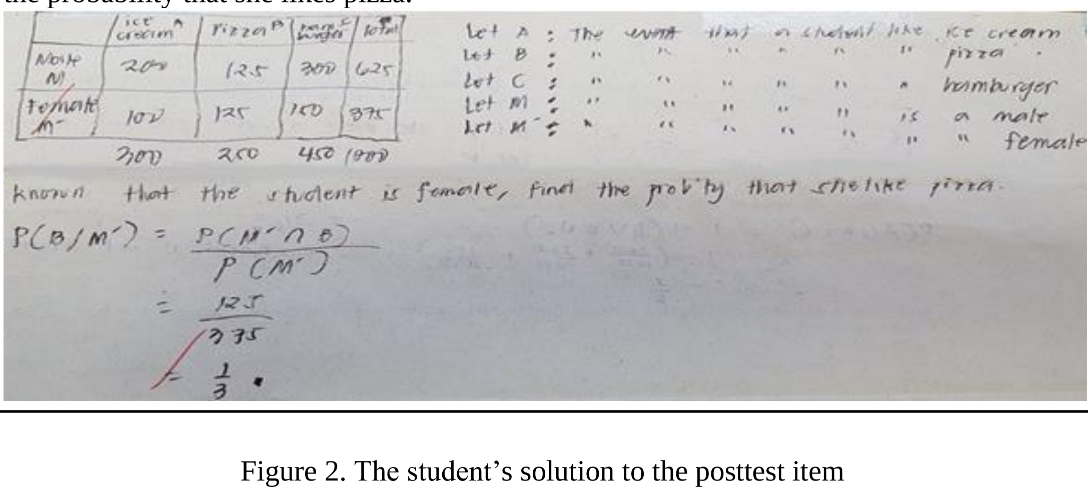

Figure 2. The student's solution to the posttest item

table corresponding to the female, with a total of 375. So I used…(paused)… the formula is the probability of B which is the probability of(sic) she likes pizza given of(sic) M prime for female. So probability of M prime intersection B over Probability of M prime, so the answer is 1/3 because 125/375. 125 for the intersection of M prime and the pizza and 375 for the population of female."

The results of a focused group discussion indicated that these pre-service students believed that classroom discussions and activities have, to a certain extent, prepared them for their eventual teaching responsibilities, especially for probability. They specifically mentioned the use of keywords and visual representations as important pedagogical tools.

This study corroborates in part the results of the study conducted by Dollard (2011). He emphasized that mathematics teacher educators, specifically those teaching probability, cannot assume that the pre-service mathematics teachers enter their classrooms with adequate knowledge of the subject matter. This is true in this study where the knowledge level of the students concerning some concepts of probability as measured by the pretest was almost nil or at most intuitive.

CONCLUSION

The pre-service students had very little knowledge of probability at the beginning of the semester as represented by very low scores in the pretest. The use of keywords and visual representations was limited.

Posttest results showed increase in the number of students who got correct answers for each item as well as the number of correctly-solved items per student. As observed from the written output as well as from the interviews, the methods used in solving the problems were more appropriate, and there was intentional use of the keywords, Venn Diagrams, and tables. They have improved in terms of the category of errors that were committed (from lower- to higher-level). Making the students more aware of the importance of keywords contributed to the movement towards higher-level (and no) error categories.

In general, the students achieved a fairly good grasp of classical, relative frequency, complementary probabilities and the addition rule for mutually exclusive events. They had a moderate understanding of the concepts of the additive rule for non-mutually exclusive events, the multiplicative rule for independent events, and conditional probability. However, there still seems to be a struggle to understand the concepts of dependent and sequential events.

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