EXTENDING INTRODUCTORY PROBABILITY THROUGH GAMES

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US undergraduate honors programs (and school gifted programs) often supplement mathematics and statistics instruction through small topic-specific seminars. Games of chance are a good supplemental topic, since games have historically driven innovation in probability, yet many concepts do not require calculus or heavy prerequisites. This paper summarizes lessons learned from offering a one-hour-per-week seminar to honors undergraduates simultaneously taking introductory statistics. The seminar included lessons on counting methods, expected value, player choice, player interaction, and game theory. Each lesson in mathematical probability was introduced through a game played by the students. After the lessons, the final group project involved analyzing a game. Games played are included, along with adaptation suggestions for other school levels and cultures.

BACKGROUND ON HONORS PROGRAMS

In the United States, honors degree programs have different requirements than in many other countries. Instead of awards based on examination grades, or additional study after completing a non-honors bachelor’s degree, most US honors colleges incorporate extra requirements within the bachelor’s degree plan. At least 90 US universities offer honors degrees (Powell, 2017), including the authors’ current university, Oklahoma State. Universities benefit by attracting students who might otherwise attend more highly ranked colleges, students with high achievement levels and high future donor rates (Campbell, 2005).

Honors programs have been found to lead to increased student success both in and out of the classroom (Hébert & McBee, 2007; Seifert, Pascarella, Colangelo, & Assouline, 2007). Students receive multiple benefits. Many students appreciate the benefit of early enrollment, allowing students to secure seats in high demand classes quickly and find a schedule that works best for them. Some classes offer sections open only to honors students, often taught by a professor from the honors program—generally selected due to relatively high teaching ability. Many universities, including Oklahoma State, provide honors students a unique housing option, with private study areas, quieter environments, and like-minded peers. Graduating students that have completed the program receive valuable notations on their diplomas.

Honors programs incorporate a few requirements. The programs have higher initial eligibility standards than the general university. At Oklahoma State, new first-year students are assured admission with a 3.0 high school grade point average (GPA) out of a possible 4.0 and a 21 ACT test score (“Admissions Requirements – Freshmen Students”, n.d.), but the honors college requires a 3.75 high school GPA and a 27 ACT test score (“Eligibility Information for New Freshmen”, n.d.). Having entered the program, a minimum college GPA must be maintained each semester. Additionally, students must complete a certain number of honors credits each semester to maintain active status. At Oklahoma State, students must complete at least 3 honors credits each semester (“Active Status in The Honors College”, n.d.).

Honors credits can be earned in several ways. Some three-credit seminars are offered within the honors college. In larger courses, students can take a section open only to honors students, earning honors credits equal to the number of course credit hours. Another way to receive honors credit is to take an honors add-on that complements a typical course. Honors add-ons meet for one hour per week, but successful completion earns 3 honors credits. Many students choose this option because they can take the non-honors course and then simply attend one additional honors class. Finally, if a course doesn’t offer honors sections or add-ons, a student may work with the professor to create an honors contract; the professor will assign the student an additional project, with successful completion earning honors credit.

This paper describes our experiences with a honors add-on course within the Statistics department; the first author was the instructor and the second author a student in fall 2017. Before 2016, the Statistics department offered a special section of introductory business statistics. Because

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most students take non-business statistics, the department had to manage many honors contracts. The honors college and the dean's office asked the department to switch to an add-on model that would complement any introductory statistics course. In fall 2016, a special seminar was offered on polling and political surveys, described elsewhere (Molnar, 2018). Although the polling course was very successful, the political polling activity of an US election year would not always be available. The honors add-on needed a topic that would work at all times for all variants. The Statistics department offers multiple introductory courses—business, engineering, general, and scientific fields. Topics differ, but each variant includes a small section on probability, to support the study of randomness specified in the 2016 GAISE college guidelines. Games of chance are a classic historical application of randomness (Bernstein, 1996). Although studying games of chance might suffice, the honors college would like an add-on course to extend beyond typical examples in the regular class. One possible extension was game theory, since expected value is a statistical topic and academic game theory expands on expected value. Combining probability and game theory would allow participants to study board and card games where players interact with other players. This had mathematical and entertainment potential, leading to the add-on titled "Games of Chance and Skill."

STRUCTURE OF "GAMES OF CHANCE AND SKILL"

Games studied generally involve both probabilistic chance elements and skill-based decisions during the game. Many common games are not included. Coin flips, roulette, and many other probabilistic games do not require in-game decisions; chess, go (igo), mancala, and many other interactive games do not include substantive elements of chance. Additionally, to increase the influence of game theory, skill decisions were divided into decisions without inter-player interaction, such as finding expected value given known probabilities, and decisions with interaction, such as bets in poker. The most interesting games, including those in class projects, include chance, decision-making skill, and player interaction.

The add-on course has been offered in the spring 2017, fall 2017, and spring 2018 semesters. Depending on the semester, the course has consisted of 12 to 15 50-minute class meetings on Friday afternoons, with a 110-minute exam session at the end. The honors college requires that add-ons have no more than 22 students; the class has had 18, 12, and 20 students. Grades are assigned on the US A, B, C, D, F scale, with grades of A and B receiving honors credit. Marks are determined based on the best 8 of 10 short weekly assignments (5% each, 40% total), a group project (35%), and an end-of-course exam (25%). The grading scale is deliberately tilted towards high grades, because participants are relatively strong students taking an optional program. Through the first two semesters, all students have received honors credit.

Major content topics appear as bullet points below, in the current course order. Although it might make sense to begin with probability, the course begins with objective decision-making for two reasons. First, accompanying courses begin probability around week 4; covering probability at the same time reinforces topics for the students. Second, the games played in objective decision-making illuminate the key parts of the course.

- Objective decision-making: Expected value, optimal strategy, fairness
- Probability and counting methods: Counting principles, permutations, combinations, conditional probability
- Game theory: Pure vs randomized strategy, payoff matrix, dominant/dominated strategies, cooperation, utilitarian decision making, Pareto equilibrium, pure-strategy Nash equilibrium
- Player interaction: Subjective probability, bluffing, betting
- Analysis of board and card games using earlier tools

Probability Games

Each major content topic has at least one in-class game associated with that topic. Choosing topical activities is tricky, because the game must play within 20-30 minutes to allow discussion time, and have some entertainment value. Additionally, the instructor has tried to choose games similar to games played outside the classroom. The current list of games by course topic follows; more information on any game can be obtained from the first author.

- Objective decision-making: Inside-Outside (based on Acey-Deucey), SKUNK (Brutlag, 1994)
- Probability and counting methods: One card poker (based on Gordon, n.d.)
- Game theory: Snowdrift game (Doebeli & Hauert, 2005), Number Choice game (competition game in "Nash equilibrium", n.d.)
- Player interaction: Liar's Dice (simplified version of Perudo), four-card double-draw lowball poker (simplified version of Ace-to-five triple-draw lowball poker)
- Analysis of board and card games: Love Letter (Kanai, 2012), UNO (Robbins, 1971)

Example game: Inside-Outside

This is a variant of a card game played on the US television show Card Sharks (Alter, 1978). Similar games are played as gambling games, including a version in Liaoning province, China ("Acey Deucey", n.d.). In the course structure, this game is the first activity played, in week 2.

Students play in pairs, one as dealer and one as guesser. Each pair of players has one standard US deck of playing cards, with 4 cards for each of 13 ranks (from low to high—Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King). The dealer randomly places two cards on the table, such as a 7 and a Queen. The guesser must then choose from three options for a randomly dealt third card. Correctly guessing the result earns 1 point.

Tie: Match the rank of either card, in this example another 7 or Queen
Inside: Have a rank between the two cards, here 8, 9, 10, or Jack
Outside: Have a rank not between or tie, here Ace, 2, 3, 4, 5, 6, or King

In class, the guesser plays 10 rounds, then dealer and guesser switch and the former dealer guesses for 10 rounds. Explaining the game takes about 5 minutes; having pairs play takes about 15 more minutes. After all students have taken a turn as guesser, the student(s) with the highest score receives a very small prize from a discount store, such as a candle, dog toy, or pack of crayons. Students find the prizes highly amusing.

Class discussion occurs after awarding the prize. The first question is about strategy – how the winner made choices, then what other players did. It's important not to criticize someone for being sub-optimal; participation is more important. The next question is about determining optimal strategy for a specific situation, the highest probability choice in the example that introduced the game. In the example above, there are 50 cards left after dealing 2 cards. Six cards will tie – the remaining three 7s and three Queens. Since randomized cards follow classical probability, P(tie) = 6/50. Sixteen cards will win with a guess of Inside – four 8s, 9s, 10s, and Jacks. P(inside) = 16/50. Laws of probability then show P(outside) = 1 – 6/50 – 16/50 = 28/50. Outside is optimal.

Sometimes the optimal strategy emerges quickly; other classes need more hints. The optimal strategy for Inside-Outside is based on the difference in ranks between the first two cards. For the version where every correct choice is worth 1 point, the key value is rank difference 7. If the two cards are less than 7 ranks apart, like Queen (12) – 7 = 5, Outside is optimal. For 7 or more ranks of difference, Inside is optimal. Tie is never best. With this knowledge, it is possible to find the probability of guessing correctly under optimal strategy. A computationally-based class might receive this as an assignment, but for this add-on providing P(win) = 69.6% is better.

The weekly assignment, due at the start of the next class, reinforces probabilities from this class while also introducing expected value. Not all questions are mathematical, since strategy is also part of the course. Listed below are the four questions assigned in the Fall 2017 assignment.

1. When you played the game in class, what was your strategy (in a sentence or two)? Don't worry if you didn't have the optimal strategy; I just want you to explain yourself.

2. Say the first two cards in a round of Inside-Outside are a 4 and a Jack [so distance = 7]. Compute the probabilities of Inside, Outside, and Tie.

3. Imagine that I gave you the following choice: Get $10 for free OR Play a round of Inside-Outside and get $18 if you are correct, but $0 if you are incorrect. Which one would you choice, and why? (There's not a right answer here. This asks about a person's risk tolerance, and people have different risk tolerances.)

4. In the class version of Inside-Outside, Tie is never the optimal choice. Say that the value of a correct choice for Tie becomes 5 points; correct Inside and Outside choices

are worth 1 point. Under this new scoring system, when is Tie the optimal choice?

CLASS PROJECTS

Because games have interaction between multiple players, the class project is completed in groups (though groups of size 1 are allowed). Each group must select a card or board game that includes all three areas explored in the course—chance, decision-making skill, and player interaction. The group must play through the game at least once. The group then prepares a poster with basic information about the game; one image from the game in action; how the game uses chance, skill, and player interaction; the most enjoyable and least enjoyable parts; and a recommendation on what type of person would enjoy the game. An accompanying written report includes two worked examples of probability that occurred during the game, detailed analysis of a key decision made during the game, and more on the relationship between area components. During one of the last class sessions, each group brings their poster and presents on their game for about 5 minutes.

The fall 2017 class chose a mix of card and board games: card game Go Fish, card game Mille Bornes, card game Munchkin, board game Clue (also known as Cluedo), board game Monopoly, and board game Catan. The second author's group played Mille Bornes (Dujardin, 1954). Since the full posters were quite large—close to 3 feet (90 cm) by 4 feet (120 cm), about ISO A0 size—Figure 1 displays only the chance and enjoyability sections of their poster.

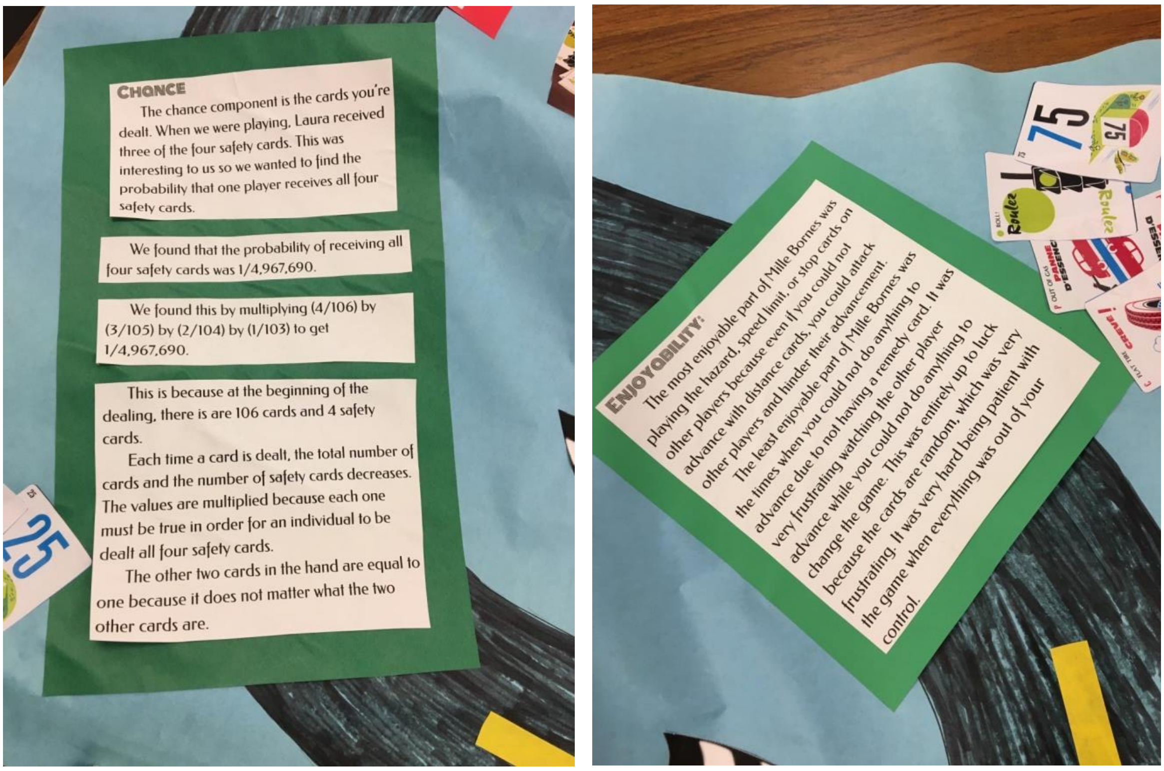

Figure 1: Portions of the second author's Mille Bornes poster.

DISCUSSION

The 2016 GAISE guidelines for teaching statistics emphasize active learning, problem solving and decision making. This add-on course does all those things. Games are active learning with decision making; having students search for optimal results is problem solving. Student participation is high; enjoyment levels are high; the Honors College is pleased. This active learning approach comes with cost, of course. Because the weekly assignments reinforce points from class discussion, they cannot be designed in advance. Creating assignments is done after each class session, which requires substantial instructor time late each Friday afternoon. The first author, the instructor, spends more time each week on this 1-hour class than on a 3-hour introductory general statistics course with typical textbook and homework support.

Logistics and Tactics

Classroom flexibility requires excellent detail management, as military strategist Eccles (1997) wrote: “The essence of flexibility is in the mind of the commander; the substance of flexibility is in logistics.” Logistical errors have consequences. For example, the spring 2017 section met in a classroom with movable chairs, but the fall 2017 section was assigned a room with fixed seating. For games with more than two players, groups often wound up sitting on the floor, a less-than-ideal position. The spring 2018 section was moved back to a movable chair classroom.

Tools of randomness also need to be considered. Distributing cards, dice, and betting chips takes time, so an instructor should determine how to distribute and collect tools efficiently. Students might need to collect the day’s tools as they enter a classroom, for instance. Tool size also matters. The instructor purchased 12 decks of cards and 100 dice sized for classroom use. The initial betting tokens, on the other hand, were heavy and difficult to carry across campus to class. A lighter set of tokens had to be purchased.

An important pedagogic decision concerns demonstrations before student game play. Doing so reduces the amount of discovery learning, but increases coherence and time for discussion. The Oklahoma State class has shifted towards demonstrations, because discussion time has increased in value over gameplay discovery. When playing, students tend to focus on trying to win, not reflective analysis. It’s been better to allow students to play first, then move into strategy discussion. This has required more discussion time, and thus less un-demonstrated discovery.

Whole-Class Reflection

Students and the instructor have found the class enjoyable, despite the substantial preparation time commitment. More importantly, some students have made the desired connection to life. To quote a student comment from fall 2017, “I think it allowed me to think about statistics in a way that is more applicable to my life. I definitely think it helped me in my regular statistics course by being able to think about the concepts more concretely with my own examples.”

Nevertheless, there is room for improvement. One problem is topic order. Originally, topics progressed from objective decision making to interaction to probability to game theory. There was no connection between game theory and interaction, plus probability appeared too late to help students in their statistics classes. Probability was shifted earlier, but game theory content still felt separate and incoherent. Attempts to fix this gap continue. For instance, in spring 2018 the game theory section added maximin selection to connect cooperative games to Pareto equilibrium. Later, bluffs in Liar’s Dice, an interactive game, were modeled with a decision matrix. It may take another semester or two to find a coherent topic list.

Any new initiative, such as this class, can be questioned on sustainability and scalability. Can this class transfer to new instructors, and more instructors, or will it fail when the initial instructor leaves? Once the topic list is settled, other instructors should be able to lead the add-on, making the course sustainable and scalable on demand. Plans exist to publish a course plan booklet to help other teachers. (For now, materials are available from the first author.) On the other hand, extending this add-on model to classes of 50, 100, or 500 students is difficult. Distributing game tools and conducting strategy-style discussion would take either a substantial commitment to smaller recitations, or a great deal of time and control inside a large hall.

Adaptations

Students at Oklahoma State tended to choose project games familiar to them; students in other cultures will prefer other games. Nonetheless, there is substantial capacity to include games from other cultures. In the current course, Inside-Outside is similar to a Chinese card game (“Acey Deucey”, n.d.); Perudo, the basis for Liar’s Dice, was developed from a Peruvian game (“Perudo History”, 2007); Mille Bornes was developed in France. Student projects could include the Korean game Yutnori, Egyptian game Senet, or Japanese game Jinsei. Modifying individual in-class games is also possible, such as using months from Japanese Hanafuda cards as ranks in Inside-Outside.

Secondary school teachers could use most of the games when teaching probability concepts. For instance, Brutlag’s 1994 article on SKUNK appears in a journal for middle school teachers, grades 6–9. Several games for elementary-aged children have been successfully used as project games, including Go Fish, Battleship, and I Doubt It. The most complicated activity is lowball poker, where complicated conditional probabilities make it difficult for children below grade 8 or 9. The game theory section stands alone; even though the mathematical computations are less complicated than probability counting principles, many classrooms don't have time for optional topics. Secondary schools with time could easily include a unit on game theory.

Many instructors might not have class time for all the games, but are interested in a single activity. For this case, the first suggestion is to personally play the game, because a confused instructor cannot help students find their way. After personally understanding the game, thoughts about logistics should come next. The add-on course has flexibility, but most statistics courses would suffer from 20 minutes of waste on a poorly executed activity. Finally, it is important to remember to reinforce the probability lesson. Yes, games are fun, but a probability game shouldn't be only an amusement; as shown in this course, games can help students discover and learn valuable concepts.

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