YEAR 13 STUDENTS' REASONING FROM AN EIKOSOGRAM: AN EXPLORATORY STUDY

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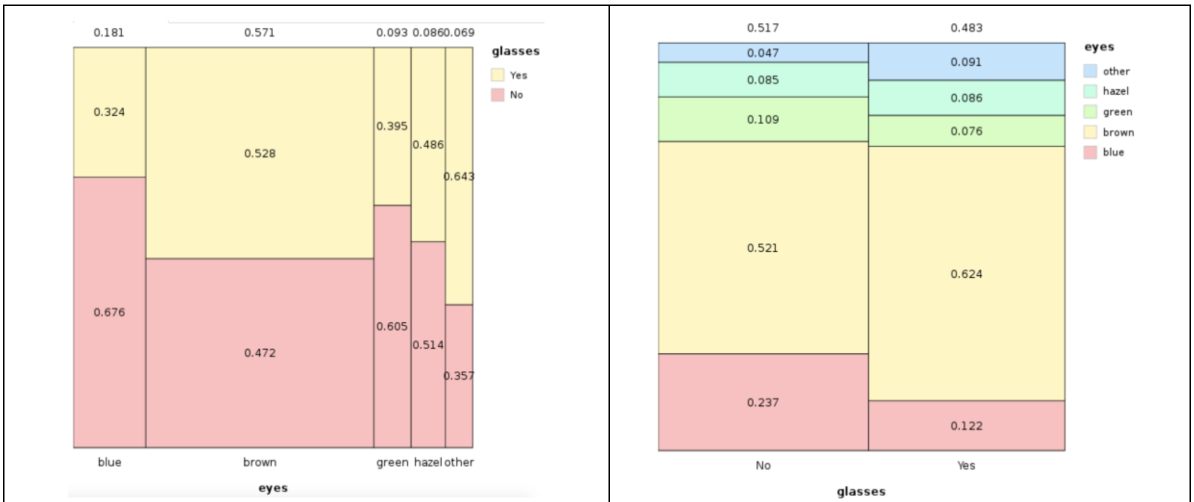
Education research on probability frequently raises concerns about the lack of effective strategies for teaching and learning probability. Using an interactive eikosogram, a visual representation of a two-way table of information, we explore Year 13 students' reasoning, interaction and comprehension behaviours associated with the display. In this paper we focus on four students as they explored data on their own. Although the students had difficulties posing investigative questions and reasoning from the eikosogram, we conjecture that the eikosogram may have the potential to assist proportional reasoning and the verbalization of simple, conditional and joint probability stories from data. To elucidate the reasoning that can be stimulated when students interact with an eikosogram we present an eikosogram graphicacy framework.

INTRODUCTION

Developments in technology are facilitating new ways of visualizing and interacting with categorical data. Traditionally, categorical data has been displayed in bar charts and tables of counts or proportions but now displays such as eikosograms (Oldford & Cherry, 2006), which are similar to mosaic plots (Friendly, 2002), are being created to answer questions, communicate information and assist in reasoning about multidimensional data. Despite the proliferation of these new representations, students' ability to reason from them has been neglected in research. Therefore, there is a need for research on how students might interact and reason from displays such as the eikosogram in order to learn what reasoning processes might be promoted or hindered.

The eikosogram was proposed by Oldford and Cherry (2006) as a way of studying probability because, unlike Venn diagrams, it was semantically consistent with the rules of probability. The eikosogram is built on a unit square representing a probability of one with non-overlapping rectangular regions representing events with areas matching probabilities. Pfannkuch and Budgett (2016) built on their ideas by creating an interactive eikosogram that could display categorical data showing visual proportions with no data, counts, or proportions conditioned on the horizontal axis with the ability to swap the other factor to the horizontal axis (see Figure 1).



Proportions conditioned on Eye Color	Proportions conditioned on Wear Glasses (Yes/No)

Figure 1. Eikosogram representations for Eye Color and Wear Glasses (Yes/No)

Pfannkuch and Budgett (2016) trialed their interactive eikosogram on six first-year university students who had completed an introductory probability course and hence the students

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were familiar with using mathematical procedures, tables of counts, or tree diagrams for calculating conditional and joint probabilities. Their findings suggested that the eikosogram might have the potential to assist students' "proportional reasoning, ability to compare proportions, consideration of proportions in both the horizontal and vertical dimensions, the unlocking and verbalization of simple, [and] conditional and joint probability stories from the data" (p. 283). They also developed a framework for these students' comprehension of the eikosogram based on Friel, Curcio and Bright's (2001) proposed construct for behaviours associated with graphicacy. Based on the findings of Pfannkuch and Budgett's (2016) study the interactive eikosogram was further developed to include one-dimensional displays, and joint proportions.

Hence the aim of this study was to trial, in a small exploratory study, an improved version of the eikosogram (online tool currently under development) on students who were not familiar with using mathematical procedures or two-way tables for conditional and joint probabilities in an effort to learn how one might introduce students to these ideas using the eikosogram. In particular we were interested in how students would interact and reason from the eikosogram, including identifying any issues that arose in the learning sequence and in the software.

RESEARCH STUDY AND PARTICIPANTS

The participants were eight Year 13 (final year of school) mathematics students, aged 16 and 17 years, from a co-educational school in New Zealand. There were two female and six male students. All students had experienced probability at the junior level, Years 9 and 10 (age 13-14), and therefore, should have had prior knowledge of some basic probability concepts. Furthermore, two of these students had experienced probability at Year 11, which covered simple conditional probabilities, and two had covered joint and conditional probabilities from two-way tables at Year 12. Hence, these students had limited understanding of conditional or joint probability.

The data were collected over a three week period in four one-hour sessions that consisted of a pre-task and a main task. The participants worked in pairs with the researcher, the first author. The purpose of the pre-task questions was to facilitate students' engagement in the eikosogram task by asking them to make predictions and to lead their thinking to the types of questions, context, reasoning and representations required. The questions were about eye-color and gender proportions in the New Zealand population. The main task involved students interacting with the eikosogram under the researcher's guidance then exploring data on their own. The students were encouraged to think aloud when discussing answers to questions or telling stories from the eikosogram.

The focus of this paper is on the second part of the main task when students were asked to explore data *on their own* using the eikosogram. Note that this part is an initial exploration where the research method is similar to a pre-clinical trial. The expectation was that the students would choose two available variables (sex, eye color, glasses, purchase alcohol, tattooed, pierced) from data collected in a web survey of 408 university statistics students, pose an investigative question or make a prediction, and then answer their question. Hence the research question addressed in this paper is: How do these students reason from an eikosogram when exploring data *on their own*?

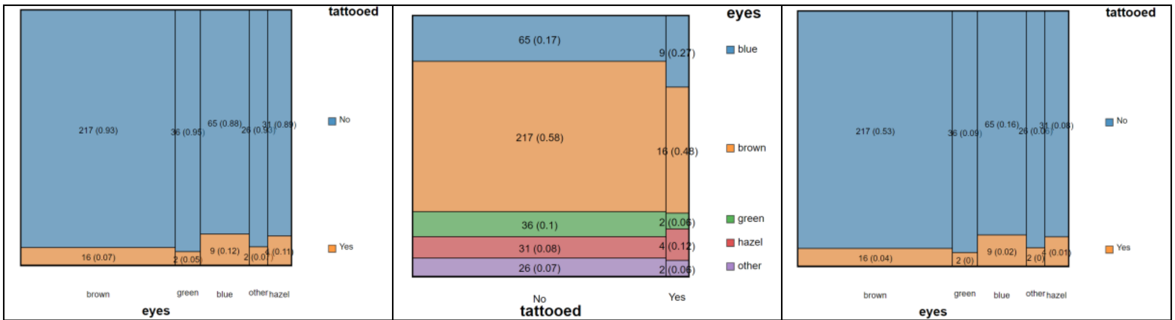
RESULTS

The second part of the main task found many issues related to students exploring the data on their own using the eikosogram. When the researcher posed the question and guided them to look at the correct representation, the students seemed to be able to interpret and verbalize the stories in the data. However, without guidance, gaps in their understanding became evident which included posing investigative questions, identifying the correct eikosogram representations and values to answer the questions, and using proportional reasoning. In order for the reader to understand these three issues experienced by all the students we focus on two pairs of students, Dan and Poe, and Sid and Kat.

Two examples of how students explored data

Students made their predictions or posed questions based on beliefs using language related to frequencies. Students' beliefs were from their own personal experience of everyday life. For example, Dan and Poe were exploring the variables 'Eye Color' and 'Tattooed'. Dan stated: "brown-eyed, they're probably Polynesians and most Polynesians have their cultural tattoo on them". He was associating brown-eyed people with Polynesian ethnicity and cultural practice. He then predicted, "I reckon brown-eyed color will have more tattoos". However, Poe predicted that more blue-eyed people would be tattooed. Figure 2a shows the first eikosogram representation they displayed. Poe's first reaction was, "See, I told you [pointing at the blue eye column]. Blue-eyed people." He noticed that out of all the eye-color columns the 'Yes' bar for blue eyes had the largest proportion (0.12) therefore he believed that out of those who have tattoos the majority have blue eyes. Dan immediately responded, "I don't get it that way", and he quickly clicked on 'Swap factors' to change the condition to 'Tattooed' (Fig. 2b). As soon as Poe saw the frequency of nine blue eyes compared to 16 brown eyes he said, "Nah jokes, never mind." His answer to his prediction now changed and blue eyes was no longer his answer.

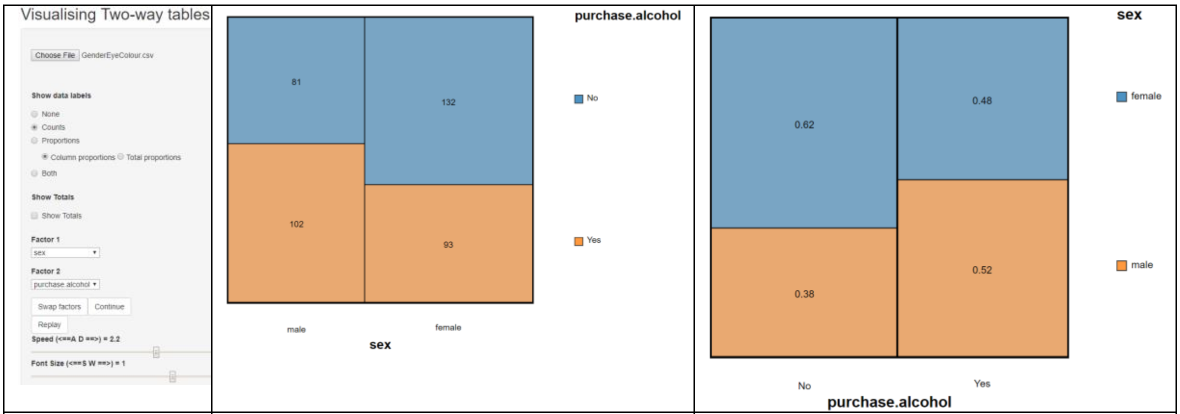
Dan was not happy with the area proportions of 'Yes-Tattooed' to 'No-Tattooed' and began to argue about the unequal number of brown eyes in the Yes/No category. However, as he saw that the proportion of 'Brown Eyes/No' to 'Brown Eyes/Yes' was 58% to 48% he stopped. It seemed that he did not know what to make of those values. Dan and Poe again changed the representation to the one in Figure 2a. Poe pointed out again that the proportion 0.12 was why he thought blue eyes were bigger. Dan then removed all proportions in Figure 2a to show the counts only as he did not seem to trust the proportional values. Lastly, Dan showed the joint proportions (Fig. 2c). This time he concluded, "2% to 4%, yeah. I said brown eyes", as he compared the 'Yes' bars of brown eyes to blue eyes to confirm his original prediction that more brown-eyed people would be tattooed. Hence, confusion about the actual question asked and about what representation to choose to answer the question is evident.



(a) Conditioned on Eye Color | (b) Conditioned on Tattooed | (c) Joint proportions

Figure 2. Eikosogram representations used by Dan and Poe

Sid and Kat chose to explore the variables 'Sex' and 'Purchase Alcohol'. Kat's question was, "Like do more males purchase alcohol than female?" and Sid's prediction was, "Yes, more males". From Sid and Kat's further discussion about this question, we were convinced that their actual question was, "Of those who purchase alcohol, are there more males or females". They chose to answer their question with Figure 3b. According to our interpretation of their question, this is an incorrect representation because it is conditioned on 'Sex' rather than 'Purchase Alcohol'. Sid responded to Figure 3b by saying, "Yeah more males purchase more alcohol than females" and Kat agreed. They were both satisfied with reasoning from frequencies to answer their question and comparing the frequencies of the two rectangles, 'Yes to purchase alcohol given male' and 'Yes to purchase alcohol given female'. Kat confirmed this interpretation saying, "the survey says out of the male population there's 102 that purchased alcohol and out of the females there's only 93", but she did not realize that it was not answering their investigative question. The researcher suggested that they display proportions so they clicked on the column proportions button (Fig. 3a). Again, with the proportions conditioned on 'Sex', they were convinced that more males purchase alcohol than females. They then swapped the variables so that 'Purchase Alcohol' was the condition (Fig. 3c). The researcher asked whether this was a better representation for their question. Both Kat and Sid replied that this representation showed the same result but they preferred Figure 3b as it was easier to read. Similar to the previous example, it was evident that Kat and Sid were confused about the actual question asked and about what representation to choose to answer the question.



(a) Controls | (b) Conditioned on Sex | (c) Conditioned on Purchase Alcohol

Figure 3. Eikosogram representations used by Sid and Kat

Three main issues identified when students explored data

Students were mainly *posing investigative questions* and predictions that were comparative and conditioned on a sub-group. They were interested in comparing two or more sub-categories of one categorical variable in terms of the sub-categories of another categorical variable. For example, comparing each eye color to see who is 'more/less' tattooed. A typical prediction was, for example, Dan's prediction, "I predict that male is ahead of female." What was missing from these predictions was the condition under which they were comparing. Similarly, posed questions used the same type of wording and incomplete verbalizations, for example, "do more males purchase alcohol than females?" This type of question was unclear as to which relationship they were referring to: a particular category or the whole sample. Only when students started discussing the data, it became apparent that their questions were conditional. That is, they were wanting to compare sub-categories under a certain condition. They were most interested in the prominent group under a conditioning variable. Because the verbalizations of questions were incomplete, it was difficult to determine whether their questions or predictions were conditional or joint. Students also made predictions based on frequencies in the sample of data. For example, Poe predicted that there would be more female than male out of those purchasing alcohol because "it's just because there's more females in there" referring to the number of female in the data.

Students had problems in *identifying the correct eikosogram representation to answer their questions*. To answer their questions, they needed to make three choices (i) which variable to put on the horizontal axis; (ii) which eikosogram representation to use (column/total); and (iii) which value(s) to reason with in the representation (frequency/proportion). When answering their posed questions, students arranged the variables on the axes in the way that was, according to them, "easier to read". Some students preferred the arrangement with a fewer number of columns on the horizontal axis and some with a fewer number of rows on the vertical axis. There were also some students who preferred to always place the sex variable on the horizontal axis as it was, according to them, easier to see things clearly. As a result, students displayed a representation they saw as being easier to read but not necessarily the correct one to answer their question. In the eikosogram the conditioning variable is always on the horizontal axis so when students swapped axes the conditioning variable changed resulting in values changing for the column proportions, an aspect the students did not seem to understand. With incorrect arrangement of variables on the axes and choice of values according to their posed questions incorrect interpretations were evident.

*Choosing the correct values to reason from and using proportional reasonin*g was difficult for students. They did not seem to know what values (frequency, conditional or joint proportions) to reason from and which ones could give them the answer to their questions. In the first part of the main task it was noted that the students lacked proportional reasoning skills. Some students seemed to prefer to reason with frequencies rather than proportions, as shown in the following excerpt.

> Researcher: So which one would you look for, for comparison? Do you look at the numbers, do you look at proportions?

Dan: Proportion.
Poe: The number.
Dan: No I go more proportion.
Poe: I look at numbers like the first thing I read, left to right.

As they explored data on their own, they continued to reason with both frequencies and proportions. Those who were reasoning with proportions often reverted back to frequencies when interpretations got complicated such as interpreting conditional proportions. They seemed to be more comfortable with whole numbers and did not seem to realize that reasoning with whole numbers led to incorrect reasoning when comparisons were involved. As a result, incorrect reasoning and conclusions were made and some questions were left unresolved.

The eikosogram graphicacy framework

The consequence of these findings was to redefine graphicacy for the eikosogram as "the ability to derive meaning from the eikosogram representations as well as constructing appropriate eikosogram representations for questions posed and making meaning from data that are explored." Two key comprehension components and student behaviours to allow for exploring of data were added to the beginning of the framework (Table 1).

Table 1: Abbreviated framework for eikosogram graphicacy (cf. Friel et al., 2001)

Comprehension Components	*Student behaviours*	*Eikosogram descriptors with main code*
1. Posing investigative questions	Recognize there are different types of questions	Types of questions (Q): Simple, Conditional, Joint
2. Selecting appropriate representation for questions	Identify representations that will answer questions posed	Types of representations (R) :Arrangement of variables, Column proportions/frequencies, Total proportions/frequencies
3. Visual decoding	Recognize the components and their interrelationships	Visually Decode (VD) the variables in: one-dimension, two-dimensions
4. Judgment tasks	Verbalize the language specific to the display when reasoning about information	Verbalize Language (VL) for: Simple probability, Conditional probability, Joint probability
	Select appropriate values for reasoning about the questions posed	Types of values (V): Frequencies, Proportions
	Interpret information at four interrogation levels	Levels of Interpretation (I): Extract information, Find relationships, Infer relationships, Suggest contextual factors
	Understand the relationships between and within displays	Link Representations Between (RB) and Within (RW) eikosogram(s)
5. Context	Be aware of one's relationship to the context and understand the constraints of the context when making an interpretation	Contextual and Statistical (CS) knowledge used together for sensible interpretations: Some or little evidence of contextual knowledge but lack detail, Comparing data evidence against beliefs

The first comprehension component is *posing investigative questions* of which there are three types: simple, conditional and joint. The second comprehension component is *selecting appropriate representations for questions posed*. Depending on the type of question posed, variables for two-dimensional eikosograms first need to be selected in the correct arrangement. Subsequently, the correct representation needs to be selected: column proportions/frequencies, if the question is conditional; and total proportions/frequencies, if the question is joint or simple.

The next three components are based on the work of Friel et al. (2001). Hence, the third component is *visual decoding* which refers to how people make sense of what they are looking at when seeing visual displays. Student behaviour that demonstrates evidence of visual decoding is the ability to recognize the components and their interrelationships in one-dimensional and two dimensional eikosograms. The fourth component is *judgment tasks* that requires students to engage with the eikosogram to seek information, and involves four student behaviours (see Table 1). Hence, students are expected to use specific language and appropriate vocabulary associated with the task and the visualisation tool. The fifth component is *context*, which requires students to be aware of and question data issues (e.g., data quality, survey questions, and to what population an inference can be made) and their interpretation of data in context and their beliefs about the context. As stated in the framework, contextual and statistical knowledge should be used together for sensible and meaningful argument as it can provide evidence against beliefs.

CONCLUSION

The research question addressed in this paper was about how these students reasoned from an eikosogram when exploring data on their own. What we found was the central importance of posing suitable and productive investigative questions, a finding similar to Arnold (2013) and Gould, Bargagliotti and Johnson (2017). Typically, students are presented with a two-way table of data and asked questions posed by a teacher or assessor such as "Suppose a person is drawn at random, what is the probability of A, given that B has occurred?" Students are not often asked to articulate and verbalize all the possible investigative questions they could pose when confronted with a two-way table or asked to explicitly write down clear unambiguous questions. Furthermore, our findings confirm both Arnold and Gould et al.'s call for more attention to be paid to developing teachers and students' skills in posing questions. Explicit questions can then feed through to facilitating students' ability to draw and construct statistically sound conclusions. The students in this study had difficulty fully verbalizing their conclusions as did the students in Pfannkuch and Budgett's study who had completed an introductory probability course.

These students showed confusion of the inverse, a well-documented misconception (e.g., Gigerenzer, 2014) and difficulties differentiating between conditional and joint proportions. There was also evidence that they had not made a key transition in their statistical reasoning to understand that proportions rather than frequencies should be used for comparing groups. From this small exploratory study trialing the eikosogram on novice students, we realized that a much longer learning trajectory needs to be created to develop students' reasoning with categorical data displayed in two-way table formats. Central to the learning trajectory will be posing unambiguous questions. More modifications are being made to the eikosogram such as the ability to break it down into bar graphs and to visually show proportions with one variable morphing into proportions with two variables. Despite these students' difficulties with the eikosogram, we still think that it has the potential to enhance student learning. More research, however, is needed on how students interact with categorical data displayed in eikosograms and mosaic plots.

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