STATISTICALLY DEEP LEARNING BASED ON THE INVESTIGATIVE CYCLE OF A SIXTH-GRADE ELEMENTARY SCHOOL STUDENT

Mitsuru Matsushima
Kagawa University, Japan
m-matsushima@ed.kagawa-u.ac.jp

This paper considers what kind of competency is enhanced in a PPDAC cycle for sixth-graders. PPDAC (Wild and Pfannkuch, 1999) is a statistical inquiry process introduced on the revision course of study in Japan in March 2017. By analyzing lessons in statistical analysis, I explored sixth-graders' competencies to compare two data sets using average value, and to discuss other analysis features as well, including spread of data, mode, and maximum and minimum values. Students also compared reasoning based on data with reasoning not based on data. Further, they recognized the importance of reasoning based on data.

INTRODUCTION

The most recent national course of study for elementary and junior high school students in Japan was released in March, 2017. It contained a number of important outlines for mathematics education, including applying mathematics to everyday life, expressing and processing problem solving mathematically, and metacognitive reflections on the process of mathematical thinking. These outlines are designed to improve deep learning and practical application of mathematics, for example in understanding statistics (Japan Society of Mathematical Education, 2014). The statistical research process "PPDAC" (Wild and Pfannkuch,1999) was applied to more than fifth-grade elementary school students, but Japan's mathematics educational research does not have much accumulation of practice research on the PPDAC cycle (for example, Aoyama and Ono, 2016). Elementary school mathematics teachers in Japan aim to enhance students' competencies. In this paper, I consider what kind of competency the unit "how to investigate data" enhances in sixth-graders in an elementary school.

THE VIEWPOINT OF A LESSON DESIGN

In the lesson study of this paper, I aimed to have all the children experience a series of PPDAC cycles and achieve mathematical deep learning. The acronym PPDAC stands for Problem in school days, Plan to solve it, Data collection, Analysis of data, and Conclusion of a problem-solving result based on their analysis (Wild and Pfannkuch,1999). I established a lesson plan based on the following four points.

1. Children identify an issue with school life, and experience a series of PPDAC cycles in the class.
2. When children compare two kinds of data, the teacher sets up time for them to consider comparison methods other than average values.
3. The teacher sets up an activity in which students compare reasoning based on data and reasoning not based on data.
4. The teacher sets up time for children to discuss the validity of data-based and non-data-based reasoning.

THE OUTLINE OF A LESSON STUDY

Sixth-graders sometimes use their independent daytime periods to develop committee activities, such as performances and events. One year, the life improvement committee planned and carried out a "flying airplane convention." The convention was held in the gymnasium and designed to provide younger students with activities they could perform on rainy days.

Some days after the event, the sixth-graders decided to perform the same event. They too did most of their work during classroom activity time, and organized the convention mainly through the life improvement committee. They explored various questions, such as: "Which flies a longer distance, a complexly structured paper airplane or a simple paper airplane?" Further, "Does the angle that the plane is thrown at change the distance it is able to fly?" I then constructed an instruction unit around the first question and used this to expose the students to the PPDAC cycle. The lesson study plan of the unit was as follows. Ten mixed-gender groups were created in the class, with each group comprising four

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children. The creation of these groups did not consider differences in ability. Groups worked together every day, and each one addressed the same question: whether a paper airplane flies further depending on whether it has a simple or complex structure. Following is the time schedule of the PPDAC cycle.

Plan: The 1st hour as classroom activity: Children wrote down the plan to fly and tested two kinds of paper airplanes.

Data: The 2nd hour: Collecting data on the distances the airplanes flew.

Data: The 3rd-4th hour: Creation of a frequency table and a histogram, which expresses the results of the investigation.

Analysis & Conclusion: The 5th hour: Analysis of the frequency tables and histograms, and conclusion.

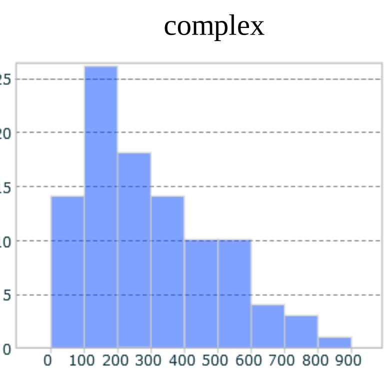
The children in the class hypothesized that a simpler airplane would fly a longer distance. In the first hour, they discussed and determined a plan based on the following two points.

1: Develop a common understanding of a complex and simple method of making a paper airplane, and a unified understanding of how to throw an airplane. Subsequently, a few children create 20 airplanes by the common method.

2: Throw each of the two kinds of airplanes 100 times, then collect and analyze data on how far the airplanes flew.

In the 2nd hour data were collected. Between the 2nd hour and the 3rd hour, the teacher entered the data in a table. The table was shown to the children at the beginning of the 3rd hour. In the 3rd and 4th hours, students used calculators to create the average value, frequency table, and histograms of two kinds of data. At the end of the 4th hour, they had decided to use average value to prove their hypothesis. In the 5th hour, they produced an analysis without using average value in each mixed-gender group which was organized for the 1st hour. They argued about results in the group. The following figure shows the frequency tables and histograms, which the students created in the 3rd-4th hour.

distance(cm)	Frequency
0-100	14
100-200	26
200-300	18
300-400	14
400-500	10
500-600	10
600-700	4
700-800	3
800-900	1
total	100



distance(cm)	frequency
0-100	0
100-200	2
200-300	9
300-400	18
400-500	11
500-600	17
600-700	27
700-800	12
800-900	2
900-1000	0
1000-1100	2
total	100

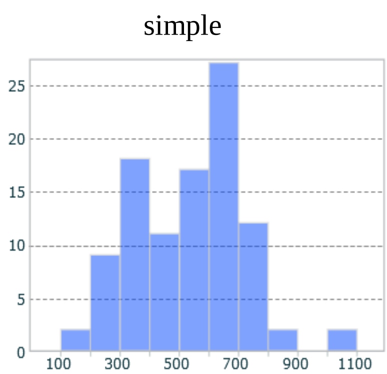


Figure 1. Frequency table and histogram of a complex plane and a simple plane

In the 5th hour, when students produced an analysis not based on average value, they focused instead on mode, spread of distribution, maximum value, minimum value, and median. These view-points were not provided by the teacher but were arrived at by the students themselves. Below is an example of the conversation the students had regarding maximum and minimum values and spread of the distribution.

Child A: I looked at complex data first. Data mainly inclines toward left-hand side --

Teacher: What is inclined?

Children: Graph? Scale? What looks like a stick? Number of times? Pillar-shaped graph? Stairs? Mountain? (Many children asked these questions)

Teacher: Can you have an image of a mountain? Please explain using the word "mountain."

Child A: Yes. The complex graph has a summit of a mountain, which is on the left-hand side of the graph. Next, the simple graph has a summit of a mountain, which is on the right-hand side as compared with the complex graph. In a graph, since the right-hand side numerical value is large, I think that the simple paper airplane flew further.

Teacher: This is how you see the graph. Then, let's listen to the explanation of the 2nd and 10th groups. Child B please.

Child B: I will explain. Please see the data on the 100 flights. A complex airplane most often flows 816 cm, whereas a simple airplane most often flies 1050 cm. I think that if we assume a simple airplane will fly the same distance in the future, a simple airplane will fly a longer distance than a complex one.

Teacher: Please describe your explanation once again. How would you answer if you were explaining based on a histogram?

Child B: Let me see...

Child C: I will help B. At first, the largest flying distance is 800 cm to 900 cm, on the right-hand side of the complex graph.

Teacher: What was the exact distance?

Child C: It was 816 cm. Then, the right-hand side of the simple graph was 1000 cm to 1100 cm, specifically 1050 cm. Since the maximum for the simple paper airplane was 1050 and the complex one was 816, I think that a simple paper airplane flies farther than a complex one.

Teacher: What kind of view is this view, in other words?

Children: When it will be next flying, think by the case of the most flying.

Child D: What? If it is right, I may be able to compare also with the minimum.

Teacher: This is very interesting. Do you compare the minimum value? Now, you check the minimum value in graphs and tables.

Children: The minimum of a complex airplane is 0 cm. A simple airplane is 120 cm.

Teacher: The simple airplane with the greater minimum value and the greater maximum value flies further than the complex one. It is possible to compare by the minimum value, too.

At the end of the discussion of each group, it was found that the 8th group of students had resorted to reasoning that was not based on data – they hypothesized that “the airplane with the largest wingspan will fly the greatest distance.” The presenter for the 8th group began by saying, “Although not based on data…” The other groups who heard this suddenly erupted in noise. Several of them said at once, “Is it related to an experiment?”, “Something is strange,” and so on. These exclamations show that the class had become less confident in reasoning that was not based on data. Then the teacher carried out the following discussion with the students:

Teacher: What should we do in order to prove the 8th group’s reasoning that “the airplane with the largest wingspan will fly the greatest distance.”

Children: It is necessary is to fly an airplane with a large wingspan, and an airplane with a small wingspan, and then compare them.

Teacher: That is correct. This opinion of the 8th group has not been proved as yet.

DISCUSSION

I think that the results of this study indicate two main points about mathematical deep learning.

(a) Children can compare two data sets easily using average value. It is also possible for them to conduct such comparisons through an analysis based on other methods, such as maximum and minimum value or distribution spread, and most effectively through dialogue based on the PPDAC cycle.

(b) Children can be conscious of the importance of reasoning based on data. An important step in cultivating this awareness is having them critically compare reasoning based on data with non-data-based reasoning.

Arguably, sixth-graders are able to easily compare two data sets based on average value because they have used this value extensively in previous grades. In their next course of study, the sixth-graders will learn about analysis in which average value is not as efficacious a tool; either the teacher will do this or the students will be asked to come up with examples themselves. This research indicates that even if a teacher does not provide any examples, such as the maximum or minimum value, the children will be able to develop the analysis themselves. Thus, one of the research-result is that, through the unit "how to investigate data", students’ competency for developing a new measure was improved. Also, it is important that children recognize the value of developing hypotheses based on data. In this lesson study, some students were willing to present reasoning not based on data. In such cases, the teacher should not criticize them but should encourage them to analyze arguments for and against various methods to prove the proposed hypothesis. If the argument for the data-based method succeeds, the children will gradually recognize the importance of reasoning based on data. This perception on the part of the children is one competency that creates a new measure for considering data, such as the maximum or minimum value.

It is not difficult for elementary school statistics students to apply the PPDAC to real-world problem-solving processes. While children can understand the mathematical contents, they can also develop their data-analytical competencies.

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