DEFINING THE RELATIONSHIP BETWEEN STATISTICAL THINKING AND STATISTICAL COMPUTING

Victoria W. Woodard
Department of Mathematics and Computer Science
Meredith College, Raleigh, NC 27607
webervic@meredith.edu

As the demand for data scientists to analyze data has increased, using advanced statistical computing tools in the classroom to teach appropriate statistical techniques has grown more popular. As such, introductory statistics classrooms are replacing the tools designed to "teach" statistics in favor of the tools meant to "do" statistics. As with any technology change, this may affect how a student learns the concepts in their field. In particular, we have little knowledge of how students will think statistically while using these new tools. Through a qualitative study of written work and task-based interviews of students in a second course in statistics, this research study begins to define the relationship that exists between a student's ability to think statistically while utilizing statistical computing tools.

BACKGROUND

With the greater demand for data scientists, statistics professors have been teaching and requiring more computing in their classrooms (Baumer, Cetinkaya-Rundel, Bray, Loi, & Horton, 2014; Horton, Baumer, & Wickham, 2014). While previous research has shown that using software that is menu driven and dynamic, such as TinkerPlots, helps students to be able to think and reason statistically (delMas, Garfield, & Zieffler, 2014; Watson, 2014), there has been little research done to see how command based statistical computing tools, such as R, will affect a student's ability to think and reason statistically and how students use the tools to solve statistical problems. As such, the purpose of this research was to start defining these relationships and open the door for other researchers to expand upon this work.

FRAMEWORK

A framework was needed to classify student work that would allow us to evaluate the relationship between statistical thinking and statistical computing. Using the work of Wild and Pfannkuch (1999) which addresses five types of statistical thinking, supplemented by the work of other authors (American Statistical Association, 2016; Ben-Zvi & Garfield, 2004; Chance, 2002; Lee, et al., 2014; Makar & Rubin, 2009; Pfannkuch & Wild, 2004), a framework for identifying statistical thinking was created.

No such framework was readily available to identify statistical computing in students' work. Current research in the field mainly focuses on what could be taught in a statistical computing course (Gentleman, 2004; Hardin, et al., 2015; Nolan & Temple Lang, 2010), but no succinct collection of the affordances that the tools give to students was found. Thus, a framework to help define students' statistical computing actions was needed. One of the main ideas that this framework tries to convey was inspired by the work of Nolan and Temple Lang (2010), which is that the true purpose of statistical computing is about the journey, not the destination. Both the statistical thinking and the statistical computing frameworks used in this research can be found in Table 1.

METHODS

To begin understanding the relationship between statistical thinking and computing, an IRB approved qualitative study was conducted. 14 students from a second course in statistics participated. As is the case in many situations, students came into the course with different backgrounds. Specifically, some students had no knowledge of the program R, so it was necessary throughout the semester to teach the basics of the software, while also teaching the statistical materials.

As part of the assessment for the course, students completed 8 writing assignments where they needed to use data and statistical computing software to argue their decision about a statistical question. Students used Woodard's (2016) four step structure (answer the question, give relevant facts, state the implications that the facts imply, state the conclusion) to complete the assignments.

Table 1. Framework for identifying statistical thinking and statistical computing in students' work

	Framework Aspect	*Actions the students might take*
Statistical Thinking	Recognizing the need for data	• gives a statement about the data in the interpretation of the results • is able to identify if proper data collection techniques are used • recognizes that anecdotal evidence is not enough to conduct statistical inference
	Transnumeration	• is able to think about the variables in the study and is curious of ways to represent them • use exploratory data analysis tools for exploration, and not just prescribed methods • uses multiple representations to interpret and make sense of the data
	Considerations of variation	• is conscientious that all data is variable • is able to redesign an experiment to reduce or eliminate variability • is able to quantify and explain variability to help them understand statistical inference
	Reasoning with statistical models	• knows which type of model to use for specific types of data • knows when a specific model is appropriate to use for a given data set • understands theories underlying statistical models
	Integrating statistical and contextual	• is able to plan data collection and analysis based on contextual knowledge of the data • is able to make more reasonable conclusions using the context of the data. • avoids unnecessary statistical jargon in their interpretations of the data
Statistical Computing	Automate computational procedures	• creates multiple graphs or summaries to make sense of the data • uses the technology to conduct the analysis, then makes proper inferences
	Increase computational thinking	• creates a solution strategy and can communicate it to the computer • is able to think critically or abstractly about concepts the technology is demonstrating
	Offer new methods to explore concepts	• does not follow a prescribed set of procedures to answer a question • comes up with their own methods and reasons for using the technology in analysis
	Aid in pattern recognition and decision making	• recognizes patterns, either in the way things are coded or in how the results behave • uses the output from the technology to help decide where to go next in the analysis

After the course was completed, 5 students also volunteered to participate in task-based interviews that necessitated the use of a statistical software to solve a problem utilizing ANOVA. The statistical and computational backgrounds of these students beyond the statistics 2 course are described in Table 2, below. Students are ranked by what the researcher believes is the most experienced (1) to the least experienced (5) as far as total knowledge of computing and statistics. These rankings are based on information gained from the interview and a demographic survey as well as observations made by the researcher while the students were in class.

ANALYSIS

To analyze the data for the writing assignments, students' work was segmented by the elements of the four-sentence structure. Instances of statistical thinking and statistical computing from the framework were then identified in students' work and were coded using whole segments of their work. Relationships between segments were then identified, in particular, those that identified a link between statistical thinking and statistical computing. These helped to identify the ways in which the different statistical thinking and statistical computing constructs worked with and led to each other. Five themes were identified from this process and will be discussed more below.

The task-based interviews were also coded for instances of statistical thinking and statistical computing as defined in the framework. Additionally, using Lee and Hollebrands's (2006) problem solving phases framework, students' work was segmented into partitions by these phases. This was done to see how students use statistical computing in problem solving. In addition to the phases cited by Lee and Hollebrands, a 7th partition was also created: *research*, which was used to account for the time students spent on the Internet, researching how to solve a problem. Figure 1 shows how the five students spent their time in the different problem solving phases.

Table 2. Interviewee experience, and statistical/computational knowledge rankings. Pseudonyms used.

Name	*Experience*	*Statistical Rank*	*Computational Rank*
Allison	Statistical Computing in R, tutored Intro Stats, DataFest	1	1
Cora	Statistical Computing in R, Intro Statistics grader	2	3
Genevieve	Psychology Statistics course which included SPSS.	3	4
Moira	Computer science major with SQL experience	5	2
Samantha	Intro Stats independent study to learn R	4	5

Figure 1. Time that students spent in the different problem solving activities during the task-based interviews

RESULTS

In the writing assignments, five main themes were identified. These themes shed some light on how students use statistical computing to solve statistical problems and the ways in which statistical thinking and statistical computing impact each other.

Students' written work does not always provide evidence of statistical computing.

Some students did not have any evidence of statistical computing in their written work. This was found because of two different reasons. The first was that students were not using the computer to help them solve the problem. In particular, this occurred in the third writing assignment, which was the first that students completed on their own. The other artifacts that did not demonstrate statistical computing were because of a misclassification pitfall in the framework. Often times it was obvious that students were using the computer to provide graphics to help with the analysis (potential computational thinking), but these graphics were coded as transnumeration. This was because students would supply graphics for their arguments but did not discuss the use of the computer in their creation.

Students' arguments use statistical computing to provide Facts, not Implications.

The use of Woodard's (2016) four step structure made it possible to see how students view the computer when using it to help in creating statistical arguments. At no point while coding the students' written documents did students utilize statistical computing in the implications of the argument. It was often found when students were presenting facts and occasionally in the conclusion.

Students' written arguments may demonstrate correct statistical thinking and statistical computing.

We would ideally like our students to fall into this theme – appropriately utilizing the computing tools and thinking statistically about the results. 30 of the 91 documents collected from students fell into this theme, giving us a chance to see how students properly use statistical thinking and statistical computing in conjunction with each other to solve statistical problems. It was found that statistical thinking led to, worked with and was the product of statistical computing, demonstrating that good statistical thinking and computing are highly intertwined.

Students sometimes apply incorrect methods or have incorrect computational thinking in their evidence for arguments.

There were only two types of incorrect statistical computing identified in students' work. These were incorrect computational thinking and incorrect new methods. Students were found to have incorrect statistical computing when they found an example program and were able to use it properly but were unable to accurately describe what the program was doing. Students were also coded as having incorrect computational thinking if they had a strategy for programming a solution to the problem but had an error in their program that they did not realize was there. In both instances, students were able to think and reason about the results they found, even if they weren't completely capable of utilizing the software to produce results. When incorrect new methods were observed, students did not know what they needed to program to solve the problem and went online to find code they did not know how to use or what it did. This resulted in the students creating incorrect solutions and having incorrect statistical thinking about the problem.

Each of these cases had something in common. None were preceded by any statistical thinking. It appears that if students do not think statistically, or at least do not write about thinking statistically, before they utilize statistical computing software, they are more likely to produce incorrect statistical computing in their work.

Students' correct application of statistical computing does not always lead to correct statistical thinking due to pitfalls in student's work.

Not all students who had correct statistical computing produced correct statistical thinking. Though, it was not the correct computing that caused the incorrect thinking. There were several factors that were identified between the time the output was produced by the computer and the thinking that took place that influenced these students. These pitfalls included students relying heavily on the capabilities of the computer to get a solution, being too trusting of the results they obtained, interpreting the output from the software so that it would confirm suspicions they already had about the data, or having a poor statistical knowledge base and subsequently drawing improper conclusions from the output that was obtained.

The interviews gave a different perspective on how students use statistical computing technology to solve statistical problems. While the writing assignments provided an introduction to how statistical thinking and statistical computing work together, the results of the interviews focused more on how the students solved problems, considering the students' computational and statistical backgrounds. We can think of the results presented below as a ladder for solving statistical problems.

Students used the software to automate computational procedures

The highest rung of the ladder involves students being able to *automate the computational procedures* necessary to solve the problem. It was quite clear from the results that the interviewee with the most experience both statistically and computationally, Allison, was able to use the software with the most ease to solve the problems. While her method wasn't perfect, she was able to solve the problem, and in some places automate the computational procedures to do so. In fact, she was the only student who was able to automate procedures in any capacity during the interviews. While Allison is no expert in solving statistical problems, her interview suggests that she is on her way to becoming one.

Students created a problem solving strategy when unsure of how to program

When students were initially unsure of how to program the software to reach a solution to the problem, some were able to come up with a strategy that would eventually lead them to a solution. Evidence of this process can be seen in Figure 1, when reviewing the problem solving steps that Moira and Cora, (ranked 2 and 4 in overall knowledge, respectively) took to solve the problem. Unsure of how to convey to R to run certain statistical procedures, or of how to format the data properly in order to do the procedures, these students would first research a method on the internet, think about how they could convey that solution to R, implement that strategy and then use the error codes or output that the software gave them to assess what they needed to do next. They were using their incorrect *computational thinking* in a guess and check process to eliminate potential programming strategies to solve the problem. In regards to the problem solving phases, these two students spent a lot of time in in a cycle which included *research, planning, implementing* and *assessing*. While sometimes one of the steps was omitted, they tended to use this strategy a lot and in rapid succession, and it eventually helped both of them to come to a solution for the problem.

Students spend a lot of time researching how to do a problem

The lowest tier of the ladder involves students that did not know how to program the software to solve the task-based interview problem and had no problem solving strategy to aid them in finding a solution. This was the case for both Genevieve and Samantha (ranked 3 and 5). Reviewing Figure 1, we can see that both of these students spent a lot of time on the internet *researching* a strategy to solve the problem. This resulted in these students making little forward progress for the problem on their own. In a completely opposite dynamic from Moira and Cora, it was as if they were afraid to try something on the computer and learn from it. This could be due to a lack of experience in using the computer, or it may be that their problem solving strategy in a general sense was to ask for help when they were stuck, as opposed to figuring it out themselves. Both students asked a lot of questions in their interview, and Genevieve was verbally distraught that I would not answer those questions.

DISCUSSION

Recall that a new framework for assessing statistical computing in students' work was created for this research. When comparing the results of the written work to that of the interviews, it was noted that there were some discrepancies in the type of statistical computing that were identified. In particular, in the results of the written work, automation for computational procedures was by far the most commonly cited affordance of statistical computing with computational thinking being a distant second. However, when reviewing the interviews, computational thinking was by far the most commonly cited affordance of statistical computing and automation was the least cited.

This tells us that the statistical computing framework may not always be accurate when reviewing students' work that has been completed in hindsight. When students write about what they did, they can leave out key details of the process that might help to assess how the statistical computing tools were used. However, the framework did seem to accurately identify students' use of statistical computing affordances when it was utilized in real time, with the task-based interviews. This confirms Nolan and Temple Lang's (2010) understanding of statistical computing in that it is not about the destination it's about the journey. We can see the journey students took to solve their problem in the task-based interviews, but we lose much of what the students did when we assess their writing.

Even with the discrepancy in coding for written work, the results found above should still hold in a general sense. When students utilize both good statistical thinking and statistical computing they can provide sound arguments to statistical problems. Incorrect statistical computing does not necessarily mean that a student is unable to think statistically about a problem. Even with minor flaws in their *planning*, or *implementing*, then they can still think statistically about the problem and the results they found. It is when the student is lost and has no strategy to solve the problem that they struggle to think statistically. This was evidenced in the writing assignments when the students didn't show any statistical thinking prior to showing poor statistical computing, and in the interviews when Samantha and Genevieve became frustrated with programming, hindering their ability to think statistically about the problem.

This research demonstrates that when students are expected to learn statistical theory and the computing techniques to solve those statistical problems in the same course, some students may struggle to succeed. Teaching statistical theory and computing simultaneously is not impossible though. Students that had strong statistical or computational backgrounds prior to the course were able to demonstrate a method to solve the statistical problems while using technology, after the course was over. Requiring these courses as a prerequisite may help students to succeed. An alternate solution comes from Horton et al. (2014) as they showed that teaching statistics with technology can be done, but at the cost of teaching less of the statistical content and focusing on activities that help students increase their statistical and computational knowledge.

This research also shows us that statistical thinking and statistical computing are not independent of each other. In order for our students to be successful problem solvers in statistics, they need to be strong and confident in both statistical thinking and computing. If students are uncomfortable with the technology or have weaker statistical backgrounds, statistical problem solving may be confusing and frustrating, which means enforcing the use of statistical computing software before students are ready to see it may be more of a hindrance to their ability to learn statistics, than it is a help.

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