

THREE PARADIGMS TO DEVELOP STUDENTS' STATISTICAL REASONING

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This article is a reflection on my statistics education research that aimed at understanding the emergence of young students' statistical reasoning in authentic data investigations in 'Connections' – a technology-enhanced learning environment. Three main evolving paradigms that guided us are described: EDA – children investigate sample data they have collected without making explicit inferences to a larger population; ISI – children make inferences informally from sample to population; and Modeling – children use computerized tools to model the phenomenon they study and draw random samples from the model to study probabilistic ideas and improve their model. In each of these three paradigms, a short rationale, an example of learning outcomes, and learned lessons are provided. Current statistics education challenges are discussed in light of these paradigms.

INTRODUCTION

The teaching and learning of statistics has been a focus of research in many areas. A majority of these research studies (reviewed by Garfield and Ben-Zvi, 2007) suggest innovative ways of teaching and learning statistics that differ from the traditional classroom practices. This article offers three foundational paradigms (approaches or models) to teaching statistics in primary school: EDA, ISI, and Modeling that can be used by statistics educators to a) better conceptualize their enterprise; and b) as starting points of theory and design for meaningful learning of statistics.

Statistical reasoning is presented first, followed by a description of the 'Connections' learning environment – the setting of the studies described in this article. This will be followed by the three paradigms that guided the design and analysis of the learning trajectories and present three stages in the development of Connections. For each paradigm, a rationale, a quick outline of the intervention, empirical example, and lessons learnt from that stage are provided. Current challenges of statistics education conclude the article.

Statistical Reasoning

Statisticians and statistics educators are challenged to carefully define the unique characteristics of statistics and in particular, the characteristics of statistical literacy, reasoning and thinking (Ben-Zvi & Garfield, 2004). Statistical reasoning, which is this article focus, is the way people reason with statistical and probabilistic ideas, consider how to collect data, create, select and interpret sets of data, graphical representations, and statistical summaries and models, and make sense of statistical information embedded in context. Deep conceptual understanding of important statistical concepts, such as, distribution, variability, covariation, sample and sampling, modeling, and inference, is essential for statistical reasoning that involves connecting one statistical concept to another, understanding and being able to explain statistical processes, or may combine ideas about data and chance to make inferences and interpret statistical results (Garfield, 2002).

The Connections Project

The design and research Connections project is an inquiry- and technology-enhanced learning environment built upon the foundations of socio-constructivist theory (e.g., Cobb, 1994). The project began in 2005, with the goal of developing young learners' in primary school statistical reasoning in Israel (Ben-Zvi, Gil, & Apel, 2007). Students actively experience some of the processes involved in statistics experts' practice of data-based inquiry. Students conduct data and statistical modeling investigations through peer collaboration and classroom discussions using TinkerPlots (Konold & Miller, 2011). The learning trajectory consists of a series of open-ended real data investigations that provide students with rich and motivating experiences in posing statistical questions, collecting, representing, analyzing and modeling data, and formulating informal inferences in authentic contexts, which result in meaningful use of statistical concepts (Ben-Zvi, Aridor, Makar, & Bakker, 2012; Ben-Zvi & Ben-Arush, 2014). The data the students investigate are obtained from a questionnaire designed by the research team, teachers and students, and administered by students in their school and in neighboring schools. The Connections classroom is conceptualized

and organized as a learning community (e.g., Bielaczyc & Collins, 1999) that supports collaboration, argumentation, sharing and reflection. This is done physically in the class and virtually in a website that includes all educational materials and scaffolds, students' reflective diaries, and peer and teachers' feedback. Students are highly motivated to present and discuss their work in short presentations during the project and at the statistical happening, a final festive event for the students and their parents.

In the Connections learning environment, statistical concepts are initially problematized—that is, rather than first teach students directly about these concepts, then ask them to apply them in investigations, the investigations themselves are designed to raise the need to attend to these concepts, hence deepening students understanding of both their relevance and application. Additional strategies are used in the design of the learning trajectory such as growing samples (e.g., Bakker, 2004) to sensitize and slowly introduce students to the decreasing variability of apparent signals in samples of increasing sizes. The growing samples heuristic combined with “what-if” questions not only helped Connections students make sense of the data at hand, but also supported their informal inferential reasoning by observing aggregate features of distributions, identifying signals out of noise, accounting for the constraints of their inferences, and providing persuasive data-based arguments (Ben-Zvi, 2006). The growing awareness of students to uncertainty and variation in data enabled students to gain a sense of the middle ground of ‘knowing something’ about the population with some level of uncertainty, and helped them develop a language to talk about the grey areas of this middle ground (Makar, Bakker, & Ben-Zvi, 2011).

Connections students gained a considerable fluency in techniques common in exploratory data analysis, use of statistical concepts, statistical habits of mind, inquiry-based reasoning skills, norms and habits of inquiry, and TinkerPlots as a tool to extend their reasoning about data (e.g., Ben-Zvi, Aridor, Makar, & Bakker, 2012; Gil & Ben-Zvi, 2011). In a longitudinal mixed methods study (Gil & Ben-Zvi, 2014), long-term impact of teaching and learning was found among ninth graders, three years after their participation in the three-year Connections intervention. During the more-than-a-decade project, our academic passion is focused on the following global research questions: 1) what can young students understand about data and do with data? 2) how does students' statistical reasoning develop? and 3) how can we nurture students' statistical reasoning? These questions were explored throughout the years through the gradual employment of three paradigms, each expanding on its predecessors. The three paradigms are detailed in the following sections, as well as the necessity that motivated each expansion.

THE EDA PARADIGM

The Connections project was initially based on the exploratory data analysis (EDA) pedagogic approach (Shaughnessy, Garfield, & Greer, 1996). This paradigm is inspired by Tukey's thought and innovation (1962, p. 2):

For a long time I have thought I was a statistician, interested in inferences from the particular to the general. But as I have watched mathematical statistics evolve, I have had cause to wonder and to doubt ... All in all I have come to feel that my central interest is in data analysis.

According to the EDA paradigm, students are encouraged to become “data detectives” who use critically the PPDAC cycle (Problem, Plan, Data, Analysis, and Conclusion) in order to make sense of data (Wild & Pfannkuch, 2009). These stages follow the logical order of an investigation starting from understanding and defining the problem, making a plan, and proceeding to data collection, organization, and interpretation in order to come to a conclusion. Real research, however, seldom proceeds in this orderly fashion due to all sorts of reasons, a main one being the interdependency of these research phases (Konold & Higgins, 2003, p.194):

In these respects, data analysis is like a give-and-take conversation between the hunches researchers have about some phenomenon and what the data have to say about those hunches. What researchers find in the data changes their initial understanding, which changes how they look at the data, which changes their understanding, and so forth.

We designed the Connections EDA learning trajectory keeping this dynamic and complex view of data analysis in mind. This meant that we wanted students to stay focused on the data and what the data have to tell, be attentive to the context from which the data were taken, and use data

analysis tools for collecting, graphing (or data modeling), analyzing, and summarizing in a creative and critical way.

A typical example from our EDA studies is brought from a ten-hour experiment with second graders. The students studied data they had collected on the baby teeth lost by children in kindergarten to grade 3. The students were capable of making sense of the problem, the data collection and the organization of the sample data in appropriate inscriptions, some more concrete including verbal notations (Fig. 1) and some more abstract (Fig. 2). To develop their statistical reasoning, we used the growing sample strategy and “what if” questions.

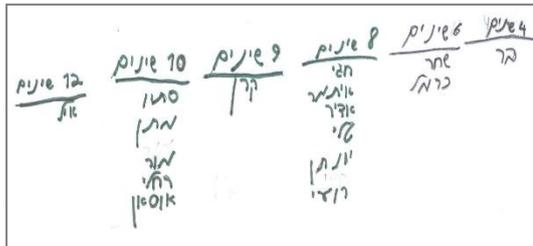


Figure 1. An inscription of baby teeth lost in grade 3

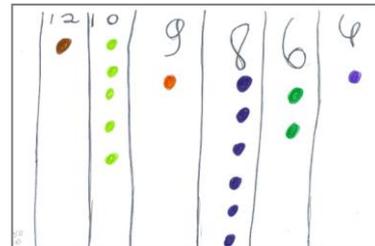


Figure 2. A graph of baby teeth lost in grade 3

Some challenges have been identified in students’ statistical reasoning during years of study in the EDA learning environment, mainly: 1) global (aggregate) views of data versus local (pointwise) views of data (Ben-Zvi & Arcavi, 2001); 2) seeing the signal in the noise (Konold & Pollatsek, 2002); and 3) viewing distribution as an entity (Ben-Zvi & Amir, 2005). To respond to these challenges, we brought statistical inference to the center of the Connections educational arena. The reasons for this change were: 1) doing statistics without inference misses the main goal of the discipline; 2) inferential reasoning requires aggregate view of data, considering uncertainty, and identifying a signal in the noise.

THE INFORMAL STATISTICAL INFERENCE PARADIGM

One ultimate goal of statistical reasoning is to enable students to make sound statistical inferences. More specifically, a statistical inference is a statement about a population or process, which is inferred from a sample, along with an explicit level of confidence. Researchers in statistics education have been studying the foundations of students’ reasoning with statistical inference for several years (Garfield & Ben-Zvi, 2008, pp. 261–288). More recently, they have focused on students’ use of Informal Statistical Inference (ISI) in shaping these foundations (Pratt & Ainley, 2008).

While several definitions have been offered for informal statistical inference, its use is still fairly ambiguous. Makar and Rubin (2017) identified three features of both an informal and formal statistical inference: 1) a statement of generalization beyond the data, 2) use of data as evidence to support this generalization, and 3) probabilistic (non-deterministic) language that expresses some uncertainty about the generalization. The word ‘informal’ is used to “a) make it clear that statistical inference is a broader concept than what is typically presented as hypothesis testing or estimation in an introductory statistics course; and b) emphasize that students are not expected to rely on formal statistical measures and procedures to formulate their inference” (Makar, Bakker, & Ben-Zvi, 2011, p. 153). This attribution of informality agrees with studies of learning, which highlight the importance of informal, situated, and contextually rich instructional activities for supporting development of more formal knowledge (Hershkowitz et al., 2002).

Overall, students tend to have more difficulty with formal ideas of statistical inference than almost any other statistical concept (Garfield & Ben-Zvi, 2008). Therefore, informal inferential reasoning (IIR) leading to an ISI may be an important goal on its own, especially with young children. Thus, a decade ago we have chosen in the Connections project to transition from focusing mainly on EDA to focusing on a combination of EDA and ISI. Students drew informal inferences from real samples they had collected and investigated, and made informal inferences about a larger population.

One example is a pair of sixth grade students (age 12) studying the question, “How far do sixth and seventh graders jump?” To respond to this question, they analyzed several graphs of random

samples taken from an unknown population of six and seventh graders in their school (an example of such a graph is shown in Fig. 3). A typical informal inference they made was, “based on these samples, it is possible to infer that six graders usually jump farther than seventh graders.”

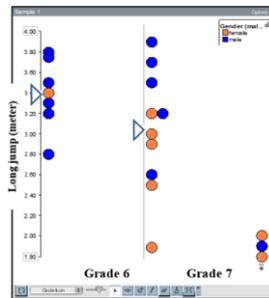


Figure 3. Long jump by grade (color – gender, blue triangles – mean values)

Guiding students from informal to formal reasoning may be an effective way to help them build better foundations of statistical inference (Garfield & Ben-Zvi, 2008). However, asking students to formulate an ISI, they are challenged to 1) integrate data and chance ideas, 2) quantify uncertainty using probability, and 3) understand sample-population relations. To address these challenges, we have sought a new direction by integrating statistical modeling in the learning trajectory.

THE STATISTICAL MODELING PARADIGM

To foster students' appreciation of the power and limitations of their informal inferences, a model-based perspective has been recently added to the Connections learning trajectory. Students build a model (a probability distribution) for an explored (hypothetical) population and generate random samples from their model using the TinkerPlots Sampler. An Integrated Modelling Approach (IMA) was developed to guide the design and analysis of a learning trajectory aimed at supporting students' IIR (Manor & Ben-Zvi, forthcoming). It is comprised of data and model worlds to help students learn about the relationship between sample and population. The data world is designed to foster in-sample reasoning (the reasoning involved when exploring real samples), and the model world is designed to foster between-samples reasoning (the reasoning involved while drawing repeated collections of samples from the population or from a model of the population).

In the data world, students collect a real sample by a random sampling process to study a particular phenomenon in the population. In this world, students choose a research theme, pose questions, select attributes, collect and analyze data, make informal inferences about a population, and express their level of confidence in the data. However, they may not account for probabilistic considerations, such as the chance variability that stems from the random sampling process. In the model world, students build a statistical model (a probability distribution) for the explored population (the same one explored in the data world), and generate random samples from this model. They study the model and the random process that produces the resulting simulated samples from this model. The details vary from sample to sample due to randomness, but the variability is controlled. Given a certain distribution of the population, the likelihood of certain results can be estimated.

In the IMA learning trajectory, students iteratively create connections between the two worlds by working on the same problem context in both worlds. Another means of connecting between the two worlds is using TinkerPlots, which includes the Sampler that allows learners to design and run probability simulations to explore relationships between data and chance, by means of one tool.

Our hypothesis is that the IMA approach can support students' development of reasoning with uncertainty when making ISIs, by experimenting with transitions and building connections between the two worlds. Two main features of the IMA that may support students' reasoning with uncertainty are: 1) working on the same problem context in both worlds, and 2) the support of the learning trajectory guiding questions, e.g., what is the minimal sample size needed to draw conclusions about the population with certain confidence, or “what if” questions on optional real data results while exploring model generated random samples. By analyzing generated random samples

and comparing them with the suggested model, students can learn about the relationships between samples and populations.

DISCUSSION

The goal of this article is to follow the development of an experimental curriculum that aimed at developing young students' statistical reasoning. We have started from the pedagogical paradigm of data analysis (EDA), in which the emphasis is on exploring authentic real data at hand. We have integrated the ISI paradigm, in which the relationship between sample and population is emphasized while making informal statistical inferences. The goal was to tighten the relations between data and chance (statistics and probability), but based on our observations, we concluded that a third paradigm had to be added, data modeling, to enable students better achieve this goal. Thus, a new design emerged (IMA) that integrated reasoning about models and modeling with reasoning about inference.

The three paradigms, EDA, ISI and Modeling, present the progress we made in our study, theory and practice of developing young students' statistical reasoning in the Connections project. Conceptually, we strive for better understanding of young students' abilities to develop statistical reasoning. We show the power of informal ideas of statistics in building students' conceptual basis for future development in their studies at later age. Pedagogically, we wish to develop sound design ideas for a learning environment that changes the way we treat the content, pedagogy and assessment.

The foundational paradigms of a learning environment can guide statistics educators and researchers to view, design and assess statistics teaching and learning. A learning environment is a complex and dynamic educational system, composed of multiple factors: key statistical ideas and skills (content), engaging tasks, real or realistic data sets, technological tools, classroom culture including modes of discourse and argumentation amongst students and between students and teachers, norms and emotional aspects of engagement, and assessment methods (Ben-Zvi, Gravemeijer, & Ainley, 2018). Integrating all these factors in order to reform the way statistics is learnt and taught is a challenging endeavor. The foundational paradigms of a learning environment can support the intentional transformation of an educational setting based upon conjectures about how the integration of features of the designed setting will support learning statistics.

The field of statistics education faces nowadays complicated challenges worldwide. Some of them are "classical": Poor teachers' statistical knowledge, lack of appropriate curricular materials and time at all age levels, and shortage of technology and access to data and resources. Recent developments in the emerging discipline of data science establish new kinds of challenges regarding the goals of statistics education, the interdisciplinary nature of our discipline. Current research in statistics education must consider these new challenges posed by data science by elucidating the foundational paradigms that can help us better prepare our students to the information-based society of the 21st century.

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