

## QUESTIONING THE WORLD – STATISTICAL INQUIRY

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*This paper contributes further insight into the new paradigm of ‘questioning the world’ and study and research paths and proposes that students work with important and meaningful ‘big’ questions. The design of the project proposes ways for teachers to engage students in statistical investigations. In the lessons, the 6 grade students develop critical stance about data, but, in the statistical investigations, it transpires that the questions posed by the students and teacher are insufficient to make the students explain or justify their choices of statistical methods. The students and teacher do not appreciate the potential for statistical inquiry in their own questions, and the students do not question or justify the choice of statistical descriptors to calculate or connect these choices with their answers to the ‘big’ question.*

### INTRODUCTION

In this paper, I report and analyse students’ reasoning processes in a sequence of statistics lessons, ‘Youngsters and ICT’. The lessons are built on Cobb and McClain’s (2004) argument, that to teach statistical reasoning, students must reason about methods and data rather than automatically apply procedures for manipulating numerical values. However, when teaching statistical reasoning, most teachers tend to teach concepts and procedures and hope that reasoning will develop as a result (Cobb & McClain, 2004). Some of the constraints are related to what has recently been called the ‘monumentalistic’ paradigm (Chevallard, 2016). My purpose is to explore whether and how study and research paths (SRP) can be used as an analytical tool to analyse classroom processes. The emphasis of ‘Youngsters and ICT’ is to create new statistical reasoning habits for classroom interactions (Skott & Østergaard, 2016) in which students work with ‘big’ questions, dissect those questions, explore an authentic context, develop new questions, and then answer them.

### ‘YOUNGSTERS AND ICT’

‘Youngsters and ICT’ is a 15-lessons sequence of statistics for grade 6 (13 years old students) and designed to improve students’ statistical reasoning. Together with the teacher, the students must initiate, negotiate, and establish two overall classroom statistical practices: 1) to be critical towards the use of statistics and 2) to investigate tendencies and patterns in data sets. One central and general teaching practice is to include and build on students’ statistical contributions in classroom discussions. Before teaching the ‘Youngsters and ICT’ the teachers participate in a 6-hour professional development workshop and are introduced to a website with lesson plans and descriptions about ways in which teachers can engage students in statistical investigations; formulate statistical problems; generate, analyse, and reason about data; interpret results; and disseminate the data both inside and outside of a school context – all this to help the students make decisions based on data. The overall question in ‘Youngsters and ICT’ takes as its starting point a newspaper article and the conclusion of the article: ‘Children spent eight hours a day on social media’. The use of data in the newspaper article is questionable and the presented charts misrepresentative. In ‘Youngsters and ICT’, we ask whether ‘children and young people spend too much time on media ... or?’

### STUDY AND RESEARCH PATHS

Researchers and practitioners agree that teaching should not only focus on direct transmission and application of knowledge, but also allow students to undertake inquiries in authentic situations. This is what Chevallard (2015) refers to as moving from the paradigm of ‘visiting monuments’ with no place to raise ‘what for?’ or ‘so what?’ questions (Chevallard, 2006), towards the paradigm of ‘questioning the world’. The aim in ‘questioning the world’ is to focus on important and meaningful ‘big’ questions, e.g., ‘how to predict population dynamics’ (Barquero & Bosch, 2015) or ‘how many people have lived on Earth?’ (Haub, 2002). SRP are a construct in the Anthropological Theory of the Didactic (ATD) and were originally introduced as a design tool for teaching within the paradigm of ‘questioning the world’ (Chevallard, 2006; Winsløw, 2011). Winsløw et al. (2013) re-introduce SRP

as a modelling tool for analysing didactic processes. SRP require new teaching strategies and modification of the ‘old’ didactical contract that currently exists: teachers need to supervise the inquiry process and avoid the temptation to pose questions and impose possible answers (Barquero & Bosch, 2015); students must decide on questions to pursue and defend their answers; and the milieu emerges in the classroom throughout the SRP process and during validation of the answers. Barquero and Bosch (2015) describe the need to develop new mathematical and didactic infrastructures to support self-sufficient SRP activities. The starting point of an SRP is a generative question, referred to as  $Q_0$ . Elaborating on  $Q_0$  is the purpose of the study, and mapping the questions ( $Q_k$ ) and answers ( $A_k$ ) in a tree diagram allows visualisation of the possible paths the students can follow. The answers to the derived questions,  $Q_k$ , generate temporary answers,  $A_k$ , that can be helpful in developing a final answer,  $A_n$ . The numbers next to the questions and answers in the tree diagram indicate the order of the questions, and the colours – black, white, and grey – indicate whether the question is posed or the answer is provided by the teacher, the students, or in collaboration.

### QUESTION POSING

In the process of changing to the paradigm of ‘questioning the world’ the complex learning of teachers and students meaningful question posing, reformulation and extending existing of questions is important. Zapata-Cardona and Rocha-Salamanca (2016) categorize four emerging categories of teachers’ questions: ‘closed questions’, ‘procedural questions’, ‘monitoring questions’ and ‘analysing questions’, in their study the teachers posed closed questions 52 % of the time. Also students question posing is categorised. Stoyanova and Ellerton (1996) describe tree forms of question posing situations, free (naturalistic situation); semi-structured (the students explore an open situation and apply existing knowledge) and structured (a specific problem) and Pittalis, Constantinos, Mousoulides, and Pitta-Pantazi (2004) describe the specific types of question posing tasks as; Editing quantitative information (students pose problems or questions that are appropriate to given, specified answers); translating quantitative information from one representation to another (e.g. diagrams, graphs, or tables); comprehending and organizing quantitative information by giving it meaning.

### STATISTICAL REASONING

Statistical reasoning is making sense of statistical information using methods from statistics – how we dissect a data set; what we look for; what methods are applied; and how we interpret the results. Statistical reasoning means understanding, being able to explain statistical processes, and being able to interpret statistical results – explaining how and why (Garfield, 2002). Statistical reasoning can be learned through the process of working cyclically with data, from formulating a problem to constructing conclusions based upon the data analysis; this is also described as an investigative cycle (Wild & Pfannkuch, 1999), which often requires an “unpacking of the ideas into components that begin with informal notions and build gradually toward formal notions” (Garfield & Ben-Zvi, 2008, p. 120). Statistical reasoning can be difficult to distinguish from statistical thinking, and when working on a specific task, the two types of activities cannot necessarily be separated. delMas (2004) however, writes that it is possible to distinguish them through the nature of the task: “a person who knows when and how to apply statistical knowledge and procedures demonstrates statistical thinking. By contrast, a person who can explain, why a conclusion is justified demonstrates statistical reasoning.” (delMas, 2004, p. 85). Mathematical reasoning and argumentation is generally characterised as a social phenomenon (Krummheuer, 1995; Stylianides, 2008). What is accepted as genuine in the class depends on the milieu.

### RESEARCH QUESTIONS

My main aim is to address two research questions. First, I am interested in analysing the content of the students’ and teachers’ questions and answers in ‘Youngsters and ICT,’ and second, I want to create a description of how we can analyse students’ development of reasoning as SRPs, while assessing not only the formal structure but also the quality of the reasoning.

### METHODS AND DATA

I employ a micro-ethnographic design, well-suited to describing, analysing, and interpreting a specific aspect of a group’s shared behaviour (Garcez, 1997). Over a two-year period, we observed a

teacher and her students (two classes) in 31 classroom lessons – 16 observations from the course ‘Youngsters and ICTs’ and 15 from before or after the course. We further conducted four audio-recorded semi-structured interviews with the teacher. Semi-structured interviews were chosen to get an understanding of the background of the classroom milieu. We transcribed the interviews and 15 selected lessons from the observations.

To address the research questions, I use SRPs and tree diagrams as analytical tools to analyse classroom practices. I focus on the teacher’s and the students’ questions and answers, and the knowledge thereby gained of the students’ development of statistical reasoning. The aim of using the tree diagram as an analytical tool is that the visible structure of the tree diagram can give new in depth knowledge about the students’ development of statistical reasoning and the structure of the lessons.

## ANALYSING CLASSROOM SITUATIONS

In the present paper, I analyse a smaller part of ‘Youngsters and ITCs’; 1) teacher and students questions and answers when analysing a newspaper article and discussing the question (Q<sub>2</sub>) ‘do children spend 8 hours on media every day?’ and 2) a group of students’ oral presentation of their answers to ‘how much time did we spend on media’ and ‘do students spend too much time on media?’. The sequence of statistics lessons was not originally designed as an SRP course, but it contains the same inquiry elements and is centred around a ‘big’ question. In the analysis, it makes no sense to judge whether the teacher avoids posing questions and imposing possible answers, but it is of interest if she changes the ‘old’ milieu to a milieu with whole-class discussions. The following tree diagram is constructed after the sequence of lessons was implemented in the classroom and not during the design process.

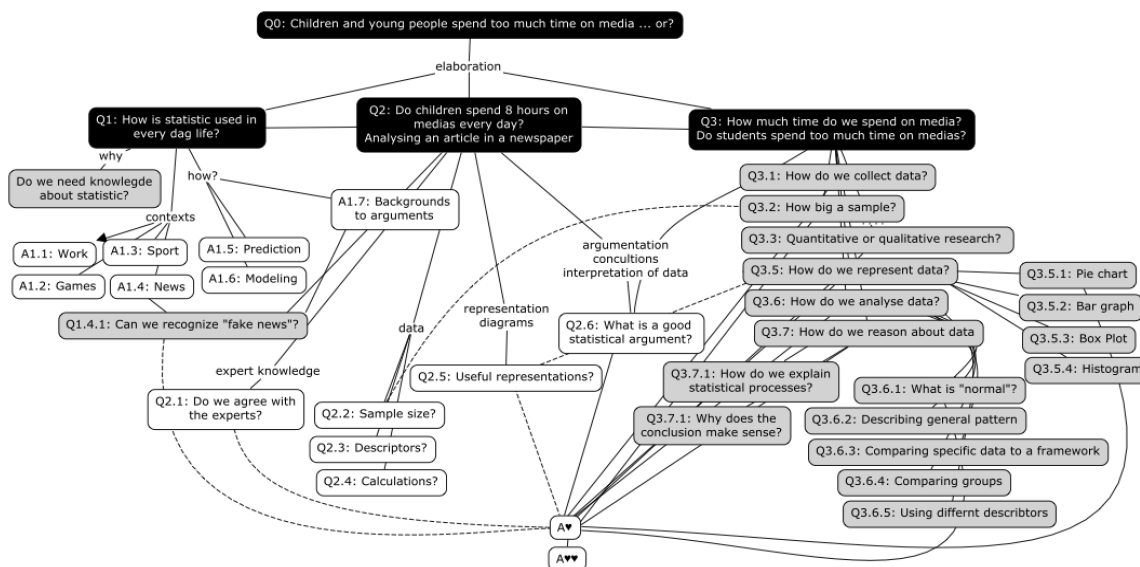


Figure 1. Tree diagram of the imagined potentials of ‘Youngsters and ICT’.

In the constructed tree diagram (Figure 1), we imagine the potential for the students to consider and ask many questions in collaboration with the teacher. The imagined potentials are that the students’ answers will include an explanation of their results and of their choices of representations, and that the students will use statistical reasoning to justify their claims when they generalise about conjectures. To answer Q<sub>0</sub>, the students explore Q<sub>2</sub> and a type of media that is not common in classrooms: a newspaper article. The students might hold the impression that a conclusion in a newspaper must be true. Studying the media places a special focus on analysing data, representing data, and statistical reasoning. In answering Q<sub>0</sub>, we expect the derived questions about representations (Q<sub>3.5</sub>), analysis (Q<sub>3.6</sub>), and reasoning (Q<sub>3.7</sub>) to support the students in exploring the data sets, instead of supporting the students in applying statistical descriptors.

### Analysis I: Discussion of a newspaper article

In the a posteriori tree diagram (Figure 2), we see questions and answers from the students' elaborative work with the  $Q_2$  'do children spend 8 hours on media every day?'  $Q_2$  is a 'free situation', where the students can analyse a naturalistic situation. It is apparent that the teacher poses a lot of questions that may help the students dissect the overall question:  $Q_{2.4.1.1}^T$ : "Why do we need to calculate the average?" and  $Q_{2.3.2.1.1.1.1.1.1}^T$ : "Yes because? No because?" [Asking the students to reason about their answers]. Ea supports and encourages the students to reasoning more deeply about the data and the newspaper's conclusion. The teacher's questions can be characterised as a combination of 'closed –', 'procedural –' and 'analysing questions'.

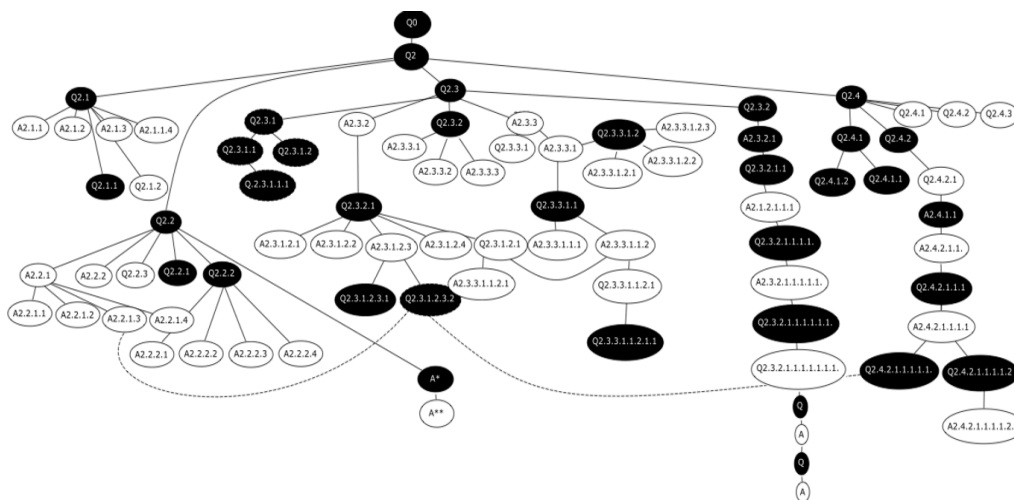


Figure 2. The a posteriori tree diagram for  $Q_2$ .

We also see students answer these questions, but the answers are often not directly connected to the analysis of data, but rather associated with everyday experiences and somewhat non-statistical moralisation:  $A_{2.1.2}^S$ : "I spend 10 hours" and  $A_{2.2.3}^S$ : "Even though it is okay to play computer games, it is not okay only to play games. You must also get outside, and be together with other people" (Larsen & Østergaard, in press). Some of the questions and answers focus on the trustworthiness of the conclusion of the article:  $Q_{2.1}^T$ : "Can we trust the conclusion of the newspaper?" and  $Q_{2.3.2.1.1.1.1.1.1.1.1.1}^T$ : "Are pie charts helping you read and understand the article in the newspaper?" It is not possible to connect the questions to a single 'question posing task', but there are elements from both 'comprehending' and 'organizing quantitative information by giving it meaning'. The students agree that the conclusion in the article is not reliable and the diagrams misrepresentative:  $A_{2.3.3.1.1.1.1.1.1}^S$ : "No. It is not possible to see how many minutes [a whole pie chart is]. If it is half an hour, or...".

In the a posteriori tree diagram, we see examples of how the teacher and students have posed smaller questions, such as  $Q_{2.3.3.1.1.2}^S$ : "How do you interpret the average?" and  $Q_{2.3.3.1}^S$ : "How many have they asked?"; questions about the use and understanding of pie charts, such as  $Q_{2.3.1}^T$ : "How can you describe the charts?"; and questions about how to manipulate charts so the chart gives a distorted picture, such as  $Q_{2.3.1.1.1.1.1.1.1.1.1.1.1.1.1}^T$ : "Why has he [the reporter] made them [pie charts], if you can't understand them. What will he [the reporter] show us?". The sum of the smaller questions ('analysing questions' and 'comprehending questions') enables the students to see a bigger picture of statistics, e.g., how it is possible to use data in an argument, manipulate data, and represent data in different ways. During the study of the article, the students develop critical stance, but they do not explicitly use the data from the article nor explain their conclusions. The dialogue between the teacher and the students is the fulcrum of the lesson, but questions are sometimes posed without anybody answering them, and other answers are accepted without any link to the data or any explanation as to how or why. In comparing the two tree diagrams, it is visible that implementing the paradigm of 'questioning the world.' changes the milieu to include a larger focus on questions, answers, and communication. In the tree diagram (Figure 1) I explain the potential for students' use of statistical arguments to justify

their claims. The potential is not realised in the a posteriori tree diagram (Figure 2) the students pose meaningful questions, but in the structure of the tree diagram we see how they often do not explore and elaborate on the questions. The aim of establishing new norms that include an aspect of explaining or arguing for the proposed examples or concepts is not achieved.

*Analysis II: A group's presentation of answer A<sub>▼</sub> to Q<sub>0</sub>: "Children and young people spend too much time on media ... or?"*

To answer question Q<sub>0</sub>, the students study Q<sub>3</sub>. They split into several groups to address the new question, develop a digital survey, plan data collection, collect data, and analyse the data in Excel. One of the groups communicates their interpretation and conclusions of Q<sub>0</sub> to the rest of the class. Subsequently, other students and the teacher comment on the oral presentation of A<sub>▼</sub>.

- Gry (student from the group): Is the average different on weekend days and weekdays? There is a very big difference between the days we have studied. If we look at the iPads, there is a huge difference. Saturday it's 21 hours, Sunday it's 19.5 hours, Monday it's 28 hours, and Tuesday it is 83.5 hours. And the average is: Saturday 4 hours and 6 minutes; Sunday 1 hours and 42 minutes; Monday 1 hours and 52 minutes; Tuesday 4 hours and 11 minutes. And the greatest value for Saturday is 8 hours. And the highest value for Sunday is 6 hours...
- Elsa (student from the group): Okay. Do we spend too much time on media? The average of all days – we spend 9 hours and 16 minutes on media. And maybe that's a bit too much, per person per day...
- Frederik (feedback): There were too many words. So I did not listen. Because it was just numbers, numbers, numbers, Monday, Tuesday, it became frustrating...
- Ea (teacher): Somehow there is no reason to all the talking, if you do not explain anything with the numbers. You just tell me the numbers, but what are you telling with the numbers? It's great that you've calculated all that. But there is no need to calculate if we do not use the calculations.

The above presentation is in line with other groups' oral presentations of A<sub>▼</sub>. The students have, in the process of reaching A<sub>▼</sub>, followed the investigation cycle (Wild & Pfannkuch, 1999). In the presentation, we see how the students have applied their knowledge of statistical descriptors, such as average, minimum, and maximum, and we see how the students mention the average when they present their answer to 'What do the parents think?' The students have examined and analysed their collected data, in the process, they have applied already known statistical descriptors and treated the data as numbers with only little connection to the 'context' – all this instead of investigating tendencies and patterns in the data set. We see no explanation as to how or why, neither do we see any connection to the students' work with the cyclical investigation, e.g., they do not compare their own data with their survey, nor question whether their sample is big enough. In the comments from Frederik and Ea, we see new expectations for the answers. Both Frederik and Ea call for an alignment between the presented data and the context, which can be seen as a way to change the milieu and norms for answering A<sub>▼</sub>. The dialogue does not indicate any expectation for students to justify their working process, their choices of statistical descriptors, or how or why their conclusions make sense.

## REFLECTIONS ABOUT SRP AS AN ANALYTICAL TOOL

It was rewarding to use SPR and the tree diagrams as an analytical tool to understand the students' development of statistical reasoning; it helped to distinguish different strategies for posing and answering questions and to interpret the content of the questions and answers. The tree diagram illustrates classroom patterns, new emerging milieus, answers with no statistical data, and how smaller questions lead to an overall answer – but not an answer that is justified statistically. In the analysis, it is evident that the new media – the article – gives new perspectives when we want students to develop critical stance, but zooming in on the questions and answers makes it apparent that the questions do not encourage the students to use data in their reasoning or to justify their answers.

## CONCLUSION

A comparison of the two diagrams provides a rich view of the questions and answers in the classroom, and the students' development of statistical reasoning. Nevertheless we need more information about the processes between the questions and the answers and it might be constructive to include media aspects as feedback, explanations, and introductions in the SRP analysis.

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