® REPRESENTATIONS OF DATA AND MANIPULATIONS THROUGH REDUCTION – COMPETENCIES OF GERMAN SECONDARY STUDENTS[1]

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Statistical literacy encompasses competencies regarding the use of mathematical representations and the manipulation of data through reduction. Dealing with data referring to authentic situations, basic activities of modeling are linked to this domain of statistical literacy. Based on the recently introduced German standards emphasizing the importance of representations and modeling, our study aims at assessing competencies of middle graders in German classrooms. Therefore, a pilot study with more than 180 fifth- and eight-graders in upper secondary schools was conducted with the additional aim of testing the properties of a set of assessment tasks. The results support on the one hand typical misconceptions of students and specify on the other hand the status quo of the domain of statistical literacy in question. Using Rasch-analysis we can support the hierarchical concept.

INTRODUCTION

Statistical contents have become increasingly important in mathematics school education in recent years (Batanero, Godino, Vallecillos, Green & Holmes, 1995): On the one hand, more statistics has been integrated in curricula in some countries, on the other, more attention is given to the notion of statistical literacy. Particularly in Germany, statistics had played a minor role in education at the lower secondary level. International assessment studies like Programme for International Student Assessment (PISA) 2003 (PISA-Konsortium, 2004) revealed that the achievement of German students in the domain of "uncertainty" was slightly below the international average, with scores which were lower than the scores for the other three so-called overarching ideas measured in PISA 2003. However, the situation seems to have been improved by the recently introduced German standards for school mathematics (Kultusministerkonferenz (KMK), 2004) and new curricula, emphasizing also contents from statistics in regular mathematics education. Correspondingly, research initiatives are undertaken in order to evaluate and to assess competencies related to these standards. In particular, the domain of "data and chance", with its relationships to statistical literacy, is examined. In comparison to the related field of chance and probability, research dealing with the data analysis part of statistical literacy is not as extensive (e.g., Batanero et al., 1995).

The core aim of the study presented here is to summarize the first results from a test development related to modeling in statistical contexts, the use of mathematical representations, and the manipulation of data through reduction.

In the following, we will give an outline of the theoretical background of the study with the objective of specifying a competency model in this domain.

STATISTICAL LITERACY, READING COMPETENCIES, AND COMPETENCY MODELS

The notion of statistical literacy encompasses a certain range of dispositions and abilities. Wallman (1993) provides a rather broad definition of statistical literacy: "'Statistical Literacy' is the ability to understand and critically evaluate statistical results that permeate our daily lives–coupled with the ability to appreciate the contribution that statistical thinking can make in public and private, professional and personal decisions." This definition focuses on the intended learning outcome in the field of statistics and probability, which should enable citizens to participate actively in daily life.

Regarding the contents related to statistical literacy, Holmes (1980) identified five main components: data collection, data tabulation and presentation, data reduction, probability and the field of interpretation and inference. Gal (2004) included this set of contents in his notion of a statistical knowledge base. The statistical knowledge base, together with the additional components of mathematical knowledge, context knowledge and critical knowledge as well as basic literacy skills, form the knowledge elements which are needed in order to be statistically literate. Dispositional elements such as beliefs and attitudes towards statistics are added, as they can play a role in competency development.

Findings related to the measuring of statistical literacy were reported by Watson, Kelly, Callingham and Shaughnessy (2003). Using the chance and data curriculum conceived by Holmes (1980) as a framework, this study focused on students' understanding of statistical variation. Watson et al. (2003) showed that the idea of variation could be used to develop a test instrument covering a broad range of content domains referring to statistical literacy. This test conformed to the one-dimensional Rasch model, and was used to verify the corresponding competency model empirically. Moreover, the four levels of competency concerning the understanding of variation are in line with the statistical literacy hierarchy described by Watson (1997). Watson and Callingham (2003) characterized statistical literacy as a complex construct comprising six levels, which again could be seen in close relationship to the tiers of the statistical literacy hierarchy of Watson (1997).

Findings by Reading (2002) concerning profiles of statistical understanding also fitted into a one-dimensional Rasch model and covered different sub-domains of subject matter in the area of statistical literacy.

Other competency models derived from a different theoretical perspective refer to the domain of reading as a social practice but can be regarded as related to these considerations on statistical literacy (cf. Luke & Freebody, 1997; Curcio, 1987). The competency levels suggested by Luke and Freebody (1997) can be seen in analogy to the competency models cited above (Watson et al., 2003). Similarly, in the PISA test (Organisation for Economic Cooperation and Development (OECD), 2003) there were items assessing reading competence which could also be used in a statistical literacy test for the focus on comprehension of data (cf. Curcio, 1987). This highlights the importance of statistical literacy as a competency domain that is relevant for social life.

In conformity with the overarching idea of mathematical education leading to mathematical literacy, as for example proclaimed by the PISA study (OECD, 2003) or the standards of the National Council of Teachers of Mathematics (NCTM, 2000), relevance for social life was also one of the criteria underlying the conception of the recently introduced German standards (KMK, 2003). In the following section, we will suggest a competency model focusing on the framework of these standards and referring to a sub-area of statistical literacy.

COMPETENCY LEVELS FOR USING MODELS AND REPRESENTATIONS IN STATISTICAL CONTEXTS

"Modeling" and "using representations" are two competencies included in the German standards which are considered to play an important role for the students' competency development in mathematical contexts and in particular in the content area of "data and chance" (KMK, 2003). The conception of the German standards is close to the standards of the NCTM (2000) and the competencies tested in PISA (OECD, 2003). To work mathematically, learners often need a combination of such skills. Taking a closer look at particular mathematical content areas, it is possible to concentrate on competencies crucial to this content area. As a consequence of the curricular situation in Germany, we focused on the content area of graphical representation of data and basic manipulations through reduction, examining the related competencies of "modeling" and "using mathematical representations".

Results found by Reading (2002) suggest that achievement in these two content fields of representations and reduction varied over the whole scale of students' understanding, such that, under the presumption of a unidimensional competency of statistical literacy, items from these areas could be used to map a considerable range of students' statistical abilities.

Curcio (1987) developed three different hierarchical levels in working with graphical representations of data:

Level 1: Reading the data in which interpretation is not needed. Only facts explicitly expressed in the graph or table are required.

Level 2: Reading within the data, which requires comparisons and the use of mathematical concepts and skills.

Level 3: Reading beyond the data where an extension, prediction or inference is needed.

Difficulties of the learners are mainly found in the upper two levels (Friel, Bright, & Curcio 1997). Such difficulties or misconceptions can often be explained as difficulties related to the use of models and representations. For example, “reading within the data” requires more than a one-step finding of a value in a diagram: Comparisons of values and discussing relationships between them corresponds to a higher level of complexity (cf. Batanero et al., 1995) and can be described as a two- or multi-step use of models and representations. At this level, manipulation of data through reduction (cf. Kröpfl, Peschek, & Schneider, 2000) can play an important role. Finally, “reading beyond data” is often associated not only with a more-step use of representations but also requires the use or construction of a model (cf. PISA 2000, p. 144) not initially given in the problem. As this corresponds to a higher level of complexity, difficulties like a lack of identification of patterns, inappropriate use of information and errors in predictions have been observed (cf. Pereira-Mendoza & Mellor, 1991). Aspects of manipulation, reduction and restructuring of the given data are very often at the centre of problems at this competence level.

Summing up, it seems possible to develop a competence model based on the aspects of using models and representations in statistical contexts focusing on the German standards (KMK, 2003) and referring to an important sub-domain of statistical literacy. As suggested by Curcio (1987), three levels of competence can be distinguished from a theoretical point of view. An important underlying principle of distinction between the levels consists in the number of steps of using representations and models. In this approach, we used results from our previous work on competency models and their empirical evaluation (cf. Reiss, Hellmich & Thomas, 2002). A short description of the levels of the competency “using models and representations” in the statistical context considered is given in Table 1.

Table 1
Levels of competency for using models and representations in statistical contexts

Level I	One-step use of a representation or work within a given model (e.g. reading a given value from a diagram, completing a given diagram for a given table)
Level II	Two- or multi-step use of representations or changing between two given models (e.g. comparing data including a transformation step or a mathematical concept)
Level III	Multi-step use of representations including the use of a non-given model (e.g. own modeling activities supporting a cumulative interpretation of data given in diagrams)

One of the aims of this study is to verify empirically whether the competency model provided in Table 1 can be used to develop of a corresponding test. According to the results reported by Batanero et al. (1995), we expect differences for the competency “using models and representations” in statistical contexts with respect to the students’ age and their grades. Furthermore, we suppose that difficulties of the students are linked especially to the levels II and III of the competence model presented in Table 1. From the achievement perspective, it is interesting to have the possibility of making comparisons to mathematical competency as measured in international studies like Third International Mathematics and Science Study (TIMSS). Consequently, in order to have a rough indicator for the difficulty of the test on using models and representations in statistical contexts, we included TIMSS items which fitted into the levels of competence presented above.

RESEARCH QUESTIONS

Based on the theoretical background described above, the study aimed at providing evidence for the following research questions:

1. Which abilities and misconceptions of the fifth- and eight-grade students can be observed?
2. Is it possible to construct a test of statistical literacy which fits the one-dimensional Rasch model? In particular, will the empirical item difficulties confirm the competency levels conceived according to theoretical considerations?

DESIGN AND SAMPLE

According to the competency model suggested in Table 1, a test was developed which comprised 10 items. Three of these items were identical with items used in the TIMS Study. These items served as anchoring items allowing for comparisons. The items were designed to correspond to the three levels as shown in Table 1. The choice of the three TIMSS items was done as to fit as good as possible the competence model (Table 2).

Table 2
Overview on items and levels of competence

Level	I	II	III
Items	A1, A2	A3, A4, A5	A6,A7
TIMSS-Items corresponding to the levels of competency	T1	T2	T3

The test was administered in grades five and eight of two German upper secondary schools (students aged 11/12 and 14/15 years resp.). The total of n=187 children (88 boys) split up in 112 fifth-graders (60 boys) and 75 eight-graders (28 boys). As an example, one of the items conceived for level III is shown in Figure 1.

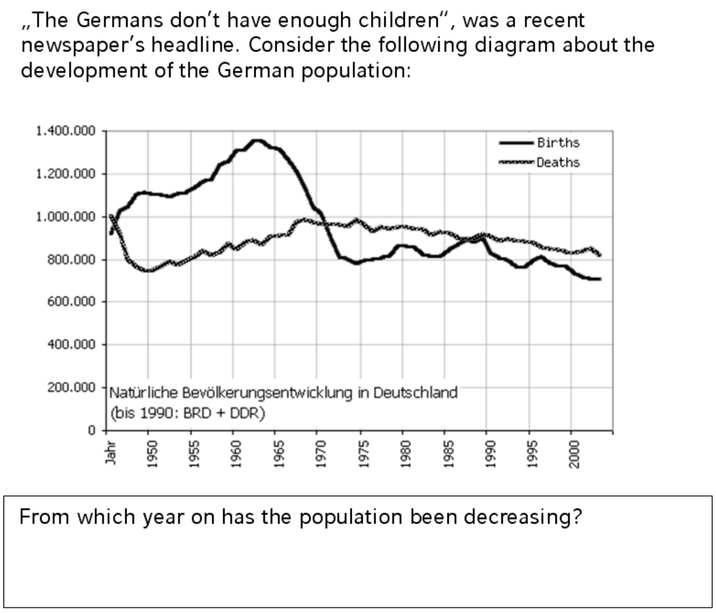


Figure 1: Item A6

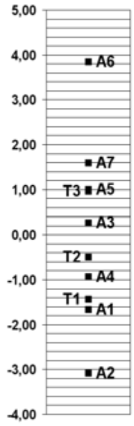
The line graph shows the development of births and deaths in Germany from 1940 on and it is asked, from which year on the population had been decreasing. This problem requires not only a multi-step use of the data given in the two line graphs, but the learners have to establish a link between the two graphs using a model not given in the diagram. In addition to the data in the representation of the diagram, the students have to consider deaths as a negative and births as a positive influence on the number of inhabitants, in order to find the values essential for the solution. On the base of this model, relevant information contained in the line graphs could be identified.

The students' answers were coded in a dichotomized way separately by two raters. A total score was calculated for each student summing up the scores of the ten items.

RESULTS

According to the first research question, it was the aim to verify whether the results can be described by the one-dimensional Rasch model. For this purpose, the fitting of the model was tested by a parametric bootstrap procedure with 300 iterations and an accuracy criterion of 0.00075. Hence, the item fit was satisfactory. The item location on the corresponding logit scale is shown in Figure 2.

In the empirical results, the levels of



Item	Item Location	Level of Comp.
A6	3.84095	III
A7	1.59096	III
A5	0.99056	II
T3	0.95492	III
A3	0.26664	II
T2	-0.49533	II
A4	-0.94128	II
T1	-1.44145	I
A1	-1.67462	I
A2	-3.09135	I

Figure 2: Item locations and levels of competence

competency are reproduced, however, there is one exception. The item T3 taken from the TIMS Study turned out to be less difficult empirically than the items of level III and showed an item location similar to the most difficult item of level II.

The frequency of the correct answers for the individual items (Table 3) verifies again the hierarchical order of the levels of competency. However, the rates are high. Especially a comparison to the data concerning the TIMSS items indicates that the students in this sample performed better than the students in the TIMSS sample.

Table 3
Percentages of correct answers

Level of competence	I			II				III		
Item	A1	A2	T1	A3	A4	A5	T2	A6	A7	T3
Total sample	97.9	99.5	97.3	87.7	95.7	78.6	93.6	25.1	68.4	79.1
Results in TIMSS for 7th			89				83			68
and 8th graders resp.			87				82			70
5th grade students	97.3	99.1	95.5	81.3	94.6	66.1	94.6	14.3	63.4	73.2
8th grade students	98.7	100.0	100.0	97.3	97.3	97.3	92.0	41.3	76.0	88.0

As a first answer to the second research question, the results in Table 3 show that an overwhelming majority of the students performed very well on the items at level I, and also at level II. These results are also reflected in Figure 3, showing the percentages of students having attained different total scores (maximum: 10 points) for 5th- and 8th graders. The distributions are biased to the right. The answering frequencies in Table 3 also show that difficulties of the students mainly occur for items at level III. This result seems to replicate findings by Curcio (1987) indicating that misconceptions and difficulties of the learners mostly concern the upper levels of competency. Taking a look at the results for different grades of the students in Figure 3, differences between the distributions become apparent. The mean difference between the grades is significant (5th grade: M=7.8, SD=1.26; 8th grade: M=8.9, SD=0.90; t-Test: T=6.868, df=184.2, p<0.001, d=0.996).

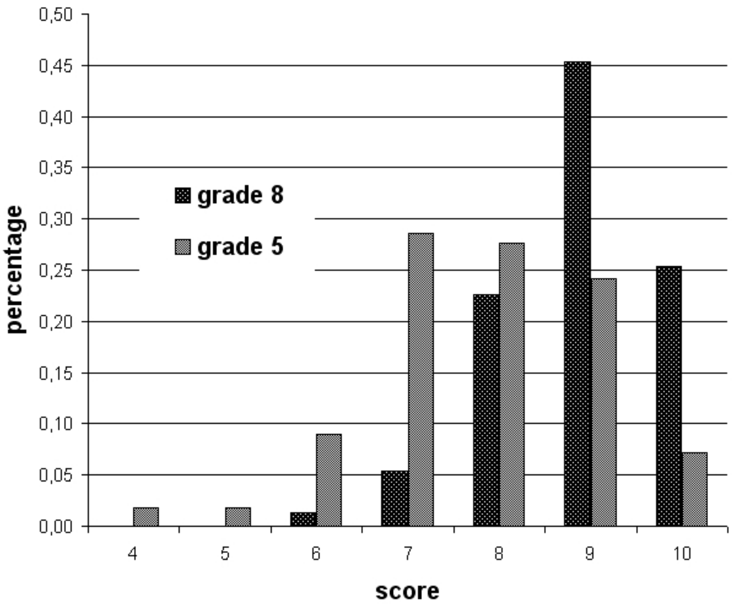


Figure 3: Distribution of total scores for 5th- and 8th-graders

DISCUSSION

The results show that the one-dimensional Rasch model could be verified for the test and the underlying competency model for “using models and representations” in statistical contexts. Previous research findings on competencies in the domain of statistical literacy and on difficulties of the learners could be replicated. However, the results also highlight possibilities for improving of the test instrument. In the first place, it seems necessary to develop more complex items. To distinguish a potential fourth level of competency, the introduction of the basic idea of variation as a criterion should be considered. A second question connected to the interpretation of the findings concerns the sample. The good results might be due to a particular school culture or school environment of the students participating in the study.

To provide evidence for these questions, follow-up studies could be conducted with larger samples, an enlarged and modified set of items as well as qualitative investigations as next step of our research project. The development of test instruments for components of statistical literacy as well as empirically verified competency models are important prerequisites for undertaking research on the variables that influence these components of statistical literacy as well as for evaluating learning environments with the aim of fostering statistical literacy.

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