TEACHER TRAINING PROGRAM ON TEACHING ARITHMETIC MEAN BY USING THE VISUAL APPROACH- A CASE STUDY

Hanan Innabi
United Arab Emirates University, United Arab Emirates
hinnabi@uaeu.ac.ae

This paper describes an action research carried out over eight days on an elementary mathematics teacher in the United Arab Emirates. The effectiveness of a short teacher training program using the visual approach in teaching the concept of arithmetic mean was examined. The teacher was trained to teach a sixth grade class using a visual approach that focused on teaching the conceptual understanding of the arithmetic mean. Results showed some positive effects of the short training program on students' conceptual understanding of arithmetic mean. Some misconceptions related to arithmetic mean were found among students in pre and post testing.

BACKGROUND

Statistics education in the United Arab Emirates (UAE) has been going through significant change. A comparative look at statistics and probability in the old and new mathematics curricula shows some deep gaps. Students, for example, will study statistics and probability from the first grade instead of the sixth and eleventh grades respectively. Many statistics concepts that were not included in the old curriculum are represented in the new. In the revised outlines of the mathematics curriculum, a new approach to learning statistics and probability can be found: formulating questions, collecting, organizing, representing, analyzing, and interpreting data (MOEU, 2001). This promising picture is a consequence of Data Analysis and Probability becoming one of the ten standards upon which the new mathematics school curriculum in the UAE is built (Innabi, 2007). To achieve this new ambitious vision, the teaching of statistics has to be shifted from procedural knowledge and drills using meaningless data to constructing statistical knowledge using problem solving techniques that require students to formulate questions and collect real and meaningful data (Mvududu, 2005).

Little research on statistics and probability has been conducted in the Arab world including the UAE. For this reason, there is an urgent need in the UAE for research on teaching and learning statistics, particularly concerning the tools and strategies that can help teachers help their students. This study supports the ongoing statistics reform being carried out in the UAE regarding one of the important concepts in statistics, the arithmetic mean.

The importance of this concept is related to both descriptive and analytic statistics. The following standard is stated in the outlines of the mathematics curriculum in the UAE: "students have to apply and use central tendency measurements in order to interpret and compare data" (MOEU, 2001). It is clear that this standard requires high cognitive processes and deep understanding. To achieve this standard, this study assumes that teaching the arithmetic mean should not be only through procedural knowledge of the algorithm but also through seeking conceptual understanding. Teaching this concept as an algorithm (add the observations and divide by their number) and preventing students from constructing it, ultimately results in not developing conceptual understanding of the concept. This assumption, which is based on results found in previous research, concludes that because teaching the arithmetic mean is based on computing not understanding, students' understanding of this concept is usually framed by its formula (George, 1995). Mokros and Russell (1995) emphasized that children who learned the arithmetic mean algorithmically, face problems in recognizing the meaning of this concept as a representative value.

Generally speaking, while students at all levels (i.e., elementary, secondary and college) may succeed in computing the value of the arithmetic mean, they usually lack understanding of this concept as a representative of the data that has some characteristics (Cai, 1995; Mokros & Russell, 1995; Pollatsek, Lima, & Well, 1981; Strauss & Bichlier, 1988). Evidence shows that students should be exposed to experiences on how to collect and deal with data, requiring answering questions before teaching them the algorithm to compute the arithmetic mean. Such

© ICMI/IASE 2008. In C. Batanero, G. Burrill, C. Reading & A. Rossman (Eds.), *Joint ICMI/IASE Study: Teaching Statistics in School Mathematics. Challenges for Teaching and Teacher Education. Proceedings of the ICMI Study 18 and 2008 IASE Round Table Conference.*

experiences shaped a background that helped students build their understanding of this concept. Teaching procedural knowledge before conceptual understanding will cause students to resort to rote learning and hold back their understanding of the arithmetic mean in the future (Russell & Mokros, 1996; Zawojewski & Shaughnessy, 2000).

Studies related to characteristics of arithmetic mean started in the 1980s (Leon & Zawojewski, 1993; Mevarech, 1983; Mokros & Russell, 1995; Watson & Moritz, 1999). Based on these studies, a need emerged for research on the pedagogy related to the nature of experiences students should be given in order to construct the concept of arithmetic mean. Many ideas were provided regarding this issue (Bremigan, 2003; Meyer, Browning & Channell, 1995).

The *Principles and Standards for School Mathematics* from the National Council of Teachers of Mathematics (2000) focused on the necessity of representing ideas in different ways and encouraged the use of different approaches including a visual approach. In the early 1990s, Bennett and Foreman (1991) presented a visual approach to teaching the arithmetic mean as an alternative to the traditional algorithmic method. In this method, students use squares or cubes to represent each observation by building a column (bar) and then leveling-off these columns. The height of leveled-off columns is called the arithmetic mean. This method reinforces the concept of the mean because students are forced to consider the relationship between the observations and the mean itself (George, 1995). Substantial studies supported this approach (i.e., Cai & Moyer, 1995; Friel, 1998; George, 1995; Kamii & Warrington, 1999).

THE PROBLEM

It can be noticed from previous experimental studies (i.e., Baker & Beisel, 2001; Bennett & Foreman, 1991; Strauss & Bichlier, 1988) that examined alternative methods (versus the traditional method) in teaching the arithmetic mean, that most of these studies were applied to small samples and in clinical situations with volunteer students, aimed at examining one specific issue. Because teachers need to see alternative methods being applied in practical and realistic situations, these methods have to be examined in real situations with regular students and classes. This study aimed to examine the effectiveness of a short training program to teach the arithmetic mean in a regular classroom by a regular teacher. In particular, this study answered the following question: Does the teaching method (traditional versus visual) affect sixth grade students' understanding of the arithmetic mean beyond the procedural knowledge of the algorithm?

The visual approach for teaching arithmetic mean depends on students' realization of this concept as a balancing or as an equalizing value for all other values. This happens by visualizing the process of getting amounts from the larger values, giving it to the smaller values until all values become equivalent. The visual method was applied in this study, using problem-solving activities that dealt with the values, utilizing concrete examples (money or cubes) or images (bars figures). The traditional method depended on the use of the textbook which concentrates on how to compute the arithmetic mean using the formula without any attention to conceptual understanding.

METHOD

To answer the study question, a paper and pencil test was used as pre- and post-test. This test examined the students' understanding of the arithmetic mean concept. This was done to examine the understanding of the mean as a representative of the values (question 1 in Table 1), located between the highest and lowest values (question 2 in Table 1), the variations above and below it are equal (question 3 in Table 1), and influenced by the extreme values (question 4 in Table 1). The test had four questions each examining one characteristic. Three questions were adapted from Strauss and Bichlier (1988). The test was examined for its validation and reliability. It involved a group interview process where the researcher went through each question by clarifying and giving instructions to all students, allowing them to move together to the next question.

Two sections of sixth grade in a female school were taught the concept of the arithmetic mean, using two different teaching methods, taught by their regular teacher in May 2005. In one section (control group), the traditional method was applied with a focus on computing the mean using the formula. In the other section (experimental group), the visual method was applied where the focus was on understanding the meaning of the arithmetic mean.

Discussions with one of the mathematics education supervisors resulted in selecting the school, the teacher, and the two sections. The school is a public urban elementary school; the teacher holds a bachelors degree in mathematics with 10 years experience in teaching fourth to sixth grades. The supervisor emphasized that this teacher, like most other teachers, usually follows the textbook. The two sections contained 23 and 24 students respectively. Based on the supervisor, the principal, the teacher, the students' Grade Point Average (GPA), and according to the pre-test that was administered to the two section to assess students' understanding of the arithmetic mean ($t = -4.8$, $p = 0.63$), these two sections were considered equivalent and homogeneous. The duration of the experiment lasted for eight days. The following is a clarification of the sequencing of events.

Day 1. The pre-test was administered to the two sections. It was imperative at this point not to inform the teacher about the test and the aim of the experiment. A meeting was conducted with the teacher to know how she would teach the lesson on the arithmetic mean. In this meeting, the teacher showed an understanding of the lesson's objectives. These objectives were: 1) know the concept of arithmetic mean; 2) to compute the value of the mean; and 3) to apply this concept to real life situations. When the teacher was asked about how she would know that her students had achieved these objectives, she said that by being able to compute the mean, being able to use the formula and being able to compute their GPA, the students would demonstrate this achievement. In this meeting, the teacher was asked to teach this lesson to one of the two sections, and to delay teaching it to the other section for three more days. No information about the alternative method was given to the teacher.

Day 2 and Day 3. The traditional method was applied in the control group. Knowing and using the formula for computing the mean was the focus of the lesson, and the formula was applied to find the students' GPA. Students were informed to be ready for a test. By the end of day 3, a meeting between the teacher and the researcher was carried out. In this meeting, the teacher confirmed that she and her students have achieved the lesson objectives because her students were able to find the correct answers. At the end of this meeting, the teacher was given a booklet to read for next day. This booklet displayed the meaning of some of the basic concepts and ideas in statistics (mean, median, quantitative and qualitative data, and representing data by using tables and bars). It also contained activities based on the problem solving approach where students need to formulate questions and collect data. This booklet was prepared by the researcher and aimed at showing teachers how to teach statistics in meaningful ways that focused on the conceptual understanding of the subject matter.

Day 4. The post-test was administered to the control group. A workshop type of meeting was carried out with the teacher, the researcher and the supervisor. The purpose behind this meeting was to discuss and clarify the booklet in order to present the alternative method for teaching the concept of arithmetic mean. After this meeting, the teacher recognized the difference between calculating the mean and understanding the mean. The teacher agreed to teach the lesson about the mean for the second section in a different way.

Day 5. Two meetings were conducted with the teacher; each lasted for about three hours. The aim was to prepare the lesson for the experimental section. Even though this preparation was guided by the researcher to focus on the visual approach, the teacher was encouraged to participate and suggest ideas; especially in determining the lesson objectives. Interestingly, the suggested objectives were the same as those for the control group but with different meanings. For the teacher, the objective related to knowing the arithmetic mean had a different meaning from realizing the arithmetic mean as a representative value that has specific characteristics.

Day 6 and Day 7. In two sessions (45 minutes each), the teacher taught the experimental section using the alternative method. First, two activities focused on helping the students construct the meaning of the arithmetic mean. One of those activities required dealing with real money to find the distributed equal share. The other activity required using wooden cubes to represent different amounts. By using structured questions, the students in these activities found the mean (without paying any attention to the term arithmetic mean at this stage). The term was presented, and the formula to compute the mean was suggested by the students at a later time. Some extra activities were given as an assignment and later discussed. Summaries were presented several times during the teaching process (from both the teacher and the students) to focus on the characteristics of the mean. The summaries were also used to clarify the different methods to find the mean, i.e., moving the cubes or squares, distributing the total amount, and using the formula.

Day 8. The post-test was administered to the experimental section. An assessment meeting was conducted with the teacher to discuss the teachers' impressions about the alternative method in teaching the arithmetic mean.

RESULTS AND DISCUSSION

Two sources of data were considered in this study; one from the teacher and the other from students' answers in pre- and post-tests. The qualitative data collected from the teacher emphasized that the students in the experimental section learned the arithmetic mean in a meaningful way and understood the concept in a way that was superior to the procedural knowledge. The teacher noticed that the students who participated in the experimental group were more active and excited and that the general learning atmosphere of the classroom was more positive than the section taught using the traditional approach. The data collected from the students were qualitative data where students explained their answers. The analysis of these data showed different patterns in the students' understanding between the two groups, with better quality answers from the experimental group. For example, in the question that assessed the students' understanding of the mean as a representation of the data, the number of students in the control group who thought there is a value that could represent all other values increased from nine (in pre-test) to 12 (in post-test). However, this number increased from 11 to 16 in the case of the experimental group (see Table 1).

Some misunderstandings were found in the students' answers in the pre- and post-tests. Less of these misunderstanding were found in the experimental students' answers compared to answers of the control group. These misunderstandings were: 1) the biggest (or smallest) value is the best value to represent the data; 2) the value located in the middle is the best and the only value to represent the data; 3) no specific value can represent many different values; 4) arithmetic mean could be bigger than the biggest value because adding numbers will increase the mean; 5) arithmetic mean could be bigger than the biggest value because this depends on the number and the quantity of the other values; and 6) what has been given is more than what has been taken (in the item that measured that variations above and below mean are equal).

In the second stage of the analysis process, students' answers were scored according to specific rubrics, and each question was given a score out of three. T-tests for related groups and for independent groups were conducted to compare means in the pre and post-tests in each group and to compare the means of the two groups on the post-test. Results showed that, in general, students' means in the experimental group were significantly higher than the means of the control group.

Taking into account all the limitations of this action research, especially in relation to the school, teacher, students, training program, and the pre/post-test, one can say that in general, the visual method for teaching the arithmetic mean was effective in helping students understand the concept. There is a need to extend the frame of this study and apply it to more classes, teachers, and students. It is hoped also that the misconceptions that students showed in this study will be considered in teaching and in textbooks. The significant point that this paper showed is the success of the short training program that the teacher was given. Although we cannot make any generalizations about this success, we can deduce from this study that explicit, convincing, clear, practical, and realistic instructions given to teachers has the potential to positively affect the teaching and learning of statistical concepts.

Table 1. Number of students for answers on the pre/post test questions

Question		Student answers	Control pre	Control post	Experiment pre	Experiment post
1. During the last 5 days the temperature degrees in Dubai were as follows: day1= 28, day2= 29, day 3= 30, day 4= 32 day 5= 36. Do you think that there is a one single number that can represent the temperature degree during the last 5 days? (yes/no; Why?)	Yes	lowest/biggest	4	1	2	1
		an answer related to the median	3	3	6	4
		an answer related to the mean	1	7	1	8
		don’t know/no answer	1	1	2	3
		Total	9	12	11	16
	No	values are different	6	5	3	0
		don’t know/no answer	5	3	5	4
		Total	11	8	9	4
2. Friends decided to share the cookies they brought to their party. Each one brought a different number of cookies, but Yasmeen brought the biggest number (6 cookies). When they were handed out all cookies, each one received 8 cookies. Do you think this could happen? (Yes/no, Why?)	Yes	depends on the values/its number	4	4	2	1
		add small amounts gives a big amount for each	3	3	1	1
		the sum is the multiplication of all amounts	2	1	1	0
		don’t know/no answer	3	4	7	3
		Total	12	12	11	5
	No	impossible to be bigger than the biggest value	1	3	4	12
		don’t know/no answer	7	5	5	3
		Total	8	8	9	15
3. Friends brought cookies to a party. Some of them brought many, and some brought few. Those who brought many gave some to those who brought few until everyone had the same number of cookies. Was the number of cookies given by those who brought many the same as the number of cookies received by those who brought few	Yes	the given amount = the taken amount	0	1	0	7
		depends on the values /its number	4	7	3	0
		don’t know/no answer	6	3	7	8
		Total	10	11	10	15
	No	depends on the values/its number	6	5	3	3
		values above average should be less/more	2	4	2	2
		don’t know/no answer	2	0	5	0
		Total	10	9	10	5
4.On Monday, each kid brought a small number of marbles. When they passed them out among themselves so that everyone would have the same number, it turned out that each child received 2 marbles. On Tuesday, each kid brought exactly what they brought on Monday except for Fatema, who brought a lot of marbles. When the kids passed out all of their marbles so that everyone would have the same, everyone received 2 marbles. Do you think this could happen?	Yes	don’t know/no answer	7	5	8	3
		Total	7	5	8	3
	No	it should be more	5	5	5	11
		don’t know/no answer	8	10	7	2
		use algorithm or bar graph	0	0	0	5
		Total	13	15	12	17

REFERENCES

Baker, J .D., & Beisel, R.W. (2001). An experiment in three approaches to teaching average to elementary school children. *School Science and Mathematics, 101*(1), 23-31.

Bennett, A., & Foreman, L. (1991). *Visual mathematics course guide.* Volume II. Salem, OR: Math Learning Center.

Bremigan, E. G. (2003). Developing a meaningful understanding of the mean. *Mathematics Teaching in the Middle School, 9*(1), 22-26.

Cai, J. (1995). Beyond the computational algorithm: Students’ understanding of the arithmetic average concept. In L. Meira, & D. Carraher (Eds.), *Proceedings of the 19th International Conference for the Psychology of Mathematics Education* (Vol. 3, pp. 144-151).

Pernambuco, Brazil: Universidade Federal de Pernambuco.
Cai, J., & Moyer, J. C. (1995). Middle school students' understanding of average: A problem solving approach. In D. T. Owens, M. K. Reed, & G. M. Millsaps (Eds.), *Proceedings of the Seventeenth Annual Meeting of the North American Chapter of the International Group of the Psychology of Mathematics Education* (Vol. 1, pp. 359-364). Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education.
Friel, S. (1998). Teaching statistics: What's average? In L. Morrow & M. Kenney (Eds.), *The teaching and learning of algorithms in school mathematics* (pp. 208-217). Reston, VA: National Council of Teachers of Mathematics.
George, E. A. (1995). Procedural and conceptual understanding of the arithmetic mean: A comparison of visual and numerical approaches. In D. T Owens & M. K. Reed (Eds.), *Proceedings of the International Group for the Psychology of Mathematics Education. North American Chapter XVII* (Vol. 1, pp. 204-209). Ohio State University.
Innabi, H. (2007). Factors considered by secondary students when judging the validity of a given statistical generalization. *International Electronic Journal of Mathematics Education, 2*(3). Online: www.iejme.com/.
Kamii, C., & Warrington, M. (1999). Teaching fractions: Fostering children's own reasoning. In L. Stiff & F. Curcio (Eds.), *Developing mathematical reasoning in grades K-12* (pp. 82-92). Reston, VA: National Council of Teachers of Mathematics.
Leon, M. R., & Zawojewski, J. S. (1993). Conceptual understanding of the arithmetic mean. Paper presented at the *Annual Meeting of the American Educational Research Association*, Atlanta GA, April 12-16.
Meyer, R., Browning, C., & Channell, D. (1995). Expanding students' conceptions of the arithmetic mean. *School Science and Mathematics, 95*(3), 114 -117.
MOEU (2001). *National document on mathematics curriculum.* United Arab Emirates: Ministry of Education and Youth.
Mokros, J., & Russell, S. (1995). Children's concepts of average and representativeness. *Journal for Research in Mathematics Education*, *26*(1), 20-39.
Mevarech, Z. R. (1983). A deep structure model of students' statistical misconceptions. *Educational Studies in Mathematics, 14*, 415-429.
Mvududu, N. (2005). Constructivism in the statistics classroom: From theory to practice. *Teaching Statistics, 27*(2).
National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
Pollatsek, A., Lima, S., & Well, A.D.(1981). Concept or computation: Students' understanding of the mean. *Educational Studies in Mathematics 12*, 191-204.
Russell, S., & Mokros, J. (1996). Research into practice: What do children understand about average? *Teaching Children Mathematics, 2*(6), 360-364.
Strauss, S., & Bichlier, E. (1988). The development of children's concepts of the arithmetic average. *Journal for Research in Mathematics Education, 19,* 64-80.
Watson, J., & Moritz, J. (1999). The development of concepts of average. *Focus on Learning Problems in Mathematics, 21*(4), 15-39.
Zawojewski, J., & Shaughnessy, M. (2000). Mean and median: Are they really so easy? *Mathematics Teaching in the Middle School, 5*(7), 436-440.