

ELEMENTARY SCHOOL TEACHERS' UNDERSTANDING OF THE MEAN AND MEDIAN

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This paper presents results from a case study that explored elementary school teachers' understanding of essential topics in statistics. Teachers' understanding of the mean and median is presented in light of the suggestions by the GAISE document at Level A. It is important to consider where inservice teachers' understanding currently lies as we explore issues related to improving the sophistication of teaching and learning statistics in elementary schools.

INTRODUCTION

Over the past twenty years several studies have been conducted on students' understanding of average (e.g., Russell & Mokros, 1991; Cai & Moyer, 1995; Zawojewski & Heckman, 1997; McGatha, Cobb, & McClain, 1998; Watson & Moritz, 1999; Zawojewski & Shaughnessy, 1999; Batanero, Cobo, & Diaz, 2003; Garcia & Garret, 2006). However, only one of these studies (Russell & Mokros, 1991) has included teachers as part of their investigation. In light of recent reform movements in the United States of America (USA) to include more sophisticated statistical topics in the K-12 setting, the study presented in this paper explored three elementary school teachers' understanding of the mean and median in relation to what is now expected of students in the USA as described in the Guidelines for Assessment and Instruction of Statistics Education (GAISE) (Franklin, Kader, Mewborn, Moreno, Peck, Perry & Scheaffer, 2007).

One of the primary concerns that motivated the creation of the GAISE document was that "statistics...is a relatively new subject for many teachers who have not had an opportunity to develop sound understanding of the principles and concepts underlying the practices of data analysis they are now called upon to teach" (Franklin et al., 2005, p.5). With these expansions to the K-12 curriculum, it is important to examine what teachers know about the subject matter. The mean and median are considered for the purposes of this paper since these topics have been present in elementary schools in the USA for years. If elementary school teachers have difficulties with these topics, then extreme caution should be used when increasing the level of sophistication at which statistics is covered in schools without addressing teachers' preparation to teach statistics effectively.

The authors of the GAISE document (Franklin et. al., 2005) identify three levels of statistical development (Levels A, B and C) that students must progress through in order to develop statistical understanding. It is paramount for students to have worthwhile experiences at Level A during their elementary school years in order to prepare for future development at Levels B and C at the middle and secondary levels. "Without such experiences, a middle (or high) school student who has had no prior experience with statistics will need to begin with Level A concepts and activities before moving to Level B" (Franklin et. al., 2005, p.13). The GAISE document indicates that students at Level A should understand the mean as a fair share (Franklin et. al., 2005, p. 30) and the median as the middle point (Franklin et. al., 2005, p. 29).

Since elementary school teachers do not typically receive much training in statistics during their teacher preparation programs, it is safe to assume their understanding of the mean and median is similar to that of students. As indicated previously, research focused specifically on elementary school teachers' understanding of statistics is sparse, at best.

PREVIOUS RESEARCH ON STUDENTS' UNDERSTANDING OF AVERAGE

Watson and Moritz (1999, 2000) discussed the complexity of the concept of average in relation to the three measures of center – mean, median, and mode. They emphasized that these concepts should be developed in isolation of one another over time. "Teachers need to explore actively the three ideas associated with average in various contexts throughout the years of schooling in order to build up experiences which will point to appropriate measures for different

contexts” (Watson & Moritz, 1999, p. 36). Russell and Mokros (1991) found that “the introduction of (an) algorithm as a procedure disconnected from students’ informal understanding of mode, middle, and representativeness causes a short-circuit in the reasoning of many children” (p. 313).

Garcia and Garret (2006) categorized students’ understanding of average into the following areas: average as mode, average as algorithm, average as something reasonable, average as mean point, average as mathematical equilibrium. These categorizations were very closely related to the work of Russell and Mokros (1991). Garcia and Garret also discussed the importance of using both multiple-choice and open-ended questions in assessing understanding of average, because “many students who choose the correct answer for multiple-choice questions are not able to demonstrate reasonable methods for solving open-ended questions” (2006, p. 3). This may be related to students possessing a procedural versus a conceptual understanding of measures of central tendency.

Cai and Moyer (1995) found that students lack conceptual understanding of the arithmetic average. However, their study provided evidence that appropriate instructional activities could improve students’ understanding of average. Batanero, Cobo, and Diaz (2003) reported, “it is through repeated activity of solving significant problems related to the concept that the student progressively acquires and widens his/her understanding” (pp. 6-7). Particularly in the USA, since more sophisticated statistical topics are now included throughout the K-12 curriculum, it is important to examine elementary school teachers’ understanding of various statistical topics. If teachers do not have the knowledge expected of students, based upon the fact that they did not have worthwhile statistical experiences during their schooling, then it is unfair to be able to expect them to be able to help students develop a conceptual understanding of various statistical topics.

Research in the USA has shown students’ knowledge of the mean and median to be lacking. In their analysis of the Sixth Mathematics Assessment of the National Assessment of Educational Progress (NAEP), Zawojewski and Heckman (1997) found that students in 7th and 11th grade do not understand the mean and median. A study conducted by McGatha, Cobb and McClain (1998) indicated that when students are asked to find the “center” of a set of data, they most often choose the mean regardless of the context. Furthermore, students do not realize that in some contexts the median may be a more appropriate measure of center (Zawojewski & Heckman, 1997). The most likely reason why students often calculate the mean without thinking of the specific context is that they have been exposed to only non-contextual situations where the objective is to correctly perform a calculation, rather than use a statistic to analyze a set of data (Zawojewski & Shaughnessy, 1999).

In light of the suggestions offered by the GAISE document for the increased level of sophistication at which statistical topics should be covered, this paper presents the results of a study that sheds light on the current status of elementary school teachers’ understanding of the mean and median. The realization of where their knowledge lies should influence the way future teachers are prepared to teach statistics at the elementary school level.

METHODOLOGY

A case study involving three elementary school teachers was conducted in a school district in the USA. The results presented in this paper are from a larger study that explored elementary school teachers’ understanding in other areas of statistics as well as how their interaction with a particular curriculum and assessment instruments influenced their awareness of their knowledge (see Jacobbe, 2007). Three teachers were involved in the study – Ms. Alvin, Ms. Brown, and Ms. Clark. Ms. Brown and Ms. Clark taught Grade 3 while Ms. Alvin taught Grade 4. Ms. Alvin was the most senior of the three with nine years experience. Ms. Brown and Ms. Clark had five and four years experience, respectively.

The study was limited to three participants because of convenience sampling and the length of time necessary to visit, interview, and assess the teachers at the depth involved in this study. All three teachers were highly recommended by their principals and district supervisor as they were viewed as exemplary teachers of mathematics. Although generalizations cannot be

made because of the small sample size involved in this study, the results provide a glimpse into what may be expected of exemplary elementary school teachers.

Over the course of 18 months, each of these teachers participated in interviews, completed questionnaires and assessments, and allowed the researcher to observe their classroom at least 12 times. The results reported in this paper are from teachers' responses to questions on some of the nine instruments involved in the case study.

RESULTS AND DISCUSSION

The three teachers performed similarly on multiple-choice/short-answer questions involving the mean and median. Of the eight questions in this area, Ms. Alvin, Ms. Brown, and Ms. Clark correctly answered 63%, 63%, and 75%, respectively. It is interesting to explore Ms. Alvin's and Brown's responses to questions that involved finding the median of a data set. All three teachers were successful in answering one of the items related to the median. However, two of the three teachers computed the mean rather than finding the median. The mean of the numbers is 21.4, and the median of the numbers is 21. Figure 1 shows the question as well as Ms. Alvin's approach to answer the question.

There are five 4th grade classes at Taft School. The number of students in each of these classes is given below.

21, 19, 20, 24, 23

What is the median number of students for these classes?

A 19
B 20
C 21
D 24

$$\begin{array}{r}
 21 \\
 19 \\
 20 \\
 24 \\
 \hline
 23 \\
 107 \\
 \hline
 21 \\
 5 \overline{) 107} \\
 \underline{10} \\
 7 \\
 \underline{5} \\
 2
 \end{array}$$

Figure 1. Ms. Alvin's response to median question

Ms. Alvin and Ms. Brown were able to select the right answer among the choices provided; however their work showed a lack of procedural knowledge for finding a median. These errors are consistent with Garcia and Garret's (2001) categorization of understanding average as an algorithm. These errors also support the suggestion of Russell and Mokros (1991) that the introduction of an algorithm causes a short-circuit in students' thinking.

On a similar question, Ms. Alvin and Ms. Brown's performance differed. This particular question was administered at a later time in the study.

2. The high temperatures in degrees for 7 days last week are shown below.

~~70, 71, 68, 71, 62, 73, 68~~

What is the median temperature?

A 68
B 69
C 70
D 71

Figure 2. Ms. Brown's response to a subsequent median question

Ms. Brown's thinking appeared to be changing. In this particular problem, Ms. Brown knew that the median involved the middle number; however she neglected to order the numbers from least to greatest. On this question, Ms. Alvin repeated the same error she made in responding to the question shown in Figure 1.

In addition to being asked multiple-choice questions, the teachers were asked to respond to various prompts during interviews. These types of questions are important to assess understanding of the mean and median at a deeper level as described by Garcia and Garret (2006). During these interviews, all three teachers were able to describe an appropriate method for determining the mean and median. It was surprising that Ms. Alvin was able to provide an appropriate description for finding the median, as it contrasted with her approach to answering the questions that appear in Figures 1 and 2.

Teachers were also asked to describe the difference between the mean and median as well as provide examples of where the median would be more informative than the mean for a given situation. In this line of questioning, two of the three teachers had difficulty explaining what these measures of center represent. Ms. Clark indicated how to calculate such measures rather than explaining what they represent. Her response is shown in Figure 3 and provides further evidence of how these teachers understood average as an algorithm.

Researcher	What does the mean represent?
Ms. Clark	The mean is another word for average. So if you had the numbers, 1,2,3,4,5, then the mean would be $1+2+3+4+5$ divided by 5.
Researcher	What does the median represent?
Ms. Clark	The median is the middle number, so if the numbers were 1 through 5 the median would be 3.

Figure 3. Ms. Clark's response

Ms. Brown was confused regarding the difference between these two measures of center as shown in Figure 4. Her thinking moves toward understanding average as something reasonable as described by Garcia and Garret (2006). However, Ms. Brown does not realize that there is a difference between the mean and median.

Researcher	What does the mean represent?
Ms. Brown	The average.
Researcher	What is an average?
Ms. Brown	The usual amount over a range of numbers.
Researcher	What does the median represent?
Ms. Brown	The number that is in the middle...but wait...they are the same. No, not really
Researcher	So, what is the difference between the mean and the median?
Ms. Brown	I don't know the difference.

Figure 4. Ms. Brown's response

Based on the responses presented in Figures 1 through 4, Ms. Brown and Ms. Clark possessed a procedural knowledge of the mean and median; however they did not possess conceptual knowledge of these topics.

The most conceptual response to these questions came from Ms. Alvin. The initial response indicates that Ms. Alvin could calculate the measures of center but could not provide more of an explanation. However, upon further probing, Ms. Alvin seemed to be working toward a conceptual description of the difference between the mean and median. Ms. Alvin's response was surprising due to her inability to adequately respond to the questions that appear in Figures 1 and 2. These comments show that it is possible for someone to possess some conceptual knowledge of a particular topic without possessing the associated procedural knowledge.

Researcher	What does the median represent?
Ms. Alvin	I don't know... I don't know how to explain it. I guess I just know how to do it. I think that explaining it is difficult. It is easier just to show.
Researcher	What is the difference between the mean and the median?
Ms. Alvin	The median is finding the middle of all the data you have collected. You are not...I do not know...you have all the information there, but you do not manipulate the numbers to get one number. I don't know. All I can say is...the difference is when you are finding the average you are taking all of the numbers and manipulating them to get one number that represents the whole group and to find the median you still have that information, you're just finding the one that falls in the middle.

Figure 5. Ms. Alvin's response

Another interesting response was provided by Ms. Clark in regard to the usefulness of the median and the mean as shown in Figure 6.

Researcher	What is the median useful for?
Ms. Clark	The median, being the middle number, would be useful...I don't really know. I guess just to know what a median is. I don't know why you would really need to know what the middle number is, but I guess to know how many times something is done or halfway.
Researcher	Could you give me an example of a set of data where the mean would be more useful than the median?
Ms. Clark	Since I really don't know what the purpose of the median is, the mean would be more important to me in any situation.

Figure 6. Ms. Clark's response

Ms. Clark's response in Figure 6 again reveals that although she possessed a procedural knowledge of the median, she was unable to provide a reason why anyone would be interested in finding a median. This response in particular further emphasizes the importance of using both multiple-choice and open-ended questions in assessing the understanding of average as Ms. Clark was able to answer several multiple-choice questions correctly but clearly did not have a deep understanding of the mean and median.

The results presented in this paper show that the teachers involved in this study lacked a connection between the procedures for finding the mean and median and what these measures of center actually involve within a particular context. Once students have been exposed to the different measures of center in isolation of one another, then they should be presented with varying contexts where one measure of center is more appropriate to use than another. This will lead to students developing both conceptual and procedural understanding. Similar to the findings of Russell and Mokros (1991), these findings also speak to the importance of students (and teachers) possessing an understanding of average that is not dominated by an algorithm but focused on the underlying concepts those algorithms represent.

CONCLUSION

The three teachers involved in this study do not possess knowledge of the mean and median as outlined at Levels A and B of the GAISE document. It is important to note that this lack of knowledge is not due to the three teachers' inability to understand statistics but due to a lack of content exposure as described in the GAISE document. If the sophisticated level of understanding described by the authors of the GAISE document is to be realized by K-12 students, it is important that teachers are prepared to teach statistics at this level. Since these

expectations are relatively new, most preservice teachers likely have not had sufficient experiences during their K-12 schooling to develop such an understanding. As a result, teacher preparation programs should develop courses to ensure the objectives identified in the GAISE document are realized by the teachers who will be called upon to educate students in such a manner. The GAISE document clearly emphasizes the importance of students progressing through each level of statistical understanding. In order for preservice teachers to develop conceptual understanding of the statistics topics they are now called upon to teach, they must progress through these levels in a similar manner during their teacher preparation programs.

The results presented in this paper reveal that even some exemplary mathematics teachers at the elementary school level do not possess conceptual knowledge of two of the most basic concepts in statistics. A course focused on statistics for elementary school teachers will not help teachers that are already in the field. Sustained professional development should be provided to those teachers in order to allow them to progress through the levels of understanding suggested in the GAISE document. The importance of having ongoing professional development throughout the year and not just several full-day inservices is expanded upon in the full report of this study (see Jacobbe, 2007). Until teachers' preparation is addressed, it is unfair to expect elementary school teachers to have the understanding necessary to teach statistics at the depth described in the GAISE document.

REFERENCES

- Batanero, C., Cobo, B., & Díaz, C. (2003). Assessing secondary school students' understanding of averages. *Proceedings of CERME 3*, Bellaria, Italia. Online: www.dm.unipi.it/~didattica/CERME3/.
- Cai, J., & Moyer, J. (1995). Beyond the computational algorithm. Students' understanding of the arithmetic average concept. In L. Meira (Ed.), *Proceeding of the 19th Psychology of Mathematics Education Conference* (vol. 3, pp. 144-151). Recife, Brasil: Universidade Federal de Pernambuco.
- García, C., & Garret, A. (2006). On average and open-end questions. In A. Rossman & B. Chance (Eds.), *Proceedings of the Seventh International Conference on Teaching Statistics*. [CD-ROM]. Salvador (Bahia), Brasil: International Statistical Institute.
- Franklin, C., Kader, G., Mewborn, D., Moreno, J., Peck, R., Perry, M., & Scheaffer, R. (2007). *Guidelines for assessment and instruction in statistics education (GAISE) report*. Alexandria, VA: American Statistical Association.
- Jacobbe, T. (2007). *Elementary school teachers' understanding of essential topics in statistics and the influence of assessment instruments and a reform curriculum upon their understanding*. Unpublished Ph.D. Clemson University.
- McGatha, M., Cobb, P., & McClain, K. (1998). An analysis of students' statistical understandings. Paper presented at the *Annual Meeting of the American Educational Research Association*, San Diego, CA.
- Russell, S. J., & Mokros, J.R. (1991). What's typical? Children's ideas about average. In D. Vere-Jones (Eds.), *Proceedings of the Third International Conference on Teaching Statistics* (pp. 307-313). Voorburg, Netherlands: International Statistical Institute.
- Watson, J. M., & Moritz, J. B. (1999). The developments of concepts of average. *Focus on Learning Problems in Mathematics*, 21(4), 15-39.
- Watson, J. M., & Moritz, J. B. (2000). The longitudinal development of understanding of average. *Mathematical Thinking and Learning*, 2(1&2), 11-50.
- Zawojewski, J. S., & Heckman, D. J. (1997). What do students know about data analysis, statistics, and probability? In P. A. Kenney & E. A. Silver (Eds.), *Results from the sixth mathematics assessment of the National Assessment of Educational Progress* (pp. 195-223). Reston, VA: National Council of Teachers of Mathematics.
- Zawojewski, J. S., & Shaughnessy, J. M. (1999). Data and chance. In P. A. Kenney & E. A. Silver (Eds.), *Results from the seventh mathematics assessment of the National Assessment of Educational Progress* (pp. 235-268). Reston, VA: National Council of Teachers of Mathematics.