STATISTICAL GRAPHS IN THE TRAINING OF TEACHERS

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The responses to four multiple-choice questions taken from delMas, Garfield and Ooms (2005) by 190 future Spanish teachers and 345 American college students are compared and then complemented with a qualitative analysis of the justifications given by a subsample of 44 Spanish teachers. Some conclusions for the training of teachers are presented.

INTRODUCTION

Statistical graphs have developed into a useful tool in many disciplines by allowing for a statistical overview of phenomena and understanding these tools is a vital part of statistical literacy (Gal, 2002). As such, one of the objectives of primary education is that students be able to use basic data gathering techniques to obtain information on phenomena and situations in their surroundings, represent the information graphically and numerically and draw appropriate conclusions. Consequently, helping future teachers acquire good graphing skills should be a main focus of pre-service teacher training in statistics, since graphical representations of data are among the main statistical topics included in the elementary school mathematics curriculum.

In many countries, statistics education for these teachers is only provided by mathematics or mathematics education courses and tends to focus on descriptive statistics. It includes a review of traditional graphs for quantitative and qualitative variables, which prospective teachers should have studied in school, such as histograms and frequency polygons and an introduction to other graphs, like stem-and-leaf and box plots, along with asymmetric, skewed, uniform or bimodal distributions. However, our own education experience has shown us that prospective teachers graduate with insufficient knowledge to adequately handle the learning requirements for statistical graphs proposed in the primary school curriculum. This experience led us to start a research project to assess the future teachers' understanding of statistical graphs as a first step in the design of educational materials aimed at improving the education of these teachers and the way the subject is taught in the future.

Background

Early research of understanding statistical graphs focused on describing different levels in this understanding (Friel, Curcio & Bright, 2001). More recently some authors have emphasized the importance of reading graphs involving out-of-school situations (Monteiro & Ainley, 2007) or tried to relate the reading of graphs to statistical literacy (Gal, 2002; Aoyama, 2007).

Our own research (Espinel, 2007) served to describe conceptual and interpretative difficulties in the way future teachers construct histograms and frequency polygons, such as not considering zero frequency intervals, inadequately labeling of real numbers on the axes, separating histogram bars or not completing the polygon. There was also a marked inconsistency between the way some of the prospective teachers constructed their own graphs and their assessment of the correctness/incorrectness of other students' graphs, which will affect their ability to evaluate their future students' learning of graphical techniques.

In spite of the aforementioned results, there is still need for more research specifically focused on teachers, since, according to Batanero et al. (2006), "research in statistics education is showing that many teachers unconsciously harbour a variety of probabilistic and statistical difficulties and errors (misconceptions) that might be shared with students" (p.3).

METHOD

To determine whether prospective teachers were able to correctly read and interpret data distributions, we used questions from a test devised for the ARTIST (*Assessment Resource Tools for Improving Statistical Thinking*) project, (delMas, Garfield & Ooms, 2005) involving

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statistical literacy and reasoning that collected data from 345 college students from the United States of America. The statistics literacy questions require knowledge of basic terminology and tools, in addition to the interpretation of data representations, while the reasoning questions entail handling and making sense of statistical information.

For our research, we selected the four multiple-choice questions from ARTIST that best suited the curriculum followed by Spanish students, with two questions involving statistics literacy and two more on statistical reasoning. Data were gathered from 190 future teachers at the University of La Laguna, hereafter referred to as “future teachers.” These students had received 12 hours of training in statistics, which included the graphs mentioned in the introduction. The test was conducted during a routine class session.

Below we list the questions and compare the results obtained by the future Spanish teachers and the participants in the delMas, Garfield and Ooms (2005) study. The interest of making this comparison is that both groups involve students of similar age, enabling us to assess whether the future Spanish teachers reached at least the same level of understanding as participants in the delMas, Garfield and Ooms study. Each table lists, in the first column, the choices available to the students, with the correct answer highlighted in italics. The next two columns, labeled “College” and “Future Teachers”, show the percentage of American students and Spanish teachers who selected each option. A qualitative analysis of a subsample of 44 Spanish teachers who were asked to justify their answers to each question serves to analyze the reasoning future teachers used and the difficulties they encountered.

ANALYSIS OF THE TEST RESULTS

Statistical literacy: Reading a graph. Question 1 (Figure 1) poses the challenge of asking for the number of students who scored above 15, a value which is not specified on the horizontal axis, and of realizing that the values for the variable can only be integer numbers. The correct choice involves assuming that the variable is discrete, that is, adding the frequency of the last two intervals (option b), since the frequency of the interval [14,16) should not be taken into account. Only 36% of the teachers (Table 1) picked this answer; moreover, this was the only question in which the teachers outperformed the college students (24%).

Question 1. Scores for a quiz were calculated as the number of correct responses. Below is a graphical display of the quiz scores. How many of the scores are above 15?

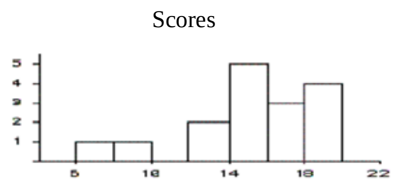

Figure 1. Question 1: Reading a graph

Table 1. Percentage of answers for Question 1

Answer	College	Teachers
a. 6	4.1	7.4
b. 7*	23.8	36.3
c. 12	30.4	50.5
d. 15	1.4	4.2
Blank	40.0	1.5

*****correct answer

Fifty percent of future teachers incorrectly considered the variable to be continuous (option c), resorting to the cumulative frequencies histogram, or to the cumulative frequency polygon, to interpolate its value, therefore keeping with the process learned in statistics classes. This answer was given by 30% of college students. Note that only 1.5% of the teachers, versus 40% of college students, provided no answer.

The qualitative analysis of the explanations to the answers (*n*=44 students) revealed that most of those teachers who picked the right answer correctly justified their choice, that is, they read the graph and added the last two columns (3+4=7). Some teachers drew intervals to justify their answers, noting how the frequencies in the interval [14,16) did not count. In a few cases the answer was correct but not the justification. For example, a future teacher selected a score of 15 on the horizontal axis but assigned a frequency of 7 on the vertical axis. Another teacher assigned a score of 22 to 4 students and 18 to 3 students, which totaled 7, and picked 7 as the answer, though he had only associated one value with the entire frequency corresponding to the interval. Yet another calculated 22-15=7, thus chancing upon the right answer.

Option a was justified by virtue of the highest frequency bar, with arguments noting that the other options (7, 12 and 15) were "off the graph," that is, interpreting the options as simple frequencies even though they were being asked for cumulative frequencies. Those choosing option c justified it by saying they added from 14 on (5+3+4=12), or resorted to the intervals, noting how there are 12 students in the interval (15,20]. There was some incoherent reasoning for option *d*, such as "a lot must have scored above 15" or "the area of the graph is greater than 12".

Statistical literacy: Interpreting a graph. The graph in question 2 (Figure 2) shows a data distribution and asks for the best statistical description from among four correct statements.

*Question 2***.** The following graph shows a distribution of hours slept last night by a group of college students. Select the statement below that gives the most complete description of the graph in a way that demonstrates an understanding of how to statistically describe and interpret the distribution of a variable.

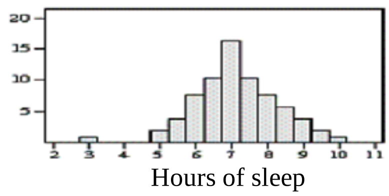

Figure 2. Question 2: Interpreting a graph

As shown in Table 2, most teachers (44 percent) picked the correct answer, d, which best describes the distribution considering the shape, center and range of the data. The percentage, however, is below that of the college students (73 percent). The written justifications (in the subsample) to the right answer were, for the most part, correct and show that this description was chosen for its use of terms such as bell-shaped, range, typical and outlier. It was also described as being the "best written," "most complete" or "most coherent" of the four choices.

Table 2. Percentage of answers for question 2

Answer	College	Teachers
a. The bars go from 3 to 10, increasing in height to 7, then decreasing to 10. The tallest bar is 7. There is a gap between three and five.	4.5	30.6
b. The distribution is normal, with a mean of about 7 and a standard deviation of about 1.	19.5	13.7
c. Most students seem to be getting enough sleep at night, but some students slept more and some slept less. However, one student must have stayed up very late and got very few hours of sleep.	3.1	11.6
d. The distribution of hours of sleep is somewhat symmetric and bell-shaped, with an outlier at 3. The typical amount of sleep is about 7 hours and the overall range is 7 hours.*	72.9	43.7
Blank	0.0	0.5

*correct answer

It is worth noting that the second choice of American students was option b, while for Spaniards it was option a (30 percent). The teachers' justifications for picking option a fell into one of two categories. Some ruled out option d for using the term "symmetric", thinking that a graph with an outlier cannot be symmetrical. The second reason involved the "photographic" description of the graph (Janvier, 1978) given by option a, that is, it describes the distribution of the “hours of sleep” variable sequentially. Those students who opted for option b did so because of its use of the terms mean and standard deviation. Those choosing option c noted the contextualized text, which describes the “hours of sleep” of students at that university.

Statistical reasoning: Matching a graph with its description. Question 3 (Figure 3) offers three descriptions that the students must match with one of the four graphs shown. The results are depicted in Table 3.

*Question 3***.** Match each description to the appropriate histogram.

Description A. A set of quiz scores where the quiz was very easy.
Description B. The last digit of the winning lottery numbers for a year.
Description C. The average weight of a healthy adult compiled monthly over the course of two years.

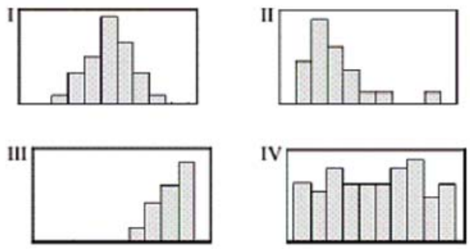

Figure 3. Question 3: Matching a graph with its description

Table 3. Percentage of answers for question 3

Graph	Description A		Description B		Description C	
	College	Teachers	College	Teachers	College	Teachers
I	5.4	23.7	26.1	16.3	24.7 *	13.2*
II	7.2	5.8	21.8	42.1	8.3	3.1
III	69.4*	33.1*	7.2	21.6	7.3	6.3
IV	18.1	30.0	44.8*	4.7*	59.7	62.6
Blank		7.4		15.3		14.7

*correct answer

*Description A***.** Note the balanced percentages for the teachers, versus the 70% of college students who correctly matched description A to histogram III. Only 33 percent of teachers chose histogram III (the right answer), justifying their choice with commentaries such as “there is a greater frequency of high scores,” “the scores bunch up at the end” or “they’re the high grades.” Almost the same percentage (30 percent) opted for IV, noting that “if the questions were easy, the results should be uniform.” A high percentage (24 percent) chose I even though this option uses an example from a teacher’s everyday life.

Description B. Option IV, the right answer, was the least chosen by the teachers, making this the most missed question. Among college students, 45 percent chose this option, with nearly half of the remaining answers being divided between graphs I and II. This result shows the difficulty of interpreting a uniform distribution by our teachers, who failed to associate each bar with a digit. On the contrary, the use of the expression “last digit” in the question led some to choose distribution II, which has an outlier and thus makes no sense. In their justifications, the few teachers who did answer correctly clearly realized there were 10 values of the variable, noting the digits go from 0 to 9. For example, one student wrote that “they all have the same chance of being picked, there is a slight margin of difference.”

Description C. This question posed a high degree of difficulty for both groups, with some 60 percent of the college students opting for graph IV. Of the few teachers who answered correctly, we note the following justification: “There is a mean surrounded by varying positions.” The results indicate that the average weight of an adult is not associated with a normal curve, which leads to choosing the uniform distribution of curve IV. Moreover, these teachers had no clear idea of what a distribution is, visualizing the data like a time series, with some of them indicating that the horizontal axis represents the months of the year and the vertical one the weight (instead of the density or frequency). Many teachers confessed in their justifications that this question confused them, that they either did not know what to answer or hazarded a guess. The question yielded very poor results among the teachers, something that also appeared among the college students to a lesser extent.

Statistical reasoning: Matching different graphs drawn from the same data. In question 4 (Figure 4), the students had to select the box plot that matches the histogram, which requires noting the lower limit, the outlier and the asymmetry to answer correctly.

Question 4. The following graph shows a distribution of hours slept last night by a group of college students. Which box plot seems to be graphing the same data as this histogram?

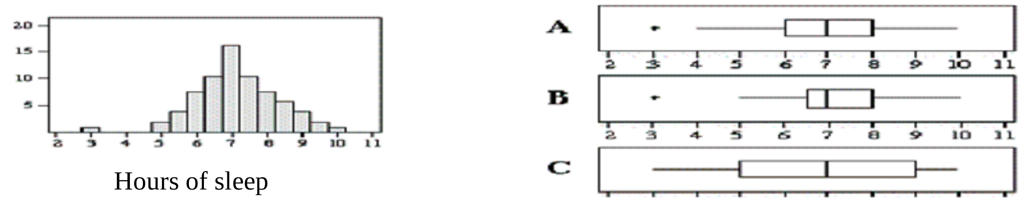

Figure 4. Question 4: Matching different graphs drawn from the same data

The results are depicted in Table 4. The differences here were not large, since 49 percent of teachers (versus 56 percent of college students) picked option B. Most teachers who chose B in the subsample focused on the mean (median), the outlier and the range; others noted that "there is a gap from 3 to 5, like in the histogram," "the numbers match" or "the data are between 5 and 10." They also stated that they constructed the box diagram with the data from the histogram. Option A is justified "by the symmetry of the box", "because the numbers 6 and 8 appear in the histogram", "because the mean is 7 and the deviation is 1", or simply "because the values of the numbers match." Those choosing C noted that "the graph goes from 3 to 10, like the box", "there are quartiles", "it encompasses the entire pyramid", "because of the range of 7", or "because it most resembles the graph".

Table 4. Percentage of answers for question 4

Answer	College	Teachers
A	30	24.2
B*	55.8	48.9
C	14.2	25.8
Blank		1.0

*correct answer

CONCLUSIONS

The results suggest that future teachers struggled when reasoning about graphs, not quite achieving at the same levels as those in the delMas et al. (2005) study, though the same questions proved difficult for both groups. The training received by the teachers was only useful in helping them understand the explicit information in the graphs but did not help them reason beyond the information provided by statistical Cartesian graphs. Though they grasped the essentials of descriptive statistics, they lacked the experience to interpret graphs, making errors involving symmetry, outliers and cumulative frequencies. They struggled with mean and median and thought mainly in terms of qualitative variables, thereby confusing histograms with bar graphs. They incorrectly identified the relevant variable and failed to interpret the data distribution as a histogram. The conclusion drawn from the qualitative analysis is that students do not take into account the distribution as a whole, focusing instead on specific aspects, such as the average or an outlier.

A description of the different variables was not enough to associate the distribution graph. Thus they failed to recognize the behavior patterns of variables, which resulted in difficulty when matching a description to a distribution in question 3. The unconventional nature of this question for teachers should be noted, as they are used to working with data distribution graphs in a Cartesian system where more information, such as scales or the names of the variables, is given. Given the complex process involved in interpreting data distributions, the students have no resources for tackling this problem (Bakker, 2004).

In conclusion, there is a pressing need to better educate primary school teachers in the area of statistics, so they can both teach it in the mathematics classroom and use it to understand statistical information in other primary school subjects, such as social studies and science. As professionals, they will likely need to either draft reports where they present a statistical analysis or read and interpret statistical summaries on education written by others. To improve their handling of graphs, it may be useful to propose activities using familiar contexts (Burgess, 2002) in which the teachers have to recognize and associate patterns with variables, as was done in question 3 of this study. The properties of distributions can also be analyzed via comparisons by using measures of central tendency, dispersion and skewness. It is also necessary to reason about distributions, one of the key concepts in statistics, as shown by the recent wide-ranging research into the issue. To effect these improvements it is necessary to increase the amount of time dedicated to the statistics training of future teachers. It is also necessary to continue the research into the knowledge teachers have of statistics. For our part, we shall continue gathering data on our project, to be presented at the Study conference.

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