STATISTICS TEACHERS AS DESIGNERS OF CONCEPTUAL SPACE

Dave Pratt
University of London, United Kingdom
d.pratt@ioe.ac.uk

STATISTICS AS A HANDLE-TURNING

Agendas for reform in the way mathematics is taught place emphasis on problem solving, relevance and child-centred activity. I am concerned that statistics education is just as culpable as that in mathematics in failing to meet those agendas, and, on the contrary, that most students' experience of statistics at all ages is essentially one of acting out procedures, which have been rendered meaningless by the teaching approach. I shall offer an example, taken from the National Curriculum for England and Wales, to illustrate the issues that press those anxieties.

Of course, groups and movements are trying to remedy these deficiencies. For example, some teachers have begun to deploy exploratory data analysis (EDA) techniques to involve students in the manipulation of data drawn from real world contexts and to avoid the procedural computation of descriptive measures of average and spread or the mindless application of formal statistical inference, which can be thought of as the mere turning of a handle to produce the statistical information. The danger, under such circumstances, is of course that the statistics produced and the techniques used are misapplied and inappropriately interpreted.

Technology can make the situation worse. Just as the unthinking use of calculators in mathematics might lead to a certain de-skilling in arithmetic, the use of software tools such as Statistical Package for Social Science (SPSS) can lead to unawareness of the limitations and constraints that should be applied to the automated techniques. I will illustrate through an SPSS example how this might happen. However, banning the use of calculators in mathematics classrooms means that many pedagogic opportunities for the effective and imaginative use of calculators are missed. Similarly, ignoring the opportunities that digital technology offers statistics teachers would be a terrible pedagogic waste.

TEACHING PROBABILITY WITH TECHNOLOGY

There is now emerging software that has been specially developed as an educational tool, a tool for learning statistics, rather than a productivity tool for doing statistics (the category I would reserve for SPSS and similar packages). Two examples stand out, Fathom (www.fathom.com) and its younger sister, Tinkerplots (www.keypress.com/x5715.xml). In a sense, these packages have perhaps been inspired by dynamic geometry software such as Cabri Geometre (www.cabri.com) or Geometer's Sketchpad (www.dynamicgeometry.com/), and I have heard them referred to as dynamic statistics software. Certainly they exploit the digital possibilities of dynamics and visualisation to provide intuitive tools for learning about statistics.

However, as with calculators, all tools, whether inspired at the design level or not, can be used in ways neither in keeping with the reform agenda around the world, nor as envisaged by the designers. As a teacher and a software designer, my work over many decades has been inspired by Logo and the corresponding constructionist movement (Harel & Papert, 1991), but I would be the first to admit that many teachers have in the past used Logo in their classrooms in ways very different from the constructionist philosophy, and that perhaps as a result, Logo has more or less disappeared from mathematics curricula, which have developed along an opposite path to that envisaged by the original Logo developers. From my research on students' thinking-in-change (Noss & Hoyles, 1996) about probability and its relationship to the design of software tools, I am able to report on some findings not only about student learning of probability but also about teaching probability.

STUDENT KNOWLEDGE OF PROBABILITY

I provide a brief synopsis here of research that has previously been widely reported (for example, Pratt, 1998; Pratt, 2000; Pratt & Noss, 2002). It is worth emphasising that this study reported not only how student thinking evolved towards a more sophisticated understanding but

© ICMI/IASE 2008. In C. Batanero, G. Burrill, C. Reading & A. Rossman (Eds.), *Joint ICMI/IASE Study: Teaching Statistics in School Mathematics. Challenges for Teaching and Teacher Education. Proceedings of the ICMI Study 18 and 2008 IASE Round Table Conference.*

also on what students already knew, in contrast to many previous studies, which had placed emphasis very much on people's fallibility through the reporting of misconceptions (for example, Kahneman and Tversky, 1982; Lecoutre, 1992; Konold, 1989).

My study used a design research approach (Cobb et al., 2003) to build a domain of stochastic abstraction, *ChanceMaker*, in which 10-11 year old students were able to edit the behaviour of broken gadgets, simulations of everyday random generators, such as coins, spinners and dice, as the students attempted to fix the gadgets so that they behaved according to the students' expectations. The students controlled the behaviour of the gadgets by modifying a workings box; for example, the workings box of a broken dice might by default read "choose-from [1 2 3 4 5 6 6 6]", which might eventually be edited by the student to read "choose-from [1 2 3 4 5 6]".

The window, provided by the iterative design process on the students' thinking-in-change about randomness, indicated that the students used a range of expert-like meanings described as *local* (in the sense that these were all situated in the short-term here-and-now); the students would recognize as random those gadgets which were unpredictable, uncontrollable, un-patterned or fair. Fairness was an important resource that the students were able to use to construct more sophisticated *global* meanings (an aggregated long term perspective on randomness).

In their interactions with ChanceMaker, the students began to articulate heuristics that were situated but nevertheless appeared to describe behaviour across the gadgets. These articulations could be characterized, for example, as "the more times you throw the dice, the more even is the pie chart" to describe the observation that fairness in the pie chart (as opposed to in the appearance of the gadget) would emerge when trials were repeated large numbers of times. This causal-like heuristic (more trials cause a more "even" pie chart) seemed to capture the essence of what the expert might refer to as the *Law of Large Numbers*. Noting, however, that this heuristic seemed to fail when the number of trials was small, or when the workings box was not fair, students began to articulate a second heuristic: "The more even is the workings box, the more even is the pie chart, provided the number of trials is large", which appears in expert terms to acknowledge the role of distribution.

TEACHING PROBABILITY

The iterative design of ChanceMaker sensitised me to the relationship between the resources I provided and the effect on students' learning. Hence, by reflecting on those design decisions, I became aware of how teachers might conceptualise their own activity, if they were to use technology, and other material resources, in a way that met the commonly espoused reform agendas. In this sense, while I might think of myself as a software designer, teachers might consider themselves also as designers; not only do they design materials, lesson plans and curricula in a direct way, but they also aspire in a more indirect way to design their students' conceptual space. When teachers seek to use educational tools for this purpose, they must seek also to recognise the design intentions, and incorporate its use in ways that are consistent with those pedagogic intentions. However, for this to be feasible, it is incumbent on designers to make explicit those intentions.

So in setting out my own design heuristics, I simultaneously offer an analysis of how those heuristics might inform the teaching of probability (reported at length in Pratt et al., submitted). In particular, I will discuss four heuristics: (i) Testing personal conjectures, (ii) Building on current knowledge, (iii) Linking purpose and utility (Ainley et al., 2006), and (iv) Fusing control and representation. In each case, I shall explain how the heuristic developed out of the research study, and the extent of its relevance, not only to the design of ChanceMaker, but also to the teacher as designer of conceptual space.

REFERENCES

Ainley, J., Pratt, D., & Hansen, A. (2006). Connecting engagement and focus in pedagogic task design. *British Educational Research Journal*, *32*(1), 23-38.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research, *Educational Researcher*, *32*(1), 9–13.

Harel, I., & Papert, S. (1991). *Constructionism*. Norwood, New Jersey: Ablex.

Kahneman, D., Slovic, P., & Tversky, A. (1982). *Judgment under uncertainty: Heuristics and biases*. Cambridge: Cambridge University Press.

Konold, C. (1989). Informal conceptions of probability. *Cognition and Instruction*, *6,* 59-98.

Lecoutre, M. P. (1992). Cognitive models and problem spaces in “purely random” situations. *Educational Studies in Mathematics, 23,* 589-593.

Noss, R., & Hoyles, C. (1996). *Windows on mathematical meanings: Learning cultures and computers*. London: Kluwer Academic Publishers.

Pratt, D. (1998). *The construction of meanings in and for a stochastic domain of abstraction*, Unpublished Doctoral Thesis, Institute of Education, University of London, May 1998.

Pratt, D. (2000). Making sense of the total of two dice, *Journal for Research in Mathematics Education*, *31*(5), *602-625*.

Pratt, D. & Noss, R. (2002). The micro-evolution of mathematical knowledge: The case of randomness. *Journal of the Learning Sciences*, *11*(4), 453-488.

Pratt, D., Noss, R., Jones, I. & Prodromou, T. (submitted). Designing for mathematical abstraction. *International Journal of the Learning Sciences*.