

“Data and Predictions” Emerging as One of the Basic Themes in the Mathematical Curriculum of the First Cycle School Level in Italy

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Abstract

In the reform of the Italian school system, a new pedagogical point of view has shifted attention away from content and skills and toward competencies. Also contributing to the proposal of a new mathematical curriculum is a new way of looking at mathematics that takes globalised societal needs into consideration.

This paper describes the role of the Italian Statistical Society (SIS) in proposing knowledge and skills of the “Data and predictions” nucleus, which has emerged as one of the basic themes in the new mathematics curriculum. We present the guidelines which have been implemented to enhance the visibility of statistics and probability in the mathematical curriculum, taking into account the results of some recent research on statistics education carried out in Italy.

Introduction

As is the case in many parts of the world, statistics and probability are taught in Italy as part of the mathematical curriculum. This has been the case since 1979 when members of the Italian Statistical Society (SIS) succeeded in introducing the contents of “Mathematics of certain events and mathematics of probable events” in middle school (ages 11-14). This was followed, in 1985, by the introduction of “Probability, statistics and computer science” in elementary school (ages 6-11).

In spite of their introduction in the curricula, statistics and probability continue to be the Cinderella among teaching themes in mathematics. This is largely due to the teachers’ lack of training in the subjects. However, as the view of mathematics in society and school is changing profoundly, the way in which statistics and probability are perceived is also changing. This seems particularly apparent during this time of deep transformation in the Italian school system and curricula. A new overall system was designed in 2003 and new official curricula were approved by the Ministry of Instruction, University, and Research (MIUR) in 2004.

Until March of 1999, curricula in Italy were laid down by a central authority (then, the Ministry of Public Education) and were mandatory all over the country, both in state and private schools. Since then, schools have had a greater measure of autonomy. At present each school has the power to organise, year by year, 20 percent of the curriculum according to its students’ socio-economic and cultural needs. In light of this history, this paper presents the old curriculum and its underlying philosophy, as well as the statistics and probability nucleus in the mathematics curriculum proposed by the Italian Mathematics Union (UMI) and the Italian Statistical Society in 2001, and the official MIUR nucleus in statistics and probability of 2004.

Stressing the mathematical reforms required by the role of globalisation, we describe the influence of new teaching methods and new mathematical curricula on the contents of statistics and probability courses.

The Former Curricula for Elementary School and Middle School

According to the 1985 curriculum, the goals in teaching elementary school mathematics were to enhance students’ development of a rigorous language used to define concepts and to enhance their ability to express hypotheses used to read and interpret reality. Teachers were also encouraged to use activities

that were problem-based and exploratory. In this context the introduction of “Probability, statistics and computer science” played an important role.

It has to be stressed, however, that it was left up to the individual teacher to choose the educational methods to be used and to identify in detail the contents of the items of the programme as well as the teaching/learning plan to be followed with the pupils. Notwithstanding the officially recognised interdisciplinary nature of teaching statistics as well as its instrumental nature as a language to communicate and express reality, the introduction of “Probability, statistics and computer science” caused some perplexity with regard to effective teaching. It had to be admitted that some teachers were never taught these notions in the course of their training, and professional expertise was often lacking, thereby diminishing the value of introducing these new topics.

The prior curriculum for the middle school (implemented in 1979) focused on three essential goals common to all subjects in the curriculum. The first goal was to provide “linguistic education” (i.e., instructional activities that develop students’ skills in using verbal and non-verbal language pertaining to all different disciplines). The second goal was to act in an “operative” manner in order to bring forth a kind of teaching/learning process where rules are not simply applied but discovered through real life experiences. The third goal was to implement the utilisation of the “scientific method” in the process of acquiring knowledge by first analysing phenomena to formulate hypothesis on their nature and origin, then verifying the proposed hypothesis by suitable experimentation (La Torre, 1996).

With reference to mathematics, the curriculum was accompanied by some discussion on how the subject matter should be taught. The guides for mathematics teachers paid heed, for the first time, to the results of psychological studies of adolescence. For instance, the guides recommended introducing new concepts with concrete examples, and moving on, much later, to the abstract and structural foundations, while always maintaining a balance between the two. This insistence on a balance between abstraction and pragmatism can be seen in the advice of Montaldo (1992) which attests to the benefits of using an inductive approach with pupils that draws on observation, easy experiments, and empirical trials to exercise their capacity for intuition and the spirit of research.

In the light of these considerations the introduction of statistics and probability in the curricula is necessary, as statistics not only allows one to develop non-verbal linguistic skills and helps in discovering rules in real life, but it is also the core of the “research method” when doing empirical enquiry (Wild & Pfannkuch, 1999). The authority who designed the curriculum introduced the sub-theme titled: “Mathematics of certain events and mathematics of probable events” within the mathematics curriculum, and also spread some basic elements of statistics within subjects such as geography, history, and technical education. There, statistics played an interdisciplinary role, and was seen as an instrument for understanding pertinent social, economic, and demographic phenomena.

However some concerns immediately arose. It was assumed that teachers were able to cope with this new subject, but in reality they had never been trained in probability or statistics, neither in schools nor in university. In addition, statistics and probability were added to an already huge curriculum, as, in Italian schools, there is only one teacher for science and mathematics in middle schools. Some teachers protested against the imposed curriculum, while others simply tolerated it. This may be why in 1981 the ministry enacted a new decree in order to consider statistics and probability as a topic for the final written examination.

Teachers, Pupils, and Teaching/Learning Problems at the Elementary Level

In 1992, a survey on 2700 fifth class pupils (10-11 years old) was carried out on behalf of the Italian Ministry of Public Education to monitor the implementation of the reform. No doubt the sample was small as it represented only 0.4 percent of the fifth class students, and 0.4 percent of the fifth classes. However this small non-random sample showed that “Probability and statistics” test items scored, on average, higher in schools where mathematics was taught by one specific teacher compared with schools where only one teacher was in charge for all disciplines (Bolletta, 1992).

As far back as 1993, Italian statisticians were producing papers regarding the teaching/learning of statistics and probability in elementary schools. These early papers related to textbooks (Moncecchi & D'Argenzio, 1994), articles in Italy's most widespread educational journals (Ottaviani, 1995), and psycho-pedagogical issues in teaching/learning statistics and probability (D'Argenzio, 1994). An early survey on elementary school mathematics teachers was carried out in the Province of Vicenza in 1995 (Canato & Luchini, 1996). This was followed by a more extensive survey on about 70 percent of primary school teachers in the Provinces of Padua, Treviso and Vicenza (Batic & Luchini, 2003). The main objectives of the research were to discover the teachers' attitudes towards statistics and probability and determine how they approached the topics. According to the analysis of the data collected and the segmentation analysis (CHAID), teachers were not all "favourably disposed" towards these two new subjects. In addition, it appeared that the objectives and the contents of statistics and probability for the children's formative process were not yet clear to teachers.

The majority of teachers received training in statistics and probability through a variety of activities in order to bring them up to date. However, they achieved unsatisfactory results even when using didactic supports above and beyond the traditional ones (Batic & Luchini, 2003). Thanks to a grant from the Ministry of Public Education, a more extensive observational study was carried out from 1999-2000 under the coordination of the CIRDIS¹ project. With reference to elementary schools, the purpose of this research was to evaluate the effectiveness of a teaching approach based on concept maps rather than on a classic instructional model.

Previous studies had already shown that many teachers and pupils learn basic statistical notions without understanding how to use them. They gather data, try to represent it in a chart, calculate the average, and so on, but are unable to analyze and evaluate it (D'Argenzio, Luchini, & Moncecchi, 1998). As a result, the CIRDIS group chose to utilize a statistics teaching model aimed at constructing conceptual connections between concepts, rather than focusing on separate notions. Another important decision by CIRDIS concerned the means of spreading this new model of teaching and learning statistics to practicing teachers.

Studies also showed that most teachers in Italy do not receive training in statistics during their pre-service education. In addition, the textbooks they have at their disposal have no serious pedagogical foundation (cf. Moncecchi & D'Argenzio, 1994), and contain many mistakes, particularly in graphical representations. Also, the educational journals often provide very poor recommendations and didactical materials on the subject (cf. Ottaviani, 1995). For these reasons, CIRDIS decided to provide a preliminary 32-hour training course in statistics and probability for teachers. The course focuses on 1) basic statistical concepts; 2) basic notions on how to critically evaluate and correctly analyse data; 3) elements of educational psychology for the teaching of statistics; 4) concept map models, class semi-structured interviews, conceptual maps and nets.

At elementary school level, the CIRDIS project involved 145 teachers and 2,129 pupils from five Italian towns. Following Dunkels' (1999) suggestions, teachers were given some teaching units for three levels (ages 6-7, 8-9, and 10) to be used in class. Teachers who adopted the concept maps strategy focused more heavily on learning processes than on the products of learning or on required skill standards. They started from a psychologically structured concept map.

For each class activity, teachers used different maps containing children's everyday terms and experiences that could be related to basic statistical notions. Pupils had two class discussions and semi-structured interviews on the meaning of words that appeared in the map used in their class. These discussions and interviews took place at the beginning of statistical work and at the end. This enabled teachers to compare the pupils' ideas had before and after the statistical activity. After the class activity in statistics, pupils demonstrated competence in collecting and manipulating data, making connections between statistical and mathematical concepts and real world, and understanding the first key ideas of statistics and connecting them in a simple network by making explicit connections that were implicit in instruction. Children also developed some critical capacities for collecting and inspecting data. Furthermore, most pupils achieved an understanding of basic statistics terms.

The results of this qualitative study confirmed that statistics education programs in primary schools could enable pupils to formulate questions that can be addressed with data collection; represent them at first with real and concrete materials, then with pictograms, and later in a slightly more abstract way with stem and leaf graphs or block graphs (where each observational units is represented by a block); organize and display data to answer the proposed questions; and use appropriate methods to analyse data and graphs.

Additional research has demonstrated that better results may be obtained in elementary school if teachers adopt a concept net approach, use class interviews as tools to highlight both the individual's and group's knowledge, and build on cognitive maps to check the progress of concept acquisition by pupils (Luchini, D'Argenzio, Moncecchi, & Giambalvo, 2000; Luchini, D'Argenzio & Moncecchi, 2002).

Teachers, Pupils, and Teaching/Learning Problems at the Middle Level

A profile of Italian "mathematics and science" teachers in middle school was compiled in 1986 via a random sampling survey concerning 1/9 of the population. The teachers who dealt with statistics and probability in the school year 1982/83 were an estimated 66 percent of the 45,800 teachers in active service. Among these teachers, many factors affected their decision to deal with the subject. The basic conditions seemed to have been a positive attitude of teachers toward their role, and the feeling of belonging to a socially relevant institution. The largest percentage of teachers of "mathematics and science" who dealt with statistics (85 percent) was found among those who attended a specific course after graduation. This put again into relief the role of attitudinal variables as determinants of the decision to teach statistics.

The presence of full-time activities in a school was also positively correlated to the teaching of statistics. The most popular parts of the statistics and probability programme dealt with the broad field of descriptive statistics (52 percent of the respondents treated it) followed by probability (41 percent). Those who treated data collection and analysis basically avoided every reference to statistical inference and, at best, they introduced a vague concept of probability through combinatorial analysis. On the other hand, more than half of those who treated combinatorial analysis neglected statistics (Fabbris, 1986).

With reference to middle schools, the complex observational research coordinated by the CIRDIS project has particularly studied teachers' perceptions of the importance of statistics learning as a means to develop students' logical abilities and problem solving skills (Milito & Marsala, 2002). The objective of this research was to evaluate the effectiveness of the teaching approach based on the Data-Oriented Approach (DOA) using a Co-operative learning (CL) model, compared to the DOA method for teaching statistics. DOA focuses on using real data related with daily life situations in which students can be personally interested or directly involved. The CL methodology was chosen because of good results obtained in other experiments, and because of the specific advantages it presents for learning statistics (Moore, 1997).

At middle school level, the experimental design of the DOA/CL study involved 86 teachers and 1514 pupils in four Italian regions. The teachers who agreed to participate in the research were trained on basic statistic topics at middle school level and on CL methodology. As is the case with most teachers, teachers in this study did not have any pre-service knowledge of statistics and had at their disposal very poor textbooks that were full of mistakes, particularly about graphical representations and the differences among qualitative and quantitative characteristics and their consequences. At the beginning and at the end of the training course, the teachers responded to a survey measuring their attitudes towards statistics. This was done in order to analyse their mental attitude and the educational value they gave to statistics.

The CIRDIS project research team prepared the materials for the teacher training and for the experimentation in the classrooms. In particular, teachers received a glossary containing statistical terms and other material with statistical contents presented in the training course. The teachers interested to the CL methodology also received a book on this matter. Every teacher in the second phase received five teaching units to be utilised in the classroom. The contents of the units were limited to descriptive

statistics, and included: 1) questionnaire and data collection; 2) table representations; 3) graphics; 4) means; and 5) variability.

Regarding the analysis of the student achievement test data, it was interesting to observe that a number of systematic errors cropped up in the results. For example, there were errors in conceptual understanding and calculation of the median and of variability. Also, there were mistakes in building histograms. Comparing the results of the two different teaching strategies, it can be seen that, on the whole, the scores of pupils' participating in the experiment were distributed in a negative asymmetric pattern, with similar variability for the CL group and the traditional group. The mean achievement test scores attained by each group were mixed. In the schools of Umbria and Veneto the mean score after CL was about four points higher than after the traditional method, but in Palermo and Rome, the highest scores were obtained by pupils taught with the traditional methods during the experiment.

Presenting statistics in class using the DOA approach offered concrete elements for pupil development and in particular, from a teaching point of view, it took advantage of the natural curiosity of the pupils and their motivation at being made responsible for their own learning. Utilising CL as a pedagogical model had, from the point of view of the pupils, some positive aspects: a) discovering new resources in themselves, b) personally managing the teaching activity, c) the acquisition of a valid working method, d) personal maturity in relationships with peers and collaborators. One aspect worthy of further investigation is whether the results of the experiment were linked to a favourable attitude of the teachers towards statistics. On the whole the training course produced a slight change in attitude towards statistics, with approximately 65 percent of teachers showing a more positive attitude (Galmacci & Mito, 2002).

From the statistics education research carried out in Italy from 1986 to 2003, some signals emerged concerning critical difficulties in teaching statistics. Mathematics teachers' lack of training in statistics and probability, and the poor instructional materials that they had at their disposal are problems that remained unsolved. In addition, larger scale research conducted by CIRDIS demonstrated positive results for an approach to the discipline that did not consider statistics as a part of mathematics but rather as a discipline in its own right. Highlighting the usefulness of statistics for understanding the real world and society appeared to motivate students in their learning process. This way of thinking about statistics is in line with the Italian school of statistics (Gini, 1962) as well as with the "contemporary statistics" view (Hoaglin & Moore, 1992). These findings permitted the elaboration of a framework useful for developing a curriculum for statistics and probability that focuses on statistical concepts and meaning much more than on calculation techniques.

A Changing School in a Changing Context

With the view toward aligning the Italian system with that of the other EU countries, an act was signed by the Italian Parliament in March 2003 that designed a complete school system from kindergarten to university. As a result of this act, two cycles are foreseen. The first cycle of eight years comprises primary school (five years) and lower secondary school (three years), the second cycle has two branches: lyceum (five years) and education and professional training (maximum four years). The main aims of this act are the promotion of continuity in the learning process through childhood and youth, the co-ordination of curriculum between the two cycles, and the possibility of shifting from the lyceum system to professional training system and vice-versa through a certification of competences.

"Competence" is the key word of the transformation process. This word is used in several different ways. It may refer to:

- a mix of knowledge and skills that can be used in professional life and that can be endorsed in the professional training system;
- a result of the learning process that needs to be assessed at the end of each school period or at the end of each cycle; or

- a personal quality developed via general or specific knowledge and skills, in accordance with personal attitudes and choices, and that is adequate for participation in social life and the workplace at the local, national, and European level.

The first meaning is more linked to particular professional situations, while the second is connected to assessment. According to the third meaning, “competence” is a long term goal that students are expected to achieve. “The competences must be possessed by the students in a way not linked to the space and the time in which they were reached, that means in contexts different from those in which they have been learnt and in times different from the school, one that is in citizen’s life” (Arzarello, Robutti, & Paola, 2002, p. 95). The shift from content topics to competence has many consequences from the pedagogical point of view both for teachers and students, and implies a new teaching/learning style relying particularly on problem-posing and problem-solving didactic methods (Roletto & Perrini, 2000).

Effects of Mathematics Changes on the Statistics Education Curriculum

Concerning statistics, the new school system and the school transformation process is posing some particular, very interesting, challenges due to the new idea of mathematics that has developed within the mathematics community. According to Arora (1992, p.1), “The most profound change in mathematics curricula in the twentieth century is synonymous with the introduction of the so-called *new maths* or *modern maths* in the 1960s.” Within about twenty years the movement pervaded almost the whole globe, resulting in new curricular materials, and in deliberations and debates at national and international forums. By the 1980s, however, some concerns began to arise regarding the mathematical competency and arithmetical skills of the high school graduates being produced. Questions then arose regarding why “the literates” from school were “so mathematically illiterate” (Arora, 1992).

This question was particularly important as unprecedented innovation in society and, in particular, globalisation put forward by modern telecommunications justified a new perceived “complexity of reality” and consequently of science (Mina, 2002). In this context “the central importance of mathematics and its applications in today’s world with regard to science, technology, communications, economics and numerous other fields” was officially recognised by UNESCO in 1997 to justify support to the World Mathematical Year 2000 initiative. To accept this point of view, however, means to recognise that mathematics has a role if it is no longer conceived only as the study of formal systems and is not perceived as superior among other sciences.

Obviously all of this has some influence on mathematics education and on research in the field. Particularly in the last decade of the 20th century, a movement has grown within the foundation of the International Commission for the Study and Improvement of Mathematics Education (CIEAEM). During CIEAEM’s 53rd Conference, in 2001, it was put forward that our civilisation has become increasingly mathematical and that mathematical education can no longer be restricted to an elite education but has to be conceptualised as a mathematical literacy education for all (Bazzini & Whybrow, 2002). At that time it was also recognised that it is possible to interest and engage a maximum number of children and adults by providing an education that starts from the phenomena and processes of their daily lives and promotes reflection and collaboration as a mode of teaching and learning.

The effects of this movement concern curriculum, classroom practice, and methods of teaching and learning mathematics. It is in precisely this sense and for that purpose that statistics has a new important role in school curriculum, particularly if we accept Gini’s (1962) definition, which asserts that statistics is a discipline suitable to quantitative investigation of mass or collective phenomena whose measurement requires a collection of observations. By this definition and with reference to daily life phenomena—for example social, economic, demographic, health, environmental phenomena—functional statistical competencies are many. They include the ability to pose and correctly solve such problems related to:

- the aim of the investigation,
- the definition of the phenomenon to be observed,

- the methods and tools to carry out observations,
- the collection of data,
- the organisation of the collected data in order to communicate a set of quantitative information,
- using technology to handle data,
- the conclusions drawn from the investigation process,
- the comparison among the results of the investigation process, and
- the *a priori* aim of the investigation itself.

If the collected data are derived from a random sampling of a population, the possibility of generating inductive inferential conclusions implies the utilisation of probability distributions. In this vision of statistics, arithmetic mean, standard deviation, frequencies, and so on are much more than formulae and numbers. They are also concepts and measures of concepts. In addition, probability is no longer a matter only of chance games.

“The Mathematics for Citizens” Project and Statistics Education

In July 2000, the need arose to adopt a mathematical curriculum based on competences and the idea of “mathematics for all.” This, along with the need of proposing a curriculum for the Italian school of autonomy, induced the President of the Italian Mathematics Union (UMI) to appoint a Committee to study and present a new mathematical curriculum for school, according to the needs of a 21st century society. The Committee put forward the “The Mathematics for Citizens” proposal and built a curriculum for primary through upper secondary school. Within the introduction to the project it is explained that “Mathematics education can contribute, together with all other disciplines, to the cultural development of citizens, so that they can take part in their society in a competent and critical way.

Particularly, mathematics education can contribute to the achievement of competences such as: reading, understanding and expressing information in an adequate way, posing and solving problems in an intuitive and imaginative way, planning and constructing models for real situations, and making choices in conditions of uncertainty” (MIUR, UMI, SIS, 2003). In such statement, when information has a quantitative nature, statistics and statistical literacy are in evidence. Participating in modern society in a competent and critical way requires citizens to know and to interpret collective/social phenomena in a broad sense, and understand the variability, dispersion, and heterogeneity which cause uncertainty both in interpreting and in making decisions useful in real life situations.

Posing and solving problems in every day life may require the collection of quantitative information and the ability to express such information in a suitable way. It is also necessary to note that citizens will seldom be able to control all the process from the definition of the phenomenon and the collection of data to data handling, interpreting, and utilising. More often, they will need to interpret data collected and organised by others, utilising indicators they do not control. In cases such as this, statistics competences encourage caution before utilising those data. Intuition drawn from experience would help the citizen plan and construct models for real situations, utilising data to make predictions in time or in space at assigned probability levels. Using statistical logic would help citizens make choices in conditions of uncertainty after collecting suitable data and interpreting them with the required probabilistic tools.

The “Mathematics for Citizens” project puts in evidence that mathematics possesses both instrumental functions and cultural functions; neither have to be left aside, as both are essential for significant cultural development of students. Statistics, too, is both a necessary instrument for understanding reality in a quantitative way, and a coherent and systematic discipline concerned with the study of collective phenomena. It does not matter if the study is done in a descriptive or in an inductive inferential way. What is important is that the study aims to investigate, in a scientific way, complex real phenomena that can be perceived only in a plurality of observations.

In “Mathematics for Citizens”, reading data represented by tables, histograms, charts or graphs, and calculating probabilities are listed among the competences linked to the instrumental aspects of

mathematics. All of these are, in reality, statistical instruments that, along with mathematical ones, promote the cultural development of students within an evolutionary process.

From the educational point of view, the “Mathematics for Citizens” curriculum aims to be coherent and show continuity, being based on the competence concept and on some learning nuclei, the content of which had to change progressively according to the teaching /learning needs of the students. In building the curricula, some foreign projects were referenced, in particular, the 2000 National Council of Teachers of Mathematics standards and the Belgian curriculum proposed in 1993 by the Centre de Recherche sur l’Enseignement des Mathématiques. The UMI Committee project was put forward in three steps, the first concerning primary school (five years) and lower secondary school (three years), the second referring to the first four years of upper secondary school, and the third regarding the fifth year of upper secondary school.

The curricula for the first eight years have been finalised since the end of 2001. The nuclei proposed are of two varieties. The first are content nuclei, and the second are transverse in relation to the previous ones. For primary and lower secondary school the content nuclei are: *Numbers, Space and Shapes, Relations, and Data and Predictions*; the transversal nuclei are: *Taking Measurements, Reasoning Methodically and Conjecturing, and Solving and Posing Problems*.

As illustrated by the examined statements of the reform project built on by the UMI, there was enough scope for statisticians to collaborate in a constructive way in building a new mathematics curriculum. In an agreement signed in 1999 among the, then, Ministry of Public Education, the UMI, and the SIS, M. G. Ottaviani—acting as a representative of the Italian Statistical Society (SIS)—was co-opted into the UMI Committee and chaired a sub-commission charged to make proposals concerning statistics and probability. The proposals, adjusted and partly modified by the overall commission, are presented in Table 1 and Table 2.

Table 1
The Mathematics for Citizens Project: Data and Predictions in Primary School

<i>Year</i>	<i>Skills</i>	<i>Knowledge</i>
1 st	<ul style="list-style-type: none"> • Collect data on him/her self and on him/her surroundings, and organise raw data according to observed characteristics • Classify data and objects • Represent collected data 	<ul style="list-style-type: none"> • Statistical population and statistical units • Simple graphic representations
2 nd & 3 rd	<ul style="list-style-type: none"> • Choose methods for data collection (observations and questionnaire) • Collect data • Classify data • Construct table and draw graphs 	<ul style="list-style-type: none"> • Qualitative and quantitative characteristics • Frequency tables • Graphical presentations • Comparison of frequencies
4 th & 5 th	<ul style="list-style-type: none"> • Observe and describe a graph utilising: mode, median and arithmetic mean • Utilise and discuss different way of graphing the same data • In real situation recognise true/false events • In real situation recognise sure, possible, impossible events • In real situations recognise equally likely, more likely, less likely events 	<ul style="list-style-type: none"> • Mode, median, arithmetic mean • True/false event • Sure, possible, impossible event • Assignment of a probability to an event in simple situations

Table 2

The Mathematics for Citizens Project: Data and Predictions in Lower Secondary School

<i>Year</i>	<i>Skills</i>	<i>Knowledge</i>
1 st & 2 nd	<ul style="list-style-type: none"> • Handle and classify continuous data • Graph data, also utilising a spreadsheet, and interpret them • Utilise and interpret measures of central tendency and dispersion • Compare two different frequency distributions referring to the same characteristic 	<ul style="list-style-type: none"> • Continuous characteristics • Frequency distribution of a continuous characteristic: classes of same or different size • Frequency histogram • Relative frequency and percentage distributions • Range
3 rd	<ul style="list-style-type: none"> • Random choice of an element from a population • Interpret the results of a large number of a repeated random experiment in real and virtual contexts (games and computer simulation) • Recognise complementary events, independent, mutually exclusive events, • List the outcomes of a simple experiment and assign a probability to each of them. 	<ul style="list-style-type: none"> • Population and sample: random and non random sample • Probability of an event: measurement in simple cases • Arithmetic mean and expected value

Characteristics of the “Data and Predictions” Nucleus

Within the *Mathematics for Citizens* project, in developing the *Data and Predictions* nucleus, one goal was to gradually develop simple data-driven problems that teachers could initially pose to students. They could then present the way to collect and represent data using suitable tables and graphs as a means to explore and interpret them in a view to solve the proposed problem. To do this it is necessary to utilise the classification concept in order to develop frequency tables. Some arithmetic skills are also needed to calculate simple measures of centre and spread, and mathematics instruments are needed to build graphical representations. In this framework the interpretation of observed and classified data may help students learn basic statistics concepts and produce simple inductive processes. As statistics education research has shown that most of the teachers in Italy have not been trained in statistics or in probability, much attention was paid to the fact that the nucleus *Data and Predictions* must not be perceived as isolated within the mathematics curriculum. Consequently the links with the other content nuclei have always been signalled.

The link with *Numbers* is justified by the classification and enumeration concepts that lead to frequencies. Following this, knowing division allows students to calculate relative frequencies and percentages. The links with *Space and Shapes* is evident when drawing graphical representations. Strong support for better understanding and utilising *Relations* is offered by comparing frequency distributions of the same characteristics in two different sets of statistical units, for example, in the distribution of the same characteristics in males and females, or in the relationship between the exams score of two subjects in a classroom. The link with the transverse nucleus *Taking Measurements* is particularly evident when collected data are measurements of continuous quantitative characteristics. In this case, problems for exploration relate to the unit of measure, the approximation error, and how to deal with it. The links both with *Reasoning Methodically and Conjecturing*, and *Solving and Posing Problems* are part of statistical competence.

Links are not only diachronic but also synchronic in time. In fact *Data and Predictions* content may be introduced when students already have at their disposal the contents of the other nuclei involved.

These are obtained by introducing qualitative characteristics in the 1st year of the primary school, discrete characteristics in 2nd and 3rd year, and continuous characteristics in the lower secondary school. This is justified by the fact that each characteristic has its own algebraic structure. In this way, the proposal merged the mathematical tools learners had at their disposal at that time, with the different structures of characteristics that statistics may involve. In this way it is possible to develop the logic of gaining information and of solving a problem in an orderly way, with the added bonus of demonstrating an application for mathematical concepts and tools that learners have (North & Ottaviani, 2002).

In this framework basic statistics terms and their meaning, as well as fundamental concepts such as center and variability of a distribution, are presented before any basic probability is introduced. This is done so that students first become acquainted with the idea of “diversity” and “variability” in real life. After this, the concepts of event, chance, relative frequency, and probability measurements can, little by little, be introduced.

Daily Life and Statistics in the Classroom

Proposing a curriculum is only one step in a reform process. Another important step is ensuring that the curriculum objectives are correctly understood and accepted by teachers. In order to clarify the meaning and scope of the project, the “Mathematics for the Citizens” curriculum also proposed some didactical activities. These materials were produced by some trained school teachers in collaboration with the project members at a working seminar funded by the Ministry of Education. These were published in the book titled *Matematica 2001* (MIUR, UMI, SIS, 2003).

With reference to *Data and Predictions*, the team, under M. G. Ottaviani’s guidance and with the cooperation of M. P. Perelli D’Argenzio, was formed by some elementary and lower secondary school teachers of mathematics—coming from different Italian regions—who had already taught statistics and probability in their classrooms. Some of them had also been trained during the CIRDIS research project. The team’s guiding idea was to identify some real situations that could offer a suitable context to present statistics in the classroom. For primary school, the titles of the proposed activities are:

- *Know each other: our favourite pets*
- *Know each other: male or female?*
- *The day of birth*
- *Certainty and uncertainty: some linguistic reflections on the “world of probabilities” and*
- *To be counted in official statistics*

These five activities utilise, as starting points, some motion and affective situations. These help the student learn how it is possible to express information, in quantitative way, showing the student how it is possible to consider him/herself as an element of a collection of people—the class—to which he/she belongs, while also recognising and respecting diversity. All of this grows, little by little, and the last activity proposed in the 5th year, shows the interdisciplinary nature of statistics with geography at an age when the students have already acquired the knowledge and skills necessary to come directly in contact with official statistics.

For lower secondary school, the title of the three proposed activities are:

- *Free time*
- *What do we eat? and*
- *Frequency or relative frequency?*

The first two activities provide hints for building questionnaires. The activities concern information to be obtained from and by students about their lives. Once this is completed, the database is proposed, data are handled, and suitable measures are calculated and interpreted. These activities excite students’ curiosity about statistics and understanding of its usefulness. In the third activity a random

experiment involves students in tossing a coin and using different kinds of evidence to make comparisons: by difference or by ratio.

These activities allow young learners to develop a research process in real life, while continuing to consider students' level of cognitive development and the mathematical tools at their disposal. These activities also offer the teachers a set of instructional steps they can follow to help students learn statistics and its concepts (see Appendix). In addition, some assessment items have been proposed to help teachers in their assessment task.

“Data and Predictions” in the Official Italian Mathematics Curriculum

The Italian national guidelines for primary schools and for lower secondary schools were approved by MIUR in February 2004. Mathematics learning objectives for the primary and lower secondary schools incorporated the *Data and Predictions* nucleus, thus recognising the appropriateness of the *Mathematics for Citizens* project. It is important to note that the official curriculum (see Table 3 and Table 4) goes more in deeply into the contents of the *Data and Predictions* nucleus than did the *Mathematics for Citizens* project.²

Table 3

Official Italian Curriculum: Data and Predictions - Primary School

<i>Year</i>	<i>Knowledge</i>	<i>Skills</i>
1 st .	<ul style="list-style-type: none"> • Iconic representation of simple data classified according to the categories of one characteristic 	<ul style="list-style-type: none"> • Collect data and information and organize them through iconic representation according to categories (pictogram)
2 nd & 3 rd	<ul style="list-style-type: none"> • Basic statistics terms: population, observational units, characteristics, qualitative and quantitative characteristics, frequency tables, graphical representations, mode. • Certain and uncertain situations. • Qualification of uncertain situations. 	<ul style="list-style-type: none"> • Pose questions about real life (preference, age in a group of people, sports activity, and so on). • Define respondents. • Collect data on one characteristic. • Classify data according to suitable categories. • Organise data through frequency tables or graphs, according to the examined characteristic. • Detect the mode for grouped data. • Recognise certain or uncertain situations depending on information. • Qualify level of uncertainty (more or less likely).
4 th & 5 th	<ul style="list-style-type: none"> • Analyses and comparisons of collected data through mode, median, arithmetic mean, range. • Utilisation of Official Statistics to gather information. • Qualification and quantification of uncertainty. 	<ul style="list-style-type: none"> • Collect data and recognise qualitative and quantitative characteristics. • Understand how graphical presentation and organisation depend on the characteristic. • Understand the necessity or the usefulness of grouping data, particularly quantitative ones (measurement error) • Qualify uncertainty. • Quantify simple uncertainty situations utilizing information, particularly equally likelihood approach and relative frequency.

Table 4
Official Italian Curriculum: Data and Predictions - Lower Secondary School

Year	Knowledge	Skills
1 st & 2 nd	<ul style="list-style-type: none"> • Statistical survey phases. • Tables and graphs. • Central tendency statistics and range. • Population versus sample. • Probability of an event: numerical measure of the likelihood of simple events. 	<ul style="list-style-type: none"> • Identify a simple problem that may be solved by a statistics survey, identify the target population, the observation units, draw a questionnaire, collect data, organise them in a frequency table. • Graph a frequency distribution and identify the adequate central tendency statistics: mode for whatever characteristic, median for arrangeable characteristic, arithmetic mean and range for quantitative characteristic. • Draw random and representative samples. • Predict probability of simple events.
3 rd	<ul style="list-style-type: none"> • Continuous quantitative variables: grouping data in classes of equal or different width. • Histograms. • Relative, percentage, cumulative frequencies • Official sources of data: utilisation. • Different approaches to probability: classical, relative frequency, subjective. 	<ul style="list-style-type: none"> • Draw histograms and read them. • Recognise distorted graphs and correct them, when possible. • Collect statistics information from different sources. • Utilise software to organise and represent data. • Calculate relative, percentage, cumulative frequencies and interpret them. • Utilise relative, percentage, cumulative frequencies to compare different distributions referring to the same characteristic. • Understand why, where and how different approaches to probability may be used.

In particular the proposals that were accepted included: introducing statistics before probability; highlighting the usefulness of statistics for collecting, organising and interpreting data; adopting a problem posing and solving strategy; giving importance to basic statistics terms and their meaning; and developing the arguments, giving due importance to the type of the examined characteristic.

Within the general objectives of the educational process, there are also very important psychopedagogical statements that enhance the teaching of statistics and probability in schools. In particular, in primary schools, these include making the most of pupil experience, enabling them to explore and discuss the world they live in, helping them to move from empirical categories to formal ones, and giving them the opportunity to do interpersonal and intercultural comparisons. At the lower secondary school level they include helping students to understand that any representation of reality is incomplete, to discover the possibility to model reality with scientific-mathematics tools, and to recognize both deductive and inductive inference process. Each of these is related to aims that statistics and probability help future citizens to achieve.

Statistics has an additional important quality that is valued within the new curriculum. This quality is that statistics is interdisciplinary. In fact, it may involve mathematics along with history, geography, natural sciences, and information technology.

Conclusions

The 2004 mathematical curriculum shows many points of contact and intersection between mathematics and statistics at school. In our opinion, mathematics and statistics are two different

disciplines; however the necessity to more closely link mathematics to competencies needed by citizens in the globalised society – such as: collecting, organising and analysing information; communicating information in appropriate ways; and posing and solving problems (OECD, 2001, p. 22) – can enhance efforts to promote statistical literacy in school.

Our experience in training teachers has shown to us that the inclusion of statistics, along with probability, in the curriculum can really cause problems for teachers, particularly when they have a university degree in mathematics. At least in Italy, the formalist idea of mathematics prevails in Mathematics Faculty and is consequently disseminated in school. This is a possible explanatory factor regarding why Italian students have difficulties in international assessment projects, particularly on those items where competences are requested, rather than just arithmetical and formal skills (cf. OECD 2001).

It is difficult to overcome the obstacles in promoting statistical literacy posed by formalist mathematicians serving as intermediaries to schools. This is the case even when good didactical activities are proposed. Another obstacle relates to the textbooks, which our research and experience have shown are often incorrect sometimes in serious ways. What we need to do, among other things, is to convince mathematics teachers of the importance of statistics as a discipline, to help the teachers understand the importance of statistical thinking and of the need of adequate didactical methods and didactical tools to teach the subject correctly, and to demonstrate to teachers that correctly presenting statistics in the classroom may also have a beneficial effect on mathematical learning processes. As some pupils involved in the CIRDIS project said, the value of learning statistics taught following a data-oriented approach is that, at the end, students understand the importance and usefulness of mathematics as well (Milito, Marsala, 2002).

To conclude, commenting on the “National suggestions for personal curriculum in primary and middle school”, the Ministry says that these suggestions concern the essential curriculum whose content has to be offered by all primary and middle schools, in order to insure the personal, social, and civil right to a quality education. Statistics education is part of this.

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Endnotes

¹ CIRDIS is an Italian Inter-University Centre of Research for the Teaching of Statistical Disciplines that includes the Universities of Padova, Palermo, Perugia and Roma, La Sapienza.

² In fact it looks more like the proposal of the statistician who took part into the UMI Committee.

Paper Appendix: Two Didactic Activities for the Data and Predictions Nucleus

The proposed didactic activities are written for teachers. Each activity refers to the school level and class, the knowledge and skills concerned, the other nuclei involved, and the external links with other disciplines. The context of each activity is presented with a comment about its didactic and pedagogical reasons. The activity is then described putting in evidence the basic statistics terms, so that the teachers become acquainted with them. After this, each phase of the activity is described both for what concerns “didactical activity” and the practical way to implement them. See below the summary of two activities

To Be Counted In Official Statistics

Level: 5th class in primary school.

Knowledge: qualitative and quantitative characteristics, graphical presentations, mode, median, arithmetic mean.

Skills: collect data, classify data, construct table and make graphs, observe and describe graphs utilising mode, median and arithmetic mean, utilise and discuss different ways of graphing the same data.

Other nuclei involved: reasoning methodically and conjecturing.

External links: Italian language, history, geography.

Context: demographic and social phenomena.

Comment (summary): the proposal pays attention to the needs of statistical literacy as a growing body of quantitative information is at the disposal of the statistically illiterate citizenry. It puts in evidence the necessity to refer to ISTAT data in order to get good quality information on our country. The proposal was implemented with the awareness that giving pupils the instruments to read official statistics means to build fully conscious citizens.

Attention points: carefully handle the discussion;

- put in evidence comparison of reality with its representation in numerical forms
- table and graphs.

Activity description (summary): two suitable data representations were chosen, one concerning the Italian resident population at October 1991 by regions (Figure 1), the other the territorial area of Italy by geographical area and altitude (Table 1). With reference to the data, the teacher is encouraged to guide students to identify: collective phenomenon, target population, observational units characteristic and its categories, source of data.

The teacher begins the discussion asking if and from where the pupils recognised the tables and representations of statistical data. After this they ask them to explore these “objects” and suggest that students collect cut-outs from newspapers and weeklies. Pupils are requested to identify statistical terms in the cut-outs they have collected. In particular the teacher asks them: “Who produced these data?”³

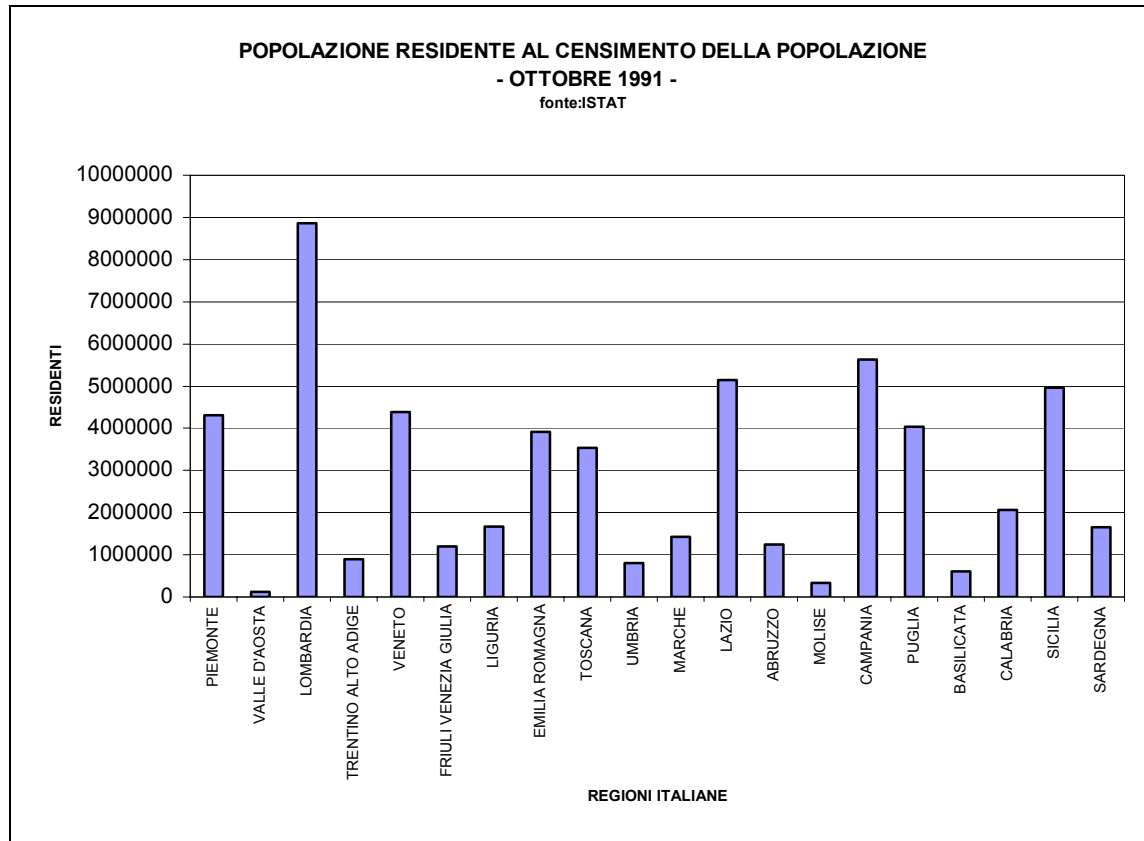


Figure 1. Residential population according to 1991 census data

At a suitable point in the discussion the teacher presents Figure 1 and asks, for example, in which column were your paternal grand-parents? And your maternal ones? And you? Roughly, how many residents were in Valle d'Aosta and in Piedmont?

About table 1: a definition of altimetrical zone is necessary. The suggestion is to read the table in different ways: by row, by column, regarding Italy or the entire Italian area, and discuss the results from each perspective.

Table 1
Superficie Teritoriale per Zona Altimetrica e Area Geografica

	MONTAGNA	COLLINA	PIANURA	TOTALE
Nord	5 531 787	2 272 918	4 187 456	11 992 161
Centro	1 576 034	3 723 859	535 469	5 835 362
Mezzogiorno	3 502 927	6 548 037	2 255 354	12 306 318
Italia	10 610 748	12 544 814	6 978 279	30 133 841

Anno 1999, ettari
Fonte: ISTAT

Frequency or Relative Frequency?

Level: 3rd class in lower secondary school.

Knowledge: frequency and relative frequency, probability of an event, measurement in simple cases.

Skills: graph data utilising a spreadsheet and interpret them, utilise and interpret measures of central tendency and dispersion, random choice of an element from a population, interpret the results of a large number of a repeated random samples in real and virtual contexts (games and computer simulations), list the outcomes of a simple experiment and assign a probability to each of them.

Other nuclei involved: numbers, relations, reasoning methodologically and conjecturing, solving and posing problems.

External links: Italian language.

Context: tossing a coin, repeated random experiment.

Comment (summary): this activity is intended to provoke a debate about some misconceptions common when dealing with uncertainty. In particular it is thought that, if an experiment is performed a large number of times, frequency (not relative frequency) “reveals” probability. This is often due to the misconception that frequency and relative frequency are not different. It is also possible to say that it is not clear enough that comparisons among two numbers may be done by difference or by ratio, and that results and meanings are different in the two cases. The activity aims to let pupils understand that it is not true that in the long run the number of heads is equal to the number of tails; ii) it is not true that absolute frequency will be the probability of an outcome in the long run.

Attention points: handle carefully the discussion with reference to the different numbers of

- repeated experiments;
- pay attention to misconceptions.

Activity description (summary): the activity has four phases and uses the didactical instruments below:

10 tosses

Figure 1

50 tosses

Figure 2

100 tosses

Figure 3

n. of tosses	$f_a(H)$	$f_a(T)$	$f_r(H)$	$f_r(T)$
10
50
100

Figure 4

As is evident in figures from 1 to 3, the three squares have the same area and each of them is subdivided in different ways producing 10, 50, 100 portions according to the number of tosses.

In the first phase the teacher poses the question: "Suppose we toss a coin 10, 50, 100 times. What do you think about the difference between the number of heads and number of tails? In the second phase groups of students are formed to perform the experiments 10, 50, 100 times. Each time a head is the outcome a portion is coloured in blue. Colour is to be put in contiguous cells. This will put in evidence that the coloured and uncoloured areas in the same square tend to have the same measure, while the difference of the number of the corresponding portions tends to augment this notion. In the third phase absolute (a), and relative (r) frequencies are written in figure 4 so that what areas say is confirmed by figures, and some reflections are done by the students. In the fourth phase the results of the simulation could be used to understand the meaning of using differences or ratios in doing comparisons. The concepts of proportions should also be involved.⁴

³ Fiini, M. G. (1996). Ritrovarsi nelle statistiche ufficiali, Tesi di corso di perfezionamento, relatore M. Perelli, Brescia.

⁴ M. Barra (2000) Probabilità e gioco d'azzardo, in *Le scienze e il loro insegnamento* , nn.5-6.