Developing Aspects of Distribution in Response to a Media-based Statistical Literacy Task

Watson, Jane M.
University of Tasmania
Private Bag 66,
Hobart, Tasmania, 7001, Australia
Jane.Watson@utas.edu.au

Moritz, Jonathan B.
University of Tasmania
Private Bag 66,
Hobart, Tasmania, 7001, Australia
Jonathan.Moritz@utas.edu.au

1. Background

Research into students' beginning intuitions about distribution has generally been associated with variation in single variable settings (e.g., Ben-Zvi & Sharett-Amir, 2005; Watson & Kelly, 2005) and has often focused on graphing attributes of students' created representations. Kelly and Watson (2002) for example found that students' graphs to represent the imagined outcomes of repeated sampling trials in a probability setting ranged from idiosyncratic drawings of the physical scenario, to time-series type graphs inconsistently justified by "more" of a certain characteristic, to informal graphing based on "middle," to a conventional distribution recognizing variation and center. In sampling or measurement investigations focusing mainly on single variable distributions, Shaughnessy (2006) described six aspects of variation as (i) extremes or outliers, (ii) change over time, (iii) the whole range, (iv) the likely range, (v) distance or difference from some fixed point, or (vi) sums of residuals. These descriptors are not seen as hierarchical and inform this study to assist in characterizing the story that students attempt to tell with the graphs they produce to represent a verbal description of covariation.

This paper, in moving from the consideration of the distribution associated with a single variable to that associated with two variables, builds on earlier research on correlational reasoning and its representation (e.g., Ross and Cousins, 1993a, 1993b). Shaughnessy's (2006) aspects of variation also apply when two variables are involved, seen as the trend in the relationship between the two variables, and seen as deviations from the trend. Various researchers asked students to create graphical distributions from data values. Brasell and Rowe (1993), for example, asked physics students to construct a graph of five paired values representing the heights from which a ball was dropped and the height to which it rebounded; they found students drew pictures, produced poorly labeled graphs, or plotted points, but rarely gave evidence of graphing to show a trend. Rather than starting with data values, Mevarech and Kramarsky (1997) asked grade 8 students to graph four different verbal claims about trend relationships of time spent studying and the marks received at school. Whereas slightly over half of students appropriately graphed a positive, negative, or no association, fewer than half graphed a curvilinear association. The most common errors observed related to graphing only a single point, to graphing only a single variable, or to graphing an increasing function regardless of the conditions set in the verbal statement. Moritz (2002, 2006) found similar outcomes in exploring this scenario with primary and middle school students. Moritz (2000, 2006) also explored upper primary students' ability to graph three verbal statements about people growing taller. Overall students understood the tasks and quite young students (e.g., grade 4) created imaginative representations to tell the story of height increase with age. Students encountered more difficulty in representing the cessation of growth after the age of 20 and in representing the difference between boys and girls.

This study seeks to explore the development of students' abilities to display hypothetical data sets involving two variables. A scenario is described but no data are presented, and students are asked to draw a graph representing a potential distribution of the data. The task is presented in a complex scenario—involving multiple variables of attributes unlikely to be familiar to students' experience of measuring—for consideration based on a two-variable descriptive cause-effect context (see Figure 1). Of particular interest in the representation of distribution is how students coordinate the variation in two variables to show the covariation present as a trend. Also of interest is how students display variation of individual values from this trend. There were two research questions for this study.

1. What developmental progression is shown in students' understanding of distribution in representing a verbal description of covariation, that is, a trend?
2. Does development occur over the middle years of schooling?

Following the analysis, suggestions are made for use of such tasks in the classroom across the years of schooling.

2. Method

Task. A survey question asked students to read the newspaper article in Figure 1 and "draw and label a sketch of what one of the researcher's graphs might look like" (Watson & Moritz, 1997; Watson, 2000).

Family car is killing us, says Tasmanian researcher

Twenty years of research has convinced Mr Robinson that motoring is a health hazard. Mr Robinson has graphs which show quite dramatically an almost perfect relationship between the increase in heart deaths and the increase in use of motor vehicles. Similar relationships are shown to exist between lung cancer, leukaemia, stroke and diabetes.

Figure 1. Newspaper article claiming an association.

Sample and Procedure. The sample for the study consisted of 1285 students from government schools in the Australian state of Tasmania: 369 in Grade 6, 312 in Grade 8, and 604 in Grade 9. Schools were chosen to be representative of all geographical regions of the state. The item was included in a statistical literacy survey of items based on newspaper extracts. It was the seventh of eight items for Grade 6 and the eighth of ten items for Grades 8 and 9. The survey was administered in class groups taking approximately 45 minutes. Students were informed that their participation was voluntary and would have no impact on their school marks. They were asked however to do their best to explain their understanding to aid the researchers to help teachers plan their instruction.

Analysis. The analysis reported in this paper is based on the general framework of Biggs and Collis (1982; Biggs, 1992; Pegg, 2002a, 2002b) in cognitive psychology. Their Structure of Observed Learning Outcomes (SOLO) model suggests five levels of performance that may be assessed in relation to a task that is set as described in Table 1 (see also Watson & Moritz, 2000). The Extended Abstract level is included in the table for completeness, although no responses were observed at this level in the current study. Within this framework, statistical appropriateness of responses was also considered.

Table 1. Summary of SOLO level expectations[1]

Level	Elements
Prestructural	No elements related to task employed in response
Unistructural	Single element of task employed in response
Multistructural	Multiple elements employed in response, usually in sequence
Relational	Multiple elements employed in a coordinated, integrated fashion in response
Extended Abstract	Response goes beyond Relational level to introduce other elements not in the initial task but relevant to its extension

[1] Summary adapted from Biggs & Collis (1982), Pegg (2002a), and Watson & Moritz (2000).

In relation to the task shown in Figure 1, the target response was to show the association of two variables: heart deaths and motor vehicle usage. A conventional representation of this would be plotted on perpendicular axes, such as a linear graph showing an increase in car usage with an increase in heart deaths, or a scatterplot that also shows deviations from this general trend. Alternative representations, however might include two graphs, one for each variable, or a single graph with two vertical scales imposed, one for each variable, and two "lines." Students without exposure to this type of representation may provide only partial responses, such as a graph only one variable, perhaps as a single variable against time. Others students were not expected to represent a trend due to lack of labelling or distraction by the multiple variables listed in the article.

The representations presented in this paper are chosen as typical of the levels of response identified from the original data set, as well as illustrating aspects of variation displayed. They demonstrate the proposed hierarchies in terms of structure and appropriateness.

3. Results

Some students drew a picture of the content in the article or in some other way refused to become further engaged in the task. This is seen in the examples in Figure 2, which are judged to be Prestructural with respect to showing the association in the statement.

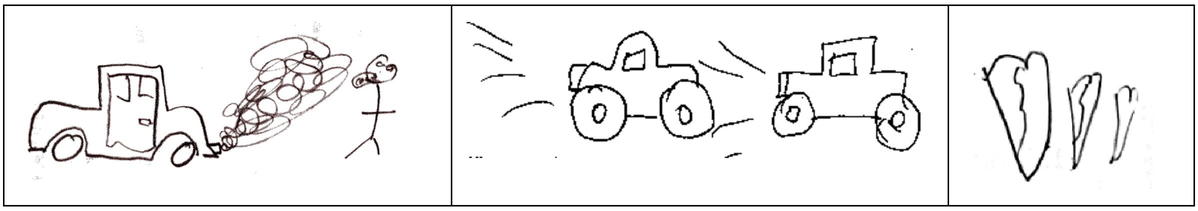

Figure 2. Pictures related to the context (Prestructural).

Other responses at the Prestructural level attempted to address the task in terms of the association as requested. On one hand, some students realized that the task was about graphing the hearts deaths and motor vehicles but could not show any values or the relationship, as is seen in Figure 3. On the other hand, some students were able to depict some type of variation in a graph, but were not able to identify it meaningfully by showing either a trend or any sense of the variables involved. Examples are shown in Figure 4.

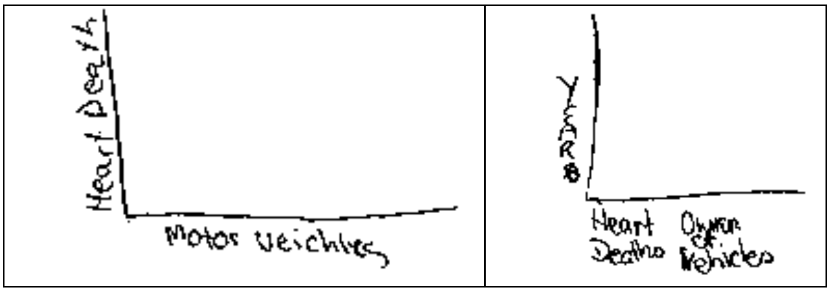

Figure 3. Labels but no representation (Prestructural).

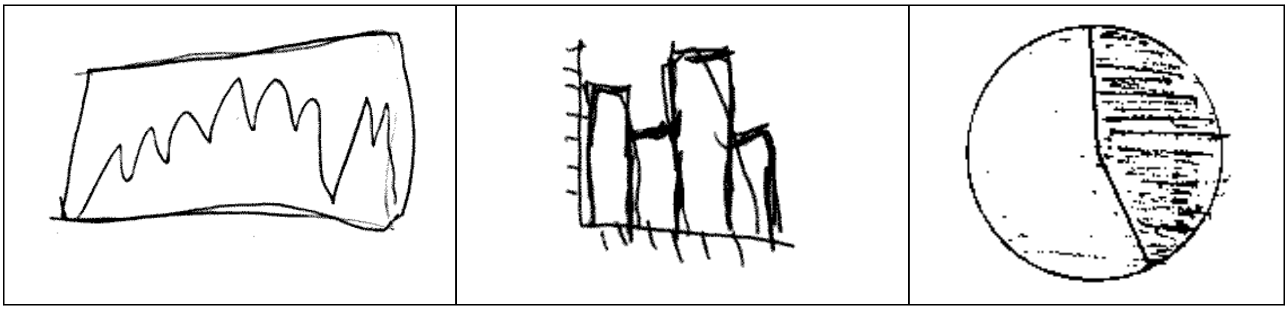

Figure 4. Variation shown in a representation but no labels (Prestructural).

At the Unistructural level, a single aspect of the task was addressed but the attempt to demonstrate the required association was unsuccessful. Examples in Figure 5 showed variation to produce a trend but there was no indication of what measures were varying. The acknowledgement of several measures was shown in the representations in Figure 6, but without a sense of variation in values of each measure. The representations in Figure 7 acknowledged both measures – heart deaths and car usage – but in showing only one value for each variable, they could not represent the covariation claimed in the article.

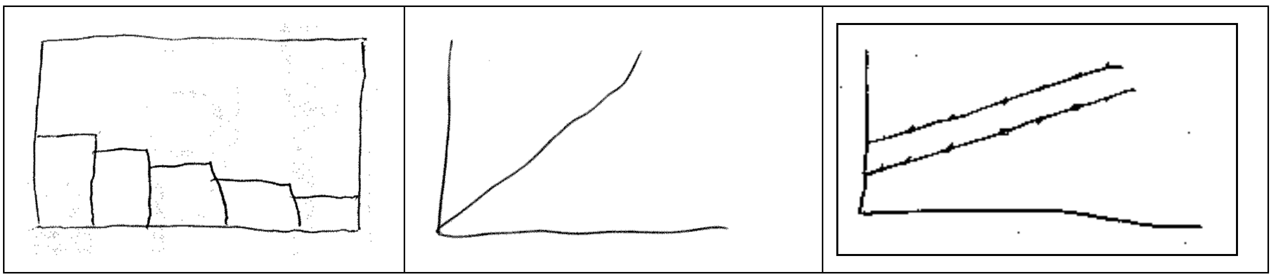

Figure 5. Trend but no labels (Unistructural).

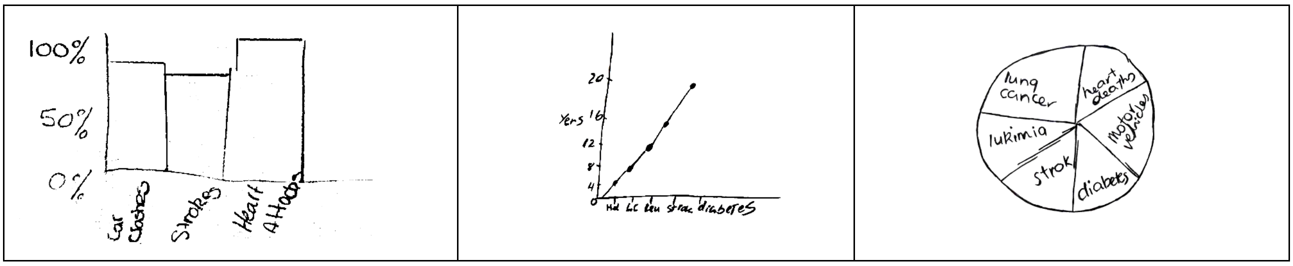

Figure 6. Single values for several measures (Unistructural).

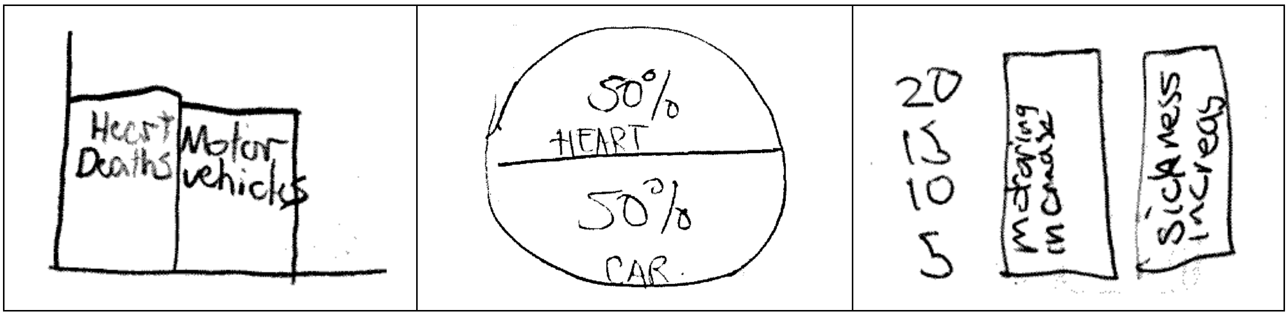

Figure 7. Single values for two variables (Unistructural).

Responses at the Multistructural Level partially addressed the task, when one variable was considered with respect to time as shown in Figure 8, or when an attempt was made to display values for different variables for two different years as in Figure 9. These reflect Shaughnessy's (2006) aspect of variation as change over time and for the graphs in Figure 8 the aspect of range. The representation at the right of Figure 9 showed multiple years and two variables but the ikonic nature of the symbols meant the relationship could not be judged and hence it was deemed incomplete. These representations appeared to recognize the nature of the claim but were incomplete either because they showed the distribution only for a single relevant variable and time or they did not show enough values to establish the association. They followed a series of steps and appeared to recognize the tension in the task but could not completely resolve it and were judged to be Multistructural. Variation was more adequately acknowledged in the latter representations but the students did not realize the full potential of showing variation over time to assist in telling the story.

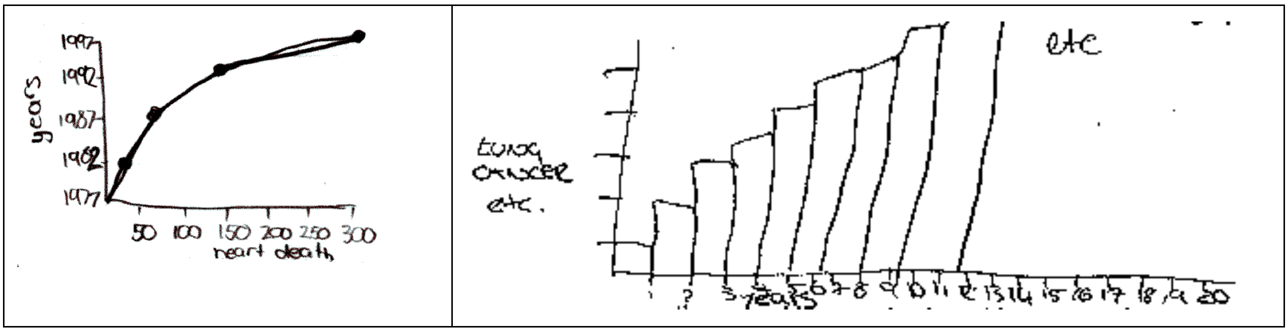

Figure 8. One variable and time (Multistructural).

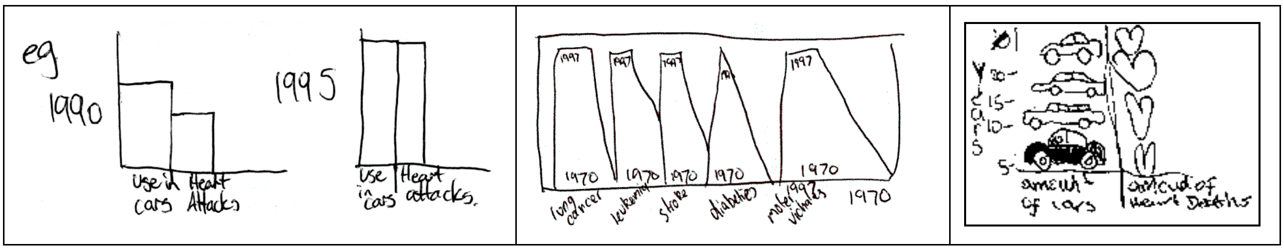

Figure 9. Multiple values for comparison (Multistructural).

Relational comparisons of two variables were demonstrated in several ways. Using two side-by-side representations with comparisons via bar or line graphs is seen in Figure 10. As well, these could be combined and represented in a single graph as shown in Figure 11. Finally the conventional bivariate representations often expected by statisticians are shown in Figure 12. The detail provided by students varied tremendously and this had implications for how judgments were made in terms of the variation shown. The graph on the left in Figure 12 shows the variation that creates the claimed association but no indication of variation about the trend line. This was often the case, with a few dots to which the trend line is "fitted" being shown in the middle graph. Again Shaughnessy's aspects of change with time and range are seen in the graphs.

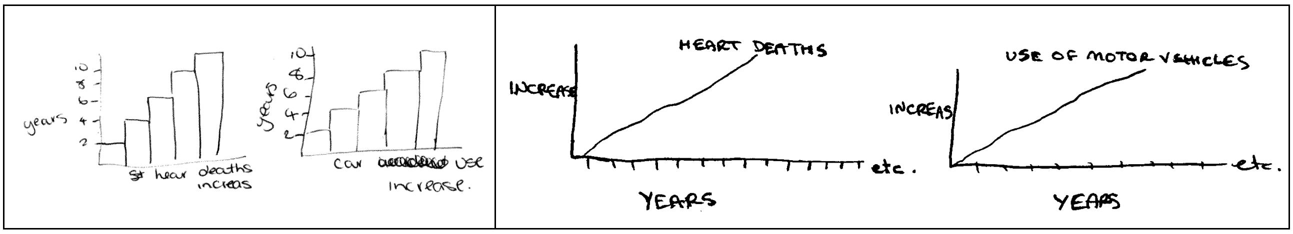

Figure 10. Side-by-side single variable representations (Relational).

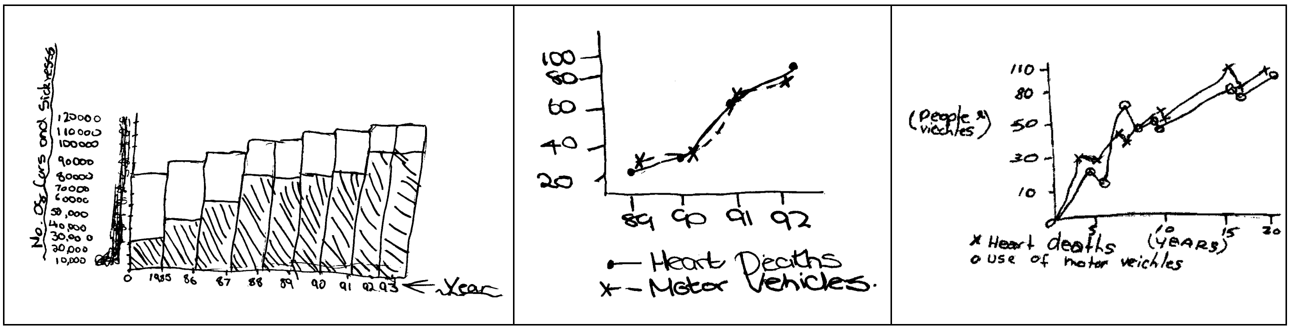

Figure 11. Two variables on the same graph with respect to time (Relational).

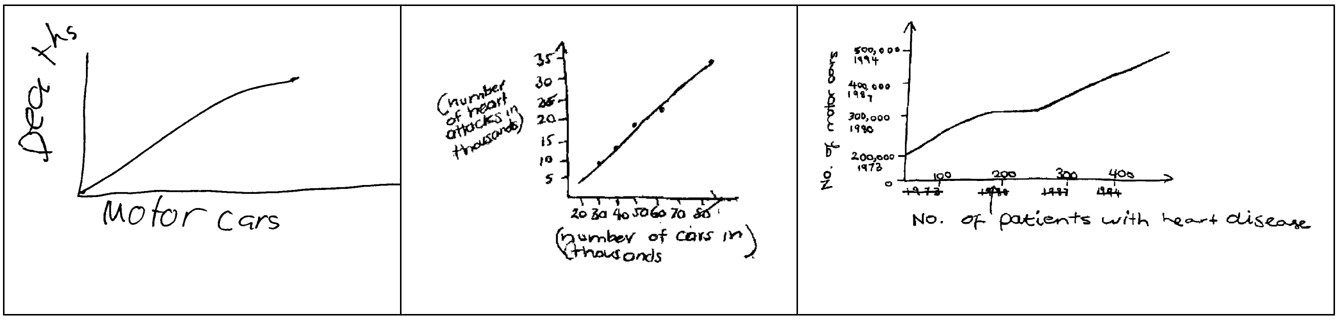

Figure 12. Bivariate graphs (motor cars and heart deaths) (Relational).

Although a person with statistical training would most likely produce a graph similar to those in Figure 12, the lack of experience with scattergraphs and correlation for most of the middle school students in this study meant that they used the methods at their disposal to tell the appropriate story of Mr Robinson's claim.

4. Summary

A summary of the performance of the students surveyed is given by grade in Table 2. Students in higher grades tended to respond at higher levels, with a lower proportion of Unistructural responses and the near doubling with grade for the Relational responses.

Table 2. Percent of responses for each grade at each hierarchical level of the SOLO model

Level	Grade 6	Grade 8	Grade 9
Prestructural – No elements of task	33	34	22
Unistructural – Single element of task	44	28	20
Multistructural – Multiple elements but single variable or incomplete comparison	13	18	19
Relational – Association as claimed for two variables	10	21	38
n	369	312	604

5. Discussion

Most of the recent statistics education research focusing on students' understanding of distribution has focused on single-variable contexts. Some has asked students to create graphs (e.g., Ben-Zvi & Sharett-Amir, 2005; Kelly & Watson, 2002), whereas other research has considered students' interpretations of graphs presented to them (e.g., Konold et al., 2002). This study asked students to create a graph from a verbal description. It added two further ingredients, however, in asking students to show a claimed association of two variables, and in choosing an authentic newspaper article as the basis for the task. In following this line of investigation, the study was addressing the issues raised by Gal (2002) for the statistical literacy needs of adult citizens: the need to interpret and evaluate critically statistical information and to communicate their reactions. A deep appreciation of distribution should be the foundation for such evaluation and communication in this context. The results of this study hence raise an awareness of the level of preparedness of middle school students to address such issues.

Most of the Grade 6 and 8 students in this study would not have been formally introduced to scatter graphs for representing bivariate data. The fact that many of the experiences of these students would have been based on bar graphs may explain the single comparison graphs in Figure 7, the single variable graphs in Figure 8, the double comparison graphs in Figure 9, and the continued use of this form including variation in Figures 10 and 11. The idea of sketching a line, as in Figure 12, may not have occurred to many students as an appropriate form of representation.

The task used in this study and its context raise the question as to the realistic expectation for what a statistically literate adult would envisage as a distribution associated with Mr Robinson's claim. Would it be a "straight" line sketched perhaps as the graph on the left in Figure 12? This would show the variation associated with the trend claimed in the article. It does not, however, show the distribution of the variation in the data set about that trend line that must have existed in the original data set. Of the examples presented, only the two graphs on the center and right of Figure 11 showed variation "about the trend" but the students then connected the points rather than suggesting a smooth trend. In teaching programs, moving from considering the variation that creates the distribution in single variable frequency graphs, it would appear to be important to stress the continued presence of variation about trend lines created by bivariate data.

Students who have wide experience with plotting scatter graphs from authentic data sets with two variables and then sketching a "line of best fit" either by eye or with a computer package, should develop a strong appreciation for the two types of variation present. Those who go on to study regression lines will appreciate the variation from the line as the "sum of residuals" referred to by Shaughnessy (2006) and the need to minimize it in determining the "best" trend line. It appears from the data in this study that explicit work in this area would be appropriate by the end of Grade 9.

The limitations of the study include that for the survey there was no opportunity to ask students to fill in gaps, for example in terms of labeling their figures. Although this is a limitation in terms of students' *potential* to explain, it also shows the need for teachers to emphasize labeling from the very beginning of graph production. The lack of opportunity to teach a unit on statistical literacy based on newspaper articles and then retest the students' understanding is another limitation but points to the possibility for further research.

6. Conclusion

What issues are involved when distributions are being judged in relation to the appropriateness of the variation displayed? Is large scale variation that leads to trends important or is small scale variation indicating expected random or error change important? These need to be distinguished for students in the classroom so they are aware of the necessity to represent (and look out for) both types of variation when creating (or observing) graphs. It seems clear that different kinds of tasks require acknowledgement of different aspects of variation and students should experience many different contexts for exploring variation.

The use of the term "distribution" in the title reflects the statistical perspective in relation to what is expected by the time students leave school and enter tertiary study. It is unlikely that students will commonly use the word before their senior secondary years. They will however hopefully draw many graphs that show appropriate variation associated with the contexts of tasks set. If they learn the importance of the word "variation," this will be an important part of the vocabulary for their later statistical lives.

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RÉSUMÉ

This paper presents an analysis of responses of 1285 Australian students in Grades 6, 8, and 9 to a survey item asking for a graph to show an "almost perfect relationship between the increase in heart deaths and the increase in the use of motor vehicles." Responses were analysed within a framework that acknowledged structural complexity and the statistical appropriateness of the response. Developing aspects of distribution feature in the description of responses and examples presented.