IS MEDIAN AN EASY CONCEPT? SEMIOTIC ANALYSIS OF AN OPEN-ENDED TASK

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In this paper we analyse the responses given by 643 Mexican students from Secondary Education and High School to a problem involving the comparison of ordinal data. Using some ideas from the Onto-semiotic approach proposed by Godino and colleagues, we carry out an analysis of the open responses, taking into account the central tendency measure used in the comparison and the students' conflicts. We use the Chi-square test to study the possible dependence between responses and school level. We observe better results in secondary school students who use median and mode more frequently, although they also tend to omit the response more frequently.

INTRODUCTION

Central tendency measures (mean, median and mode) were the focus of different research, by Pollatsek, Lima and Well (1981), Barr (1989), Cai (1995), Gattuso and Mary (1996), Watson and Moritz (2000) or Cobo (2003), who described errors and difficulties in the learning of these concepts in students of different ages. These works of research have concentrated mainly on tasks where data are measured in interval scale. However, in exploratory data analysis, that is actually recommended in the mathematics curriculum for Secondary education, order statistics, such as the median which are specially relevant with ordinal data, are given much attention. So, it would be important for statistics education in secondary and high school to take into account this kind of data. The aim of this paper is to analyse the difficulties that students might have, when comparing two ordinal data sets in a familiar context.

Theoretical framework and previous research

This paper is based on the Onto-semiotic approach proposed by Godino, Batanero and Font (2007). These researchers take the idea of "semiotic function" from Eco (1995) as a correspondence between an expression (initial object or sign) and a content (the final object; what is represented), which is fixed by a rule of correspondence that relates the expression to the content. They suggest that any possible mathematical object (concept, property, argument, procedure, etc.) may play both the role of expression and content in a semiotic function. A *semiotic conflict* appears when students' interpretation of mathematical expressions do not agree with what is expected by the teacher or the researcher. These semiotic conflicts produce errors in the students which are not due to the student's lack of knowledge, but to the fact that students establish an incorrect correspondence between the two terms in a semiotic function. The aim of this work is to determine possible semiotic conflicts related to median in secondary and high school students.

Understanding the median. Previous research suggests that the idea of median is not clear for students, who either interpret this concept as the center of "something", without understanding what exactly this "something" is (Barr, 1980) or have difficulties in computing the median. Students do not understand why we use different algorithms to compute the median, depending on the kind of data and do not perceive how this computation is deduced from the median definition (Schuyten, 1991). They also find obstacles to estimate the median when data are given in a graph, make mistakes due to the lack of proportional reasoning and are unable to handle inequalities associated to the computation of median (Estepa, 2004). Other mistakes found by Carvalho (2001) are not ordering the data when computing the median; confusing frequency with variable value and confusing the mean and median.

Comparing distributions. Konold, Pollatsek, Well and Gagnon (1997) suggested that the use of central tendency measures to compare data sets is not intuitive. Students usually focus on absolute frequencies, instead of using relative frequencies even when the sample sizes are very different. Batanero, Estepa and Godino (1997) described the following incorrect strategies when comparing two distributions: a) using only isolated values to compare the distributions; b) expecting a similar change in all cases for related samples. Watson and Moritz (2000) classified

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these strategies according to a hierarchy model. While on a first level students are able to compare equal size set, on a second level they can compare unequal size data sets using a proportional reasoning. While previous research focused on data measured at interval scale, Cobo (2003) used an item with ordinal data. In this paper, we carry out an in depth study of students' answers to this item and analyze the differences between Secondary and High school students.

METHOD

The sample was formed by 518 Mexican students from two different educational levels: 356 High school students (17-17 years old) from six different schools and 162 Secondary students (14-15 years old) from two schools. All of them had studied central tendency measures in the same year in which we administered the item for about one month and two months before doing the test. The problem included below was taken from Cobo (2003). Since data represent values of an ordinal variable we cannot compute the mean. The central tendency for this data can only be summarized by the median or mode. To solve the item students also require some knowledge about the mean (for example that the mean is not an appropriate measure for ordinal data).

Problem. A teacher classifies his students in four categories: A, B, C or D, depending on their performance. A is the best mark and D is the worst (when the student got a D he or she has not passed the exam). The following data show the results in two groups of students:
Group 1: D C C B B A A D D D C C C B A A D C C A A A A
Group 2: A A D D C B C B D D A B C A D B B
Which group has obtained better scores? Which central tendency measure would you use to represent these data?

In order to solve this problem (ordinal data; odd number of data) we should compare the median in both groups. The comparison of medians is preferable to the comparison of modes in this problem, because the median takes into account the order of data and not just the frequency of values. In order to compute the median in each group, the students must order the data and look for the central value in each group. In the second part of the item the expected answer is the median.

RESULTS

Once the students´ answers were collected, we started a cyclic process of categorization by comparing similar answers, and produced the following categories:

C1. Responses based on the median. In this category we have got the following responses: *C1.1. Correct computation of the median. C1.2.* The student tries to compute the median in both groups but cannot finish the problem. *C1.3.* Gives a correct answer to the second part of the item but does not compute the medians.

C2. Responses based on the arithmetic mean. In these answers the student transforms the data set into quantitative data. The answer would be correct if, once the data were transformed to numerical data, the student had computed the median. However, the student does not discriminate between the different types of data and is not aware that the mean cannot be computed in ordinal data. We found the following categories:
C2.1. Transforming the ordinal data and computing the mean correctly. The student assigns numerical values to the different categories, computes the arithmetic mean correctly in each group and compares the means. *C2.2. Conflict in the mean algorithm*. Similar to the previous response, but with errors in the computation of the mean. *C2.3. Applying different transformation to each group* and computing the mean. *C2.4. Computing the mean of relative frequencies.* The values obtained in both groups are almost equal, because the sum of relative frequencies is 1. *C2.5. Obtaining the expected number of students in each category if there were no differences between groups.* The student computes a mean frequency in each category of the variable, dividing the total number of students in each category in each group by two. *C2.6.* A similar strategy is computing the mean number of students in each category, dividing the total number of students in the group by the number of categories. That is, the students compute the *expected number of students in each*

category in a uniform distribution in each group. *C2.7.* The student suggests he needs to compute the mean in each group, but does not perform the computation.

C3. Responses based on the mode. The student uses the modes to make the comparison, computing the modes correctly.

C4. *The student does not use the central tendency measures* to solve the problem, showing a deficient knowledge of distribution. The same behaviour was found in Batanero, Estepa and Godino (1997); Konold, Pollatsek, Well and Gagnon (1997) and Cobo (2003). We found the following categories: C4.2. Computing percentages or relative frequencies in each category. C4.2. Computing maximums or minimums. C4.3. Comparing only absolute frequencies to solve the problem. C4.4. Giving a variable value (for example “B”) in part 2 of the problem, without taking into account the central tendency measures.

Table 1. Frequencies and percentages of responses to item

Categories of responses	Frequency	%
C1.1. Correct computation of the median	34	6.56
C1.2. Ordering the data without finishing the computation of median	1	0.19
C1.3. Correct response without justification	42	8.11
C2.1. Transforming ordinal data and computing the mean correctly	97	18.73
C2.2 Conflict in the algorithm of the mean (dividing by an incorrect number)	1	0.19
C2.3. Applying different transformation to each group and comparing means	3	0.58
C2.4. Computing the mean of relative frequencies	10	1.93
C2.5. Obtaining the expected number of students in a uniform distribution	15	2.90
C2.6. Suggesting we need to compute the mean, without actual computation	60	11.58
C3.1. Correct use of the modes to make the comparison	55	10.62
C4.1 Comparing percentages or relative frequencies in each category	20	3.86
C4.2. Comparing maximums or minimums	17	3.28
C4.3. Comparing absolute frequencies	77	14.86
C4.4. Not taking into account central tendency	12	2.32
C5. Does not justify the answer	10	1.93
C6. Does not answer	64	12.36
Total	518	100

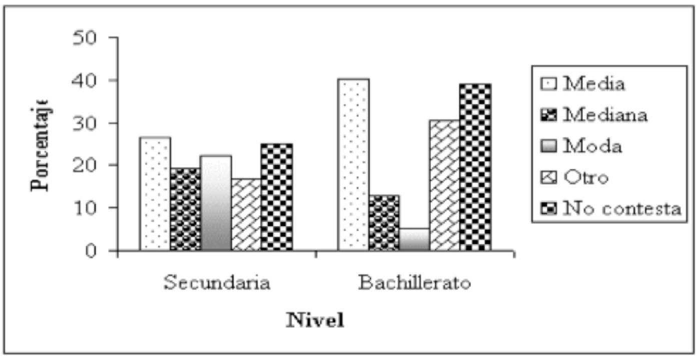

Figure 1. Results by educational level

The most frequent answer was using the mean, in general, correctly. 10.7% of the students used the mode correctly and only 7% of the students computed the median correctly, although 8% gave the correct answer, without computation. 26% of them gave answers not related to central tendency measures, a problem also described in Estepa (1993) who pointed out that this response suggests a local conception of statistical association, believing that association (or the difference) can be analysed by taking into account only a part of the data. Consequently, these students have difficulties with the idea of distribution, whose understanding implies being able to compare distributions using central tendency and spread measures (Konold & Pollatsek, 2002). If we group students by educational level (Figure 1), results are better in Secondary students, who used the median more frequently although they also gave more blank answers. Differences were statistically significant in the Chi-square test (Chi=48.04; 4 df, p=0.0001).

CONCLUSIONS

Our results show that the comparison of ordinal data, even in a familiar context, is not intuitive and it is even harder for high school students than for secondary students. Teaching does not seem to help to develop this intuition in our students. Moreover, our analysis confirms the existence of the following conflicts described in previous research: a) not using central tendency measures to compare two groups; b) computing the mean in an ordinal data set; c) not discriminating between ordinal and numerical data. We have also found the following new conflicts: Confusing central tendency measures with variable values; confusing mean with absolute frequencies; percentages with absolute frequencies; variable value with frequency; confusing ordinal variables with quantitative variables; establishing a correspondence that does not preserve the measurement scale. This list suggests the need to improve teaching that should consider the work with ordinal data, and also reinforce conceptual and procedural knowledge related to mean and median, and with more basic ideas such as statistical variables and distributions.

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