AN EXPERIENCE IN CURRICULUM DESIGN FOR HIGH SCHOOL STATISTICS EDUCATION

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This paper reports on an exercise of laying out a series of teaching sequences aimed at developing statistical reasoning in high school students. From a conceptual perspective, we examined the distinctions concerning the notions of statistical literacy, reasoning and thinking and considered three working scenarios reported in literature (data sets, sampling and probability situations) as referents for the design of information sampling and analysis. Based upon the methodology of teaching experiments, we proposed hypothetical learning trajectories, which served to guide both before- and after-design instruction in a dynamic way. Instruction results provided elements that point at reviewing the curricular design as well as the preconceptions of participants in the research.

INTRODUCTION

The experience this paper reports on took place within the framework of a research carried out by a group of teachers at National Pedagogical University, Colombia. The project, entitled “Teaching experiments for the development of statistical reasoning by high school students”, was funded by the Research Center of the National Pedagogical University and the Departement of Mathematics, at the same University. In general terms, the project was aimed at designing, developing and assessing a set of teaching experiments related to topics from the high school statistics curriculum with the purpose of promoting the development of statistical reasoning in a group of high school students. Given three different scenarios, namely, a scenario of datasets, a scenario of sampling and a scenario of probability, we assume the distinction proposed by recent research of statistical literacy, reasoning and thinking, as cognitive processes that allow a deeper understanding of the teaching and learning of statistics.

There has been progressive acknowledgment of the need to more closely relate the activities of high school mathematics teachers with those of researchers in the area of mathematical education. Such acknowledgment has given place to the emergence of methodological proposals such as the one suggested by Steffe and Thompson (2000), Lesh and Kelly (2000) and Cobb (2000), among others, and known under the title of teaching experiments.

Data collection for our study took place mainly during the second half of 2008 and the first half of 2009. The teaching experiments were carried out with ninth-, tenth- and eleventh-graders and fulfilled within the regular yearly agendas of activities in a private high school in Bogotá.

Several circumstances prevented the participation of the teachers regularly in charge of the classes, so their roles were assumed by teaching assistants and practitioners associated with our project.

CONCEPTUAL APPROACH

Rumsey (2002), Garfield, (2002), Chance (2002) and Pfannkuch and Wild (2004) have offered a first approach to the notions of statistical literacy, reasoning and thinking. Further precisions, insights and elucidations can be found in delMas (2002) and Ben-Zvi and Garfield (2004); these papers complement the first ones and offer new conceptual approaches to the subject.

In particular, the conceptual framework we adopted for designing the sequences of instruction carried out within our project followed the suggestions in delMas (2002). These can be sketched in terms of a list of tasks (see Table 1) that emphasized what is needed for students to develop and show, understanding of the corresponding processes—either statistical literacy, reasoning or thinking.

Table 1. Tasks that can distinctively characterize the three instructional perspectives (delMas, 2002)

Literacy	Reasoning	Thinking
Identifying Describing Rephrasing Translating Interpreting Reading	"Why" "How" Explaining (Processes)	Applying Criticizing Evaluating Generalizing

In other words, the goal were the development of statistical literacy, then teachers should provide tasks that require from students the identification of instances of a concept; or the description of graphical displays of a dataset or of distributions and relations among variables; or the rephrasing, translation or interpretation of results from a statistical procedure; and so on. Secondly, if tasks are designed that lead students to provide explanations of why or how have the results of a particular procedure emerged, then it is the development of statistical reasoning that is being promoted. For example, students can be asked to explain why is the median unwavering in the face of outliers values, or why random sampling tends to yield representative samples, or why can a certain conclusion be justified. Lastly, the development of statistical thinking can be distinguished from both other processes in the sense that students are encouraged to apply their basic statistical literacy and their ability for statistical reasoning within particular contexts. Therefore statistical thinking is promoted when students are challenged to apply their understanding to real world problems, to question, criticize and evaluate the design and results of already performed statistical analyses, or to generalize the knowledge derived from examples presented in the classroom to new or novel situations.

THE THREE SCENARIOS

One of the concerns that should be kept in mind throughout curricular design is that the designer must determine major areas of intellectual work that allow the attainment of proper instructional designs promoting the coverage of a broad spectrum of contents, concepts and processes in relation to statistical literacy, reasoning and thinking. In this respect, Canada (2004), suggests three scenarios or working situations that match such a concern.

A first type of scenario is provided by dataset situations. For example, Jones *et al.* (2001) claim that the deployment of statistical thinking requires understanding of a series of concepts related to the data handling that are multi-faceted and evolve in the course of time. Data handling involves "organizing, describing, representing, and analyzing data, with a heavy reliance on visual displays such as diagrams, graphs, charts, and plots" (Shaughnessy, Garfield, & Greer, 1996, cited in Canada, 2004). In other words, it entails reduction of data in such a way that their essential properties are preserved, and so paying attention not only to the mean but also to the dispersion. In particular, Friel et al. (2001, cited in Canada, 2004) notice that data reduction and the structure of graphs influence graph knowledge. In data reduction tasks, students can clearly benefit from attempts at raising their awareness of the significance of both measures of central tendency and measures of dispersion. Indeed, the crucial point within this scenario is how tasks can be stated in such a way as to lead students to understand, since issues here simultaneously demand working with statistical concepts—such as distributions, averages, dispersions—and graphs.

Sampling situations makes up a second working scenario. The focus of attention in instructional design can be placed, for this sort of situation, on the differences between repeated samplings within the same population, the size of samples and the representativeness of a sample. Sampling situations can call for multiple levels of meaning and understanding. For example, as sample size increases, statistics become less variable and give a closer look at the corresponding parameters of the population. In this respect, Well et al. (1990, cited in Canada, 2004) find that regarding the representativeness of samples, people tend to choice the bigger sample, and use information about sample size more accurately for estimating centers of the distribution than tails;

however, many seem not to understand how does the size of the sample influence the variability of the mean. In other words, when several aspects such as the notions of center and dispersion are jointly worked with graphical structure, sampling environments are truly versatile for encouraging students' reasoning and understanding. They provide opportunities to examine the effects of size sample as well as to scrutinize the ways in which different samples of the same size vary, thus offering unlike images of the same population.

A third scenario is the one provided by situations were probabilities are examined. Although underlying most work related to sampling one finds probabilistic thinking, when the sample size is limited to one the probabilistic aspects tend to become more transparent. For example, in examining a procedure involving draws from a ballot box where the size of the sample is one, Truran (1994, cited in Canada, 2004) finds that students have some awareness of the extreme numbers and the range. Shaughnessy (1997, cited in Canada, 2004) notices that when students are asked which has a higher likelihood of occurrence between two sample points in sampling and different results in coin tossing, when the coin is tossed five times, some of them try to impose the idea of sampling where it is simply not applicable. There is a sampling space in a probabilistic sense and it is different from the idea of a sample drawn from a population.

METHODOLOGICAL APPROACH

Recent research in mathematics education suggests a change in the traditional relationship between theory and practice. According to Cobb (2000), under traditional approaches such as that of positivist epistemology, practice is subordinated to theory, which is deemed to be superior; hence, teachers are considered consumers of results of research conducted outside their classrooms.

Under the new perspectives, teaching experiments are conceived of as a methodology whose main goal is not assessing the effectiveness of instructional designs previously carried out, but enhancing such designs by testing and modifying conjectures about the learning processes students undergo (Cobb, 2000). Such experiments are performed in collaboration with teachers who are associated with the research team. During them, researchers implement sequences of instructional activities or hypothetical learning trajectories in the sense suggested by Simon (1995), and then analyze processes of mathematical in students as they take place within the social situation of the classroom or within small groups (Cobb, 1999, cited in Jones et al., 2001). According to Steffe and Thompson (2000), a teaching experiment can be defined as a methodology oriented at capturing and documenting processes of thought in students within a certain period of time. Likewise, Confrey and Lachance (2000) see the teaching experiment as a planned intervention in the classroom that occurs within a significant period of time, where a continuous course of instruction is followed and where the intervention is aggressive enough to surpass traditional perspectives. Cobb (2000) hence claims that teaching experiments provide a way of exploring prospects and possibilities of changes at the classroom level.

CONCLUSIONS

The adopted conceptual approach lends to the acknowledgment of the usefulness and pertinence of assuming the challenge of attaining statistical thinking as the paramount purpose of statistical education. It also makes it possible to pinpoint different levels of complexity that gradually lead the development of a statistical education in students. Nonetheless, if developing statistical thinking becomes an actual purpose, it should be kept in mind that it will mean laborious efforts concerning curriculum design and development.

Although applications of the teaching experiments methodology in this project show some flaws, the methodology itself promises to be a powerful tool for curriculum design in agreement with of goals for statistical education at different contexts. It allows, for example, dynamically modifying the instructionally sequences upon immediate results and opens the possibility for a fuller, more systematic view of the students' processes of learning and understanding.

There are other results of the present study that can be view as related products. First, it was achieved a set of indicators that give meaning to the processes of statistical literacy, reasoning and thinking within each working scenario. Such indicators became the basis upon which hypothetical trajectories of learning were formulated in such a way that researchers made hypotheses as regards the way the students' processes of learning could evolve; on its turn, this allowed for elaborating the tasks that made up the axes of classroom activity. These trajectories are themselves a second product.

The analysis of collected information, done based on the set of proposed indicators, shows some evolution of students form statistical literacy to reasoning, but definitively not to thinking. Students were able to explain in terms of the situation the effect of dispersion of data in the mean and median. Also they talk about the convenience of a sample for a particular population depending of the way it is taken. Besides they acknowledge when a frequency probability becomes classic probability.

For the intention set by the National Ministry of Education concerning the promotion of "variational thinking" at school mathematics in Colombia, our project brought forward further insights to the need of granting the teaching of statistics and probability a stronger role within the goal of developing variation perspectives.

REFERENCES

Ben-Zvi, D., & Garfield, J. (2004). Statistical literacy, reasoning and thinking: Goals, definitions and challenges. In D. Ben-Zvi & J. Garfield (Eds.), *The challenge of developing statistical literacy, reasoning and thinking*. Dordrecht: Kluwer Academic Publishers.

Canada, D. (2004). Elementary preservice teachers' conceptions of variation. (Doctoral dissertation). Portland: Portland State University.

Chance, B. (2002). Components of statistical thinking and implications for instruction, and assessment. *Journal of Statistics Education, 10*(3).

Cobb, P. (2000). Conducting teaching experiments in collaboration with teachers. In A. Kelly & R. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 307–333). Mahwah, NJ: Erlbaum.

Confrey, J., & Lachance, A. (2000). Transformative teaching experiments through conjecture driven research design. In A. Kelly & R. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 231–265). Mahwah, NJ: Erlbaum.

delMas, R. (2002). Statistical literacy, reasoning, and learning: A commentary. *Journal of Statistics Education*, *10*(3).

Garfield, J. (2002). The challenge of developing statistical reasoning. *The American Statistical Reasoning Journal for Statistics Education*, *10*(3).

Jones, G., Thornton, C., Langrall, C., Mooney, E., Wares, A., Jones, M. R., Perry, B., Putt. I. J., & Nisbet, S. (2001). Using students' statistical thinking to inform instruction. *Journal of Mathematical Behavior*, *20*, 109–144.

Lesh, R., & Kelly, A. (2000). Multitiered teaching experiments. In A. Kelly & R. Lesh (Eds.), Handbook of research design mathematics and science education (pp. 197-230). Mahwah, NJ: Lawrence Erlbaum Associates, Publishers.

Rumsey, D. (2002). Statistical Literacy as a goal for introductory statistics courses. *Journal of Statistics Education*, *10*(3).

Simon, M. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education*, *2*(26), 114-145.

Steffe, L. P. & Thompson, P. W. (2000). Teaching experiments methodology: Underlying principles and essential elements. In A. Kelly & R. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 267–306). Mahwah, NJ: Erlbaum.

Pfannkuch, M., & Wild, C. (2004). Towards an understanding of statistical thinking. In D. Ben-Zvi & J. Garfield (Eds.), *The challenge of developing statistical literacy, reasoning and thinking.* Dordrecht: Kluwer Academic Publishers.