ENRICHING STATISTICS COURSES WITH STATISTICAL DIVERSIONS

Eric Sowey[1] and Peter Petocz[2]
[1]School of Economics, University of New South Wales, Australia
[2]Department of Statistics, Macquarie University, Australia
e.sowey@unsw.edu.au

Memorable teaching–teaching that makes memorable not only the teacher but also what the student has learned–is necessarily strong in both the cognitive and the affective domain. University statistics courses make heavy cognitive demands of students: these demands become more intense as the level of study increases. In such a setting, many students value an opportunity to break away for a while from the focus on technical aspects, to engage with a thematically-related challenge that the teacher has designed to provoke them or to pique their curiosity. We explore the design of such statistical diversions to enrich undergraduate courses at various levels. Diversions seem to be little recognized or utilized in statistics education, yet they can reinvigorate students' interest in their study of the subject. Diversions can also produce clarifying perspectives and other enlightening insight –characteristic cognitive strengths of memorable teaching.

INTRODUCTION

Teaching that illuminates structure and meaning is important for student understanding of any subject matter, but especially so in a discipline so much of which is technical. Such teaching addresses specifically the cognitive dimension of student learning. If it is done well, students will long recall their teacher with satisfaction, though their knowledge of the subject matter may fade. If the goal of education is more ambitious–that students be encouraged and inspired to adopt a deep approach to learning–then it is well recognized (see, for example, Ramsden, 1992) that teaching needs to address both the cognitive and the affective dimension of student learning. If it is done well, students will long recall their teacher *and* the subject matter.

In this vein, Ramsden (1992, p. 269) writes discerningly of teaching and of learning as an "arduous but *pleasurable* process" (italics in the original) and he continues: "There can be no excellent teaching or learning unless teachers and learners delight in what they are doing."

In the present paper (as in Sowey, 1995), we shall refer to teaching that leads to long-term retention of learning as memorable teaching. Our purpose is to bring to attention an activity that can contribute a great deal to memorable teaching. It is an activity which we have practised for many years, but one which has not to our knowledge been described in the literature. It is the enriching of statistics courses with statistical diversions.

University study of any formalized discipline makes extensive cognitive demands on students. The higher the level of study the more intense, generally, are the cognitive demands. In such circumstances, we have seen that many students value an opportunity to break away for a while from what they perceive as unremitting technicality in order to engage with a challenge (proposed by the teacher) that has a rather different 'feel'. While still related to the technical topic under study, it's a challenge that has been designed to provoke or tantalize them or to pique their curiosity. Engaging with this kind of challenge promises to be actually enjoyable!

In this context, it is clear why such challenges are referred to as *diversions*. (In a non-formal setting they are sometimes called *recreations*.) In brief, diversions are incidental intellectual challenges proposed by informed specialists that are designed to entice the uninitiated to try to find solutions, and to find enjoyment in the course of the attempt. Quite apart from their reinvigorating effect (in the affective domain) on students' interest and concentration when immersed in studying technicalities, such diversions can also generate clarifying perspectives and other enlightening insights–characteristic cognitive strengths of memorable teaching.

DIVERSIONS IN LOGIC, MATHEMATICS, SCIENCE AND LINGUISTICS

In logic, mathematics, science and linguistics, diversions have been proposed since the earliest times. One example from each field will suffice here: resolving Zeno's paradox of Achilles and the tortoise, showing that $\sqrt{2}$ is irrational, creating a mirror image that is not laterally inverted, and constructing anagrams of a given word or sentence.

In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence-based society. Proceedings of the Eighth International Conference on Teaching Statistics (ICOTS8, July, 2010), Ljubljana, Slovenia*. Voorburg, The Netherlands: International Statistical Institute. www.stat.auckland.ac.nz/~iase/publications.php

There are now countless book-length collections of diversions in these four fields, not to mention many websites. Widely-read authors in this mathematical field are Martin Gardner and Ian Stewart, in science Julius Sumner Miller and Karl Kruszelnicki, and in (English) language Willard Espy and Tony Augarde. See, for example, Gardner (2005), Stewart (2008), Miller (1978), Kruszelnicki (2008), Espy (2003) and Augarde (2003).

STATISTICAL DIVERSIONS

While diversions are plentifully documented in the fields of knowledge just mentioned, they are much more rarely noted in statistics. Why this is so is not immediately apparent. Could it be because the academic specialists take too serious a view of their field and informed popularisers of statistics (whether or not they are academics) are too few? Or perhaps because the notion of an enticing intellectual challenge in statistics sounds like such an oxymoron to the uninitiated that teachers' motivation to design statistical diversions evaporates?

Yet, for teachers who are willing to persevere, there is no shortage of intellectual kernels around which a statistical diversion may be designed. The art, in any particular course, is to identify points at which a diversion will be welcomed by learners, and then to formulate a challenge that is attractive in affective terms, as well as being constructive in cognitive terms. It will be apparent from the examples offered below that any particular diversion will appeal more to some students than to others. An important issue in designing enriching diversions is, then, for the teacher to first know his/her students. Of course, that is readily accomplished only in small classes. It does not mean that diversions are unwelcome in large class teaching. Experience will identify some diversions that are universally appealing. But it should be clear that diversions that fail to engage students will also fail to enrich the course.

DIVERSIONS AS A COMPONENT OF TEACHING

Though diversions in many fields are legion and their appeal is unquestionable (or there would not continue to be a ready market for publishing collections of them), it is quite surprising and, indeed, paradoxical how uncommonly prompts for including them are found in formal curricula in those many fields. This is even more the case in tertiary than in secondary education.

Let us, for instance, consider mathematics–and look first at secondary education. There are school textbooks (for example, Jacobs, 1994) that aim to enliven the subject matter with interspersed mathematical diversions. Nevertheless our experience, as well as casual observation over many years, suggests that high school mathematics teachers, generally, regard mathematical diversions as pleasant enough in their place, but their place is not in the mathematics classroom. It's as if getting students to engage with diversions were a faintly improper use of learning time. From this viewpoint, mathematical diversions are for keeping a class occupied when their teacher is away ill, or for filling up teaching time in the last weeks of the school year after the final exams.

What could account for such a misconceived attitude to the educational merits of *integrating* mathematical diversions into the curriculum? Perhaps it's so simple a thing as the name–*diversion*–which may conjure up thoughts of 'amusement' or 'entertainment' (which would never do), or possibly of 'distraction' or 'deflection' (which would be even worse!). There is, also the possibility that curriculum designers see no value in accommodating what they may regard solely as 'light relief' when there is so much in the way of formal mathematical technique to inculcate and so little time! Individual teachers could, nevertheless, choose to weave diversions into their teaching of difficult topics, as we are advocating. But that, too, seems uncommon. Anecdotal evidence suggests that a busy (not to say crowded) curriculum effectively discourages a teacher from 'going it alone' among his/her colleagues in offering students both the affective and cognitive fulfilments that well-designed diversions can yield.

At university level the story is even more disappointing. And the paradox is even greater, since some popular mathematical diversions (judging from their regular appearance in published collections) have generalizations that represent research challenges of the highest order. While these derived high-level challenges sustain *academics'* interest as researchers, the same academics, as teachers, seem to overlook the use of diversions for sustaining *their students'* interest.

What does the mathematics education literature have to say on a place for diversions in the curriculum? It's not a frequently discussed theme at the *secondary* level, but there is a persistent pulse that beats for a more inclusive approach. Eastaway (1997) and Zazkis and Campbell (2006) are characteristic contributions. We can find no opposing arguments in the literature. Yet, the efforts of the advocates appear to have been largely fruitless in practice.

On integration of diversions into *university* mathematics education the literature is silent. It is similarly the case with university statistics education. (A lone proposal–in a research report by Maltby, Day and Rooney (2005)–touches only very lightly on the issue.) There is clearly scope for considerably more attention to be given, both in practice and in the literature, to a role for statistical diversions in university statistics education.

STATISTICAL DIVERSIONS: SOME EXAMPLES

As already mentioned, the role of the challenges that we are calling statistical diversions is primarily affective–to reinvigorate students' interest and concentration by appealing to their curiosity or in some other way to draw them in. Where feasible, it is desirable to design the challenges so that solving them yields students a cognitive benefit as well, whether that be a novel perspective on the topic or some other enlightening insight.

On the basis of our own experience with using diversions in teaching statistics at every undergraduate level, together with parallel work we have done on devising statistical diversions for an academic readership in our regular column in *Teaching Statistics* (see the section on 'Sources and ideas for designing diversions', below), we have identified a variety of settings in which a diversion will go down well with students.

Let us consider examples in five different settings. These examples are not restricted to the syllabus of a first course.

When students are learning technical terminology

[1] The word 'stochastic' derives from a Greek word 'stochos'. What does 'stochos' mean?
[2] Why is it called 'analysis of variance'? Which 'variance' is being 'analysed'?
[3] Why is it called a 'martingale'? Has this word other meanings? Is there some connection?

When students are mastering the finer points of descriptive statistics

[4] 'Most people in Rome have more than the average number of legs.' Is this statement correct?
[5] Based on 30 continuous years of recorded temperature data, the average temperature over the 12 months in a calendar year in New York is 11.7°C, in New Delhi it is 24.3°C and in Singapore it is 27.1°C. Does this mean that it gets roughly twice as hot in New Delhi during the year as it does in New York? Does it mean that the climate in Singapore is much the same as that in New Delhi?
[6] We are comparing two symmetric probability distributions with zero means. One is leptokurtic and the other platykurtic. Which of these distributions has the larger variance?
[7] 'Imagine', says your financial adviser, 'that you have invested $100 every month over 40 years at 10% per annum, compounding monthly. At the end of this period you will have put in $48,000 of your own money. But with the wonders of compound interest these $48,000 will have grown to a staggering $632,408. That means your own capital has been multiplied about 13 times.' This calculation is numerically correct, but unfortunately the statistic in the previous sentence is wildly misleading in practical economic terms. Why?

When students are coming to grips with subtle properties of a statistical model or technique

[8] A standard normal curve is to be drawn to scale on a sheet of paper, so that its tails are 1 millimetre above the axis at $z = 6$. How tall must the sheet of paper be so that the curve's peak fits?
[9] In 1973, *The American Statistician* published a paper with the intriguing title 'How to get the answer without being sure you've asked the question'. What is the name for the type of sampling that the authors describe, and in what situations might it be useful?
[10] Think of the familiar hypothesis test on the value of the population mean of a normal distribution with known variance, against the two-sided alternative. The power curve for this test looks rather like an 'upside-down normal curve'. To what extent is this description correct?

When students begin to think of statistics as a creation of thinkers rather than as 'what's in the textbook'

[11] A statistical census was undertaken in England at Christmas 1085. What is the name of the document which recorded the results?

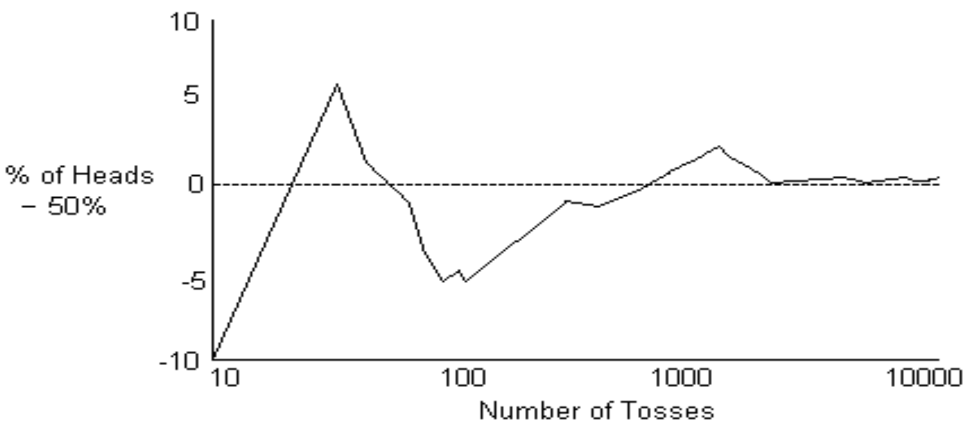
[12] In 1893, a book called *Pillow Problems* was published. It contains 72 maths problems of which 13 relate to probability. What is the name of the author as it appears on the title page of the book, and what is his real name? In which way are the author's pen name and real name connected?

[13] Which early statistician proposed testing the following statistical hypothesis (which may still be relevant today!): 'Whether of 1000 patients to the best physicians, aged of any decade, there do not die as many as out of the inhabitants of places where there dwell no physicians'?

When a puzzle with a real-life slant will make the technical subject matter more intelligible.

[14] When young children are asked about their understanding of probability, they quickly decide that the sample space for rolling a single die consists of six equally-likely outcomes. When it comes to two dice, however, they usually conclude that the sample space has 21 outcomes that are equally likely. Where does the number 21 come from?

[15] The graph below shows, on a logarithmic horizontal scale, the cumulative percentage frequency of heads in a sequence of 10,000 tosses of a coin.



These 10,000 tosses were performed by a South African statistician, John Kerrich, who went on to be the Foundation Professor of Statistics at Witwatersrand University in 1957.

(a) Where, and under what rather unusual circumstances, did Kerrich perform these 10,000 tosses?

(b) Does the information in the graph help us to define 'the probability of getting a head when a fair coin is tossed once'?

[16] In 2005, the radio news reported the story of the number 53 in the Venice lottery. The Italian national lottery is a type of lotto in which draws of five numbers from numbers 1 to 90 take place in each of 10 cities across the country (a total of 50 numbers are selected). In the Venice lottery, the number 53 hadn't come up in 152 consecutive draws over 20 months, and Italians were in a frenzy betting on 53 in Venice, ever more convinced on each occasion that it failed to appear that it simply had to appear next time.

(a) What popular name is given to the (invalid) principle which motivated the public's conviction that 53 was ever more likely to appear, the longer the continuous run of its non-appearance?

(b) What is the probability that 53 doesn't come up 152 times and then comes up on the 153rd draw (as actually happened)? Is this the probability we should actually be interested in for similar situations?

[17] A student regresses weight in kilograms on height in inches for a group of adult males. Having recorded the results, he decides that it was silly to mix metric and imperial units, and converts the heights to centimetres (using 1 inch = 2.54 cm). Then he regresses weight in kilograms on height in centimetres. Which of the following results will be the same for this second regression: the intercept coefficient, the slope coefficient, the value of R^2?

The foregoing 17 examples are chosen to show the variety of topics out of which a statistical diversion may be constructed, rather than to imply that all of these will necessarily be of interest to each of our readers, concerned with his/her own particular teaching responsibilities. (Interested readers may obtain solutions to these examples from the authors.) Our objective in making this illustrative selection is also to exhibit diversions that bring some cognitive reward to the students who respond to their challenge. The teacher should spend a little time to ensure that all students actually take in the new perspectives or other disciplinary insights that the thought-out responses of better students will uncover. Finally, it's important to reiterate that we are painting a broad picture of the utility of statistical diversions in statistics education. For reasons to be explained in the next section, our examples go well beyond the content of an introductory course.

Looking at these examples one by one, it will be clear that one may expect qualitative differences in the way they are presented by the teacher. Some may be posed as challenges for immediate attention by the class (e.g., [4], [11], [14], [17]), some will be asides mentioned during class but for students' attention out-of-class (e.g., [1], [10], [15], [16]), and some will appear in printed exercises issued to students as homework (e.g., [5], [7], [8], [12]). Similarly, there will be qualitative differences in the way they are addressed by students. Some will be tackled by pen-and-paper, some will need a little internet or library research, some will call for time with a calculator. But what is common to all of them, everyone understands, is that they are there expressly for students to cogitate on for a while *before* the teacher contributes anything at all.

USING STATISTICAL DIVERSIONS EFFECTIVELY IN TEACHING

We can report good experiences in using diversions in introductory courses, both as in-class questions that students can discuss in groups (such as [14] in an early probability class) and as assignment questions (such as [16] in a topic focusing on the statistics of gambling). Our evaluations of the positive effects of such questions on students' motivation are necessarily anecdotal, since the use of diversions is an inseparable part of our overall strategy for stimulating students to learn.

We find that diversions appeal more to statistics majors than to those doing service courses. That is understandable given that it is the majors who are investing themselves in study of the discipline. And, as they advance, it is they who encounter the heavier technicalities. Thus it is also they who are more likely to respond to the promise of intellectual excitement that a well-formulated diversion offers. It follows that the appeal of statistical diversions grows, in general, with the seniority of the course and with students' developing commitment to deep learning.

It is often remarked on that creative initiatives in statistics education, as reported in the literature, are heavily weighted towards the introductory course. Statistical diversions–so our experience with a range of courses indicates–are, in fact, no less valued, and thus no less valuable, in senior courses. Time and creativity devoted to designing diversions that fit well into senior courses are resources well spent.

SOME SOURCES AND IDEAS FOR DESIGNING STATISTICAL DIVERSIONS

Here are some sources to guide teachers in building their skill in formulating diversions.

(i) Our *Statistical Diversions* columns in each issue of the journal *Teaching Statistics* from vol. 25 no. 3, 2003 now contain in total over 100 diversions. There are five diversions in each column. Detailed solutions are presented in the following issue of the journal. The examples we have given above are largely drawn from this source.

(ii) Some striking demonstrations in statistics teaching make effective statistical diversions, see Sowey (2001). Statistical paradoxes can also serve well, see Szekely (1986).

(iii) The daily press is a fruitful source of kernels around which to build diversions. Two types of articles are particularly useful: articles involving blatant, and even not so blatant, misuses of statistics, and articles on controversial public issues that include statistics-based arguments.

(iv) Books on misuses of statistics, for example, Best (2001) and Spirer, Spirer and Jaffe (1998).

(v) Articles on out-of-the-ordinary topics in two magazines for students of statistics: *Significance* (published by Blackwell for the Royal Statistical Society) and *Chance* (published by Springer for the American Statistical Association).

Finally, it may be helpful to say something about our own approach to formulating diversions for our statistics classes.

Firstly, we have developed a turn of mind that is drawn to quirky and unusual aspects of our own professional and personal experience, and we keep a note of these as they arise.

Secondly, many of our questions are open-ended rather than being aimed at some specific answer.

Thirdly, we take into account the tools and sources that students will have at their disposal when they tackle the questions–for example, access to the internet. If we want to elicit independent, rather than internet-aided, thinking, we check the potential of a draft question by doing our own preliminary web searches. Thus, many of our diversions have no solution on the internet at the time we create them (though that may change with time, of course). On the other hand, there are many abstruse aspects of statistics which are very interesting and which can be discovered effortlessly today using the internet–a situation very different from the case just a decade ago. So there are questions we design fully anticipating an internet search. Moreover, we try to focus such questions in a way that will help students to develop useful search techniques.

And lastly, we proceed on the principle that diversions will most enrich learning if teachers get to know early something of the intellectual and the emotional temperaments of their students. It is not the careful wording of a statistical diversion that is its most valuable quality. Rather, it is the teacher's insight on how, when and to whom the diversion is offered.

REFERENCES

Augarde, A. (2003). *The Oxford Guide to Word Games*. 2nd edn. Oxford: Oxford University Press.

Best, J. (2001). *Damned Lies and Statistics*. Berkeley: University of California Press.

Eastaway, R. (1997). What is the difference between a puzzle and a maths question? *Teaching Mathematics and Its Applications*, *16*, 1-4.

Espy, W. (2003). *The Game of Words*. New York: Black Dog & Leventhal.

Jacobs, H. (1994). *Mathematics – A Human Endeavor*. 3rd edn. New York: W.H. Freeman.

Gardner, M. (2005). *Martin Gardner's Mathematical Games*. Mathematical Association of America. CD-ROM.

Kruszelnicki, K. (2008). *Science Is Golden*. Sydney: HarperCollins.

Maltby, J., Day, L., & Rooney, D. (2005). Using mathematical puzzles to aid the teaching of statistics. Research report, Psychology Network, UK Higher Education Academy. Online: www.psychology.heacademy.ac.uk/html/miniproject_details.asp?id=40.

Miller, J. S. (1978). *Why Is It So? Mechanics, Heat & Temperature, Sound and Electricity & Magnetism, Light and Modern Physics*. Sydney: Australian Broadcasting Commission.

Ramsden, P. (1992). *Learning to Teach in Higher Education*. London: Routledge.

Sowey, E. R. (1995). Teaching statistics: making it memorable. *Journal of Statistics Education*, *3*(2). Online: www.amstat.org/publications/jse/v3n2/sowey.html.

Sowey, E. R. (2001). Striking demonstrations in teaching statistics. *Journal of Statistics Education*, *9*(1). Online: www.amstat.org/publications/jse/v9n1/sowey.html.

Spirer, H. F., Spirer, L., & Jaffe, A. J. (1998). *Misused Statistics*. New York: Dekker.

Stewart, I. (2008). *Professor Stewart's Cabinet of Mathematical Curiosities*. London: Profile Books.

Szekely, G. (1986). *Paradoxes in Probability Theory and Mathematical Statistics*. Dordrecht: Reidel.

Zazkis, R., & Campbell, S. R. (2006). Number theory in mathematics education research. In R. Zazkis & S. R. Campbell (Eds.), *Number Theory In Mathematics Education: Perspectives and Prospects*. Mahwah NJ: Erlbaum.