TEACHERS' UNDERSTANDING OF STUDENTS' CONCEPTIONS ABOUT CHANCE: AN EXPERT-NOVICE CONTRAST

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This exploratory study investigated teachers' perception of students' thinking about chance. In particular, the study explored how teachers anticipated and explained students' difficulties with the idea of chance, and the strategies teachers claimed to use to help students reorganize their thinking. Two teachers, one expert and one novice, members of an AP Statistics learning community, participated in this study. They were observed in the learning community meetings, and interviewed in depth. The interviews explored four core ideas in statistics that have been associated with the source of students' difficulties about chance: sample space, randomness, independence, and the law of large numbers. The results of this study highlighted that the expert and novice teachers exhibited differences in the way they perceived students' difficulties and in the way they dealt with them.

INTRODUCTION

Although participation in modern society requires an increasing familiarity with data and chance, research shows that people have difficulty reasoning in situations of uncertainty (Garfield & Ahlgren, 1988). The research literature is full of studies that describe the difficulties students (and even teachers) have in dealing with uncertainty, but little has been revealed about the pedagogical implications of those difficulties and how teachers might help students overcome them. This exploratory study investigated, using an expert-novice contrast, how Advance Placement Statistics (a college-level course in high school mathematics offered by the College Board in the US educational system) teachers anticipate and explain students' difficulties in chance situations and what strategies teachers claim to use to assist students in reorganizing their intuitive reasoning into more formal thought. Specifically, the following research questions were addressed:

1. How do an expert and a novice teacher of AP Statistics anticipate students' difficulties in chance situations?
2. How do an expert and a novice teacher of AP Statistics explain students' difficulties in chance situations?
3. What strategies do an expert and a novice teacher of AP Statistics claim to use to assist students in the reorganization of their intuitive thinking into a more formal thinking about chance?

These questions were explored in the frame of four core ideas underlying statistics and probability: sample spaces, randomness, independence, and the law of large numbers. These ideas have been considered by some researchers to be foundational and yet very problematic for students studying statistics and probability (Gal, 2005; Garfield & Ahlgren, 1998; Garfield & Ben-Zvi, 2008; Shaughnessy, 2003). These ideas are not considered as specific topics in the AP Statistics curriculum but rather as underlying ideas across the curriculum.

The expert-novice contrast has been used in many fields of research to study what expertise is like, its effects, and how it is exhibited. Such studies have motivated much research on expertise in education and have shown that the characteristics of expert and novice teachers' performance resemble those of experts and novices in other fields (Carter, Sabers, Cushing, Pinnegar, & Berliner, 1987). There are qualitative differences between experts and novices in perception, thinking, knowledge, processing and using information, problem solving, and decision-making.

LITERATURE REVIEW

Studies in thinking about chance abound not only in statistics education but also in mathematics education and educational psychology. Most of the studies on this topic have focused exclusively on how students think, and studies on how teachers assist students to overcome their intuitive thinking about chance are limited.

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Research on thinking about chance may be classified into two approaches. The first has looked at data collected exclusively from forced-choice responses, generally given to college students, and then analyzed through statistical tools. Researchers have sought to identify whether the students reasoned properly and to identify the more common errors made by students in their solutions (Kahneman & Tversky, 1972, 1996; Lecoutre, 1992; Moutier & Houdé, 2003; Ross & DeGroot, 1982; Tentori, Bonini, & Osherson, 2004; Tversky & Kahneman, 1973, 1974, 1983). These studies have been helpful in illuminating the common errors students make in reasoning about chance, but the studies have not been successful in providing a detailed description of students' thinking or in offering explanations of the reasons that students may favor one option over others.

The second approach to research on thinking about chance has also employed multiple-choice items, but this approach has further explored students' reasoning more deeply using clinical interviews (Jones et al., 1999; Konold, 1989, 1995; Piaget & Inhelder, 1975; Polaki, 2002; J. F. Wagner, 2006). These studies have become great resources for understanding students' thinking about chance and have served as good complements to the studies using the first approach. Overall, the literature that describes students' difficulties in reasoning about chance is much more abundant than the literature that treats how to help students overcome such difficulties (Garfield & Ahlgren, 1988).

METHOD

This exploratory study followed a qualitative approach. Interviews and observations of teachers in an AP Statistics learning community were the main sources of data. In addition, the AP Statistics learning community meeting agendas, laboratories, slides, and other artifacts were collected. Interpretivism as a theoretical perspective (Crotty, 2004; Schwandt, 1994) and a grounded theory approach (Glaser & Strauss, 1967) were used to generate descriptions of teachers' understanding of students' conceptions of chance.

The learning community was a ten-teacher volunteer gathering supported by a partnership project designed to improve science and mathematics education. The purposes of the gathering were to discuss issues related to teaching, share positive teaching experiences, and encourage collaboration. Although the participants in the learning community were teachers, in this study they did not act as teachers per se but as teachers reflecting about their practice. The teachers acted as researchers. Two teachers, one expert: Eric and one novice: Natasha, from the learning community were asked for extra participation. The expertise of the participants was determined by the number of years they had taught AP Statistics, the percentage of their students passing the College Board AP Statistic examination, and their reputation based on references by key informants from the community. The extra participation involved in-depth interviews using an instrument consisting of 12 episodes to gather teachers' understanding of students' reasoning in the four core ideas. An episode consisted of two parts: one task and either a hypothetical student's incorrect reasoning or specific results from research that used the task. The interviews were conducted in a way that the teachers only were shown the task and asked for their predictions. Once the teachers made their predictions, the second part of the episode was shown to them. All the episodes used in the instrument were previously used in research.

ANALYSIS

The analysis attempts to answer the research questions in each of the four core ideas; however because of the limited space, this summary will only present an excerpt of the analysis of randomness. A deeper analysis of the other core ideas is described somewhere else (Zapata-Cardona, 2008). First Natasha's reflection will be presented and then Eric's reflection to finish with a contrast between the two teachers.

Episode – Gumball Machine

Task: A gumball machine has 100 gumballs: 20 are yellow, 30 are blue, and 50 are red. The gumballs are well mixed inside the machine. Jenny gets 10 gumballs from this machine. What is your best prediction of the number that will be red? (Taken from the 1996 NAEP, Zawojewski & Shaughnessy, 2000).

Results from previous research: Only 7% of the student from a sample of 232 students gave an extended response, and 14% a satisfactory response. The majority of the students give exact answers like 5, and only one gave a range for the answer.

In discussing students' difficulties in this task, Natasha indicated that students might struggle if they assume equal probability in the outcomes. She said, "I think that a difficulty that students might have with this is that they read that there are three colors: yellow, blue, and red. And they might assume that each one has one third chance of being selected."

Natasha also predicted that students could solve the task by drawing pictures but perhaps without being aware that the gumball machine had 100 gumballs. It was not clear what type of picture she was referring to, but in any case, it would have been time consuming to draw. This task did not need to be worked out to be able to come up with a prediction; Natasha, however, started with a sketch (Figure 1). It was interesting that her prediction was similar to the students' results in the 1996 NAEP. She did not consider a range of possible values.

Figure 1. Natasha's sketch for the Gumball Machine Task

Later, Natasha mentioned that a good approach could be to reduce the task to 10 gumballs. Note that sampling with 10 gumballs increases the variability of the results. Her goal might have been to simplify the situation for students' better understanding; but her suggestion indicated that she did not consider the implications of simplifying the task.

When Natasha saw the second part of the episode, she said: "Okay, I see. They just wrote down the number instead of explaining what could possibility happen in that situation." Her statement suggests that she focused exclusively on the fact that students did not give explanations, but she ignored the aspect that almost all the students gave exact answers and did not consider a range of possible answers. Because Natasha ignored the second part of the results, her explanations were oriented mainly to justifying why students do not give extensive explanations. To explain this phenomenon, she mentioned that the tradition in mathematics classes is that students are not asked to explain their thinking.

Natasha suggested several pedagogical strategies to help students with this task. She suggested (1) having students reread the problem, (2) reducing the task to a simpler task, (3) drawing a picture, and (4) experimenting with physical counters. Note that these strategies did not specifically address the two problems revealed in the results from research. They did not address the students' difficulties in explaining their thinking or their tendency to give exact answers. It is also notable that the strategies Natasha suggested were not associated directly with students thinking; they were instead related to teachers' actions.

In discussing the Gumball Machine task, Eric mentioned that the idea of independence could be conflicting here and that the task was not specific about that aspect. Another difficulty he predicted was related to the students' poor understanding of the concept of probability and their poor strategies for problem solving.

After Eric saw the results from the research, he was surprised by the low percentage of students that offered an extended answer but said that students' thinking is not the only factor contributing to their difficulties. He said that there were other factors outside of students' thinking that might increase their difficulties. Eric also said that the mathematics curriculum promoted the notion of expected value but left the notion of confidence intervals exclusively to statistics. He said that under those conditions, the mathematics curriculum was not contributing to overcoming students' difficulties and that it was not strange that students were not familiar with these ideas unless they had previously had some exposure to statistics.

Eric also mentioned that the NAEP is not a mandatory examination, which might have affected the results. He also said that the way the question was asked might have encouraged students to give an exact answer, but if students had been asked for an interval, the results would have been different. This suggestion is consistent with the recommendations made by the researchers that explored students' performance in this task. They found that the wording of the item may have contributed to the poor performance; "likely range" and "likely interval" would have been more appropriate wording than asking for a numerical prediction (Zawojewski & Shaughnessy, 2000).

Eric suggested several pedagogical strategies. He suggested (1) doing the experimentation with physical counters, (2) doing the simulation using technology like TI-83, (3) having the students explain their reasoning, (4) rewording the task and (5) comparing experimental probability with theoretical probability.

Note that Eric did not work out the task and his strategies were inclined to address at least one of the difficulties expressed in the results of research whereas Natasha's strategies did not address the problems directly. Eric's strategies were varied and they included different approaches: using simulations with technology, exploring the empirical distributions with different quantities and listening to students' reasoning. Additionally, Eric was reflective about the curriculum, reflection that was absent in Natasha's discussion.

CONCLUSIONS AND IMPLICATIONS

The teachers exhibited differences in the way they perceived students' difficulties and in the way they dealt with them. The more experienced the teacher, the more articulated the interventions were. The analysis also showed that the expert teacher was more accurate in predicting students' difficulties than the novice teacher was. The expert teacher was able to identify students' difficulties in the first encounter with the tasks, whereas the novice teacher needed more time and hints to be able to identify such difficulties. Several times the novice teacher identified students' difficulties only after being shown some examples of the struggles students have with the situations. The level of the difficulties described for each teacher was different. The novice teacher tended to underestimate students' difficulties and explained them in terms of students' lack of understanding of concepts related to chance. The expert not only associated students' difficulties to students' lack of understanding of ideas of chance but also described the influence of external factors like the organization of curriculum, incorrect uses of chance in the mass media, and a tradition of thinking where straight answers are privileged over answers that required exploration.

The pedagogical strategies that the novice teacher suggested to help students in the reorganization of their thinking were centered in teachers' actions and were not specifically tied to students' thinking. The expert in contrast suggested strategies that integrated the teacher's actions with the students' actions and thinking. The expert teacher's descriptions of pedagogical strategies integrated the teachers' actions with students working in teams, students revealing their ways of thinking, and students confronting their own misconceptions.

Research literature reports that experts and novices from different fields perform differently (Benner, 1984; Chi et al., 1981; Hidi & Klaiman, 1983; Swanson et al., 1990). Results from this study concur with those from previous research. Several characteristics came out from the expert teacher's reflections that were absent in the novice's reflections and that might constitute a potential framework to explain expertise. First, the teachers' conceptions of teaching and teacher's role were different. The novice teacher's reflections revealed that her conception of teaching was associated with the teachers' actions. Teaching was centered in the teacher, teaching responded to the teacher's motivations instead to the students' needs, and teacher was a provider (giver) of information. The expert teacher's conception of teaching, in contrast, was highly tied to students' thinking and the teacher was a facilitator.

Second, the teachers' familiarity with the material differed. The novice teacher had to work out throughout the tasks to be able to express her opinion about students' difficulties whereas the expert teacher's familiarity with the tasks allowed him to give reasonable predictions of students' difficulties just by reading the tasks. This finding is consistent with Swanson and colleagues' (1990) results that describe that, in solving problems, novice teachers tended to represent problems in terms of the solutions whereas the expert teachers placed priority on defining and representing the problems as well as evaluating possible strategies.

Third, the expert teacher showed a reflective attitude towards the curriculum. The expert teacher had several references to the convenience or not of certain topics in the curriculum. The novice teacher, however, did not show any indication of being reflective about the topics that should be included in the AP Statistics curriculum or even considering the structure of the curriculum as a potential source of students' difficulties.

Fourth, the way the teachers perceived students' difficulties differed. The expert teacher was more accurate in predicting students' difficulties and offered a wider range of explanations than the novice teacher. Fifth, the variety of suggested strategies differed. The strategies offered by the expert teacher were integrated and in response to the identified students' difficulties. The novice teacher's suggested strategies were similar throughout the core ideas and were less integrated to the theory. This result coincides with results from previous research that showed that novice teachers do not have a well-developed pedagogical content knowledge that allow them to construct detailed explanations (Borko & Livingston, 1989) and that expert teachers' strategies are rich in examples and representations (Leinhardt, 1989).

The results from this study suggest several directions for teacher education and professional development programs. As mentioned earlier, the novice teacher was more reflective and more open to considering students' difficulties after being shown specific students' struggles with the tasks. Providing beginning teachers with opportunities to examine real students' difficulties can assist the teachers in thinking reflectively about students' struggles and in the design of efficient pedagogical strategies oriented to help students overcome such difficulties.

Literature (Moir, 2004) has shown that beginning teachers are mainly concerned about building the curriculum by means of collecting activities to do in their classroom. Teacher preparation programs could promote the collection of resources only if that is combined with systematic reflection about the critical aspects of the activities, potential students' difficulties, contribution to the development of statistical concepts, and other teachers' experiences with the activities. If novice teachers have the opportunity to share their experiences about specific activities with other teachers and discuss the problematic features of the activity as well as the aspects that need to be redesigned, the teachers' collecting of activities would not be done just to have a repertoire of activities to pick from but also might help them to start constructing a critical pedagogical content knowledge. Deep, early, and continued reflection about students' difficulties may be a key element in teacher preparation.

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