TEACHING AND LEARNING OF THE POISSON’S MODEL: A MODEL BUILDING EXPERIENCE

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In this article, we describe an experience with the teaching and learning of the Poisson´s Model, directed to university students. We used the Anthropological Theory of the Didactic and the principles of the Mathematical Modeling Process for the elaboration of the teaching sequence, showing that it is possible to build the model without resorting to the Binomial limit, making use of the notions of calculus. In the development of the learning activities, the computer was used as a didactic tool. The analysis of results was based on the Theory of the Semiotic Functions and proved that the students learned many significant elements considered in the teaching. On the other hand, the study allowed for the identification of difficulties throughout the process: for example, the interpretation of expressions such as "at least" and "at most," the representation used, the analysis of the results of hypothesis tests and the manipulation of the software.

INTRODUCTION

This article is an excerpt from the PHD thesis developed in São Paulo (Miguel, 2005) that consisted of an interventionist study about Poisson’s Model. The application of the Mathematical Modeling Process in teaching has been growing lately, but can rarely be noted in the teaching of statistics at college level (Batanero, 2001). The several modeling schemes differ in the details of the phases of the process. Some researchers begin with observation and data collection, others start with tables with real data, with the use of the computer as a tool (Batanero, Tauber and Sánchez, 2001) and others with values obtained through computerized simulation (Henry, 2003 and Coutinho, 2001). Concerning probability distribution models, the investigation by Batanero, Tauber and Sánchez (2001) revealed that the students need to recognize several significant elements of the Normal Model in the solution of problems and that the use of the computer may provide to the students a stochastic experience that is hard to be obtained without it. In that same line of study, Henry (2003) presented a workshop on Poisson’s Model, using the computer to generate data and using the comparison between theoretical and simulated values. Coutinho (2001), however, used simulation to work with the binomial distribution in a geometric probability context.

Poisson’s Model has frequently been the subject of studies and reflections, due to learning flaws presented by the students. Since we have not found systematic studies about the issue which could indicate new strategies, and considering that the difficulty of an issue depends greatly on the teaching implemented (Batanero, Tauber and Sánchez, 2001), we decided to elaborate a didactic sequence and apply it to a group of volunteer subjects. Our questioning aimed at verifying if the use of Mathematical Modeling is favorable to teaching and learning the Poisson’s Model.

Based on preliminary studies, we developed the hypothesis that the use of the computer as a didactic tool could contribute to the development of the modeling process, giving conditions for the student to use the software to make tabulations, calculations and graphics, determine the probabilities of the theoretical model and compare experimental and theoretical results through the chi-square test. We also believed that pair-work would contribute for the faster and more confident accomplishment of the tasks. As a complement, the evaluation instruments were built and applied throughout the activities and at the end of the study, so as to identify the difficulties during the didactic interaction and those that persisted, besides detecting learning mistakes and students’ acquisitions.

THEORETICAL BASIS

The present study is based on two theories: the Anthropological Theory of the Didactic (Chevallard, 1992, 1999; Bosch; Chevallard, 1999) and the Theory of the Semiotic Functions (Godino, 2003). The use of the Anthropological Theory of the Didactic allows the organization of the study in two connected aspects – didactic and mathematical. It also allows, in each case, the description of the activities proposed by the authors of the books analyzed and of the intended teaching sequence in terms of *tasks* (evoke action: calculate, verify, etc.), *technique* (systematic and explicit way to execute tasks), *technology* (speech that explains, justifies and elaborates techniques) and *theory* (wide speech that explains and justifies technologies). Bosch and Chevallard (1999) define: *ostensive objects* as: those that have a certain materiality (sounds, graphics and gestures) and *non-ostensive objects* as: ideas, intuitions, concepts, that exist institutionally and can only be evoked or invoked by the adequate manipulation of certain ostensive objects associated to them. In the mathematical activity analysis, the joint action, ostensive/non-ostensive, is known in terms of signs (ostensive) and meanings (non-ostensive). The ostensive objects have an instrumental function, that is, are material tools for the action in the mathematical organizations, and a semiotic function, that is, a capacity to produce sense or meaning; both functions co-habit. The praxeological organization of mathematical knowledge serves to analyze, describe and study the conditions for its realization.

Our preoccupation in elaborating a teaching sequence that would favor significant learning of the Poisson's Model (in the usual meaning of the term) and evaluate its application motivated the search for other theoretical elements for its basis. Particularly, the work by Batanero, Tauber and Sánchez (2001) about the meaning of the normal distribution suggested the use of the Semiotic Functions Theory (Godino, 2003). Essentially, the creation of meaning categories of mathematical objects: *situations* (mathematical or extra-mathematical problems), *language* (representations, including the mathematical ones), *actions* (by the subject in face of mathematical tasks), *concepts* (notions that justify the adopted techniques), *attributes* (conditions to realize the actions) and *arguments* (to justify, explain and prove the solutions presented), identified in an *institutional* and/or *personal* character, allowed for the analysis and evaluation of the work developed. The confrontation between the institutional meaning (obtained through an analysis of the didactic and mathematical praxeology of didactic books) and the one presented by the students allowed for the determination of learning elements that were effectively established in the study and of others, whose difficulty was not overcome (learning mistakes).

METHODOLOGY

The choice for Didactic Engineering as the research methodology is due to the desire to carry out an experimental study based on a didactic realization, comprising conception, realization, observation and analysis of a teaching sequence, about the Poisson's Model. The first readings, related to the issue, indicated the use of Mathematical Modeling in the development of the work with the students; a preliminary analysis culminated in the decision to interpret the modeling process presented in Henry (1997) as a recurring method of the development of the intended teaching sequence, for contemplating phases that were considered essential having in mind the intended targets.

The name "Didactic Engineering" is related to the comparison with the work of an engineer, in this case, a teacher, who prepares a teaching project for a student population. In it, four phases can be distinguished: *preliminary analyses, conception and analysis a priori of the didactic situations of the engineering, experimentation* and *analysis a posteriori and validation*.

In the process of Mathematical Modeling proposed by Henry (1997), the following phases are considered: *reality, pseudo-concrete model, mathematical model, mathematical study, model/reality confrontation* and *generalization/forecasts*. The author points out that the observation and description of a real situation and the formalization of the mathematical model require different didactic contracts for demanding distinct competences. In the first, choices are made based on scientific knowledge, so as to retain what seems pertinent, having in mind the problem proposed. There is also the need for an experimentation phase that requires the elaboration of an experimental protocol, a group of precise instructions to be followed for the experience to be carried out or repeated. In the second, students need to be capable of representing and manipulating symbolically the relations obtained, so as to find an answer for the proposed problem, besides validating and generalizing the results obtained.

With the purpose of redirecting the Engineering project, a pilot experiment was carried out in the second semester of 2003, at a non-profit private university in the capital of the state of

São Paulo, Brazil. Twelve students from the second year of the College of Electric Engineering, enrolled in the discipline of Probability and Statistics, took part, voluntarily in that preliminary study, and the activities proposed were developed in pairs. The option for engineering students who had in their curriculum, in the semester in question, a study on radiation, aimed at greater friendliness with the subject of the intended experiment: emission of radioactive particles. The need to be enrolled in the Statistics course is due to the research target content, which requires previous knowledge of descriptive statistics, probability, Binomial Model, Differential and Integral Calculus besides not having studied the Poisson's Model. After a realignment of the activities, in the second semester of 2004, the experimental phase began, in the same institution, with sixteen students from the second year, being eight from the College of Electric Engineering and eight from the College of Computer Sciences, all enrolled in the discipline of Probability and Statistics and all volunteers. The option for forming the pairs with one student from each course had the purpose of enriching the exchanges within the pair that the two formations might provide, since part of the activities would be developed in the computer laboratory.

The students participated in eight two-hour meetings, and in the last meeting, they answered a final test, individually. In the first phase, in the Nuclear Physics Laboratory, the students used Geiger-Mueller counters in counting particles emitted by two radioactive sources in varied time intervals. In the second phase, in the computer laboratory, they organized and explored the data collected by means of a descriptive study using the representations in tables and graphics. In the third phase, in a common classroom, the theoretical model (Poisson's) was built, based on the experiment made. In the fourth phase, in the computer lab, a mathematical study of the Poisson's Model was made for different values of the parameter. In the fifth phase, also in the computer lab, the students made a comparison between the results obtained experimentally and those of the theoretical model, by means of a chi-square test, considering the estimate obtained experimentally as a parameter. Finally, in the last phase, in a common classroom, the students used the model created in different situations, including the approximation to the Binomial Model. At the end of every phase, the students were evaluated, and, as needed, modifications were made in the initial project, always referring back to the analyses a priori of the activities.

ANALYSIS

Among the elements of personal meaning, intended in the first phase, it can be said that the objectives were fulfilled, pointing out difficulties in the adequate use of specific radiation vocabulary. Some characteristics of the idea of "random" could be identified in the answers of most students, but not in all, giving evidence of its complexity, already emphasized by other authors (Batanero, 2001). It is believed that the experiment chosen worked to motivate and elevate the interest of the participants for the study being developed.

In the second phase, the graphic representation proved favorable for the emergence of conjectures about the properties of the model, but the use of the computer, although helpful for obtaining the intended representations, showed that students need to dominate the software to avoid compromising the intended teaching, a fact already noted by Coutinho (2001).

The construction of the theoretical model proved its complexity and the importance of the details in the mathematical passages used, and favored the learning of Poisson's Postulates and their necessity. On the other hand, the representation used was not enough for the determination of the parameter, although the dependence of the variable time and type of material had been identified. There was also an explicit interest for historical aspects related to the study that could be aggregated during third phase.

In the individual comments about the fourth phase, the work at the Computer Laboratory was praised by everyone as an instrument that favors learning, just like the work in pairs, emphasizing the advantages of the debate about doubts, more confidence in the conclusions and more agility in the resolution of the proposed situations. Some subjects explicitly declared that the graphic representations facilitated the visualization of the results and that the activity helped clarify the parameter of the model.

In the fifth phase, it was evident that the fact that students had already used software did not guarantee that it could be used in the development of new notions. From the elements of personal meaning declared by most of the students, the synthesis in the conclusion of the statistical test could be considered a learning mistake. The application of the Poisson's Model to different situations showed that the process developed was favorable for the learning of many meaningful elements considered in the teaching, and for the identification of some difficulties, such as the interpretation of terms like: "at least," "at most," or, still, that the representation of the parameter of the model adopted generated confusion.

CONCLUSION

This research allowed for the conclusion that the use of the Mathematical Modeling Process may favor the learning and teaching of the Poisson's Model, allowing for several of its meaningful elements to be put into action. It favors the development of competences in obtaining and application of techniques, and the understanding of the object being studied as a progressive mental, social and interactive process. The analysis of the results supports the conclusion that all six phases were crucial for many of the elements considered in the study to be part of the knowledge acquired by the participants. Among them, the most important are: determining the parameter of the model; identifying the value of the variable with maximum probability; determining the expected frequency; calculating the critical chi-square and observing and comparing them; identifying situations in which the Poisson's Model can be used; knowing the relationship between average, variance and parameter of the model.

Those acquisitions may have been favored by the choices made in this research, some of which were pointed out by the participants themselves: the discussion and correction of each task at its end; the explanation of the doubts during the process, the detailing in the formal demonstrations, the change of the work environment and the work developed in the Nuclear Physics and Computer Laboratories. In relation to the pair-work, the students mentioned the possibility for exchanging information, the need to discuss with each one's partner the solutions guaranteeing greater reliability and the appearance of new questions.

The study detected, also, that some elements were more difficult to understand. Among them, it is worth mentioning: identification of the elements in the symbolic representation used; interpretation of expressions like "at least two," "more than two," "a maximum of three;" the adequate expression of the conclusion of a hypothesis test; the determination of the interval of the values of the variable with non-negligible probabilities; use of the software.

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