IDENTIFYING CONTENT KNOWLEDGE FOR TEACHING STATISTICS

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The purpose of the study reported herein was to identify important aspects of statistical knowledge needed for teaching in the middle school grades. A systematic study of the current literature, including state and national standards, was conducted to identify these important aspects and to measure the degree of emphasis or importance suggested for the content. Results show that state and national standards differ greatly in their expectations of what topics in data analysis and statistics students and teachers should master. The variation is also large in the degree of emphasis given to the content. The majority of the documents analyzed suggest giving greater emphasis to the selection and proper use of graphical data representation and measures of center and spread. Additionally teachers' standards also suggest as important the proper selection and use of teaching strategies and inference of students' understanding from their work and discourse.

INTRODUCTION

More than ever before, data analysis and statistics are an important part of the school mathematics curriculum in the United States. Because new curricula are challenging teachers not just with new teaching approaches but also with new content, the preparation of teachers and teachers' knowledge in this field have also become a special interest. The purpose of this study is to contribute to the current discussion and analysis of *content knowledge for teaching* by identifying important aspects of content knowledge for teaching data analysis and statistics.

The central research question is, what are the important aspects of statistical knowledge needed for teaching at the middle school level? In particular, what aspects of content and pedagogical content knowledge do middle school teachers need in order to teach data analysis and statistics?

METHODOLOGY

Several approaches were taken to investigate this question. First, the analysis of policy documents, (e.g., national and state standards, books, and reports) as a reflection of experts' perspectives giving insight on what it is that they value or consider important. Second, the analysis of students' mathematical curricula (e.g., mathematics textbooks and teacher's guide) as a reflection of what teachers are supposed to teach with an intended curriculum.

More specifically, to identify the important aspects of content (i.e., "the big ideas"), documents at the student and teacher level were considered. At the student level, ten sets of standards from those states that have middle grades certification, *Principles and Standards for School Mathematics* (NCTM, 2000), and a standardized middle school assessment (PRAXIS II) were reviewed and analyzed. At the teacher level, the report, *The Mathematical Education of Teachers* (CBMS, 2001) was also analyzed. To identify the other important aspects of knowledge for teaching such as pedagogical knowledge, knowledge of students as learners and knowledge of assessment other document were reviewed. These include, the National Board for Professional Teaching Standards, the National Council for Accreditation of Teacher Education (NCATE) Standards, professional standards from several states (Florida, Georgia, Missouri, and North Carolina), *Professional Teaching Standards* (NCTM, 1991), *Knowing and Learning Mathematics for Teaching* (NRC, 2001). The Unit Data About Us from the Connected Mathematics Project (Lappan *et al*., 2002) was reviewed to examine how all these aspects relate to each other.

A *content matrix* originally developed by Porter (2002) to measure agreement between standards and assessment in mathematics was adapted to the context of knowledge for teaching. The matrix crosses content areas and level of cognitive demand. The levels of cognitive demand were inspired by frameworks in Garfield (2002) and delMas (2002). Garfield and delMas suggest three categories of learning outcomes: Statistical Literacy, Statistical Reasoning, and Statistical Thinking. Literacy refers to recognition or computation; reasoning refers to explaining why and how a specific process works; and thinking refers to applying statistics to a context, critiquing, generalizing. In terms of the topics, Statistical Literacy category is mainly associated with creating graphs or plots, finding measures of center and spread. Statistical Reasoning category is associated with the appropriate use and selection of graphs and measures. The matrix was then used to quantify the frequency of occurrence of each aspect and to measure the agreement between documents. Based upon this analysis the most important and ubiquitous aspects were identified. Documents (most of them state standards) were coded given a number for the topic and a letter for the level of cognitive demand. To assure reliability of the coding process, one rater was trained to code the specific standards. The trained rater and the researcher coded the specific standards independently. The researcher adjudicated when there was disagreement between them. A total of 67 specific student standards were coded and since each specific standard could receive one or more codes, there were a total of 171 codes recorded.

ANALYSIS

Contour maps were created for each set of standards and when all matrices are put together the content map of Figure 1 is created. This map allows one to “see” these documents as a whole and identify the statistical content that students in middle grades are expected to know. The map indicates that the least covered topics are shapes of distribution and the process of statistical investigation. Instead, emphasis is placed on representation of data, particularly numerical representation and measure of center at the level of statistical reasoning. That is, 9% or more of the content in middle grades described in these documents is dedicated to communicating understanding with graphical displays of numerical data. Whereas between 7.7% and 9% of the content is dedicated to communicating understanding with measures of center. In general, the main focus of the standards is at this middle cognitive demand. Only for the topic of process of statistical investigation is the highest level of inferring and generalizing emphasized.

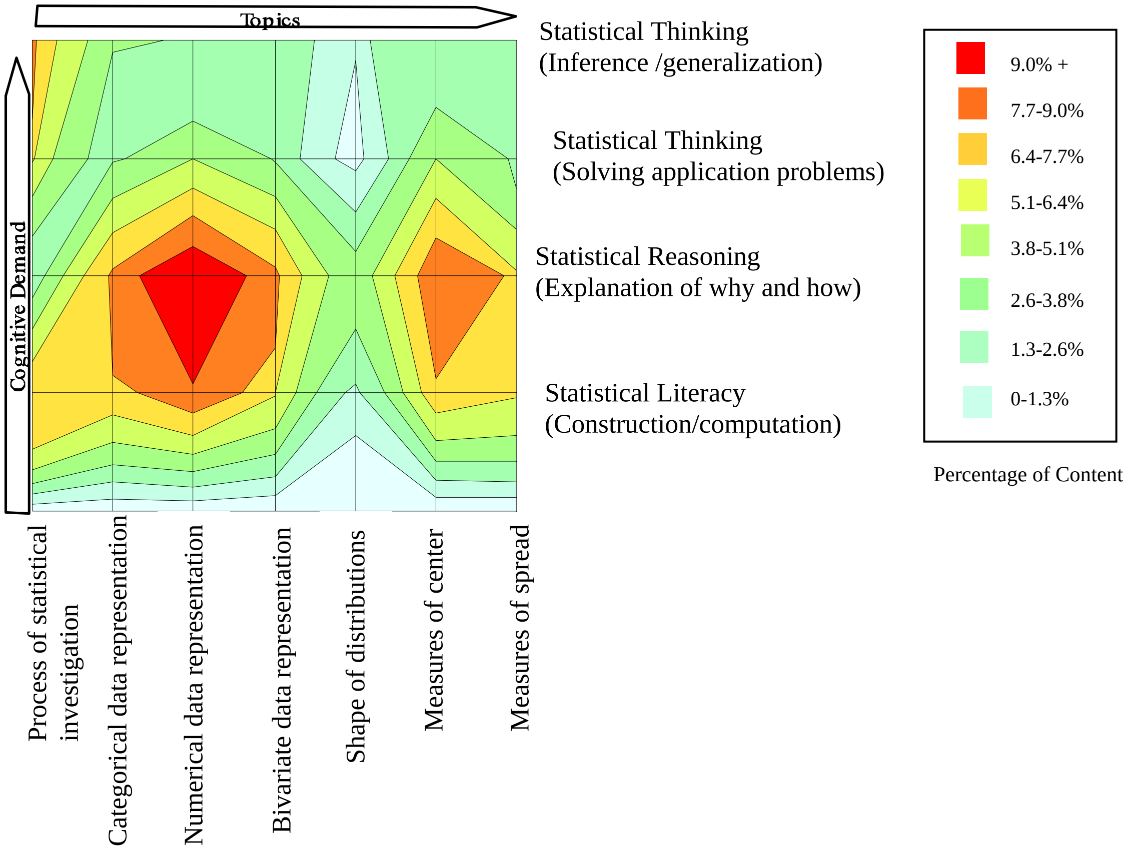

Figure 1: Content map for the ten states, the Mathematical and Problem-Solving Goals in *Connected Mathematics* Teacher’s Guide Grade 6 and 8 Textbooks and *Principles and Standards for School Mathematics* (NCTM, 2000)

Looking carefully at the individual state standards, it was found that they were surprisingly different. The variation was considerable in both the topics they each choose to cover and the level of cognitive demand. Figure 2 shows three different states, the first shows more uniformity across topics and levels but with emphasis on data representation; the second shows well defined clusters around categorical data representation and measures of center and spread at lower levels; and the third state shows emphasis at all levels for different topics. Furthermore, when the common content was sought (i.e., the intersection of content) it was found that it was almost empty. The only topic in common was the proper use of measures of center.

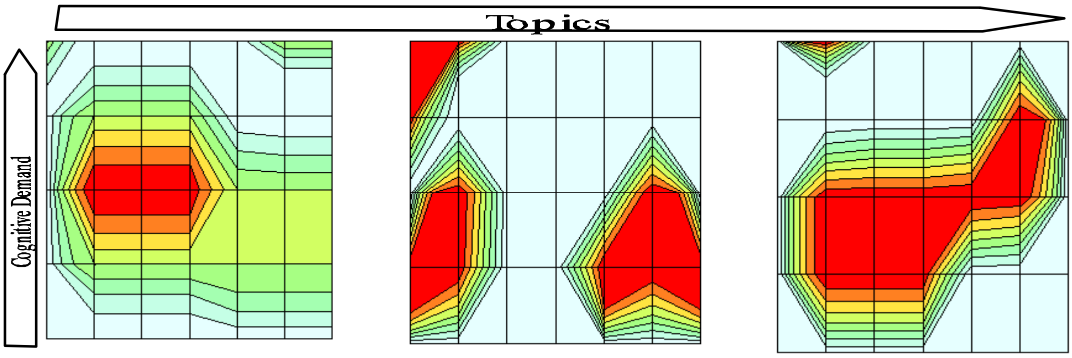

Figure 2: Content maps for three different states.

Documents related to teaching do not have the same structure as the documents related to content for students. Hence, their analysis was more challenging and less structured. Nevertheless, documents were classified into three different categories: those documents that provide general recommendations about teaching, those documents that suggest teachers' knowledge of mathematics, and those that suggest recommendations about specific content area within mathematics. The first category of documents suggests that teachers should have knowledge of the subject matter, pedagogy, students as learners, and assessment. The second category of documents suggests teaching tasks which required the use of mathematics, managing class discussions, establishing a classroom culture for mathematical reasoning, designing and selecting tasks, analyzing student thinking and work, planning instruction, and assessing student learning. Finally, the third category suggests more content specific recommendations such as the need of teachers to know how to respond to students who want to use an inappropriate type of graph or to know the advantages and limitations of physical models to introduce the concept of the mean as the "evened out" number. Table 1 shows a summary of these recommendations related to the content that showed more percentage of coverage in the content analysis. For more recommendations in a more generic form see (Sorto, 2004).

IMPLICATIONS

Several aspects of statistical content knowledge were identified as very important for middle grades teachers. The choice of aspects was based on a systematic integrated analysis of several documents such as students' state and national standards, which in part, are based on theoretical and empirical work on student learning. This approach of identification of content is of particular interest to those who are trying to make hard choices on what to include – or not to include – in curriculum guides and assessment for pre-service teachers and professional development developers for in-service teachers.

The aspects of statistical knowledge for teaching were not identified in the same fashion as the content. This kind of knowledge is much more complex and has too many dimensions to be analyzed the same way that students' content is. However, several pieces of work were examined and integrated: teachers' state and national standards, research and theoretical work on teachers' knowledge and its role in teaching, and teachers' guide to a statistics textbook. These documents provided a general framework to view teachers' knowledge as well as specific aspects that come from the actual practice of implementing curriculum. They are all important as we need the "big picture" to create vision in teacher preparation programs and the "little picture" to make it happen in the classroom and to create authentic assessment instruments. The identification of these aspects for teaching is a starting point for a discussion of what do middle grade teachers need to know about statistics in order to teach it well and a continuation of how to measure this knowledge in the other areas of mathematics.

Table 1: Summary of Recommendations about Knowledge for Teaching

	Representation of Data	Measures of Center
Students as learners	• Know how to create questions that middle school students can ask in order to collect data and might involve using a specific type of graph; • Know how to respond to students who want to use an inappropriate type of graph	• Understand how students are thinking about the data when they use different strategies and models to find the mean; • Respond to students who think that is impossible to have many data sets with the same mean, and • Anticipate students' answers or interpretation to an investigation question and be able to pose questions to students that lead them to see the effect of outliers or/and new data values have on the distribution and the mean.
Pedagogy	• Engage students in the exploration of data by having them suggest questions that might have originated the data and methods of collecting the data; • Lead students in the process of constructing a stem-and-leaf plot, and; • Pose questions that lead to "read the data", "read between the data" and "read beyond the data."	• Know the advantages and limitations of physical models to introduce the concept of the mean as the "evened out" number; • Know how to make connections between the physical model and the line plot; • Know how to create data sets with the same mean but different distribution using physical models; and • Lead students to the discovery of the algorithm of the mean and why it works.
Assessment	• Assess students' responses making judgments about their reasoning.	• Assess proper statistical reasoning for justifying students' strategies.

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