

UNDERSTANDING HIGH SCHOOL STUDENTS' IDEAS ABOUT PROBABILITY: SOME FINDINGS FROM FIJI

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This paper reports on data from a study which explored form five (14 to 16-year-olds) students' ideas in statistics (probability, descriptive statistics, graphical representations, investigations). This paper discusses the ways in which students made sense of probability tasks used individual interviews. The findings revealed that many of the students used strategies based on beliefs, prior experiences and intuitive strategies. Additionally, some students' interpretations of the question tasks were different than those intended by the interviewer. As a result, students constructed responses based on these unintended interpretations. While students showed more competence on the formal item, they were less competent on the question involving an everyday context. This inconsistency could be due to contextual or linguistic issues. The paper concludes by suggesting some implications for further research.

INTRODUCTION

Over the past 15 years, there has been a movement in many countries to include probability and statistics at every level in the mathematics curricula. In western countries such as Australia (Australian Education Council, 1991), New Zealand (Ministry of Education, 1992) and the United States (Shaughnessy and Zawojewski, 1999) these developments are reflected in official documents and in materials produced for teachers. In line with these moves, Fiji has also produced a new mathematics prescription at the primary level that places a stronger emphasis on statistics at this level (Fijian Ministry of Education, 1994). Despite its decade long presence in mathematics education, a number of research studies from different theoretical perspectives indicate that students tend to have intuitions which impede their learning of probability concepts. Some prevalent ways of thinking which inhibit the learning of probability include the following:

Equiprobability bias: Students who use this bias tend to assume that random events are equiprobable by nature. Hence, for instance, three fives or one five on three rolls of a die are viewed as equally likely events (Lecoutre, 1992).

Beliefs: Research shows that a number of children think that their results depend on a force, beyond their control, which determines the eventual outcome of an event. Sometimes this force is God or some other external force such as wind, other times wishing or pleasing (Amir and Williams, 1994; Truran, 1994).

Human Control: Research designed to explore children's ability to generalise the behaviour of random generators such as dice and spinners show that a number of children think that their results depend on how one throws or handles these different devices (Shaughnessy and Zawojewski, 1999; Truran, 1994).

Concerns about students' difficulties in statistical reasoning determined the focus of my study. Overall, the study was designed to investigate the ideas that form five students have about statistics and probability, and how they construct them.

OVERVIEW OF THE STUDY

The study took place in a co-educational secondary school in Fiji. The class consisted of 29 students aged 14 to 16 years. Fourteen students were chosen from the class and this constituted the research sample. The criteria for selection included gender and achievement.

To explore the full range of students' thinking, open-ended questions addressing probability and statistics constructs were selected and adapted from those used by other researchers. In the discussion below, findings relating to two of the probability items are discussed. Item 1A attempted to explore students' understanding of proportional reasoning in everyday setting and Item 1B was used to elicit students' ideas about comparing probabilities embedded in a formal setting. Responses demanded both numerical and qualitative descriptions.

- Item 1A: Black and white marble problem

Meena and Ronit have some marbles. Meena is 10 years old. In her box, there are 10 white marbles and 20 black ones. Ronit is only 8 years old. In her box there are 20 white marbles and 60 black ones. They play a game. The winner is the child who pulls out a white marble first. If both take out a white marble at the same time then no one is the winner and the game has to go on. Ronit claims that Meena has a greater chance of pulling out a white marble because she is older, and more clever. What is your opinion about this? Please explain your answer.

- Item 1B: Red and blue marble problem

Box A and Box B are filled with red and blue marbles as follows:

Box A	Box B
6 red	60 red
4 blue	40 blue

Each box is shaken. You want to get a blue marble, but you are only allowed to pick out one marble without looking. Which box should you choose? Please explain your answer.

Each student was interviewed individually by myself in a room away from the rest of the class. The interviews were tape recorded for analysis and data analysis was conducted using the transcripts which were read and re-read by myself. Analysis of the interviews indicated that the students used a variety of strategies for solving the problems. I created a simple four category rubric that could be helpful for describing research results relating to students' statistical conceptions, planning instruction and dissemination of findings to mathematics educators. The four categories in the rubric are: non-response, non-statistical, partial-statistical and statistical. The non-statistical responses were based on beliefs and experiences while the students using the partial-statistical responses applied rules and procedures inappropriately or referred to intuitive strategies. The term statistical is used in this paper for the appropriate responses. However, I am aware that students possess interpretations and representations which may be situation specific and hence these ideas have to be considered in their own right. A response categorized as "Statistical" simply means it is one that is usually accepted in standard mathematics text-books.

RESULTS AND DISCUSSION

For Item 1B, two students' comments amounted to non-responses. For example, student 3 said that she would choose Box B but explained that she had just guessed the answer. A minority of responses were statistical in nature. Two students in the study showed a firm grasp of the proportional reasoning concept on the first task and four on the second. For example, student 2 and student 12 were not only able to say that age did not matter in chance games, but were also able to work out the correct probabilities for Meena and Ronit. Additionally, four students were not only able to say that it did not matter which box with red and blue marbles one should choose but were also able to work out the correct probabilities for the two boxes.

In contrast, the majority of student responses were considered non or partial statistical. Prior experiences and intuitions figured prominently in the ideas of these students. In 11 cases, students did not use a statistical model but based their reasoning on their beliefs and personal experiences. Two common beliefs identified on the two tasks related to causality and outcomes being controlled. Four students tried to seek a cause for an action on 1A and two on Item 1B. The students said that because Meena was older, she had more chance of pulling out a white marble. Others claimed that Ronit had more chance of pulling a white marble because she had more whites in her bag.

A few students missed the point of the question by focussing on whether the game was fair. This is reflected in the comment made by student 26:

This is not a fair game; for this the game should be played by same aged people and there should be equal number of marbles in the box.

Another student interpreted the problem involving Ronit and Meena as a game involving competition and a winner.

Eh ... Meena should be fast, so hands will be fast so she would be able to take it out first and Ronit is 2 years younger than her so he will be not really fast.

It is clear that these students, instead of performing adequate proportional calculations, used their personal experiences to deal with the tasks. Student 6 used the control strategy for both the tasks. For Item 1B, the student offered the following explanation:

Now the marbles are in a box. It depends on hands which one it pulls.

There were nine partial-statistical responses across the two tasks, four of these being for the first question and five for the second. These students based their thinking on the unpredictability bias or applied rules for calculating probabilities inappropriately. Other students adapted the rule $[P(E) = n(E)/n(S)]$ for calculating probabilities. For instance, one student explained that probability for Meena was a half (10/20). This confusion could be due to students learning that 3:4 is the same as 3/4.

With respect to students' experiences, beliefs and learning, it is evident that other researchers have encountered similar factors. Amir and Williams (1994) note that children's reasoning appeared to be related to their religious, superstitious and causal beliefs. The results suggest that in any particular context provided in the classroom, students' individual learning is influenced to a certain extent by their prior experiences and beliefs. This may be problematic if students' prior experiences and beliefs conflict with the statistical concepts that teachers are trying to teach them. For instance, if students believe that games should be played by same aged people then they need help to overcome a reluctance to attend to proportionality information.

The findings indicate that in some cases the meaning intended by myself on the interview tasks was not that constructed by the students. As a result, students constructed responses based on these unintended interpretations. For instance, one student interpreted the problem involving Ronit and Meena as a game involving competition and a winner. It must be noted that the wording of this question completely permits this interpretation. Perhaps the student thought that the phrase "at the same time" meant that speed and action was part of the game.

The interview results show that although contexts may help students use prior knowledge, such situational knowledge is diverse and can also cause misinterpretations of the information. For instance, student 26's personalisation of the context brought in various interpretations of the task (Item 1A) and inconsistency in his explanations. Given how statistics is often taught through examples drawn from "real life," teachers need to exercise care in ensuring that this intended support apparatus is not counterproductive. This is particularly important in light of current curricula calls for pervasive use of contexts (Meyer, Dekker, and Querelle, 2001; Ministry of Education, 1992) and research showing the effects of contexts on students' ability to solve open ended tasks (Cooper and Dunne, 1997). For instance, the study by Cooper and Dunne found that some pupils have a greater facility in recognising whether they are being asked to play a 'school maths' game or an 'everyday life' game.

Conversely, in spite of the importance of relating classroom mathematics to the real world, the results of my research indicate that students frequently fail to connect the mathematics they learn at school with situations in which it is needed. For instance, while four students used a statistical model on Item 1B, only two did so on Item 1A. The results support claims made by Carraher and Schliemann (2002) and Lave (1991). Carraher and Schliemann were intrigued by the ways Brazilian street vendors did not employ school prescribed procedures but used alternative flexible strategies to solve problems. This discrepancy could also be due to the question statement. The absence of any mention of "random drawing" in Item IA could have caused misunderstandings and the students based their explanations on everyday experiences.

IMPLICATIONS FOR FURTHER RESEARCH

It must be acknowledged that the open-ended nature of the tasks and the lack of guidance given to students regarding what was required of them certainly influenced how students explained their understanding. The issues of language use are particularly more important for these students, who face schooling in a second language that is not spoken at home. Although the study provides some valuable insights into the kind of thinking that high school students use, the conclusions cannot claim generality because of a small sample. Some implications for future research are implied by the limitations of this study.

One direction for further research could be to replicate the present study and include a larger sample of students from different backgrounds so that conclusions can be generalised.

Secondly, if context is important for probabilistic reasoning, then one needs to consider several elements when designing tasks. First of all, researchers cannot make appropriate assessments without also having some knowledge about the range of embodied experiences in the real world of the learner. The interview results show that personalisation of the context can bring in multiple interpretations of tasks and possibly different kinds of abstractions. At this point it is not clear how a learner's understanding of the context contributes to his/her interpretation of context based data. There is a need to include more items using different contexts in order to explore students' conceptions of probability and related contexts in much more depth.

While two pupils answered Item 1A with reference to proportionality information, others used their prior knowledge about games. It would be interesting to see how easily (or whether) students who argued on the basis of prior knowledge on this question could be persuaded to argue purely on the basis of the proportional reasoning. Future research could incorporate this into the interview procedure to explore this issue in more depth.

Another implication relates to culture. Unlike Watson and Callingham (2003), none of the students in my study used ratio construct on Item 1B. One explanation for this could be the cultural context. Additionally, Watson and Callingham (2003) note that students in 'other cultural settings' may respond differently to their Australian counterparts, particularly to context-based items used in their studies. It would be interesting to determine how cultural practices influence conceptions of probabilistic reasoning.

Finally, the meaning intended by myself on the questions was not that constructed by the students. This misunderstanding could be due to question statements. One must be exceptionally careful when wording questions for students.

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