CURRICULUM DESIGN: TO ENHANCE STUDENTS' LEARNING

Li Jun and Zhao Xiaoping
East China Normal University, China
lijun@math.ecnu.edu.cn

China's Ministry of Education is pushing a reform to include statistics and probability in its national elementary and secondary curriculum. First, the history of introducing statistics and probability into school in China is examined in this paper. Then, the intentions and characters of the new National Curriculum Standards are summarized. Finally, we take the teaching of "average" as an example to illustrate what we could do to make statistics and probability seem coherent by better design of textbooks.

INTRODUCTION

The Chinese education system is a national system governed by the Ministry of Education (MOE). Prior to 1986, only one syllabus and one series of mathematics textbooks were used at the same time in all schools in Mainland China. In the 1990s, especially after 2000, the system was changed and some of the central control made less rigid. For instance, in 1992, Shanghai was allowed to have its own local official curriculum and to organize separate university entrance examinations for Shanghai senior high school students because of its superior economic condition and better teacher resources. After 2000, the textbook competition becomes strong and the textbook selection policy has been widely adopted. For mathematics teaching, approved by the National Teaching Material Authorization Committee, 6 series of primary school textbooks, 9 series of junior high school textbooks, and 6 series of senior high school textbooks are prepared for selection nationwide. In 2005, authorities of fourteen provinces, municipalities and autonomous organized their own separate university entrance examinations.

The education system in China is 6-3-3 system. Grade 1 through Grade 9 education is compulsory. Like many other countries in the world, China examined its school mathematics curriculum critically at the turn of the millennium and decided to start a curriculum reform after 2000. The MOE of China issued its new national curriculum standards recently: *The Standards of Mathematics Curriculum for Compulsory Education* (MOE, 2001) and *The Standards of Mathematics Curriculum for Full-time Senior High Schools* (MOE, 2003). By now, both of the two Standards have been put into experimentation for a few years.

THE HISTORY OF INTRODUCING STATISTICS AND PROBABILITY INTO SCHOOL IN CHINA

Influenced by the growing trend worldwide to introduce elements of statistics and probability into the school curriculum, the MOE is pushing the reform to include statistics and probability in its national school curriculum (MOE, 2001, 2003). Historically, though, this is not the first time that China has introduced statistics and probability into its school education.

In the 1940s, probability was introduced in an entire chapter in some senior high school Algebra textbooks used in China (Wei, 1979). The textbooks were Chinese translation or adaptation of Western textbooks. In 1949, the People's Republic of China was established. Due to political reason, the Soviet Union Model was imported. Almost all of the school textbooks used during that period of time were adapted from the Soviet Union's school textbooks. Probability was removed completely from our school curriculum simply because probability was not taught in the schools in Soviet Union.

The world-wide "new math" movement did not really occur in China during the 1960s and the 1970s. But during 1958-1963, some "reformed" textbooks were put into experiments in a small scale. The reformers intended to modernize the school's mathematics education. For example, a textbook named *Probability and Statistics* was prepared for senior high school students in Shanghai. Unfortunately, the textbook was written basically in a university textbook style and was not suitable for the younger students. Its theoretical requirements were excessively high, and practical applications were far from the daily life of the students. The experiment had to be ceased soon after.

Another rise of statistics and probability education occurred in 1978. Recovered from the 10 years' Culture Revolution, the government planed to develop education at all levels in a short time. The three ambitious syllabuses, which were issued in 1978, 1980 and 1982 arranged 10-16 total teaching hours in grade 9 to study descriptive statistics and 10-20 teaching hours in grade 12 to study classical probability. The concept of population and sample, frequency distribution and relative frequency distribution, cumulative frequency and cumulative relative frequency, mean, variance, standard deviation were included in junior curriculum. This time, the introduction of statistics succeeded but that of probability failed. Probability teaching was reduced as an optional topic (12 teaching hours) in national syllabus in 1983. In 1996 syllabus, probability was reintroduced again as a required content with 10 teaching hours. In 2000 syllabus, the teaching hours was increased to 12. Optional topics are not included in national university entrance examination, so until the late 1990s, most of Chinese students had no probability learning experiences at the secondary school level.

In 1992, Shanghai issued its own curriculum standards and arranged the teaching of probability following that of permutation and combination. Both the classical and the frequentist definition of probability, the probabilities of independent and exclusive events, the summation formula of probability, conditional probability, normal distribution and binomial distribution, estimation of population means by sampling were included. But in 1995, the topics such as the probabilities of independent and exclusive events were reduced to optional. In each of the 1998 and 1999 National University Entrance Examinations in Shanghai, only four out of 150 marks were allocated to probability.

It is fair to say that the position of teaching statistics and probability before 2000 was low. The teaching approach was theoretical and oriented toward numerical calculation. Without activity, data collection, and actual experiences of variance, the teaching of statistics and probability became very similar to the teaching of algebra. Students were unable to apply what they have learned in classrooms to probabilistic situations in daily life, and their misconceptions for statistics and probability were remained (Li, 2000). For the two most important streaming examinations in China, senior high school entrance examination and university entrance examination, only about 2%~3% of the total score was allocated to statistics and probability.

Since mid-1990s, the voices calling for major redesign of statistics and probability education have become stronger and stronger.

CURRENT SITUATION OF STATISTICS AND PROBABILITY EDUCATION IN CHINA

Major changes of statistics and probability teaching took place after 2000. Similar to many other countries, the main aim of enhancing statistics education in China is to develop public statistical literacy to meet the students' and future employers' needs. Concretely, the MOE outlined the following three components of statistical literacy (MOE, 2001):

1. Familiarity with using statistical thinking to deal with problems containing data.
2. Appreciating the role that statistics plays in decision making by going through the process of collecting, displaying, analyzing data, and making reasonable decisions.
3. Being able to critically read data resources, data analyses, and summarized information.

With the previous experiences of introducing statistics and probability into school, the new curriculum allocated much more teaching time to the topic in each stage of schooling and emphasized the process of collecting, displaying, analyzing of data in its teaching. The teaching contents are summarized as follows.

In grades 1-3, all students are expected to be able to compare, arrange and sort objects according to some criterion; have some experiences of collecting, recording, describing, and analyzing of data; learn to use pictograms, bar graphs and tables to display data and use mean to summarize a group of data; learn to obtain information from media (newspapers, magazines and television programs); explain reasons to other students based on data; identify deterministic phenomena and chancy phenomena; list all possible outcomes of simple experiments; understand that different events can happen with different chances and be able to describe chances qualitatively.

In grades 4-6, all students are expected to be involved in the process of collecting, organizing, describing, and analyzing of data; design simple questionnaires; learn to use bar graph, line graph, pie chart to display data appropriately; understand the meaning of mean, median and mode, and use them appropriately in different contexts; learn to read statistical tables and graphs appeared in media and know that graphs and data can sometimes be misleading; learn the concept of equal probability and fairness of a game; be able to calculate probabilities in simple situations and reversely, design a game to let an outcome has the given chance to happen.

In grades 7-9, all students are expected to develop techniques for organizing and displaying of data. They are required to learn construct pie charts, learn the concept of the weighted average, range, variance, frequency, relative frequency and frequency distribution. They are expected to use calculators to process data so they can pay more attention to learning to select appropriate measures, such as mean, median and mode, in given situations. They are expected to learn to distinguish a sample from a population, to understand the necessity of sampling, to see variations in sampling. Also, they are expected to use their knowledge to solve some simple practical problems. Both theoretical and experimental approaches to probability are used in this stage. All students are expected to learn to calculate probabilities by analysis of equally likely events and learn to estimate probabilities by long-run relative frequencies. They are also expected to go through the whole process of solving some practical problems, encouraged to give presentations to other students, and expected to become critical readers of reports based on data.

In grade 10, all students are expected to know the importance of sampling and learn some sampling methods such as simple random sampling, stratified sampling, and systematic sampling; learn to use mean and variance from samples to estimate those of the populations; learn to use stem-and-leaf plots, scatter plots, relative frequency distribution to display sample's data; use the least-squares linear regression technique to develop linear regression equations. The students are also expected to learn different definitions of probability (classical, frequentist, geometrical) and know about randomness and stability of frequency; know the law of addition for mutually exclusive events and be able to estimate probability with simulations.

It is evident that the new curriculum has made its great efforts on developing students' understanding of "big ideas" of statistics and probability. For example, randomness, variations, stability of frequency, estimate population parameters with sampling, estimate probability by simulations, and so on. Teachers are strongly recommended to provide sufficient opportunities for students to conduct activities and make use of new technology in teaching and learning.

DESIGN A COHERENT CURRICULUM: TAKE THE TEACHING OF "AVERAGE" AS AN EXAMPLE

Statistics is a methodological discipline and is often arranged in a method-oriented approach. However, there is a danger when statistics was taught in a "recipe book" manner. It could lead students to have an incoherent view of statistics and learn statistics by memorization without understanding.

Some concepts, such as "average," "probability," and "variance," are defined or mentioned several times in different grades. In order to make connections between old and new knowledge, teachers could start from the earlier descriptions of the concepts and lead the discussions to the new definitions. Now, we would like to take the teaching of "average" as an example to illustrate our plan of designing a coherent curriculum.

"Average" is usually a synonym for "arithmetic mean," but it also has several other common forms, such as geometric mean, harmonic mean, mode, and median. Mathematical expectation is also a kind of average.

In the first stage of learning (primary 1-3), students have learned natural numbers and their operations. We can give the children the following task.

Example 1. Here are five boys and five girls. Each boy and each girl have some pencils in their hands. The numbers of pencils are:

Boys: 2, 2, 6, 2, 3 Girls: 3, 3, 6, 3, 5

Do the girls have more pencils than the boys? Explain your reason.
Possible answers given by students could be:

a) Yes. The girls have 20 pencils in total, but the boys only have 15 in total.

b) Yes. 3 is greater than 2, 3 is greater than 2, 6 is equal to 6, 3 is greater than 2, 5 is greater than 3. Obviously, the girls have more pencils.
c) Yes, because most of the girls have three, while most of the boys only have two.
d) Yes. If collect all boys' pencils together and distribute them equally among the boys, then each boy would have 3 pencils. Do the same thing among the girls. Then each girl would have 4 pencils. So the girls have more.

The task is to compare two data sets. We think all the four explanations above are reasonable. Responses a) and b) are appropriate for the young children but could be questioned by teachers in the next stage (primary 4-6). The students who give response c) or d) seemed are trying to use a measure to represent a data set (mode and mean, respectively) in their comparisons and reflect a better understanding of "average."

In the second stage of learning (primary 4-6), a general formula of mean would be given. But we still could start our teaching from the discussion of the response d) mentioned above.

Where the answers "3" and "4" come from? Actually, the distribution could be recorded in this way:

$$(2+2+6+2+3)\div 5=3;$$

$$(3+3+6+3+5)\div 5=4.$$

Then the concept of mean and its visual representations would be introduced. In this stage, students have learned fractions. Tasks related to the real world or graphs and tables could be arranged. Ask students to find the daily temperatures in the past week from media and solve the following problem.

Example 2. What is the average temperature in the past week?

Since the students have learned how to use letters to represent numbers, a general formula of mean could be given:

$$\bar{x}=\frac{x_1+x_2+\cdots+x_n}{n} \tag{1}$$

In the third stage of learning (junior 1-3), weighted average could be introduced by the following example.

Example 3. Make a survey in your class to find what is the number of television sets owned by a household on average?

After the survey, suppose we have obtained the data as shown in Table 1.

Table 1: The number of television sets owned by a household

# of TV sets owned	0	1	2	3	>3
# of households	2	10	20	8	0

According to the formula (1), the mean can be calculated as

$$\bar{x}=\frac{\overbrace{0+0}^{2}+\overbrace{1+1+\cdots+1}^{10}+\overbrace{2+2+\cdots+2}^{20}+\overbrace{3+3+\cdots+3}^{8}}{2+10+20+8}=1.85$$

i.e., the students in our class have 1.85 television sets per household on average.
Actually, it could be simplified as

$$\bar{x}=\frac{0\times 2+1\times 10+2\times 20+3\times 8}{2+10+20+8}=1.85$$

Generally, it could be written as

$$\bar{x} = \frac{x_1 f_1 + x_2 f_2 + \cdots + x_k f_k}{f_1 + f_2 + \cdots + f_k}. \tag{2}$$

Let

$$p_i = \frac{f_i}{f_1 + f_2 + \cdots + f_k},\ i = 1, 2, \cdots, k.$$

We could introduce the concept of weighted mean and rewrite equation (2) to

$$\bar{x} = x_1 p_1 + x_2 p_2 + \cdots + x_k p_k. \tag{3}$$

Equation (3) agrees well with the mathematical expectation formula that will be taught in the fourth stage of learning (senior 1-3).

$$E(x) = \sum_{i=1}^{\infty} x_i p_i \text{ and } E(x) = \int_{-\infty}^{\infty} xp(x)dx$$

During the fourth stage, the students also learn to use mean and variance from samples to estimate those of the populations. Why does the method work? We could organize an activity like this:

Example 4. What's your height? Collect data on every student. Once the data collection is completed, each of the students was asked to select a random sample from the population with a suitable sample size, and work out the sample mean.

The students can easily find that sample means are various. But most of the sample means concentrate around the population mean. Only very few go far from the population mean. Therefore it is reasonable to use sample mean to estimate population mean.

Further, we could ask the students to estimate the population variance but this time we need to help the students to find that using the mean value of sample variances

$$s_m{}^2 = \frac{1}{m}\left(\sum_{i=1}^{m}\left(x_i - \bar{x}\right)^2\right)$$

is not as well as using the mean value of

$$s_{m-1}{}^2 = \frac{1}{m-1}\left(\sum_{i=1}^{m}\left(x_i - \bar{x}\right)^2\right)$$

Overall, we try to provide more opportunities for the students to start learning from their prior knowledge and actual experiences.

DISCUSSION

The new curriculum in China is designed in a spiral way. Probability and statistics teaching is also arranged in many years of schooling. But we believe that the problem how to make connections between related conceptions taught in different years is not resolved yet. Students at different ages have different knowledge backgrounds and cognitive abilities. If textbook writers are aware of the available research information of students' learning and make use of the knowledge in their design, it might change students' incoherent view of statistics and probability.

REFERENCES

Li, J. (2000). *Chinese Students' Understanding Of Probability*. Unpublished doctoral dissertation. National Institute of Education, Nanyang Technological University, Singapore, http://www.stat.auckland.ac.nz/~iase/publications/dissertations/dissertations.php)

Ministry of Education of China. (2001). The standards of mathematics curriculum for compulsory education (Trial Draft). (In Chinese). Beijing: Beijing Normal University.

Ministry of Education of China. (2003). The standards of mathematics curriculum for full-time senior high schools (Trial Draft). (In Chinese). Beijing: People's Education Press.

Wei, Z. (1979). Some suggestions to the teaching of probability in high school. *Mathematics Teaching.* (In Chinese), 2, 4-5.