

MEANINGS' CONSTRUCTION ABOUT SAMPLING DISTRIBUTIONS IN A DYNAMIC STATISTICS ENVIRONMENT

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This paper presents an analysis of the meanings of sampling distribution as supplied by some undergraduate students in a dynamic statistics environment (Fathom). The paper identifies stages in the simulation process where multiple and dynamic representations were crucial to students' understanding of the relationships among sample size, the behavior of sampling distributions and the probabilities of some sample results. One of the foremost difficulties observed in the simulation process was linked to the use of symbolic representations in the software, mainly at the formulation of the population model stage.

INTRODUCTION

Sampling distribution concept is a key in studying statistical inference; nevertheless, theoretical-probabilistic approach does not always supply the conceptual elements that students require to understand the logic and methods that underlie this concept. To understand it, it is necessary to relate properly concepts like sample, population, distribution, variability and sampling (Chance, delMas and Garfield, 2004). A teaching alternative to this is supplied by statistical dynamic environments (v. gr., *Fathom*). In these environments it is possible to visualize sampling processes and to participate actively in the process of constructing a sampling distribution.

Research works made by Lipson (2002), Saldanha and Thompson (2003) and Meletiou-Mavrotheris (2004), suggest that a dynamic and multiple representations of an environment, even if not free from difficulties, can help students comprehend and apply the concept of sampling distributions in their reasoning about inference. However, a lot of research is needed to be done in order to know how students build their meanings about that concept in computer environments.

In our research, we have specifically posed the questions: What is the effect of dynamical statistics environment on the meanings that undergraduate students attribute to sampling distributions? Do students' meanings evolve as a result of working in such an environment?

In order to carry out the study, we selected *Fathom* (Finzer, 2002) as a representative software of dynamic statistics environment. This software allows the visualization of the whole process of building sampling distributions and of linking the concepts involved through multiple simultaneous representations (graphs, tables, and formulae). So, a corresponding change in a datum or parameter in a representation can be visualized immediately in the other representations.

THEORETICAL FRAMEWORK

The theoretical framework is based on two components: The first one is related to the meaning and understanding of mathematical objects as proposed by Godino and Batanero (1994; 1998). The second is about the use of computers as cognitive tools, seen as capable of generating structural changes in students' mental activity when exploring concepts and problem solutions (Dörfler, 1993; Pea, 1987).

In Godino and Batanero model (1994; 1998), the meaning is conceived as a practical system used by people to solve series of problems where the concept arises. This system can be observed by means of various elements involved in mathematical activity developed by people in solving problems.

There are five meaning components:

1. *Problem-situations*: This refers to a set of problems or situations where the under study concept arises.
2. *Language*: This is any verbal or written representation used to describe or refer to concepts and features involved in a problem.

3. *Actions*: These are the procedures or strategies used in solving problems.
4. *Concepts and properties*: These are concepts, properties and their relationships with another concepts involved when solving a problem.
5. *Argumentations*: They are chains of propositions used to convince others of the validity of solutions to the problems or the truth of the properties related to the concepts.

From this point of view, the understanding of a mathematical object consists of a continuous and progressive process where students acquire and connect different elements involved in the meaning of a concept.

Regarding to cognitive tools, Pea (1987; p. 91) defines them as “any environment that helps mind to go beyond its limits in thinking, in learning and in solving problems activities.” The cognitive tools constitute, particularly in the case of computers, extraordinary and powerful instruments useful in learning to think mathematically and to operate not only with numbers, but also with symbols. They also allow the storage and manipulation of symbols dynamically and the interaction with users in actual time. From Pea’s point of view (1987), intelligence is not a trait of mind alone but also a product of relationships among mental structures and intellectual tools supplied by culture.

Based on Pea’s ideas, Dörfler (1993) identifies various forms in which insertion of a computational tool in teaching is capable of causing reorganization in students’ cognitive system:

1. Change of activities at a higher level (meta-level)
2. Change of objects to execute activities
3. Focused activities in transformation and representations’ analyses
4. Support situated cognition and problem solving

THE STUDY

The study was organized around nine activities some of which relate to a particular type of population distribution such as binomial, uniform, normal, and irregular. Data were collected over a period of two months during a workshop of 20 sessions each of which was 1.5 hours.

The teacher played a role of a leader by reading the activities’ texts and assisting the students only when it was absolutely necessary. Some activities were executed individually, others were executed in pairs. Physical simulations of the first three activities were carried out before the computational simulations..

The respondents are 11 students who were taking a course in Statistics Inference in the *Instituto Politécnico Nacional*, Mexico City. They had already studied the topic- Sampling Distribution, earlier.

The instruments for gathering data were two questionnaires- pre and post test activities, answer sheets and diskettes containing the work done by each student. Another set of data were collected, towards the end of the workshop through interviews.

The concepts investigated are:

- Sampling variability
- Sample size effect on the form, center and variability of a sampling distribution (central limit theorem);
- Estimation of probabilities of sample outcomes, and
- Sample size effect on the values of those probabilities.

RESULTS AND DISCUSSION

In every stage of the simulation process, students brought into play different meanings components and established relationships among them. According to the theoretical model that guides this investigation, the meaning of a concept is made up of the five components earlier mentioned. Next, the most representative components are described and some episodes that show parts of the meaning built by students.

- *Problem-situations*: All situations posed to students belong to a field of sampling distributions, the aim of which is to calculate probabilities of some sample values through a deductive process in which the population distribution and its parameters are supposedly known.

- *Language (Representations)*: Graphical and numerical representations were undoubtedly the most used meaning components as against the symbolic. Along the simulation process, students established relationships among population, sample and sampling distribution through various graphical representations such as histograms, data tables, tables with descriptive measures, formulae to calculate sample statistics and proportions of sample results. An example is shown in Figure 1, which illustrates the use of different representations of the population and a sample made by Mónica, at the first stage of the activity.

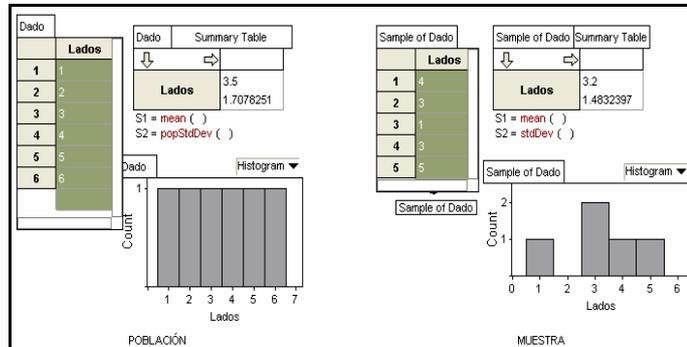


Figure 1: Simultaneous representation of populations and samples

The relationship between the population and samples could be visualized dynamically and simultaneously during sampling because of the multiple representations feature of *Fathom*. This helped Mónica to get adequate information on sample variability and to correct some misunderstandings shown in the diagnostic questionnaire.

Another representation used by most of the students in activities that involved calculating probabilities of sampling outcomes, was linking a table with a sampling distribution graphic, filtering the outcomes of interest and, in some cases, shading the corresponding area, as Monica and Coral did on an activity they worked together (Figure 2). Such a representation relates numerical, graphical and symbolic aspects.

- *Actions and procedures*: In dynamic statistics environment, students focused their actions and procedures in building sampling distributions by defining the population, drawing out samples repeatedly, and calculating statistical data from each sample in order to form the distribution and calculate proportions (empirical probabilities) of certain sample outcomes. These, obviously, with some difficulties that were gradually eliminated as they “took possession” of the software.

The hardest stage of the study for the students was the formulation of the population model because they needed to identify outstanding features of the population, such as the type of variable and the values of parameters. Usually their actions and strategies at formulation stage were not guided in most of the activities. As an example, one activity was about a binomial population in which there were a 30% of defective bearings. Most of the students considered both defective and non-defective events have the same probability. Only two students established the model without help. This happened in most of the activities.

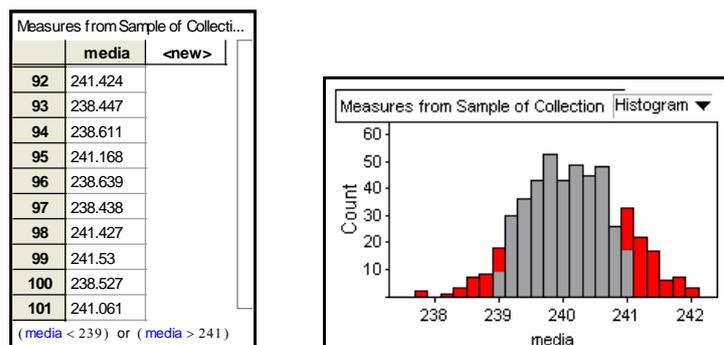


Figure 2: Representations used by Monica and Coral in an activity

- *Concepts and properties:* Different concepts and properties were brought into play in each stage, such as sample variability, the effect of sample size in the behavior of sampling distributions and the calculation of probabilities of sample results. For instance, in studying the effect of central limit theorem on the behavior of sampling distributions, an emphasis was placed on building sampling distributions for different sample sizes and making comparisons among them. Monica, in an interview with the researcher, made a graphical representation (see Figure 3) that was essential in understanding the properties of sampling distributions as well as in interpreting the effects of sample size on probabilities.

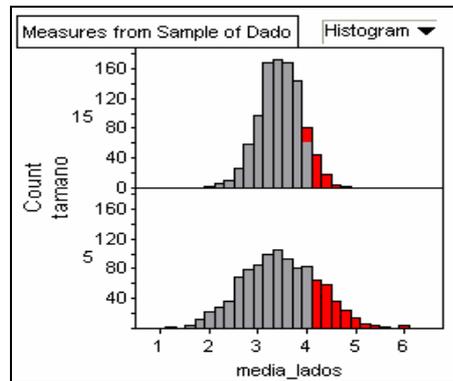


Figure 3: Sampling distributions for $n=5$ and $n=10$.

Part of the interview between the Researcher (R) and Mónica (M) is as follows:

R: *What happens with the mean of the sampling distribution when compared to the mean of the population?*

M: *Usually the mean tends towards the population mean.*

R: *What happens with the standard deviation?*

M: *The standard deviation varies; it becomes smaller as sample size increases.*

In calculating and interpreting probabilities, Monica compared a table of values of means with the histogram and used filter option to shade interesting portions.

R: *In which of both distributions is more likely to appear a mean equal to or greater than 4?*

M: *In the distribution which has a sample size of 5.*

R: *Why?*

M: *Because we have many values towards the right hand side of 4.*

R: *If it were necessary to bet; to what values would you bet?*

M: *Between 3 and 4.*

R: *What values would you choose if you want to increase your probability of winning?*

M: *I would choose between 2 and 5.*

Another aspect we were interested in was to investigate the interpretation that students give about sampling distributions and their outcomes. For instance, in an interview with Omar, during an activity involving a population with 30% of defective bearings, he answered as follows:

R: *What does the sampling distribution graph illustrate or how would you describe it?*

O: *Well, this graph contains all the defective bearings that appeared in every sample.*

R: *Between what values are the most of defective bearings found in the graph?*

O: *Between 2 and 4*

R: *Which are the values with less defective bearings?*

O: *8, 9 and 0*

It can be seen that Omar has no difficulties in interpreting the outcomes of a sampling distribution because he pointed out that all defective bearing were part of the information represented in the graph. He explained further about the most probable values. This was not however the case with other students, who could not interpret their graphs.

In continuation of the researcher's interview with Monica, she was asked to explain the relationships that exist among population, sample size and sampling distribution

R: Do you think there are some relationships between population and the sampling distribution?
 M: Yes; the population is the values of the dice and the sampling distribution contains the means of the samples.
 R: Then, isn't there any reason for them to look alike?
 M: I think not. The values in the population are original while those in the sampling distribution are already processed values.
 R: If there were a variable population like a swing instead of a uniform population like this, do you think that there would be a repercussion on the form of the sampling distribution?
 M: No; usually the form of sampling distributions are normal distributions.
 R: Is that true always, regardless of the form of the population?
 M: Yes.

As could be observed, in a dynamic environment, Monica and Omar were able to:

- Create fundamental notions concerning sampling distributions theory and sample variability.
- Establish the relationship between the sample size and the form of sampling distribution.
- Establish that the sampling distribution converges towards the normal distribution.
- Identify the effects of sample size on the distributions variability and on probability of sample results.

Nevertheless, many students were not able to adequately bring to play, the different meaning components and in post-test they showed difficulties in understanding the studied relationships and their features. This is could be seen in the Table 1 that shows students' answers to the pre and post tests.

Table 1: Showing students' answers to the pre and post tests

<i>CONCEPT OR FEATURE SEARCHED</i>	<i>PRE-TEST (RIGHT ANSWERS)</i>	<i>POST TEST (RIGHT ANSWERS)</i>
Variability of a data set in a graphic	3	5
Sample variability	5	5
Effect of sample size on variability of a sample distribution	3	7
	2	4
Mean of sample distribution centered on population mean	4	9
Difference between sample mean and population mean can be due to sample variability	3	9
	4	9
Sample size effect on a sample outcome probability	1	8
Sample size effect on sampling distribution shape	2	5

- *Argumentations and validations:* One of the merits of dynamic environments in statistics is that it provides students with a control mechanism while solving a statistical problem. For instance, the formulae inspector shows syntax mistakes in formulas, a shaded area to confirm that they are calculating the required proportion, and a feature to adjust empirical distribution to theoretical distribution. Furthermore, the results obtained through the dynamic environments can be used to compare the accuracy or otherwise of the theoretical results.

CONCLUSIONS

It could be observed that the actions and procedures used by students in the dynamic environment to solve the problems were different from those that are normally used in a theoretical approach.

Also, some crucial stages in theoretical approach such as the standardization and the use of probability, which many students usually use as a routine while solving problems with the attendant mistakes the often make were not required in the dynamic or simulation approach.

The different representations, the dynamical links among them and the immediate feedback offered by the software when some data or parameters are changed, had an important

effect on the students' meanings. This was the case of Monica and Omar, who could construct fundamental notions concerning sampling distributions theory: sample variability, the convergence of sampling distribution towards a normal distribution and the influence of sample size on the variability and on the probability of sample results.

Students got involved in the management of different concepts and their relationships while still focusing on the processes as well as on the final result, in contrast to the theoretical probabilities which is but a routine.

Finally, the resources provided by the software to validate their results helps students to detect their mistakes during the simulation process. The resources also permitted them to solve many problems or many stages in a problem independently of the teacher.

Students' meanings were evolving as the study progressed, refining gradually their strategies and centering their attention on the conceptual aspects of problems. The post test results showed an increase in questions answered correctly, not only in the number of right answers but also in quality of such answers. They made more scientific statements as the process progressed than in the diagnostic examination.

In conclusion, the results suggest that students need some learning experiences that permit them to explore concepts like sampling distributions and their properties, the central limit theorem and the effect of sample size on probability of a certain sample outcome in dynamical statistics environment, supported by multiple representations before they are initiated into studying statistical inference.

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