

## **USING EXPERIMENTAL APPROACHES FOR TEACHING PROBABILITY: WORKING ON A PROJECT USING FACE TO FACE AND VIRTUAL SESSIONS**

Cileda de Queiroz e Silva Coutinho  
Pontifícia Universidade Católica de São Paulo, Brasil  
cileda@pucsp.br

*In this paper we discuss the teacher's role in the introduction of probability to students aged from 11 to 18 years old. Coutinho (2001) has shown that model-building approaches enable students to attend to the duality of the probability concept. However, Gonçalves (2004) argues that teachers do not easily appropriate such teaching situations, because their conceptions are associated with their own practices, built from classic approaches to probability. In this paper, we discuss a teacher education project on teaching and learning probability problems, in which teachers and researchers collaborate during face-to-face and virtual sessions, to reflect upon and about teaching practices and especially about the possibilities associated with a model-building approach to probability.*

### **INTRODUCTION**

The construction of the concept of probability has been part of our concern for more than ten years. Our researches and reflections direct us to pre-service and in-service training teachers of Mathematics. Starting from the study of that concept construction process for 14-18 year-old students, we could notice that teachers didn't feel comfortable when working on this field of Mathematics. In Coutinho (2001) we worked with the hypothesis that a first contact with probability and, consequently, a first contact with randomness, can be favored by the use of experimental activities proposed in a modeling context. That is, proposed activities aiming to interpreting and modeling real situations. Such situations hold a high level of complexity due to the intervention of chance, being therefore necessary to simplify it for a probabilistic apprehension. Let us mention an example of that apprehension type: a dice thrown on a table represents the manipulation of a generator of the chance, inside a random experiment. Such experiment needs to be well defined so that we can simplify the situation complexity. By defining the goal of the observation we give a first step in that direction. We can choose that the course of that dice on the table surface does not interest us, nor the number of times that it rotates on itself before immobilizing. We were just interested on observing the superior face after immobilization. That choice already allows us a probabilistic glance: among the six possible results, which will come? It is that probabilistic glance that allows the topic to have a meaning? This way, the need to unchain a simplification and an abstraction process of real objects is what leads to the modeling context aforementioned.

In this process, we used the probabilistic context of the situations that can be acted and interpreted by the Binomial model (Bernoulli's Urn), but with the premise that the students of that age group do not necessarily possess the mathematical knowledge in order to conclude the process: to present the model that was chosen to represent and to interpret the real situation we want to analyze. In other words, we have worked with the premise that, in this school level (Junior High School) student still does not know what is neither a random variable, nor what a function of probability would be and, consequently, he does not know about probabilistic models. Our choice is based on the fact that the Binomial Model is the least complex for a first familiarization with the process of probabilistic modeling. According to Raymond (1997):

most of the discreet laws can be introduced starting from models of urns. These models have the advantage of its simplicity and they are rendered easily to a practical accomplishment. They model situations found in practice very well (...). (p. 251)

The consequence of that lack of mathematical tools to complete the modeling process is that the passage from domain of reality to theoretical domain cannot be ended. The mathematical knowledge required for students to be engaged in this process holds a low level of complexity: number knowledge, fraction, proportionality, basic knowledge of the language of sets. This lack of formal knowledge can become source of difficulties for the students engaged in that process,

source of difficulties for the meaning attribution for the solution of the proposed problem. According to Girard and Parzysz (1998):

in general, we do not say to the students that we are in a model when we are using mathematics to find the solution to a concrete problem. It is however important to know that we are not any more in reality. (p. 576)

In order to minimize this didactic difficulty, students can work in an intermediate domain, in which all elements are already an abstraction of reality, they are idealized objects though, which are designated by the same names (by the same labels) used for the objects of the reality that these models represent. In this intermediate domain the elements are still contextualized and it can be noticed and worked by students: “(...) in a current vocabulary, the objects of the model are endowed with well defined characteristic properties.” (Henry, 1997, p. 79). We will call it Pseudo-Concrete Domain. Teachers, in classroom practice, usually get their students to work in that domain, even if they are not aware of that or they have not planned this kind of work.

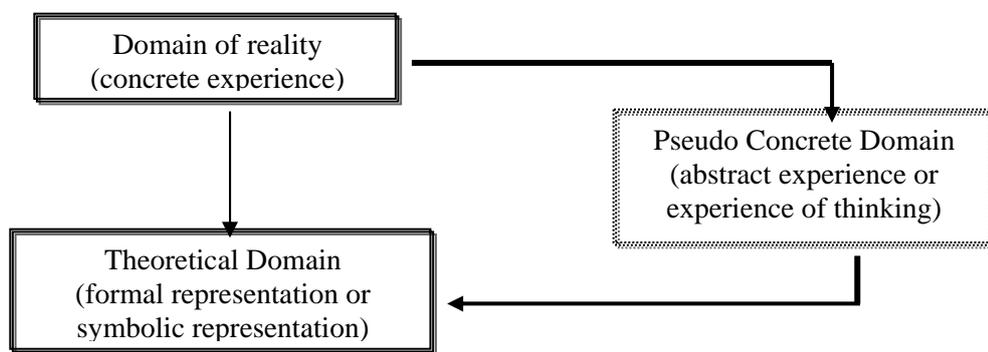


Figure 1

According to Raymondaud (1997):

the urn model describe in pseudo-concrete terms certain random experiences composed by one or more accomplishments, in which the possible results are summarized in a certain number of qualitative characteristics. (p. 251)

In the case of the Bernoulli’s Urn, these results can be classified in two classes that present a dichotomy, success and failure.

Let us return to the modeling process in order to understand better the work into Pseudo-Concrete Domain. We can verify the existence of a specific stage in this process: the student is not working in the domain of the reality anymore when he just manipulates mental objects (in a certain way, idealized). In the case of random situations, the student should abstract the concrete experience (a game of dice, for instance) to represent it by the appropriate probabilistic model: the fictitious game with a perfect dice (perfect cubes, but that the student continues to designate as “dices,” in reference to the concrete object that was effectively manipulated).

The modeling process begins this way with the observation and description of reality: a description guided by a glance that it is already filtered by a theoretical knowledge, which immediate consequence is the interpretation of perceptive properties of the situation for a local description of this reality in ideal terms. The student is still in the domain of the reality, but s/he is already in the limits of a sub-domain to which this characteristic group and local properties belong. It is a first step for the abstraction. The activities proposed to students aged 14-15 years old were inspired in that process, in that possibility to work on the Pseudo-Concrete Domain, aiming to introduce them to the first steps of a modeling process in a probabilistic context, illustrated in the Figure 2:

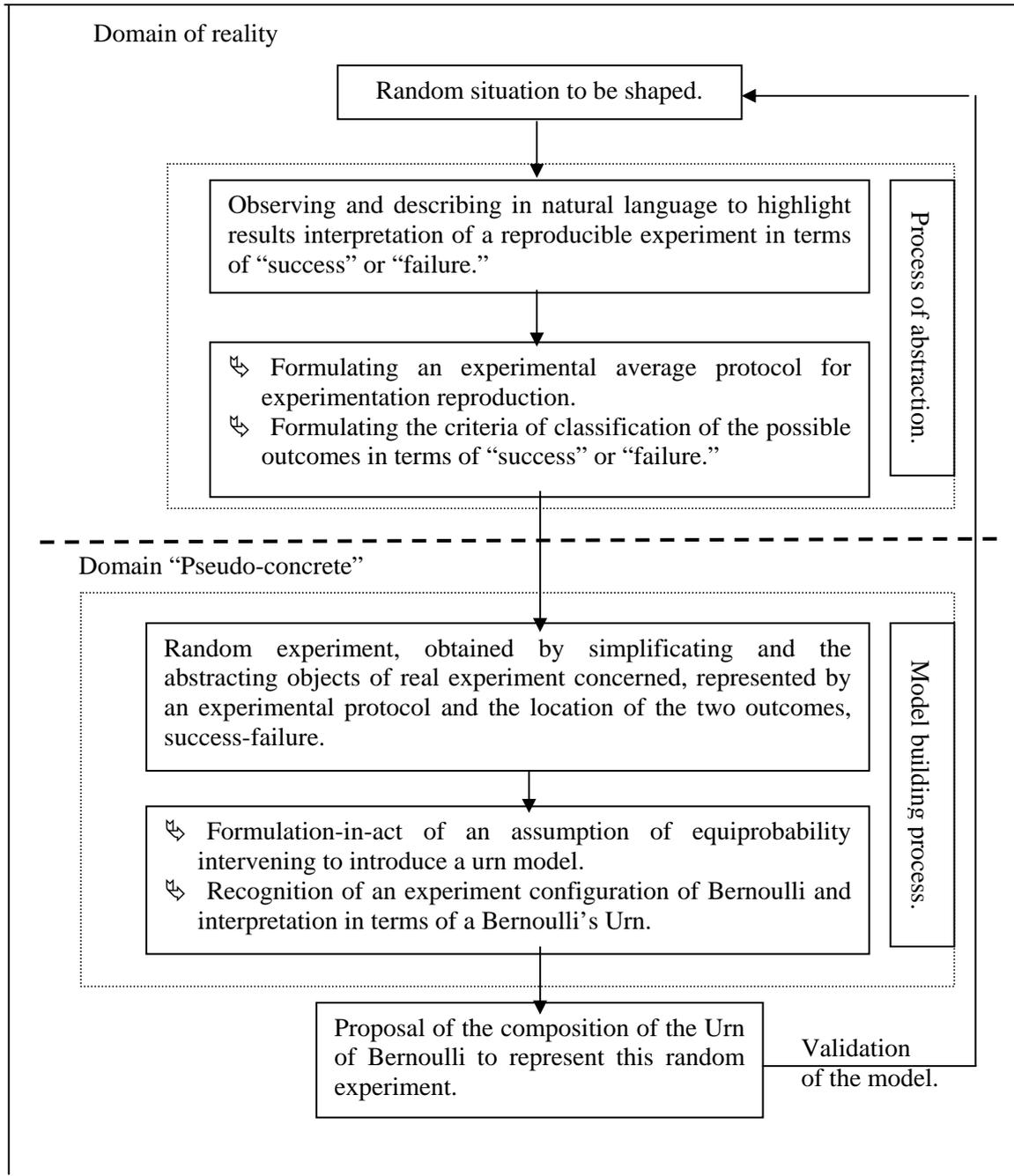


Figure 2

Coutinho (2001) has made a didactic choice: students would have to effectively accomplish the experiments that they wanted to model. For instance, if they wanted to build a model representing the distribution of the colors of the cars that were parked in a certain parking lot at a certain moment, they would have to undergo the experience: *“to observe cars that are leaving the parking and to note their color.”* This activity type is not part of the teachers' repertoire. They are not used to this activity type and some researches show that what the teacher thinks about teaching determines what s/he does in practice. According to Borovcnik and Bentz (1991), it is very important that the work with primary intuitions be able to influence the work with the “official mathematics,” and they show that a conventional teaching establishes few relations between these two aspects (intuition and formal knowledge):

This is critical for probability as there are no direct experiences which help learners to establish these links on their own. But, if learners are to understand mathematics and apply it to other situations, teaching has to start from the learners' intuitions and to change them gradually. The learner needs to see how mathematics will reconstruct his/her intuitions. (p. 75)

#### TEACHER'S CONCEPTIONS ON PROBABILITY

Even if the results obtained in Coutinho (2001) encourage the work with concepts of probability using experimental focus and modeling process, Gonçalves (2004) shows that the probabilistic conceptions identified in Mathematics teachers correspond to that categorization done by Goded (1996, in Gonçalves, 2004) when studying teachers of the initial series. It means that the identified conceptions coincide with both generalist teachers and teachers that study mathematics to teach in the final grades of the Junior High and in the High School. These categories are:

- *Non Probabilistic of Reality* that is characterized by the absence of the understanding of chance and of random success. The answers are based on beliefs, with determinist models of reasoning and its explanations are based on the occurrence of simple and immediate successes.
- *Intuitive Probabilistic* that it is characterized by the presence of some understanding of chance and its connection with random successes, however, partially and closely to the concrete models. The heuristic judgments are fundamental in the outline of different situations.
- *Emerging Probabilistic* that it is characterized by the acceptance and understanding of the multiple mathematical representations of chance. There is an understanding of some probabilistic models and certain capacity of application in certain cases, the most common ones. This conception suggests the presence of some probability and statistics instruction, although it is an initial character.
- *Normative Probabilistic* that it is characterized by the deep understanding of probabilistic models and their application in various situations. They present abilities to compare and to contrast different random situations, according to different models. (Goded, 1996 in Gonçalves, 2004, p. 108)

It can be inferred that teacher, in order to work with the concept of probability in its duality of focuses and to use the experimental approach with modeling process needs to mobilize a conception type at least Emerging Probabilistic or preferentially Normative Probabilistic. In other words, those identified teachers who have conceptions in the first two categories would need a professional formation that would develop them to the last mentioned category. Such formation should discuss theoretical aspects, because we can also observe that most of the surveyed teachers in the area do not feel comfortable with working on that content. According to a research accomplished in the second semester of 2005 by students of Professional Master's degree course in the Program of Postgraduate Studies in Mathematical Education of PUC-SP, among 292 surveyed teachers, more than 90% allege to have studied Statistics in his/her initial formation, but approximately 50% of these 292 teachers allege not to work these contents for lack of self-confidence. According to Showers, Joyce and Bennett (1987):

what the teacher thinks about teaching determines what the teacher does when teaching. In training teachers, therefore, we must provide more than "going through the motions" of teaching. (p. 79)

In the same page, these authors remarks that:

almost all teachers can take useful information back to their classrooms when training includes four parts: (1) presentation of theory, (2) demonstration of the new strategy, (3) initial practice in the workshop, and (4) prompt feedback about their efforts. (p. 79)

We can then detach the importance of the training design to be offered to in-service or pre-service teachers, so that students can attribute one meaning to probabilistic concepts they had construct, what is also confirmed by Showers, Joyce and Bennett aforementioned in this article.

### THE PROJECT: USING VIRTUAL SESSIONS

The project in which we developed our research joins teachers of Junior High and High School of public schools in the State of São Paulo, Brazil. Those teachers have already been participating in that group, working with other themes of their classroom practice. They proposed to work with Probability, alleging to have difficulties for preparing instruction because of the gaps in their probabilistic knowledge. These teachers were also participants in the diagnosis instrument applied by Gonçalves (2004), whose probabilistic conceptions follow the identified categories previously mentioned in that text. At a first moment, we tried to familiarize those teachers with the virtual tool “forum” organizing a debate on what their understanding of random would be. During the period they worked in this virtual forum, the “face to face” meetings treated the theme “Fractions” and the teachers were constantly motivated to participate in the virtual forum. Our objective was to turn it a supportive tool for the meetings. The discussion about the perception of chance was planned because it would not demand, in a first moment, a larger investment of the teachers’ time, and because perception is the starting point for the work with the concept of probability, according to Coutinho (2001).

We intended to discuss that, in a way, perception also introduces the idea of random experiment, detaching the three basic characteristics (reproducibility, chance interference and possibility to identify all possibilities of results). According to the stages proposed by Showers, Joyce and Bennett (1997), we have chosen to work firstly with the theoretical aspects of the concept of probability so that, in the sequence, we would be able to approach the new focus for educational work: the experimental focus and the use of the modeling process as didactic resource. Those teachers’ testimonials were fundamental to our choice, once they alleged not to work with this content because of lack of self-confidence on their own knowledge. In this sense, it was proposed to allow students not only to investigate and to distinguish ways to solve the proposed questions, but also to acquire new knowledge and strategies of problems solving, that is: to aid student and teacher in the construction of knowledge in a constructive way and to develop certain abilities, for instance, to know how to read, to interpret and to use mathematical representation.

In order to identify beliefs and integral conceptions of the teachers who take part of the continuous formation project to which we are linked, we have put a discussion in an Internet forum, which theme was Chance. Some questions were proposed to us in order to open the debate, and among them we worked with the following: “What’s your idea of randomness?” Although we tried to get them to explain the differences they talked about, teachers expresses opinions in a very synthetic way and, in many times, just reproducing texts of textbooks they use and it prevented us to identify the type of belief that was behind the speech.

In the sequence of the discussion, we tried to identify differences among randomness and randomness experience, and after some discussions, we have come to the consensus that randomness is a component of an experience or of a phenomenon that brings as a consequence of its existence an uncertainty degree. In other words, there is an impossibility of knowing the result of that experiment or phenomenon accurately. When we proposed the link of that with probability, there was also consensus about the possibility of this theme to already be approached in the 5th grade of Junior High (11-12 years). Such consensus is in agreement with the proposals that can be found in the National Curricular Parameters (PCN) for the Junior High. According to Coutinho (2003), at the end of the first two cycles, the teacher can already work with the concepts below, that we presented in a kind of net that allows the introduction to the concept of probability.

Most of the time, the teachers who have participated in our forum were limited to placements that could find in textbooks. That hindered the debate on randomness, since that aspect is little approached in such books. Books define randomness experiment just to identify the events, without a discussion on how to identify the intervention of chance in these experiments. Not even a discussion about the distinction among result (concrete experience, domain of the reality), and the result of the outcome (I model abstract of the concrete experience, already in the

pseudo-concrete domain) and still the distinction as for the term event, that it is a subset of the sample space that allows the accomplishment of such or such result.

## CONCLUSION

In this paper, we have tried to raise the discussion about the teachers' continuous formation and the perception of the chance when this teacher is engaged in a teaching-learning process involving stochastic reasoning. Previous studies show that students possess, already in the third cycle of the Junior High (fifth and sixth year of education), the necessary readiness for a formalization of these concepts. The study of textbooks, that was part of these researches, shows that several authors already present whole chapters dedicated to the study of the combinatory and probabilistic thinking, and in the initial graded the focus is the work with the students' spontaneous conceptions on the ideas of chance, possibilities.

It can be noticed, however, that both PCN and the analyzed textbooks are insufficient in the teacher's instrumentation to work with those concepts in a way to guide the students through a significant learning of the basic probabilistic concepts. This way we can realize the need of a specific educational formation for the work with randomness, with experimental focus on the introduction of the concept of probability, in such a way that teacher acquires autonomy to formulate and execute a mathematical and didactical organization of that content aiming the Junior High School and our proposal considers the use of e-learning tools in order to make a more closely attendance possible, in the phase of instruction planning as well as in the phase of execution of that planning and discussion of their results.

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