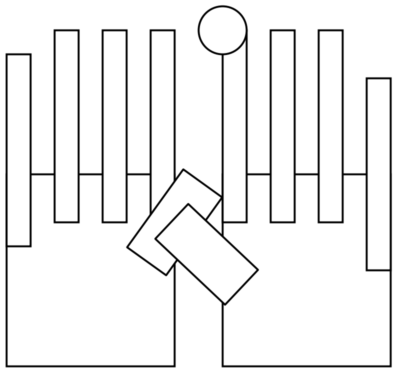
TEACHING PROBABILITY AND STATISTICS TO 10 YEARS OLD CHILDREN

Marie Berrondo-Agrell (I.I.S.)
Pôle Universitaire Léonard de Vinci
France

Here is the description of a presentation of probability-statistics science to 10 years old children. This pedagogical experiment is based on a reasoning with images rather than direct simulations and can be divided into 7 parts:
1) Demonstrating continuity with what they already knew in math.
2) Showing with easy graph theory that a drawing can be math.
3) Presenting the basis of Boolean algebra thanks to our Boolean Bingo.
4) Introducing measurement theory on areas, using generalized Venn diagrams.
5) Throwing 3 coins and analysing the results.
6) Throwing 2 dice and analysing the results.
7) Using their great new ability to win chewing gums, images, sweets, or cookies while playing simple dice or coins games with other children.

The Bilingual School EAB in Paris offered me to teach probability and statistics to a group of ten *10-years old children*, as *a pedagogical experiment*. The session lasted one hour per week, from October 1999 to June 2000. The children who had chosen this activity were usually good in math (others were not interested in applying). The *introductory part of our program* was to make them enjoy it, while demonstrating continuity with what they already knew in math**.** *Funny arithmetic* seemed quite suitable, as for example the fingers-multiplication by 9:

Put your 2 hands on the table. Hide your nth finger from the left. The result will be shown by the number of the fingers staying on the left and the number staying on the right of the hidden one. Example: 9 x 7 = 63.



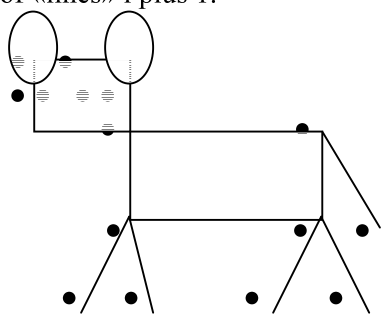
At that age, every proof has to be done through images. Then, it is necessary to show that drawings are also a form of mathematics, not only through geometrical analysis but also through Graph Theory. This is why *Graph Theory* was the subject of the *second part of our program,* presented of course through a simplified or childish vocabulary:

A drawing is a set of «*Big points*» connected by «*Lines»*.
If you use a scissors instead of a pencil for your lines, you obtain different pieces of paper called «*Surfaces*».
To cut along all the lines and big points, you may be have to perforate the paper several times. The minimal number of such perforations is called the «*Connexity degree».*

And now*,* let us consider *the EULER'S FORMULA: b + s = c + l + 1*
«beeplussessequalceeplusselplussone»
which means: The number of «big points» b added to the number of «surfaces» s gives us the «connexity degree» c added to the number of «lines» l plus 1.

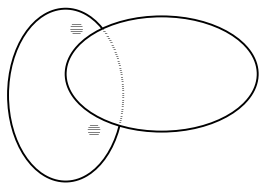
Let us consider 2 different examples

b = 14; s = 5; b + s = 19
c = 3 ; l = 15 c + l + 1 = 19



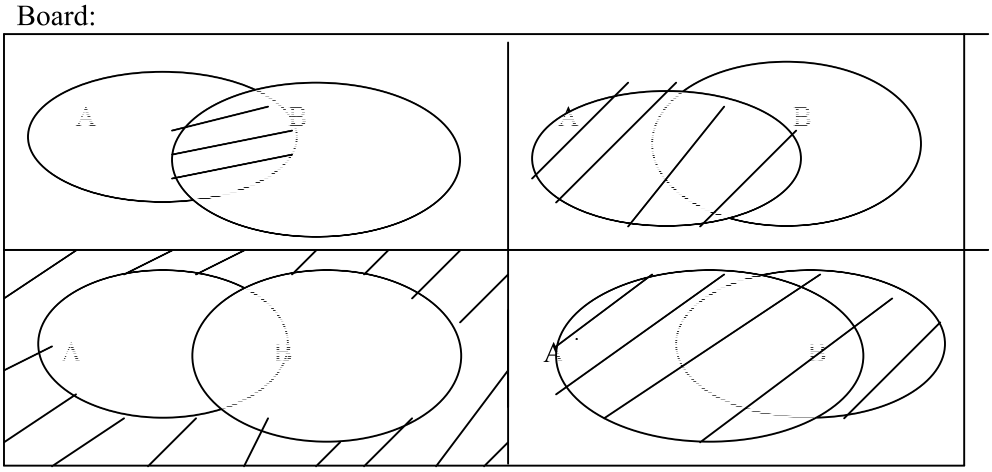
b = 2
s = 4
c = 1
l = 4

b + s = c + l + 1

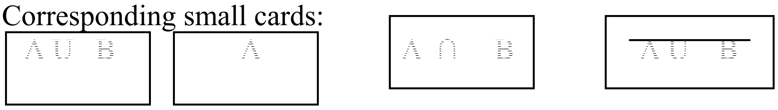


With this formula, we hope to develop the idea of the beauty and the universality of mathematical theorems! More precisely, we hope to have conveyed a mathematical vision of drawings. *The third part* of our teaching program involved the main operations of *Boolean Algebra***.** We use classical Venn diagrams to define the sets operations U, and . We also define the relation of complementarity. The pedagogical tool was a *BOOLEAN BINGO* with «boards» and «small cards». Each board represented 4 drawings. Each drawing was a classical two-dimensional Venn diagram with a coloured zone, i.e. a coloured subset of the corresponding graph-surfaces. Here is an example:

Board:



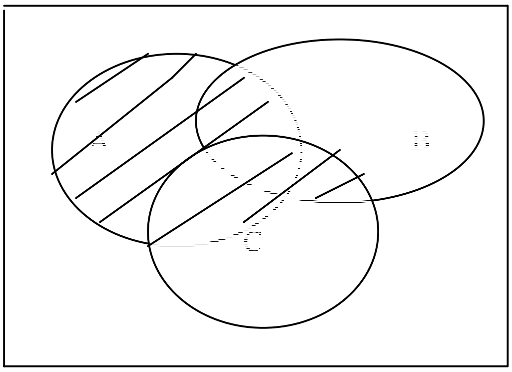
Corresponding small cards:



This game was played like any bingo. Each child received a board with 4 Venn diagram configurations. The teacher mixed the small cards and called them out. Each child recognized as quickly as possible the card he needed and placed it on its board .The child who covered its board had finished. The first who had finished won.

A second more complex bingo followed , with 3-dimensional Venn diagrams, and 6

Here is a card as an example : A U (B ∩ C)



The fourth part of our teaching program introduced a measurement. We chose the simplest one[1]: the area.

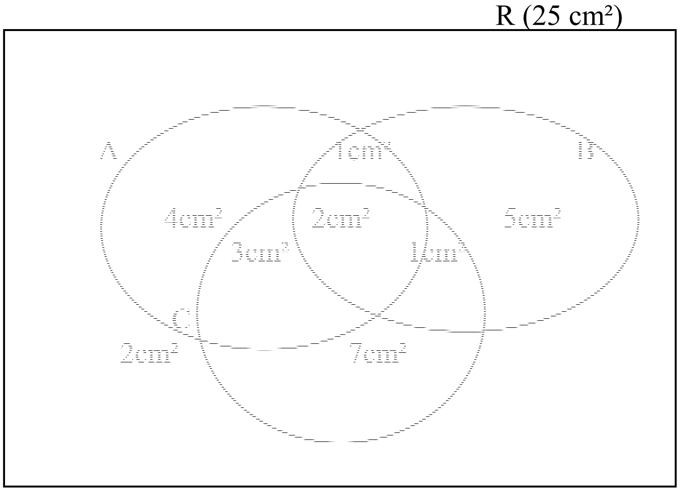
Area (A U B) = Area(A) + Area (B) – Area (A ∩ B)

And Area (Ǿ) = 0

Human mind automatically uses theses propositions.

Each child received a set of amusing exercises, each one consisting of drawing 7 or 10 arrows, connecting the names of subsets and their own measurements. This is done by studying a « Completed Venn Diagram » [1], which means a Venn diagram with the represented areas of each surface. Here is an example:

R (25 cm²)



Area (A)	3 cm²
Area (A ∩ B)	13 cm²
Area (A U $\overline{B}$)	18 cm²
Area (A U B U C)	23 cm²
Area (A ∩ B ∩ C)	1 cm²
Area (A ∩ B ∩ $\overline{C}$)	2 cm²
Area (A U C)	10 cm²

We then presented another kind of diagram as possible basis for measurement: Partitive Diagrams. They look like matrices and the child could do the same arrows exercise. Here is an example (without precising cm² or other numbers):

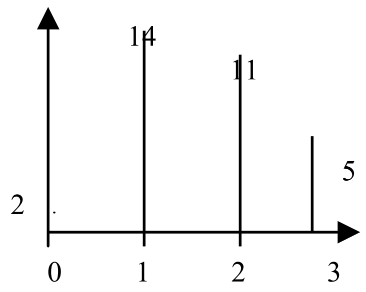
R(20)

	A	B	C
X	2	6	3
Y	1	3	5

« Connect each subset on the left to its measurement on the right with an arrow » :

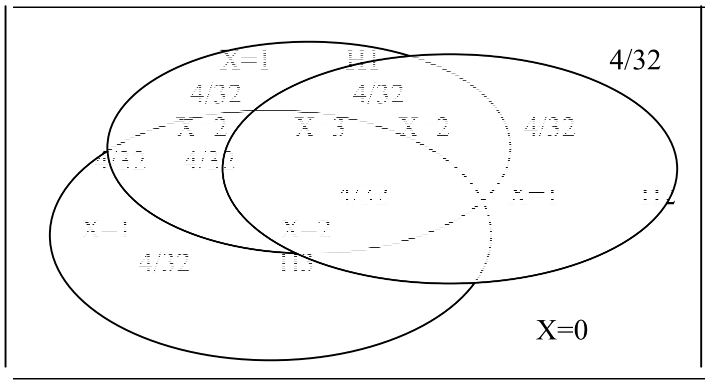
Area (A)	0
Area (X)	3
Area (A ∩ B)	20
Area ((A U B) ∩ X)	5
Area (A U B U C)	15
Area (C ∩ Y)	8
Area (X U B)	11

In the *fifth part of our teaching program, we tackled a real probabilistic experiment.*
Each child takes *three coins* and throws them, once with his right hand, once with his left. The teacher throws the coins again until a total of 32 throws, heads or tails.
For each 3 coins throw, we count how many heads we have. We then present the result in a standard bar-graph.
Here is an example:

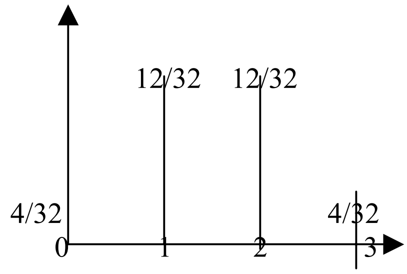


(We must emphasize that a result of 0 head is not «worse» than the result 4).

Let us now consider the same experiment in probabilistic terms, which means a 3 dimensional Venn diagram, the 3 closed curves[2] representing the events «head in the first throw» H1, «head in the second» H2, «head in the third» H3. This gives 8 intern surfaces within the rectangle, each giving the same «percentage if chances» to happen, which can be presented as follows (with percentage measurements instead of areas):



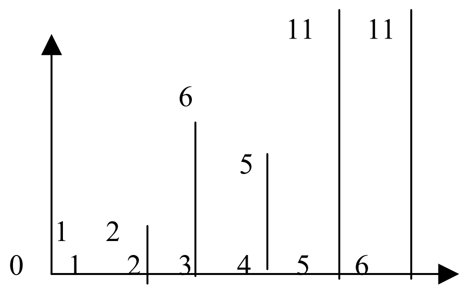
For each number of heads X, we count the corresponding measurement (area or percentage) . Here is our new bar-graph :



Children are amazed by the similarity of the 2 bar graphs, one being entirely the statistic, analysing past events, the other one being entirely probabilistic, analysing future, without knowing anything of the personality of each child and the way he throws each coin!

In the *other probabilistic experiment, sixth part of our teaching program*, each child throws *two dice*, let us say a red one and a blue one. He does it 3 times and writes on a special card each time the smallest appearing result of the 2 dice, the biggest result of these 2, and the sum of these 2. (The teacher completes the experiment to arrive to a total of 36 two dice-throw, which means 12 little cards).

We then do summaries of all these results on 3 bar graphs , one for the smallest result, one for the biggest result and one for the sum of these 2 . Example for the biggest result:



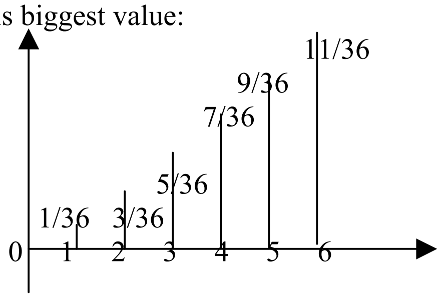
Let us compare them to the theoretical ones. We'll reach them by drawing the partitive diagrams: 36 cases appear. We write in each either the sum of the 2 corresponding results, or the biggest one, or the smallest one. Here is the example for the biggest one:

Blue result: ↘
Red result: ↘

Red \ Blue	1	2	3	4	5	6
1	1 1/36	2 1/36	3 1/36	4 1/36	5 1/36	6 1/36
2	2 1/36	2 1/36	3 1/36	4 1/36	5 1/36	6 1/36
3	3 1/36	3 1/36	3 1/36	4 1/36	5 1/36	6 1/36
4	4 1/36	4 1/36	4 1/36	4 1/36	5 1/36	6 1/36
5	5 1/36	5 1/36	5 1/36	5 1/36	5 1/36	6 1/36
6	6 1/36	6 1/36	6 1/36	6 1/36	6 1/36	6 1/3

We can then draw the bar-graph, for this biggest value:

The similarity between the statistics bar-graphs and this probabilistic one strikes again the children.



Then we do the same bar graphs research for the smallest value of the two dice throw and for their sum. Similarities appear again! We generalize our explanations, talking about *individual freedom and group prevision, about statistics and probability.*

The last part of our teaching program was the presentation of 3 games. They use their new knowledge *to win chewing-gum, images, sweets or cookies.* The point is just to play with somebody who has not followed our session, and to play many times (36 for example).

I. «Let us *throw 2 dice.* Let us bet one piece of chewing-gum each. If ever the sum equals 6, 7, 8 or 9 I win. In all the other cases, you win the 2 pieces of chewing-gum».

II. «Let us *throw 2 dice*. Let us bet one piece of chewing-gum each. If ever the biggest number is 5 or 6, I win. In all the other cases, you win the 2 pieces of chewing-gum».

The other child believes he has a big advantage in each of these 2 games. But our clever pupil wins in the long run. He then proposes a third game:

III. « Let us *throw 3 dice* now. Let us bet again one peace of chewing-gum, each. One of us wins if at least two throws are equal. The other one wins if the 3 throws are different. Do you prefer betting on 2 throws being the same or the opposite?»

The ignorant player having just mysteriously lost twice thinks he should bet this time on throws being the same. And *our clever pupil wins again … «Just great!» What conclusion could be better?*

REFERENCES

[1]. Agrell. P., & Berrondo-Agrell, M. (1992). Towards a syntax of Venn diagrams, fighting a myth. *Journal de la société de statistique de Paris, n 1/2*.

[2]. Grunbaum, B. (1984). The construction of Venn Diagrams. *College Math Journal, 15.*