INTRODUCING THE STUDY OF RANDOM SITUATIONS AND THEIR MODEL BUILDING PROCESS TO 14-15 YEARS OLD STUDENTS

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This paper proposes a first approach with random situations by using a modeling process within the model of Bernoulli's Urn. This way of learning is accessible to 14-15 years old pupils. The software Cabri-géomètre II is used as an empirical computation environment for simulation of the game of "Franc-Carreau", principal activity proposed to pupils in our didactical engineering.

INTRODUCTION

The main of this article is to discuss about introduction to probabilities with 14-15 years old pupils. Some research on their probabilistic thinking shows that, depending on the context, some of the spontaneous conceptions they have constitute obstacles to the building of the relationship between probability and randomness. Consequently, these conceptions constitute obstacles to the learning process of probability. We understand here as obstacle a knowledge that is mobilized outside of its domain of validity (Brousseau, 1997). Among others, we can invoke the research of Maury (1986), Coutinho (1994), Coutinho et al. (1996) and Cañizares (1997). Those researches identify some of the misconceptions that, resisting the application of a didactic sequence, become obstacles the construction of the concept of probabilities. Some of these obstacles, particularly the "idea of equiprobability", become more resistant with the age (Fischbein). Consequently, many works in didactics of probability converge to an anticipation of a first contact with the random one and the calculation of probabilities. Beyond the research on conceptions and pre-conceptions of students on probability, we also find some researches about the construction of this concept. Particularly, on the construction of the relation between the calculation of a probability and the probabilistic model that justifies this calculation. Among others, we invoke the research of Zaki (1990), Girard and Parzysz (1998), as well as diverse publications of the Inter-IREM Commission of Probability and Statistics.

In this article we will focalize especially the didactic conditions that allow an anticipation of a first contact with probabilities, leading in account the construction of the meaning of relation between calculation and probabilistic model justifying it.

A FIRST CONTACT WITH RANDOMNESS: WHICH ARE THE DIDACTICAL CONDITIONS FOR IT?

Our research is based on the hypothesis that a first contact with probabilities and, consequently, a first contact with the randomness, can be favored by experimental activities proposed inside a modeling context. That is, activities proposed in the aim of shaping realities' situations. We intend to present some of the results that we got in Coutinho (2001). Particularly we will present and analyze one of the activities that had formed our didactic engineering, and that was developed in computational environment: the game of "Franc-Carreau". Before presents this game we will present the modeling process that we are pretending as one of the main didactic conditions for the anticipation of the first contact with probabilities. As a consequence of this anticipation, we must assume that the pupils still do not own necessary mathematical knowledge to finish the modeling process: to present the model that was chose to represent and to interpret the reality's situation we want to analyze. In consequence, the transfer from the domain of reality to the theoretical domain cannot be concluded. This absence of a formal knowledge can become a source of difficulty in the process, for the students, of the construction of the meaning of the calculation that it is requested.

> … in general, we do not say to the students that we are in a model when we are using mathematics to find the solution to a concrete problem. It is however important to know that we are not any more in reality ... (Girard & Parzysz, 1998, p. 576).

In order to minimize this didactical difficulty, pupil can work in an intermediate domain in which all elements are abstract, ideal, however they are assigned by same labels used for the objects of the reality they represent. In this intermediate domain there exist still a context for the abstracted elements, context that is perceived and worked for the students: "… in a current vocabulary, the objects of the model are endowed with well defined characteristic properties." (Henry, 1997, p. 79). We will name this domain by *Pseudo-Concrete Domain*.

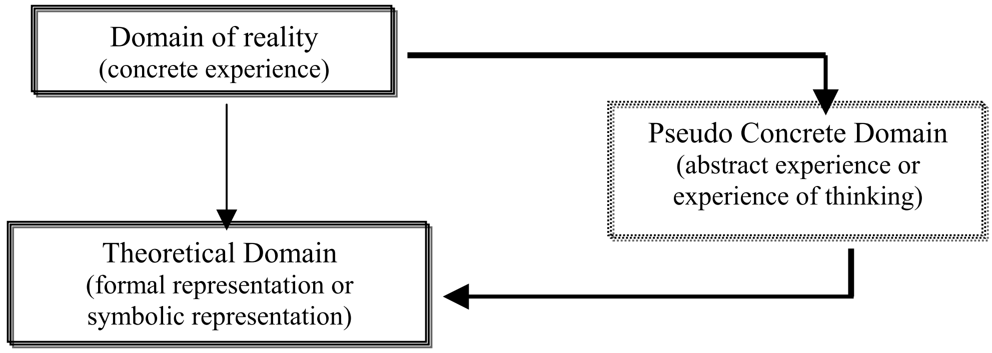

Figure 1. Scheme 1.

THE MODELING PROCESS

In order to construct a context for a first contact with a probabilistic model, we pretend that the Bernoulli's Urn offers to the pupils a bigger wealth of instruments of the domain pseudo-concrete, as well as a bigger simplicity for the abstraction process. The Bernoulli's Urn pseudo-concrete model is filled with balls of two colors and this is the only characteristic that allows the distinction of these balls. In a random drawing, all these balls have exactly the same chance to be drawn, and there are only two possible outcomes: success or failure. The probability of an event resulting from a chance experiment is comparable with the ratio between the number of balls representing this event in that urn and the total number of balls in the urn. We can note that Coutinho (2001) shows that pupils mobilize this spontaneous conception of probability without difficulty. We expect that pupils he can identify, at a first moment of the model building process, the possibility to reproduce a given random experiment that represents the situation that we want to shape. This identification follows the scheme below in figure two:

The context we retain, from a didactical point of view.

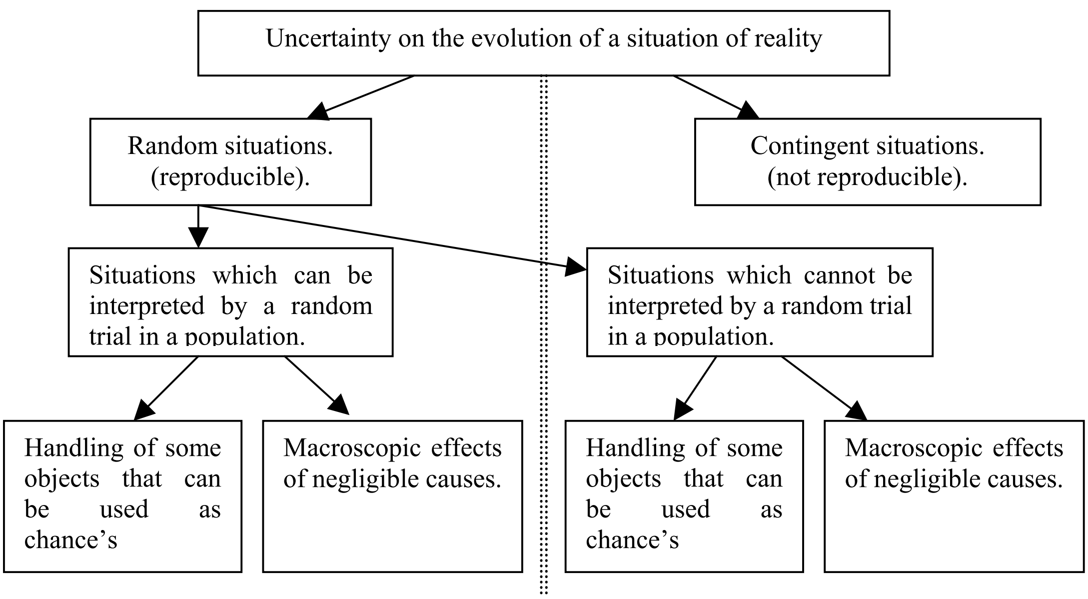

Figure 2. Scheme 2.

Modeling this kind of situations – the ones that can be reproduced in the same conditions by being interpreted as a random trial in a given population – can activate some psychological schemes to classify and compare various interventions of chance. After this identification of reproducibility, the modeling process develops as the scheme below:

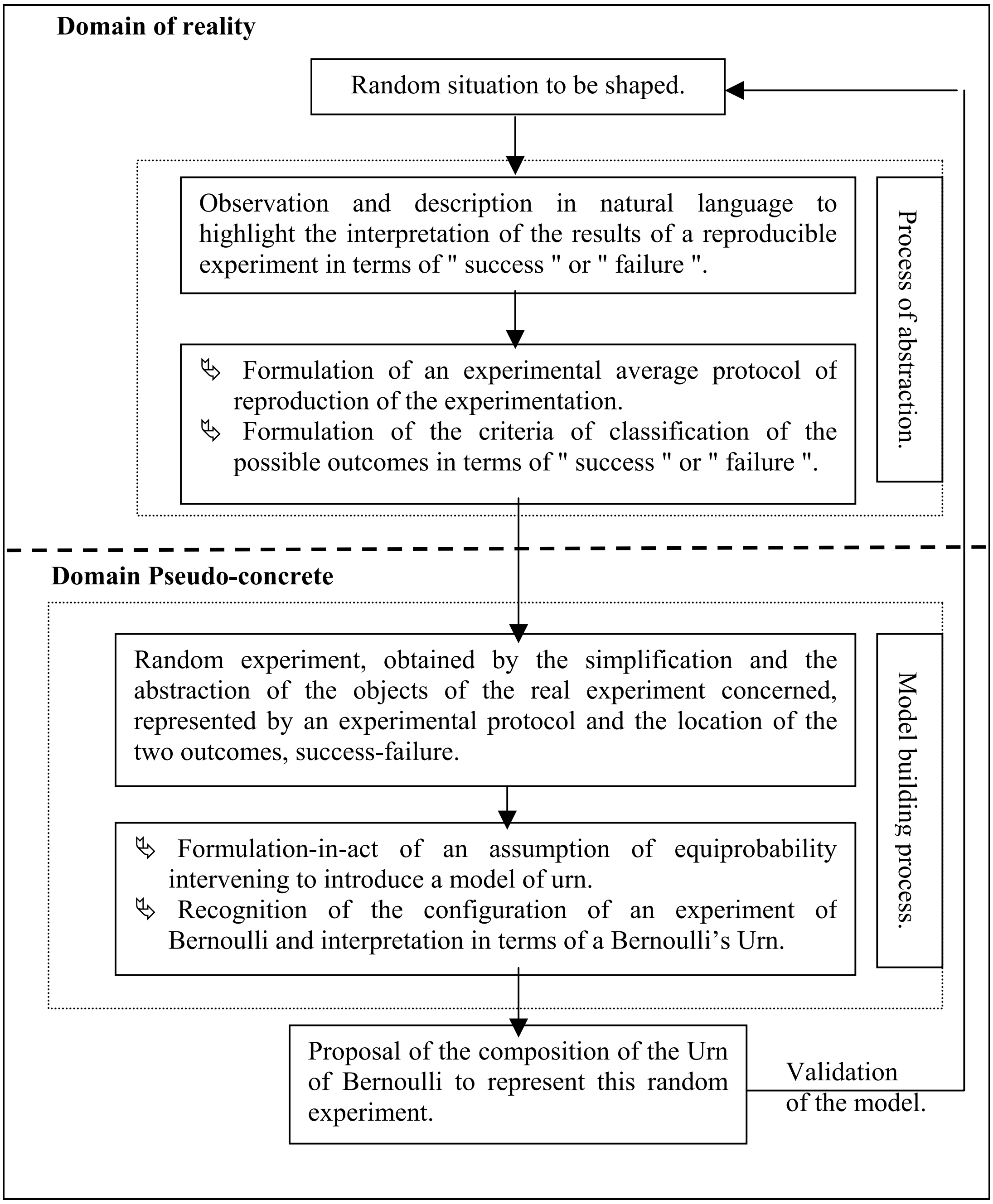

Figure 3. Scheme 3.

The activities we proposed to pupils aged 14-15 years old were carried out inspired on the process given by the scheme 3, in order to initiate them to the first probabilistic modeling steps. We made a didactical choice that guided this work: pupils have to do, themselves, the experiments they want to shape. For instance, if they want to build a model to represent the distribution of the colors of the cars in a parking of a supermarket at a given moment, they will need to make the experience: "*to observe cars that are leaving the parking and to note their color*". We suppose that any car will arrive in this parking while they are making the experiment. Pupils have to give "instructions" for someone who wants to make this experience in the same conditions: the experimental average protocol. Then, they can recognize and describe the possible outcomes using a criteria to classify these outcomes as a “success” or a “failure”. Assuming that all cars that are in the parking have the same probability to go out during the experiment, and estimating the rate between the number of outcomes that give them a “success” and all the outcomes that arrived during the experiment, pupils can give a “Bernoulli’s urn” that represent this experience. Note we assume the statistical point of view to estimate the composition of this urn.

A SPECIAL ACTIVITY: THE GAME OF “FRANC-CARREAU” AND THE DYNAMIC GEOMETRY SIMULATION

A French naturalist and mathematician, George-Louis Leclerc, count of Buffon, studied this game for the first time in 1733. The game consists in throwing a coin above a tiling. The players bet on the final position: will it fall entirely on only one tile (franc-carreau), on one borderline of two tiles, or on two, three or four borderlines?

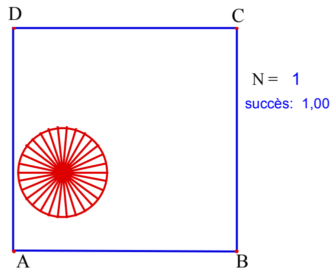

Figure 4. Tiling Reduction.

In our “didactic engineering”, this game is presented initially by the teacher who launches a coin on a paving. Then he shows the game representation in a computation environment: Cabri-géomètre II. On the screen, he can move the circle in a random way by the use of a specific macro-construction in Cabri II. Figure 1 displays a reduction of the tiling to only one square with sides "a". A circle with radius r represents the coin in this model. Let us consider as successes of this chance experiment the following event: *the coin falls in the position “Franc-Carreau”*. The task for the pupil is to determine a Bernoulli’s Urn that represents *the game of Franc-Carreau*. This requires to determining the probability of obtaining a success in this chance experiment as the ratio between the number of white balls and the total number of balls in this Urn of Bernoulli.

There are two possible methods, available to pupils of age 14-15: the geometrical resolution (ratio of surfaces) giving a theoretical value of probability or the estimate of this probability by experimental method (simulation in the Cabri environment). In Coutinho (2001) we noted that the choice made by the pupils depends on their familiarity with the manipulation of the environment and the degree of difficulty of this manipulation: we observed pupil manipulating a TI-92 and the computer with a Windows version of Cabri II. The possibility of parameters manipulation and of returning to the same screen of the beginning of the activity by the tool “Revert” of Cabri-menu provides feedback which can help the students during the construction and validation of their conjectures.

There are two values to be obtained: a theoretical value obtained by the ration between the surface of the square ABCD and the surface of the square described by the center of the circle, and a experimental result obtained by the simulation. The comparison between these two values allowed pupils to validate the simulation. That consequently gives to Cabri a status of a random generator robust for further use.

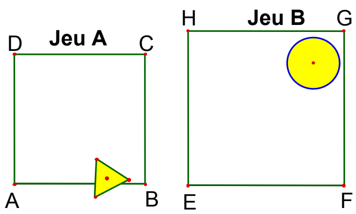

Figure 5. Franc-Carreau” Squares.

To offer an opportunity for reinvesting on this concept of estimate of a probability, pupils were asked, in a new activity, to choose the game they expected being most successful. Further they were asked to justify their answer by describing an Urn of Pixels representing their choice. We understand here Urn of Pixels as a didactical instrument to transform surfaces in the screen into a discrete set of pixels that can be chosen in a random way, and that have the same probability to be chosen. This instrument was introduced in Coutinho (2001).

The games to be analyzed are games A and B (see Figure 2), and in both of them, the aim is to obtain the position *"franc-carreau*". Squares ABCD and EFGH have different size, as well as the radius of the circle and the side of the triangle. We have changed them in a non-proportional way in order to prevent solutions based on a comparison between the surfaces of triangle and circle. The geometric solution is not available to pupil aged 14-15 years old in the case of the triangle. Consequently, it remains to pupil to carry out a simulation in order to obtain an estimate of the probability of the outcome "*franc-carreau"* by the analysis of the experimental results. The Urn of Pixels is, in this case, the solving tool of the problem.

CONCLUDING REMARKS

The data collected in the observation of the teaching sequence are analyzed in Coutinho (2001). The experimental teaching took place in two French schools in year 1998/1999 and 1999/2000. The results showed that 14-15 years old pupils have already the requisite knowledge for the introduction to the modeling of the random situations, when an experimental approach is proposed as a problem-solving tool in estimating probabilities. The possibility of manipulating simulation parameters as a tool for operational validation is very important in the construction of pupils' reasoning. During the interviews we carried out two months after the teaching sequence ended, pupils used this method to validate the possibility of comparison between theoretical results, obtained by the comparison of surfaces, and experimental results, obtained by successive simulations. This validation strategy made reliable the device created to carry out simulation. As a result, when solving the problem to establish a Bernoulli's Urn for the game of *Franc-Carreau* without knowing any measure, neither of square ABCD nor of the radius of the circle, the pupils hesitated to use the experimental method. We observed that the pupils could give a meaning to the result they proposed, without using sophisticated mathematical tools: it was enough for them to be aware of the concepts of surface, of ratio and proportionality, and of frequency.

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