ASSESSING AND TRACING THE DEVELOPMENT OF BASOTHO ELEMENTARY STUDENTS' GROWTH IN PROBABILISTIC THINKING ®

M. V. Polaki
National University of Lesotho
Lesotho

This paper describes two versions of a teaching experiment that traced the development of Basotho elementary students' thinking with regard to sample space and probability of an event. The instructional design phase of the teaching experiment was informed by a cognitive framework that describes and predicts Basotho elementary students' growth in probabilistic thinking (Polaki, Lefoka, & Jones, 2000). Twelve students (9-10 year olds) drawn from grades 4 and 5 of an elementary school took part in a six-week instructional program. Analysis of qualitative data revealed, amongst other things, a weak and often unstable part-part schema that was minimally effective in enabling the students to order probabilities in 1-dimensional situations, and a stronger and more stable part-part schema that made it possible for some students to experience greater success at listing complete sets of outcomes, and to order probabilities in 1- and 2-dimensional situations.

INTRODUCTION

The current worldwide reforms in mathematics education (e.g. National Council of Teachers of Mathematics (NCTM) (2000) have, amongst other things, expanded the scope of elementary school mathematics curriculum to include relatively new topics such as probability. Consistent with this worldwide trend, the new mathematics syllabus for Lesotho (Kingdom of Lesotho, 2001) not only introduces aspects of probability in the lower grades but it also advocates exposure to situations that involve problem solving and application of probability concepts. According to Shaughnessy (1992), there is a need to develop and evaluate the efficacy of research-based models that describe students' development of probabilistic ideas. To develop effective curriculum materials (e.g. worthwhile tasks, instructional activities, etc.) we need to produce detailed accounts of how students acquire increasingly sophisticated ideas in probability.

This paper reports on two versions of a teaching experiment that traced the development of Basotho (people of Lesotho) elementary students' growth in probabilistic thinking. As part of a larger study on the development of Basotho students' probabilistic thinking (Polaki, 2000), this study used the probability framework (Polaki, Lefoka, & Jones, 2000) to design, carryout, and assess the effectiveness of each version of a teaching experiment that traced the development of Basotho (people of Lesotho) elementary students' growth in thinking about sample space and probability of an event. More specifically, each version of the teaching experiment was designed to (a) identify key features of Basotho elementary students' probabilistic thinking, and (b) evaluate the effectiveness of each version of the instructional program.

THEORETICAL CONSIDERATIONS

A key tenet of this study is the notion of teaching experiment which has been described as a conceptual tool for documenting changes in students' mathematical thinking (Cobb, 2000). Consistent with Cobb (1999), each version of the teaching experiment followed the developmental research cycle that consists of two phases: (a) instructional design, and (b) classroom based analyses. The instructional design process was informed by the cognitively guided instruction model according to which research-based knowledge of students' thinking in a content-specific area can be used to guide instructional decisions (Carpenter & Fennema, 1988). Accordingly, the framework (Polaki et al., 2000) constituted a research-based knowledge of students' thinking in probability. More importantly, the framework (Polaki et al., 2000) described students' probabilistic thinking in terms 4 cognitive levels that were found to be consistent with those described by Case (1996) in his general cognitive model. Whereas Level 1 is associated with subjective reasoning, Level 2 is transitional between subjective and informal quantitative thinking. Level 3 involves informal quantitative thinking, and Level 4 incorporates numerical reasoning. This framework constituted a central component of the instructional theory that guided the design and implementation of each version of the teaching experiment. It was also used to evaluate the effectiveness of each version of the teaching experiment. Classroom-based analysis followed Cobb's (2000) Interpretative Framework for analyzing Individual and Collective Activity at the Classroom level. The interpretative framework enabled the researcher to examine students' thinking from the psychological and social perspectives.

SUBJECTS

The subjects were students in grades 4 and 5 (8-10 year olds) of an elementary school in a semi-urban area of Lesotho, Southern Africa. Using records of past achievement in mathematics, the researcher purposely sampled 12 students from each of the three achievement levels: upper quartile, middle quartiles, and lower quartiles. These students had not been exposed to formal instruction in probability. However, they had been taught how to add and subtract simple fractions, albeit instrumentally.

PROCEDURE

Following pre-assessment, the 12 students were randomly assigned to the first and second versions of the teaching experiment with equal numbers from each grade and achievement levels (6 in each group). In the *first version*, students focused on the generation of small sample experimenting (performing a random experiment 20 times) prior to analyzing sample space composition as a strategy for figuring out a solution strategy. In addition to generating small sample experimenting, students in the *second* version examined large sample experimenting (simulating a random experiment 100, 500, 1000, 5000, and 10 000 times) before examining sample space symmetry. The researcher led instructional sessions in the presence of a witness (Steffe & Thompson, 2000) who provided feedback on the extent of development or growth in probabilistic thinking. Students were assessed three times: (a) prior to the teaching experiment, (b) end of the teaching experiment, and (c) 3 weeks following the end of the instructional program. Finally, 6 of the 12 students (3 from each instruction group) were selected for case study analyses.

DATA COLLECTION, INSTRUMENTATION, AND ANALYSIS

Data was gathered from six sources: (a) students' responses to the assessment protocol, (b) researcher evaluations of each students' thinking, (c) witness' narratives on students' thinking, (d) students' journal entries, (e) audio- and video-recordings of students' interactions during instruction, and (e) individual and collective written responses to probability tasks. A double-coding procedure (Miles & Huberman, 1994) was used to assign probability levels during each assessment period. Additionally, a grounded theory approach (Strauss & Corbin, 1990) was used to discern students' thinking patterns from other forms of qualitative data sources (e.g. case study, classroom interactions, etc.). Furthermore, non-parametric tests were used to measure the significance of changes in thinking levels from one assessment period to another.

RESULTS

Analyses of the data drawn from classroom interactions and case study students revealed existence of the following thinking patterns: (a) *sample space misconception* (Jones, Langrall, Thorton & Mogill, 1999) that entailed a pervasive belief that the outcome of a random experiment was dependent upon previously observed outcomes, and (b) *a weak and often unstable part-part schema* that was minimally effective in enabling case study students to identify complete sample space and to order probabilities for 1-dimensional experiments; and (c*) a stronger and more stable part-part schema* that enabled target students to reason with greater consistency when listing the complete set of outcomes, and when ordering probabilities for 1-dimensional experiments. Whereas *part-part schema* is a conceptual structure that enables the learner to compare or order parts of a whole, *part-whole schema* is a structure that enables the learner to compare parts to a whole (Lamon, 1999).

With regard to the first and second learning patterns, two of the case study students, Mpho and Tau showed (a) a persistent reluctance to list complete sets of possible outcomes for random experiments, arguing that those obtained on the first trial could not be obtained on the second trial, and (b) limited success at ordering probabilities in 1-dimensional situations. Despite exposure to intensive instruction, both students held on to this misconception until the final assessment period. As for the second learning pattern, two of the case study students, Palesa and Tefo showed greater success at (a) listing complete sets of outcomes for 1- and 2-dimensional experiments, and (b) ordering probabilities in 1-dimensional situations, and using informal invented language to describe probabilities quantitatively. Only one of the target students, Lineo, was eventually able to show evidence of consistency in ordering probabilities 1- and 2-dimensional situations.

Table 1 summarizes gains in probability thinking levels made by the subjects in each of the constructs at the three assessment periods. The most significant feature of Table 2 is that, in general, the 12 students made gains in probability thinking levels for sample space and probability of an event. The Wilcoxon Signed Ranked Test (WSRT) for the difference between the first and middle assessment levels for Group A was significant for sample space ($p < 0.05$), and probability an event ($p < 0.03$). This test was also significant for Group B with regard to sample space ($p < 0.05$), and probability of an event ($p < 0.05$).

Table 1
Thinking Levels for Each Instruction Group at the Three Assessment Periods

Student	*First assessment*		*Middle assessment*		*Final assessment*	
	SS	PE	SS	PE	SS	PE
			Group A (n=6)			
Palesa	1	1	4	3	4	3
Thabo*	1	1	4	3	4	3
Teboho	1	2	2	3	4	3
Tumelo	1	1	2	3	4	3
Mpho	1	1	2	3	2	3
Thabang	1	1	2	3	2	2
			Group B (n=6)			
Lineo	1	2	4	3	4	4
Tefo*	1	1	4	3	4	3
Thabiso	1	1	4	2	4	2
Mampe	1	1	2	3	2	3
Lebo	1	1	2	3	4	3
Tau	1	1	2	2	1	2

DISCUSSION AND CONCLUSIONS

The observed gains in probability thinking levels made by students in both groups suggests that, in line with the perspective advocated by Carpenter & Fennema (1988), a teaching experiment informed by a research-based knowledge of students' thinking in probability can be used to evoke noticeable changes in students' thinking. However, since the 12 students enjoyed instructor attention that is not often possible in whole-class settings, it may be useful to replicate this study in real classrooms, and over an extended period of time.

Notwithstanding the size of the sample for this study and the duration of the instructional program, this study has revealed some key patterns of students' development of probabilistic thinking. Initially, students' thinking was marked by subjective reasoning as evidenced by the pervasive belief that the outcomes of a random variable were dependent upon previous trials of the experiment. Then exposure to probability problems in game-like situations enabled some students to construct a weak part-part schema that made it possible for them to describe probabilities using informal invented language, and to order probabilities in 1-dimensional situations, albeit inconsistently. Others seemed to have developed a stronger part-part schema that enabled them to consistently list complete sets of outcomes and order probabilities in 1-dimensional situations and 2-dimensional situations that required use of a part-part schema, but not for situations that required use of part-whole comparisons. Consistent with Jones et al. (1999), part-part comparisons seemed to play the key role in enabling students to quantify and order probabilities. It seems that central conceptual structures similar to those described by Case (1996) in quantitative thought exist in the case of probabilistic thinking. Apparently a part part and a part whole seem to play a reciprocal role in supporting students' ability to describe and to order probabilities.

REFERENCES

Carpenter, T.P., & Fennema, E. (1988). Research in cognitively guided instruction. In E. Fennema and T.P. Carpenter (Eds.), *Integrating research on teaching and learning mathematics* (pp. 2-17). Wisconsin: Wisconsin Center for Education Research.

Case, R. (1996). Reconceptualizing the nature of children's' conceptual structures and their development in middle childhood. *Monographs of the Society for Research in Child Development, 61*, (1-2, Serial N0. 246), 1-26.

Cobb, P. (1999). Individual and collective mathematical development: The case of statistical analysis. *Mathematical Thinking and Learning, 1 (1),* 5-43.

Cobb, P. (2000). Conducting teaching experiments in collaboration with teachers. In A. Kelly and R. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 307-334). Mahwah, NJ: Lawrence Erlbaum Associates.

Jones, G.A., Langrall, C.W., Thornton, C.A., & Mogill, A.T. (1999). Using students' probabilistic thinking in instruction. *Journal for Research in Mathematics Education, 30*, 487-519.

Kingdom of Lesotho. (2001). *Mathematics form A & B syllabi*. Ministry of Education, Maseru, Lesotho.

Lamon, S. J. (1999). *Teaching fractions and ratios for understanding: Essential content knowledge for teachers*. Mahawh, NJ: Erlbaum Associates.

Miles, M.B., & Huberman, M.A. (1994*). Qualitative data analysis: An expanded sourcebook* (3rd edn.), Newbury Park, CA: Sage.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: Author..

Polaki M.V. (2000). *Using instruction to trace the development of Basotho elementary students' probabilistic thinking*. Unpublished doctoral dissertation, Illinois State University, Normal, Illinois.

Polaki, M.V., Lefoka, P.J., & Jones, G.A. (2000). Developing a cognitive framework for describing and predicting Basotho students' probabilistic thinking. *BOLESWA Educational Research Journal, 17*, 1-20

Shaughnessy, J.M. (1992). Research in probability and statistic: Reflections and directions. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 465-494*)*. New York: Macmillan.

Steffe, L.P., & Thompson P.W. (2000). Teaching experiment methodology: Underlying principles and essential elements. In A. Kelly and R. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 267-306). Mahwah, NJ: Erlbaum Associates.

Strauss, A., & Corbin, J. (1990). *Basics of qualitative research: Grounded theory, procedures, and techniques.* Newbury Park, CA: Sage.